www.ietdl.org

Published in IET Communications
Received on 2nd September 2010
Revised on 9th January 2011
doi: 10.1049/iet-com.2010.0799

ISSN 1751-8628

Throughput analysis of IEEE 802.11-based vehicular *ad hoc* networks

N. Etemadi F. Ashtiani

Department of Electrical Engineering, Advanced Communications Research Institute (ACRI), Sharif University of Technology, Tehran, Iran
E-mail: ashtianimt@hotmail.com

Abstract: In this study, the authors propose an analytical model to evaluate the maximum stable throughput for unicast services in vehicular *ad hoc* networks. In this respect, the authors employ two queueing networks (QNs) to model the mobility patterns of the vehicles as well as the multi-hop packet transmission scenario. In the proposed model, the authors map the features of IEEE 802.11, as well as the details of multi-hop packet transmission, regarding the dynamic status of vehicles, onto suitable parameters of the QNs. In the authors' modelling approach, they take the details of MAC and routing schemes into account by classifying the vehicles based on their mobility patterns and considering their dynamic status in average. By writing traffic equations and applying the stability condition, the authors compute the maximum stable throughput of the network, that is, the maximum rate of packets successfully received at the destinations while all vehicles are in stable conditions. In the last part of the study, the authors show the effect of some network parameters onto the maximum stable throughput and confirm the validity of their model by simulation.

1 Introduction

Each year many people lose their lives because of accidents, car crashes etc. Regarding the incomputable value of human lives, any technology leading to a significant reduction in the number of victims in the roads is of crucial importance. Vehicular *ad hoc* network (VANET) is a promising technology in this respect [1, 2]. However, knowing the capability of VANETs is an important factor for its success in practice. Although there are several similarities between VANETs and conventional MANETs (mobile *ad hoc* networks), there are some features that distinguish these two types of networks. Among them is the high-dynamic nodes in VANETs, that is, vehicles. On the other hand, regarding the main goal of VANETs, that is, increase of safety level of transportation, there are two general types of packets exchanged in VANETs [1–4], that is, safety packets and commercial packets, with different levels of priority.

Up to now, several features of VANETs have been considered by the researchers. The authors in [5] have focused on mobility modelling of vehicles in a typical VANET. Some papers have considered the connectivity problem in a VANET [5, 6]. Several valuable works focused on designing routing algorithms in VANETs, have been reported in the literature [7–9], and some papers have focused on MAC layer optimisation for VANETs [10, 11]. With respect to a new standard developed for VANETs, that is, IEEE 802.11p [1, 2], there is a great need for providing an analytical approach in evaluating the VANETs based on this standard. IEEE 802.11p has been founded on IEEE 802.11a and IEEE 802.11e as its physical and MAC layers, respectively, [1]. Several analytical models have been reported for IEEE 802.11 [12–15] but regarding dynamic status of the vehicles on one hand and specific mobility patterns of vehicles on the other hand, development of a new analytical model for IEEE 802.11-based VANETs is necessary. To the best of our knowledge, only a few papers have evaluated VANETs analytically [16–18]. An *et al.* [16] have focused on collision situation only but the throughput evaluation has not been considered in the paper. Javanmard and Ashtiani [17] have focused on a VANET scenario in sparse situations and a simple slotted ALOHA as its MAC layer. Actually, slotted Aloha MAC scheme simplifies the analysis strongly. Although [18] has considered a version of IEEE 802.11e-based VANET, its focus is on the broadcast services for safety packets. Moreover, the mobility pattern considered in [18] is the same for all vehicles.

In this paper, we focus on the throughput analysis of unicast services when we have dynamic number of vehicles with different velocities. Our analysis has been founded on two hierarchical queueing networks (QNs). We employ a mobility model reported in the literature [17] that is an open Baskett, Chandy, Muntz and Palacios (BCMP) QN [19]. With respect to this model we are able to consider the dynamic location of vehicles by their spatial probability distribution. Then, we propose another open QN for each typical vehicle representing the details of IEEE 802.11 MAC, routing scheme and collision situations in multi-hop packet transmissions. By writing traffic equations of the corresponding QNs and applying the stability condition for each typical vehicle, we are able to obtain the maximum stable throughput, that is, the maximum packet generation rate (at vehicles) successfully received at the destinations, guaranteeing the stability of all vehicles. In our modelling approach, owing to the high-dynamic status of vehicles, we apply some approximations in considering the status of vehicles. Thus, we present some simulations to observe the accuracy of our model. Moreover, we present some numerical results to evaluate the effect of network parameters onto the maximum stable throughput. It is worth mentioning that when the packet generation rate at the vehicles is the rate corresponding to the case of maximum stable throughput, some of the vehicles are saturated, that is, their average queue length is infinity. Thus, in this case the average delay is infinity. To compute the average delay for the lower packet generation rates we need an analytical model to trace the number of hops traversed by the packets as well as the waiting time at each hop, that is beyond the scope of this paper.

Following this introduction, we briefly review IEEE 802.11 and its relation to IEEE 802.11p in Section 2. In Section 3, we have an introduction on the mobility model reported in [17]. Section 4 is dedicated to description of our VANET scenario, including the routing scheme. In Section 5, we describe, in detail, our analytical approach in modelling different features of IEEE 802.11 MAC layer as well as routing scheme in our VANET scenario. We will discuss how we compute the maximum stable throughput in the same section. We present the analytical and simulation results in Section 6. Section 7 concludes the paper.

2 Brief review on IEEE 802.11 standard

According to [1, 2], IEEE 802.11p is particularly introduced for communications of vehicles with each other and also between vehicles and roadside stations. IEEE 802.11p uses a modified version of 802.11a in PHY layer and 802.11e in MAC layer with some specified parameters [2]. The IEEE 802.11e enhanced distributed channel access is an extension to IEEE 802.11 distributed coordination function (DCF) that provides service differentiation and QoS [20]. Focusing on IEEE 802.11-based VANETs is actually equivalent to confining IEEE 802.11p-based VANETs to transmission of only one type of packets (e.g. safety packets) on common control channel (i.e. a continuously active channel) [1]. We believe our analytical approach provides a suitable foundation to extend the model to include more complex scenarios, that is, considering different types of services and channels.

According to IEEE 802.11 DCF, in four-way handshaking mode [12] (Fig. 1), when a packet is ready to be transmitted a backoff counter is set randomly between 0, $CW_{min} - 1$. If the channel is idle for a DIFS (distributed inter-frame space) the backoff counter counts down. Channel sensing is done at each slot. If the channel is busy, the counter will be frozen, so the slot extends until the channel is sensed idle again for a DIFS. When the backoff counter reaches zero, an request to send (RTS) is transmitted. If two nodes send RTS signals at the same slot, a collision occurs provided that one of the transmitters is within the interference range (IR) of the receiver of another simultaneously active link. We have considered IR as double size of the transmission range (TR). In a collision situation each of the collided packets should incur another backoff time, randomly selected from a contention window with a double size compared with its previous one. If the RTS is received successfully the receiver sends clear to send (CTS) signal, afterwards the packet transmission begins. During packet transmission, the backoff counters of the other vehicles, which are in the sensing range (assumed as double size of the TR [21]) of the transmitter or the receiver, are frozen. Owing to lack of space, we ignore discussing more details of the protocol in this section.

3 Mobility modelling

As we indicated in Section 1, one of the distinct features of VANETs compared with usual MANETs, is in distinguished mobility pattern of the vehicles. So mobility modelling plays a key role in the network performance. To model the mobility patterns of vehicles we have used a simple model introduced in [17]. In this respect, we have divided a two-way street into rectangular sub-regions of length L_1 and width L_2 (see Fig. 2). Each sub-region is specified with two indices, (x, y). Each direction has three lanes such that vehicles arrive at each lane with a Poisson distribution and select a velocity with a uniform distribution. In fact, the velocity distribution for all lanes is uniform, but their ranges are different. Each vehicle may change its lane and correspondingly its velocity two times along the street, first, at one-tenth part of the street and second at nine-tenth part of the street (see Fig. 2). This is equivalent to parts near the crosses for a typical street. It is worth noting that the above assumptions are for the sake of simplicity and can be extended easily to more complex scenarios [17].

To model the above mobility pattern, we consider each vehicle as a customer and each sub-region as an M/G/∞ queueing node, leading to an open BCMP QN [19]. The manner of lane transition is mapped onto the routing probabilities in the QN (see Fig. 2). We also map the sojourn time of the vehicles at each sub-region onto the

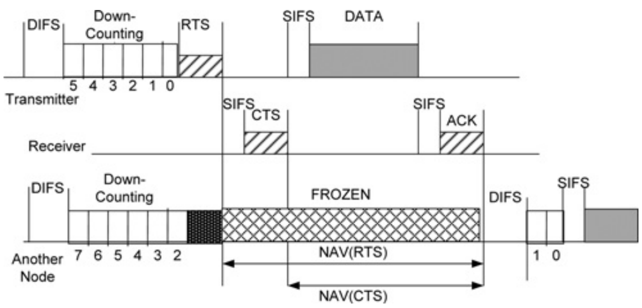

Fig. 1 *Packet transmission in IEEE 802.11*

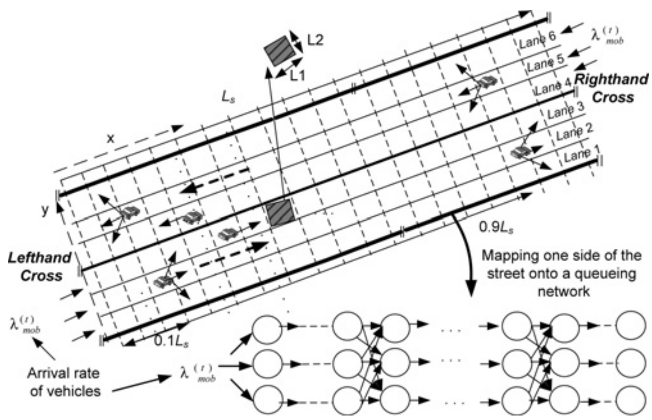

Fig. 2 *Mobility patterns of vehicles in a typical street*

service time of the customers at the corresponding queueing node. Obviously, it depends on the distribution of vehicles' velocities at different lanes and L_1 [17].

With respect to above assumptions, the vehicles arriving at the tth lane of the street have similar mobility patterns (i.e. velocities and probability of lane transition at different sub-regions). Thus, to compute the spatial traffic distribution in steady state, we classify the vehicles based on the initial lane they arrive at the street. Solving the obtained open BCMP QN will lead to the following results [17, 19]

$$P_{\text{loc}}(x, y, n) = \exp\left(-\sum_{t=1}^{6} \rho_{\text{mob}}^{(t)}(x, y)\right) \frac{\left(\sum_{t=1}^{6} \rho_{\text{mob}}^{(t)}(x, y)\right)^n}{n!}$$

$$1 \le x \le N_s\,, \quad 1 \le y \le 6\,, \quad n \ge 0, \quad N_s = \frac{L_s}{L_1} \tag{1}$$

$$P_{\text{loc}}^{(t)}(x, y, n) = \exp(-\rho_{\text{mob}}^{(t)}(x, y)) \frac{(\rho_{\text{mob}}^{(t)}(x, y))^n}{n!};$$

$$1 \le x \le N_s, \ 1 \le y \le 6, \ n \ge 0, \ 1 \le t \le 6$$

$$\rho_{\text{mob}}^{(t)}(x, y) = \frac{\alpha_{\text{mob}}^{(t)}(x, y)}{\mu_{\text{mob}}^{(t)}(x, y)} \tag{2}$$

where L_s denotes the length of the street and $\rho_{\text{mob}}^{(t)}(x, y)$ denotes the traffic intensity [22] for the queueing node equivalent to sub-region (x, y). Also, $\alpha_{\text{mob}}^{(t)}(x, y)$ and $\mu_{\text{mob}}^{(t)}(x, y)$ are the arrival and service rates at node (x, y), corresponding to the class-t vehicles, that is, vehicles with the tth initial lane, respectively. $\mu_{\text{mob}}^{(t)}(x, y)$, depends on the velocity distribution in the tth lane. For example, if the vehicles with the tth initial lane choose their velocity with a uniform distribution, that is, $U[v_{lt}, v_{ht}]$, $\mu_{\text{mob}}^{(t)}(x, y)$ equals $L_1/(v_{ht} - v_{lt})ln(v_{ht}/v_{lt})$, where v_{ht} and v_{lt} represent the maximum and minimum possible velocities of the tth lane, respectively. Moreover, $\alpha_{\text{mob}}^{(t)}(x, y)$ is obtained by solving the related traffic equations [17]. For an M/G/∞ queueing system corresponding to each subregion, the traffic intensity, $\rho_{\text{mob}}^{(t)}$, shows the average number of class-t customers at the system. Obviously, for sub-regions at one side of the street [e.g. $(x, y) = (2, 3)$] the traffic intensity corresponding to vehicles of the classes equivalent to the other side of the street (e.g. $t = 4$) equals zero. Furthermore, $P_{\text{loc}}^{(t)}(x, y, n)$ and $P_{\text{loc}}(x, y, n)$ denote the probability of n class-t vehicles and n vehicles (irrespective of their classes), to be in sub-region (x, y), respectively. Regarding the BCMP QN comprised of M/G/∞ queues, it is obvious that the probability of n vehicles at a region follows a Poisson distribution [19].

In the sequel, it is necessary to know the probability that a class-t vehicle, existed at the street, occupies a sub-region (x, y), that is, $P_{\text{loc-typ}}^{(t)}(x, y)$. In fact, it is a conditional probability, obtainable by considering the aforementioned QN as a closed one with one customer [19]. By solving the closed QN, we have the following equations [17]

$$P_{\text{loc-typ}}^{(t)}(x, y) = b \frac{\alpha_{\text{mob}}^{\prime(t)}(x, y)}{\mu_{\text{mob}}^{(t)}(x, y)}; \ 1 \le x \le N_s, \quad 1 \le y \le 6, \quad n \ge 0 \tag{3}$$

$$\sum_{x=1}^{N_s} \sum_{y=1}^{6} P_{\text{loc-typ}}^{(t)}(x, y) = 1 \tag{4}$$

where $\alpha_{\text{mob}}^{\prime(t)}(x, y)$ is the arrival rate at node (x, y) and b is the normalisation factor, which is obtained by (4). Moreover, $\alpha_{\text{mob}}^{\prime(t)}(x, y)$ is obtained by solving the related traffic equations of the closed queueing network (QN) [17].

It is necessary to mention that although there is one customer at the closed QN, there is a nice difference between $P_{\text{loc-typ}}^{(t)}(x, y)$ and $P_{\text{loc}}^{(t)}(x, y, 1)$. The former determines the steady state probability that a typical class-t vehicle, occupies the sub-region (x, y). However, the latter determines the probability that the sub-region (x, y) is occupied by a class-t vehicle. In fact, the former is independent of the arrival rate of the vehicles at the street, that is, $\lambda_{\text{mob}}^{(t)}(1, y)$, $1 \le t \le 3$ and $\lambda_{\text{mob}}^{(t)}(L_s, y)$, $4 \le t \le 6$, but the latter is strongly dependent on it [17]. On the other hand, although the arrival rate of the vehicles is time-varying in real situation, we have assumed it does not change for a sufficiently long time interval such that the status of the vehicles can be considered as a stationary random process, that is, reaching a steady state. For example, for different time intervals along a day we may have different sets of statistical parameters. For each set, our analysis is applicable distinctly.

4 VANET scenario

In the scenario considered in this paper, we assume there exist two fixed stations along the street. They can be placed at the crosses at each side of the street. Vehicles arrive at different lanes at two sides of the street with a Poisson distribution and select a velocity from corresponding distributions. During the movements along the street, the packets are generated continuously with a Poisson distribution via sensors mounted on the vehicles. The packets should be delivered to the destination, that is, the nearest fixed station. Since the distance between a vehicle and the nearest station may exceed the TR, we have to use multi-hop transmissions (see Fig. 3). To justify our scenario, we explain one example. The packets can be data that vehicles sense while traversing the street, such as accidents, traffic states, or even normal situations of the highway etc. They should send these data to the fixed stations. Fixed stations can be emergency centres, traffic control centres, police stations etc. So, these stations can use the information extracted by received data to control traffic and handle abnormal situations.

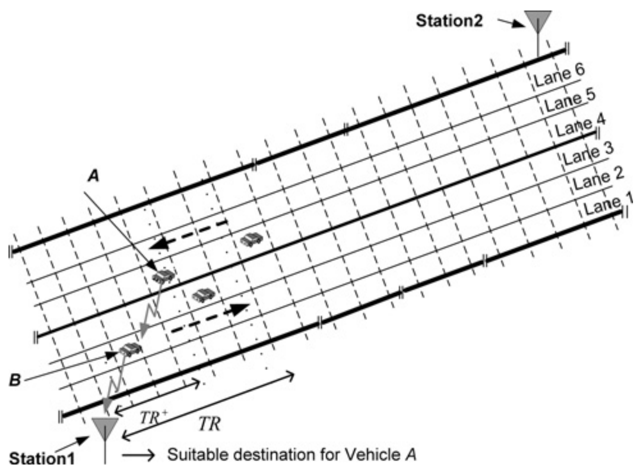

Fig. 3 *VANET scenario and a simple example of packet transmission towards the destination*

doi: 10.1049/iet-com.2010.0799

Each packet transmission is done based on IEEE 802.11 DCF in RTS/CTS mode through an omni-directional antenna. We assume that the vehicles know the location of themselves through GPS and send it to their neighbours periodically by beacons (in our analysis, we have ignored the details of the beaconing). So, the vehicles know the location of their neighbours too. Also, they know the location of fixed stations via digital maps. Based on the above information available at each vehicle, the packet routing scheme is based on packet transmission in the direction of nearest fixed station (called as its destination), towards a vehicle nearest to the destination and within its TR at both sides of the street. To clarify the multi-hop packet routing in the VANET scenario, consider a simple example. A typical class-t vehicle (vehicle A) wants to send its packet towards the destination (see Fig. 3). First it finds the nearest station, that is, station 1, and then it checks its TR to find a vehicle which has the minimum distance to the destination (station 1) at both sides of the street. As soon as vehicle A finds the most suitable vehicle (vehicle B), it considers it as its next hop. In the case that some vehicles with the same situation have been found, one of them is selected in random. For the sake of simplicity, we consider a sufficiently dense situation which means that with a high probability (nearly equal to one), vehicle A finds another vehicle within its TR closer to the suitable destination (i.e. nearer to station 1), that is, within its suitable TR(TR^+). Otherwise, it should carry the packet by itself until finds a suitable vehicle. Furthermore, if the destination is within the TR of vehicle A, it uses a one-hop transmission and sends the packet to the destination directly.

5 Analytical modelling of the VANET scenario

In this section, we propose an analytical model to represent multi-hop packet transmission among the vehicles. Regarding the mobility model introduced in Section 3, we propose an open QN for a typical class-t vehicle at the street ($1 \le t \le 6$), as shown in Fig. 4. In this model, we map different phases of packet transmission process (e.g. backoff, RTS etc.) onto different nodes of the proposed QN. Packet arrival at the vehicle is equivalent to customer arrival at the QN. At each time instant only one packet may be active (i.e. in the transmission process) in the vehicle, in a real scenario. However, it is possible to have more than one customer (packet) simultaneously at the QN that is not matched with the real scenario. Therefore by applying a condition in Section 5.3, we guarantee not having more than one customer in the QN in average. We discuss the details of the QN, including routing probabilities, service times at each node and the traffic equations in the following parts.

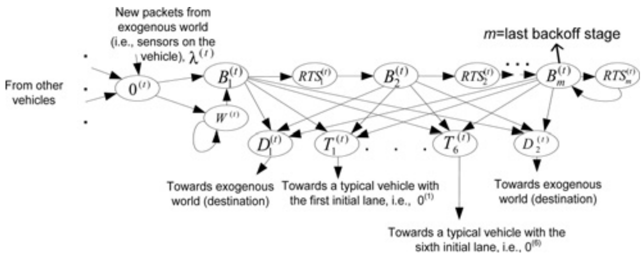

Fig. 4 *QN model corresponding to a typical class-t vehicle*

5.1 Model description and computation of the routing probabilities

The new packets arrive at each class-t vehicle ($1 \le t \le 6$), with the Poisson distribution and a rate equal to $\lambda^{(t)}$ (In this paper, the superscript within the parenthesis indicates the class of underlying vehicle.). In the corresponding QN, a packet arrived at the vehicle enters node $0^{(t)}$ (Fig. 4). Regarding the slotted structure among the vehicles because of MAC layer, node $0^{(t)}$ represents the synchronisation time for new packets as well as the time for specifying the next hop vehicle. Now consider that a typical packet (newly generated or routed from the other vehicles) has arrived at node $0^{(t)}$ in the QN corresponding to vehicle A. If vehicle A could find at least one vehicle within its TR^+, so the packet is routed to node $B_1^{(t)}$, that is, enters the first backoff stage. In the case of not finding a suitable vehicle (i.e. nearer to destination) for the next hop, the packet is routed to node $W^{(t)}$ (the corresponding probability is $r_{0^{(t)},W^{(t)}}$), then waits sufficiently such that the location of vehicles relative to each other changes in average. Then, vehicle A checks again its TR^+ to find another vehicle. Since we have assumed the mobility of the vehicles independent of each other and regarding the waiting time at $W^{(t)}$, the probability of finding a suitable vehicle when the packet departs node $W^{(t)}$ is assumed to be the same as the probability of finding a suitable vehicle when the packet departs node $0^{(t)}$. Thus, $r_{0^{(t)},B_1^{(t)}}$ equals $r_{W^{(t)},B_1^{(t)}}$, and is computed as in the following

$$r_{0^{(t)},B_1^{(t)}} = r_{W^{(t)},B_1^{(t)}} = \sum_{x_1=1}^{N_s} \sum_{y_1=1}^{6} P_{\text{loc-typ}}^{(t)}(x_1, y_1) \times \left(1 - \exp\left(-\sum_{t_1=1}^{6} \sum_{(x_2,y_2)\epsilon TR^+} \rho_{\text{mob}}^{(t_1)}(x_2, y_2)\right)\right) \quad (5)$$

$$r_{0^{(t)},W^{(t)}} = r_{W^{(t)},W^{(t)}} = 1 - r_{0^{(t)},B_1^{(t)}} \quad (6)$$

where $\rho_{\text{mob}}^{(t)}$ and $P_{\text{loc-typ}}^{(t)}$ were introduced in Section 3. In (5), the first term, that is, 'ΣΣ', indicates the probability for vehicle A to be in different sub-regions of the street, and the second multiplicative term shows the probability of finding a vehicle within its TR^+. This term has been written regarding the Poisson distribution of the vehicles at each region, discussed in Section 3.

When the packet is routed to node $B_1^{(t)}$, the packet starts the backoff process with the contention window equal to CW_{min}. If vehicle A finds more than one vehicle within its TR^+, it chooses the most suitable one as discussed in Section 4. After the packet departs $B_1^{(t)}$, it may be routed to one of the nodes $D_1^{(t)}, D_2^{(t)}, T_1^{(t)}, \ldots, T_6^{(t)}$. Nodes $D_1^{(t)}$ and $D_2^{(t)}$ represent the successful transmission of the packet (i.e. successful RTS/CTS exchange and packet transmission) such that its next hop is station 1 and station 2, respectively. Successful packet transmission to the stations 1 and 2 is modelled as routing to the exogenous world in the proposed QN. Similarly, the packet routing to $T_k^{(t)}$ denotes successful packet transmission towards a class-k vehicle. However, if there is not any response to RTS, after CTS-timeout, the next backoff stage starts (modelled as routing to $RTS_i^{(t)}$ and then to $B_{i+1}^{(t)}$, see Fig. 4). It is worth mentioning that nodes $D_1^{(t)}, D_2^{(t)}, T_1^{(t)}, \ldots, T_6^{(t)}$ include RTS/CTS as well as ACK signalling events. Moreover, for the sake of simplicity, we have ignored any degrading factor because of physical layer parameters, for example, noise, interference level, channel fading etc. except collision in our analysis. Thus, the routing probabilities are computed as in (7), where the first multiplicative term, indicates the location of the transmitter (i.e. vehicle A). The second multiplicative term, guarantees the existence of a class-k vehicle within a specific sub-region, that is, (x_2, y_2), which is considered as the location of the receiver within TR^+. The third multiplicative term, indicates the probability of not finding a more suitable vehicle for the next hop compared with one in (x_2, y_2), and the last multiplicative term shows the probability of not having a transmission or reception in the IR of the transmitter or the receiver. The last term within the parenthesis is to exclude the collision situation with respect to RTS/CTS signalling. Actually, considering RTS/CTS signalling and a sensing range equal to IR, the hidden terminal problem is removed [21]. Since in our modelling approach, $\rho^{(t)}_{\mathrm{mob}}$ plays a key role in the parameters of the QN in Fig. 4, so, the vehicles with the same class have the same status in average. Thus, the vehicles with the same class have the same transmission and reception probabilities in average. In (7), although some vehicles in A_3 may not be independent, we assume independent behaviour for all those vehicles. Thus, the probability of not sending or receiving for n_i class-i vehicles at a slot seeing by vehicle A, will be $(1 - P^{(i),(t)}_{\mathrm{tr}} - P^{(i),(t)}_{\mathrm{rec}})^{n_i}$ where $P^{(i),(t)}_{\mathrm{tr}}$ is the probability that a typical class-i vehicle sends its packet (successful or unsuccessful) and $P^{(i),(t)}_{\mathrm{rec}}$ is the probability that such a vehicle receives a packet. In the latter case, its corresponding transmitter is beyond the IR of vehicle A, otherwise vehicle A can sense it. We will give the details of obtaining $P^{(i),(t)}_{\mathrm{tr}}$ and $P^{(i),(t)}_{\mathrm{rec}}$ later. The above independency assumption is only an approximation. In fact, because of RTS-CTS signalling, if a vehicle in A_3 transmits or receives, some of the other vehicles in the same range will be frozen, that is, their behaviour are not independent. By independency assumption, we overestimate the collision probability [i.e. the complement probability of (7), see (9)], leading to an underestimate for the maximum stable throughput. Our numerical results in Section 6 indicate that this assumption does not have a very degrading effect.

For the case that vehicle A is within the TR of destination $(D^{(t)}_1, D^{(t)}_2)$ the corresponding routing probabilities, that is, $r_{B^{(t)}_1,D^{(t)}_1}, r_{B^{(t)}_1,D^{(t)}_2}$ are the same as (7) except that the second and third multiplicative terms are omitted.

The other routing probabilities will be computed as in the following

$$r_{B^{(t)}_1,T^{(t)}_k} = r_{B^{(t)}_2,T^{(t)}_k} = \cdots = r_{B^{(t)}_m,T^{(t)}_k}, \quad 1 \le k \le 6 \tag{8}$$

$$r_{B^{(t)}_1,\mathrm{RTS}^{(t)}_1} = r_{B^{(t)}_2,\mathrm{RTS}^{(t)}_2} = \cdots = r_{B^{(t)}_m,\mathrm{RTS}^{(t)}_m} = 1 - \sum_{k=1}^{6} r_{B^{(t)}_1,T^{(t)}_k} - \sum_{i=1}^{2} r_{B^{(t)}_1,D^{(t)}_i} \tag{9}$$

$$r_{\mathrm{RTS}^{(t)}_1,B^{(t)}_2} = \cdots = r_{\mathrm{RTS}^{(t)}_{(m-1)},B^{(t)}_m} = r_{\mathrm{RTS}^{(t)}_m,B^{(t)}_m} = 1 \tag{10}$$

$$r_{T^{(t)}_i,0^{(i)}} = 1, \quad 1 \le i \le 6, \quad 1 \le t \le 6 \tag{11}$$

where $r_{T^{(t)}_i,0^{(i)}}$ denotes the probability of packet transmission from a class-t vehicle to a class-i vehicle as its next hop.

Now, we compute transmission and reception probabilities [used in (7)] as in the following

$$P^{(i),(t)}_{\mathrm{rec}} = \sum_{x_1=1}^{N_s}\sum_{y_1=1}^{6} P^{(i)}_{\mathrm{loc-typ}}(x_2, y_2) \times \sum_{j=1}^{6} \sum_{\{(x_1,y_1):(x_2,y_2)\in \mathrm{TR}^+\}} \Bigg\{(1 - \exp(-\rho^{(j)}_{\mathrm{mob}}(x_1, y_1))) \times \exp\left(-\sum_{\{(x_3,y_3)\in A_4\}}\sum_{i'=1}^{6} \rho^{(i')}_{\mathrm{mob}}(x_3, y_3)\right)\Bigg\} \times P^{(j),(t)}_{\mathrm{tr-suc}}$$

$$A_4 = \{\text{sub-regions more suitable than the current sub-region, that is } (x_2, y_2)\} \tag{12}$$

where in (12), the first multiplicative term, indicates the probability that a class-i vehicle, as the receiver, occupies different sub-regions of the street. For each sub-region (x_2, y_2), the second multiplicative term, denotes the probability of having at least one vehicle, as the transmitter, at each sub-region that (x_2, y_2) is within its TR^+. For each pair of sub-regions, that is, (x_1, y_1) and (x_2, y_2), the third

$$r_{B^{(t)}_1,T^{(t)}_k} = \sum_{x_1=1}^{N_s}\sum_{y_1=1}^{6} P^{(t)}_{\mathrm{loc-typ}}(x_1, y_1) \times \sum_{(x_2,y_2)\in A_1} \Bigg\{(1 - \exp(-\rho^{(k)}_{\mathrm{mob}}(x_2, y_2))) \times \exp\left(-\sum_{j=1}^{6}\sum_{(x_3,y_3)\in A_2} \rho^{(j)}_{\mathrm{mob}}(x_3, y_3)\right) \times \sum_{n=0,n=n_1+\cdots+n_6}^{\infty} \sum_{n_1,\ldots,n_6=0}^{n} \prod_{i=1}^{6}\Bigg\{\exp\left(-\sum_{(x_4,y_4)\in A_3} \rho^{(i)}_{\mathrm{mob}}(x_4, y_4)\right) \times \frac{(\Sigma_{(x_4,y_4)\in A_3}\rho^{(i)}_{\mathrm{mob}}(x_4, y_4))^{n_i}}{n_i!}(1 - P^{(i),(t)}_{\mathrm{tr}} - P^{(i),(t)}_{\mathrm{rec}})^{n_i}\Bigg\}\Bigg\} \tag{7}$$

$1 \le k \le 6$, $\{n_i$: the number of class$-i$ vehicles, located in subregion $(x_4, y_4)\}$

$A_1 = \{$sub-regions in the TR^+ of (x_1, y_1), that is, nearer to destination$\}$

$A_2 = \{$sub-regions more suitable than $(x_2, y_2)\}$

$A_3 = \{$sub-regions in the IR of the transmitter or the receiver$\}$

doi: 10.1049/iet-com.2010.0799

multiplicative term denotes the probability of not having any other vehicle in the sub-regions more suitable than the current receiver's location within the TR$^+$ of the transmitter. The last multiplicative term, $P_{\text{tr-suc}}^{(j),(t)}$, denotes the probability of successful transmission by the transmitter. On the other hand, the probabilities $P_{\text{tr}}^{(i),(t)}$ and $P_{\text{tr-suc}}^{(i),(t)}$ are computed as in the following

$$P_{\text{tr}}^{(i),(t)} = \rho^{(i),(t)} \times \frac{\sum_{c\in A_5}(\rho_c^{(i)}/T_c^s)}{\sum_{d\in A_6}(\rho_d^{(i)}/T_d^s)};\ 1 \le i \le 6 \qquad (13)$$

$$A_5 = \{D_1^{(i)}, D_2^{(i)},\ T_1^{(i)} \ldots T_6^{(i)}, \text{RTS}_1^{(i)},\ \ldots, \text{RTS}_m^{(i)}\}$$

$$A_6 = \{D_1^{(i)}, D_2^{(i)}, T_1^{(i)} \ldots T_6^{(i)}, \text{RTS}_1^{(i)}, \ldots, \text{RTS}_m^{(i)}, B_1^{(i)}, \ldots, B_m^{(i)}\}$$

$$P_{\text{tr-suc}}^{(i),(t)} = \rho^{(i),(t)} \times \frac{\sum_{c\in A_7}(\rho_c^{(i)}/T_c^s)}{\sum_{d\in A_6}(\rho_d^{(i)}/T_d^s)},\ 1 \le i \le 6 \qquad (14)$$

$$A_7 = \{D_1^{(i)}, D_2^{(i)}, T_1^{(i)} \ldots T_6^{(i)}\}$$

where the second multiplicative terms (i.e. the fractions) in (13) and (14) denote the probability that a typical packet in the transmission process of the class-i vehicle, is in the transmission attempt phase (successful or unsuccessful) and in the successful transmission phase, respectively, at the slot seen by a class-t vehicle. Actually, since the vehicles are synchronised (regarding the structure of DCF), in computation of $P_{\text{tr}}^{(i),(t)}$ and $P_{\text{tr-suc}}^{(i),(t)}$, we need the number of slots in the corresponding phases. Since the traffic intensity of each node (c), that is, $\rho_c^{(i)}$, shows the proportion of time (in steady state) that the node is non-empty, thus, we divide the traffic intensity at each node into the average time of the corresponding slot, T_c^s, to find the number of slots at the corresponding node (i.e. transmission phase). In (13), (14) $\rho_c^{(t)}$ equals the ratio of arrival rate over service rate. These rates as well as T_c^s will be calculated in Sections 5.2 and 5.3. Moreover, customer arrivals at the nodes belonging to A_7 and A_5 indicate the successful transmission and transmission in general, respectively.

It is worth mentioning that in computing $P_{\text{tr}}^{(i),(t)}$ and $P_{\text{tr-suc}}^{(i),(t)}$, we need to multiply the aforementioned fraction by the probability that the corresponding vehicle is non-empty when seen by vehicle A. Since vehicle A sees the other vehicles only at the slots that it has a packet itself and the slots have variable durations, so, we define another probability $\rho^{(i),(t)}$ denoting the steady state probability that a class-i vehicle is non-empty at the slots seen by a class-t vehicle. For computation of this probability, see Appendix.

5.2 Service time distribution at the nodes of the proposed QN

The service time at nodes $\text{RTS}_i^{(t)}$, $D_i^{(t)}$ and $T_i^{(t)}$ in Fig. 4 are clearly computed. However, since in nodes $B_i^{(t)}$ there are several virtual time slots, to compute the service time distributions, we describe what happens when a packet enters node $B_i^{(t)}$. To this end, assume that vehicle A wants to send a packet towards vehicle B (as the next suitable vehicle). Vehicle A should wait for a number of backoff slots before it starts to transmit. Moreover, it senses the channel prior to counting down at the beginning of each slot. Now, we consider what can happen at each slot. There are three cases for a typical slot which vehicle A may consider:

(1) Vehicle A considers a typical slot as a normal slot which its duration is equal to an idle time slot (T_{slot}). Its probability is indicated by $P_{\text{nor}}^{(t)}$.
(2) Vehicle A considers a typical slot as a successful slot which its duration (T_{suc}) is equal to the summation of RTS-time (T_{RTS}), CTS-time (T_{CTS}), and transmission time of the packet (T_{packet}). Its probability is indicated by $P_{\text{suc}}^{(t)}$.
(3) Vehicle A considers a typical slot as an unsuccessful slot which its duration (T_{unsuc}) is equal to the summation of RTS-time and CTS-timeout ($T_{\text{CTS-timeout}}$). Its probability is indicated by $P_{\text{unsuc}}^{(t)}$.

The first case occurs when there is not any vehicle in the sensing range of vehicle A which is already transmitting or receiving a packet. In this case, vehicle A continues to count down. The second case happens when there is at least a successful transmitter or a receiver within the sensing range of the vehicle A which means a successful RTS/CTS signalling has been exchanged earlier, so, vehicle A has been frozen. The last case happens when there is no successful transmission or reception in the sensing range of vehicle A, however, at least one unsuccessful transmission is currently active. In the second and third cases, vehicle A postpones its counting down until the end of the virtual slot.

The following equations show how we calculate the service time for a packet in node $B_i^{(t)}$. At first, we calculate the probability for a slot to be normal, successful or unsuccessful. To this end, we define two related probabilities corresponding to each sub-region as in 15 and 16, where $P_1^{(t)}(x_2, y_2)$ indicates the probability that a typical class-t vehicle, located at sub-region (x_2, y_2), does not see a typical slot as a successful one. So, if there are n_i class-i vehicles in the IR of the typical vehicle, they should not transmit successfully or receive a packet. Regarding the mobility model in Section 3, the probability of n_i vehicles at a region follows a Poisson distribution. On the other hand, $P_2^{(t)}(x_2, y_2)$ denotes the probability that a typical class-t vehicle, located in sub-region (x_2, y_2), sees a typical slot

$$P_1^{(t)}(x_2, y_2) = \sum_{n=0, n=n_1+\cdots+n_6}^{\infty} \sum_{n_1,\cdots,n_6=0}^{n} \prod_{i=1}^{6} \left\{ \exp\left(- \sum_{(x_1,y_1)\in \text{IR}} \rho_{\text{mob}}^{(i)}(x_1, y_1) \right) \frac{(\sum_{(x_1,y_1)\in\text{IR}} \rho_{\text{mob}}^{(i)}(x_1, y_1))^{n_i}}{n_i!} \times (1 - P_{\text{tr-suc}}^{(i),(t)} - P_{\text{rec}}^{(i),(t)})^{n_i} \right\} \qquad (15)$$

$$P_2^{(t)}(x_2, y_2) = \sum_{n=0, n=n_1+\cdots+n_6}^{\infty} \sum_{n_1,\ldots,n_6=0}^{n} \prod_{i=1}^{6} \left\{ e^{-\sum_{(x_1,y_1)\in\text{IR}} \rho_{\text{mob}}^{(i)}(x_1,y_1)} \frac{(\sum_{(x_1,y_1)\in\text{IR}} \rho_{\text{mob}}^{(i)}(x_1, y_1))^{n_i}}{n_i!} \times (1 - P_{\text{tr}}^{(i),(t)} - P_{\text{rec}}^{(i),(t)})^{n_i} \right\} \qquad (16)$$

as a normal one, that is, within its IR there is not any transmission or any reception by another vehicle. We have the following probabilities corresponding to aforementioned three cases

$$P_{\text{nor}}^{(t)} = \sum_{x_2=1}^{N_s} \sum_{y_2=1}^{6} P_{\text{loc-typ}}^{(t)}(x_2, y_2) \times P_2^{(t)}(x_2, y_2) \quad (17)$$

$$P_{\text{unsuc}}^{(t)} = \sum_{x_2=1}^{N_s} \sum_{y_2=1}^{6} P_{\text{loc-typ}}^{(t)}(x_2, y_2) \times (P_1^{(t)}(x_2, y_2) - P_2^{(t)}(x_2, y_2)) \quad (18)$$

$$P_{\text{suc}}^{(t)} = 1 - P_{\text{unsuc}}^{(t)} - P_{\text{nor}}^{(t)} \quad (19)$$

Now we calculate the average service time (i.e. the inverse of service rate) for the packets at each node of the QN, as in the following

$$\frac{1}{\mu_{0^{(t)}}} = T_{\text{slot}} \quad (20)$$

$$\frac{1}{\mu_{W^{(t)}}} = N_W \times T_{\text{slot}}, \quad 1 \le t \le 6 \quad (21)$$

$$\frac{1}{\mu_{B_i^{(t)}}} = \left(\frac{W_{B_i} - 1}{2} (P_{\text{nor}}^{(t)} \times T_{\text{slot}} + P_{\text{unsuc}}^{(t)} \times T_{\text{unsuc}} + P_{\text{suc}}^{(t)} \times T_{\text{suc}}) \right), \quad 1 \le i \le m \quad (22)$$

$$T_{\text{suc}} = T_{\text{packet}} + T_{\text{RTS}} + T_{\text{CTS}}$$

$$T_{\text{unsuc}} = T_{\text{RTS}} + T_{\text{CTS-timeout}}$$

$$\frac{1}{\mu_{\text{RTS}_1^{(t)}}} = \cdots = \frac{1}{\mu_{\text{RTS}_m^{(t)}}} = T_{\text{unsuc}} \quad (23)$$

$$\frac{1}{\mu_{T_i^{(t)}}} = \frac{1}{\mu_{D_j^{(t)}}} = T_{\text{suc}}, \quad 1 \le i \le 6, \ 1 \le j \le 2 \quad (24)$$

where N_W is the number of idle slots a vehicle should wait before it checks again for finding a suitable vehicle as the next hop, in cases that the vehicle has not found anyone within its TR^+. The other terms in (20)–(24) have been defined before. According to the above equations, we can now calculate T_c^s, used in (13), (14) as in the following

$$T^s_{B_1^{(t)}} = T^s_{B_2^{(t)}} = \cdots = T^s_{B_m^{(t)}} = (P_{\text{nor}}^{(t)} \times T_{\text{slot}} + P_{\text{unsuc}}^{(t)} \times T_{\text{unsuc}} + P_{\text{suc}}^{(t)} \times T_{\text{suc}})$$

$$T^s_{\text{RTS}_1^{(t)}} = \cdots = T^s_{\text{RTS}_m^{(t)}} = T_{\text{unsuc}}$$

$$T^s_{T_1^{(t)}} = \cdots = T^s_{T_6^{(t)}} = T^s_{D_1^{(t)}} = T^s_{D_2^{(t)}} = T_{\text{suc}} \quad (25)$$

5.3 *Traffic equations and the maximum stable throughput*

Traffic equations express how we can compute the arrival rate at each node in stable conditions, that is, when the arrival and departure rates of each node are equal. These rates are dependent upon the routing probabilities among the nodes as well as the departure rate from each node. Since the number of vehicles in the street is varying, we consider a typical vehicle from each class and then consider the rates for that vehicle. As we indicated previously, vehicles of the same class have the same mobility pattern. Since, the parameters of the QN corresponding to each vehicle strongly depend on its mobility pattern, it is reasonable to consider a typical vehicle of each class instead of considering all vehicles individually. Thus, we consider an extended open QN consisting of six QNs corresponding to six classes (initial lanes) of the vehicles. In this extended QN, the traffic equations are written as in the following

$$\alpha_{0^{(t)}} = \lambda^{(t)} + \sum_{k=1}^{6} \alpha_{T_t^{(k)}} \times r_{T_t^{(k)},0^{(t)}}, \quad 1 \le t \le 6 \quad (26)$$

$$\alpha_{W^{(t)}} = \alpha_{W^{(t)}} \times r_{W^{(t)},W^{(t)}} + \alpha_{0^{(t)}} \times r_{0^{(t)},W^{(t)}}, \quad 1 \le t \le 6 \quad (27)$$

$$\alpha_{B_1^{(t)}} = \alpha_{0^{(t)}} \times r_{0^{(t)},B_1^{(t)}} + \alpha_{W^{(t)}} \times r_{W^{(t)},B_1^{(t)}}, \quad 1 \le t \le 6 \quad (28)$$

$$\alpha_{B_j^{(t)}} = \alpha_{\text{RTS}_{j-1}^{(t)}} \times r_{\text{RTS}_{j-1}^{(t)},B_j^{(t)}}, \quad 2 \le j \le m-1, \quad 1 \le t \le 6 \quad (29)$$

$$\alpha_{B_m^{(t)}} = \alpha_{\text{RTS}_{m-1}^{(t)}} \times r_{\text{RTS}_{m-1}^{(t)},B_m^{(t)}} + \alpha_{\text{RTS}_m^{(t)}} \times r_{\text{RTS}_m^{(t)},B_m^{(t)}}, \quad 1 \le t \le 6 \quad (30)$$

$$\alpha_{\text{RTS}_j^{(t)}} = \alpha_{B_j^{(t)}} \times r_{B_j^{(t)},\text{RTS}_j^{(t)}}, \quad 1 \le j \le m, \quad 1 \le t \le 6 \quad (31)$$

$$\alpha_{T_i^{(t)}} = \sum_{j=1}^{m} \alpha_{B_j^{(t)}} \times r_{B_j^{(t)},T_i^{(t)}}, \quad 1 \le i \le 6, \quad 1 \le t \le 6 \quad (32)$$

$$\alpha_{D_i^{(t)}} = \sum_{j=1}^{m} \alpha_{B_j^{(t)}} \times r_{B_j^{(t)},D_i^{(t)}}, \quad 1 \le i \le 2, \quad 1 \le t \le 6 \quad (33)$$

where $\alpha_x^{(t)}$ and $\lambda^{(t)}$ denote the packet arrival rates at node x of the QN and the packet arrival rate from the exogenous world (i.e. packet generation rate at the sensors of the vehicle), respectively, corresponding to the class-t vehicle. The routing probabilities have been computed in Section 5.1.

To solve the traffic equations, at first, we assume some initial values for the probabilities of transmission and successful transmission, that is, $P_{\text{tr}}^{(i),(t)}$ and $P_{\text{tr-suc}}^{(i),(t)}$ $1 \le i, t \le 6$. By solving (5)–(11) and (20)–(25), we find the routing probabilities from one node to another node and service times in our QN model, respectively. Then we assume a common $\lambda^{(t)}$ for $1 \le t \le 6$. Now, we should solve (26)–(33) recursively until $\alpha_x^{(t)}$ for all nodes converge to some constant values. According to (20)–(24), we find the service rates for different nodes. Then, we calculate $\rho^{(i)(t)}$ (see Appendix) and the probability of transmission and successful transmission by (13) and (14). After finding the new values of $P_{\text{tr}}^{(i),(t)}$ and $P_{\text{tr-suc}}^{(i),(t)}$, we should find the new values for the routing probabilities and service times. Then, we again solve the traffic equations. We continue this process until the values of $P_{\text{tr}}^{(i),(t)}$ and $P_{\text{tr-suc}}^{(i),(t)}$ converge to some constant values.

Now, we are able to find the maximum stable throughput of each vehicle, that is, the maximum rate of packets generated at each vehicle such that all vehicles remain stable. Stability of each vehicle means that its departure rate equals its

doi: 10.1049/iet-com.2010.0799

arrival rate. Since we have considered a common value of λ for all $\lambda^{(t)}$'s, we should increase λ step by step until $\rho^{(t)} = \Sigma_{e\in A_6}\rho_e^{(t)}$, that is, the average number of packets at the server of the corresponding vehicle, for at least one of classes (i.e. bottleneck class) reaches one. In fact, for λ exceeding this rate, the departure rate of the bottleneck vehicles does not follow the arrival rate. It is worth mentioning that keeping $\rho^{(t)}$ smaller than one guarantees the existence of at most one packet, in average, at the QN (Fig. 4). Thus, we obtain the maximum stable throughput (λ_{max}) as in the following

$$\lambda_{max} = \max \lambda|_{\exists t, 1\le t,t'\le 6, t\ne t'; \Sigma_{e\in A_6}\rho_e^{(t)}=1, \Sigma_{e\in A_6}\rho_e^{(t')}\le 1} \quad (34)$$

Obviously, for computing maximum stable throughput of the network, Λ_{max}, we should multiply λ_{max} by the average number of vehicles at each class as in the following

$$\Lambda_{max} = \lambda_{max} \times \sum_x \sum_y \sum_t \rho_{mob}^{(t)}(x, y) \quad (35)$$

Table 1 Typical values for the parameters in the numerical results

Parameter	Value	Unit
speed distribution at the first lane	U [3, 14]	m/s
speed distribution at the second lane	U [14, 22]	m/s
speed distribution at the third lane	U [22, 33]	m/s
length of the street	2000	m
length f each sub-region (L_1)	10	m
width of each sub-region (L_2)	4	m
transmission range (R)	200	m
lane transition probability	1/3	NA
time slot (T_{slot})	100	µs
RTS time	$3 \times T_{slot}$	s
CTS time, CTS-timeout	$1 \times T_{slot}$	s
packet transmission time	$600 \times T_{slot}$	S
number of backoff stages (m)	3	NA
CW_{min}	4	NA

6 Numerical results

In this section, we apply our analytical approach to show its effectiveness in obtaining the maximum stable throughput in different conditions. The typical values for different parameters have been presented in Table 1. In Fig. 5, we have changed the arrival rate of the vehicles at both ways of the street symmetrically and evaluated the maximum stable throughput. As it is observed, the maximum stable throughput increases while we increase the arrival rate of the vehicles at each way of the street. However, the rate of this increase reduces gradually and after exceeding a point the maximum stable throughput will be nearly constant. This is owing to the fact that increasing the arrival rate of the vehicles leads to more crowded street, so more collisions occur and it degrades the increase rate of the maximum stable throughput. On the other hand, the maximum stable throughput of the network has an increase with a nearly linear rate. It is worth mentioning that the rate of vehicle arrivals cannot be increased arbitrarily owing to limitation of the number of vehicles at the streets.

In Fig. 6, we have changed the TR from 100 to 400 m and evaluated the maximum stable throughput for three vehicle arrival rates. As depicted in Fig. 7, the maximum stable throughput increases while we increase the TR. However, increasing the TR will lead to more collisions and the maximum stable throughput reaches a nearly constant value.

To confirm our analytical results, we have done some simulations. We have implemented a simulation program in MATLAB environment. This program is an event-driven one consisting of the following events, packet generation, packet transmission, collision situation and vehicle movement. We consider the mobility pattern of the vehicles according to the mobility model in Section 3. With respect to the simulation setup shown in Table 1, we have varied the new packet generation rate at all vehicles of each class and evaluated the successful packet departure rate towards the destination corresponding to each class, during a sufficiently large time interval. As it is observed in Fig. 7, the arrival rate equals the departure rate for $\lambda \le \lambda_{max}$, but when exceeding this rate, the departure rate deviates from the arrival rate for the saturated vehicles. Thus, if we plot the departure rate against the arrival rate for vehicles with different initial lanes,

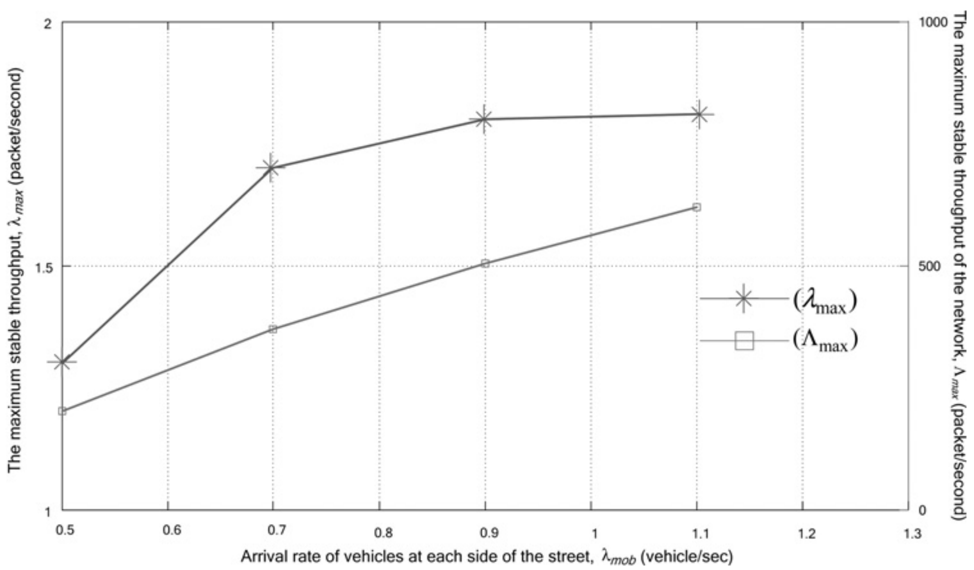

Fig. 5 *Maximum stable throughput against arrival rate of vehicles*

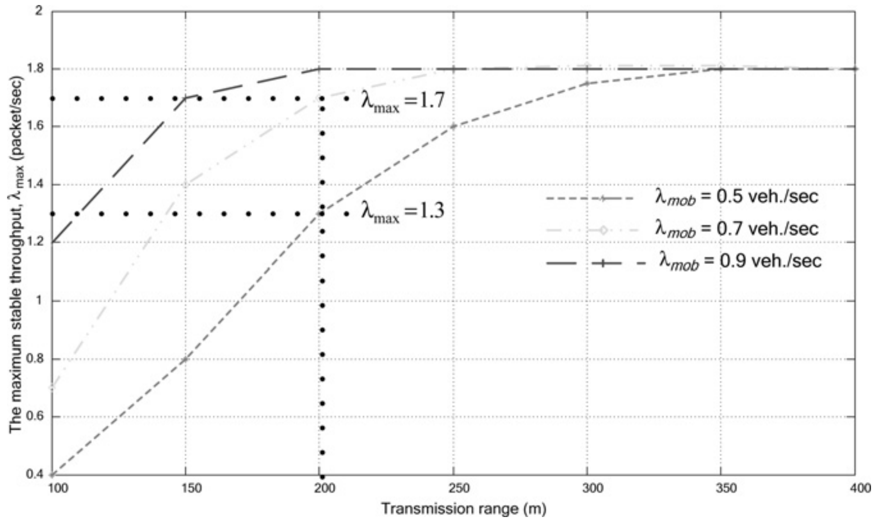

Fig. 6 *Maximum stable throughput against TR*

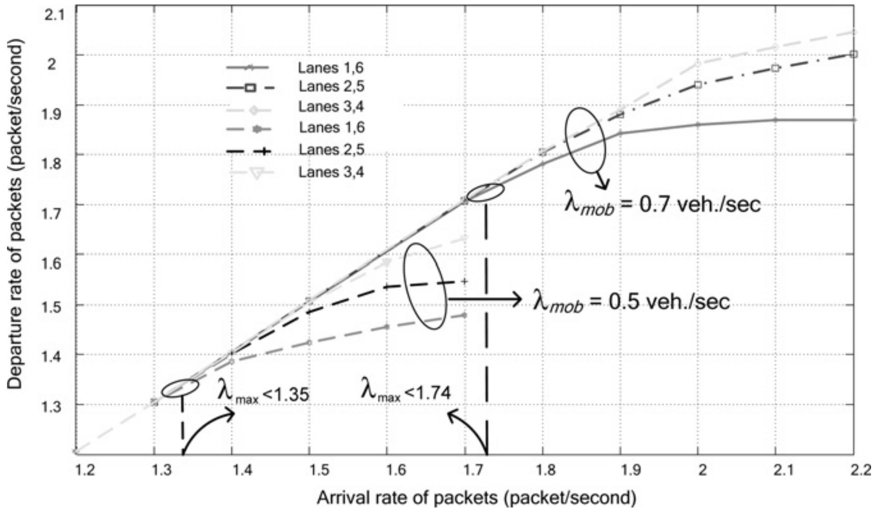

Fig. 7 *Departure rate against arrival rate (simulation)*

the corresponding curves have a slope equal to one, until at least the slope of one of the curves deviates from one. The location of this deviation (i.e. the knee of the corresponding curve) shows the maximum stable throughput. It is worth mentioning that finding the exact location of the deviation point is not easily possible, because at this point some of the vehicles are in the border of instability, thus their corresponding queues have infinite length, and the transient time will be very large (maybe infinity). Comparing the simulation and analytical results indicates that the analytical results (Fig. 5) underestimates the maximum stable throughput less than 5% when compared with the simulation results (Fig. 7) that is because of a few approximations discussed in Section 5.1. It is worth noting that the slowest vehicles are actually the bottleneck vehicles, because they remain in the street longer than the other vehicles, so their packet arrivals increases, leading to larger traffic intensity.

7 Conclusions

VANETs play a key role in future transportation systems to increase the safety level of the roads. Regarding a new standard developed for VANETs, that is, IEEE 802.11p that it fundamentally based on IEEE 802.11, in this paper we focused on a simple IEEE 802.11-based VANET scenario and proposed a new analytical approach to model the mobility patterns of the vehicles as well as the details of multi-hop packet transmission. In this respect, we exploited a mobility model previously reported in the literature and mapped the details of IEEE 802.11 MAC layer as well as routing scheme among vehicles onto suitable parameters of the proposed QN. The proposed QN represent different transmission phases of the packet at each typical vehicle regarding the dynamic status of the other vehicles. Since the number of vehicles is time varying, we focused on the typical vehicles arriving at each lane of the street. By writing the traffic equations and applying the stability condition for each typical vehicle of all classes, we were able to obtain the maximum stable throughput. At last, we showed the accuracy of our analytical approach by simulation and evaluated the effect of TR as well as the arrival rate of vehicles onto the maximum stable throughput.

8 Acknowledgment

This research work has been partially supported by Iran Telecom Research Centre (ITRC) under grant No. 500.19167 and Iran National Science Foundation (INSF) under grant No. 88114.46-2010.

doi: 10.1049/iet-com.2010.0799

9 References

1 IEEE P1609.4/D06, Wireless Access in Vehicular Environments (WAVE) Multi-Channel Operation, November 2005
2 Toor, Y., Muhlethaler, P., Laouiti, A., Fortelli, A.D.L.: 'Vehicle ad hoc networks: applications and related technical issues', *IEEE Commun. Surv.*, 2008, **10**, (3), pp. 74–88
3 Jiang, D., Delgrossi, L.: 'IEEE 802.11p: towards an international standard for wireless access in vehicular environments'. Proc. IEEE VTC'08, Spring, 2008, pp. 2036–2040
4 Menouar, H., Filali, F., Lenardi, M.: 'A survey and qualitative analysis of MAC protocols for vehicular ad hoc networks', *IEEE Wirel. Commun.*, 2006, **13**, (5), pp. 30–35
5 Mohimani, G.H., Ashtiani, F., Javanmard, A., Hamdi, M.: 'Mobility modeling, spatial traffic distribution, and probability of connectivity for sparse and dense vehicular ad hoc networks', *IEEE Trans. Veh. Technol.*, 2009, **58**, (4), pp. 1998–2007
6 Khabazian, M., Ali, M.K.M.: 'A perfromance modeling of connectivity in vehicular ad hoc networks', *IEEE Trans. Veh. Technol.*, 2008, **57**, (4), pp. 2440–2450
7 Li, F., Wang, Y.: 'Routing in vehicular ad hoc networks: a survey', *IEEE Vehi. Technol. Mag.*, 2007, **2**, (2), pp. 12–22
8 Ding, Y., Wang, C., Xiao, L.: 'A static-node assisted adaptive routing protocol in vehicular networks'. Proc. VANET'07, September 2007, pp. 59–67
9 Zhao, J., Cao, G.: 'VADD: vehicle-assisted data delivery in vehicular ad hoc networks'. Proc. IEEE INFOCOM'06, 2006
10 Karamad, E., Ashtiani, F.: 'A modified 802.11-based MAC scheme to assure fair access for vehicle-to-roadside communications', *Comput. Commun.*, 2008, **31**, (12), pp. 2898–2906
11 Choi, N., Choi, S., Seok, Y., Kwon, T., Choi, Y.: 'A solicitation-based IEEE 802.11p MAC protocol for roadside to vehicular networks'. Proc. Mobile Networking for Vehicular Environments, 2007, pp. 91–96
12 Bianchi, G.: 'Performance analysis of the IEEE 802.11 distributed coordination function', *IEEE J. Sel. Areas Commun.*, 2000, **18**, (3), pp. 535–547
13 Ghaboosi, K., Khalaj, B.H., Xiao, Y., Latva-aho, M.: 'Modeling IEEE 802.11 DCF using parallel space-time markov chain', *IEEE Trans. Veh. Technol.*, 2008, **57**, pp. 2404–2413
14 Karamad, E., Ashtiani, F.: 'Performance analysis of IEEE 802.11 DCF and 802.11e EDCA based on queueing networks', *IET Commun.*, 2009, **3**, (5), pp. 871–881
15 Ng, P.C., Liew, S.C.: 'Throughput analysis of IEEE802.11 multi-hop ad hoc networks', *IEEE Trans. Netw.*, 2007, **15**, (2), pp. 309–322
16 An, J., Guo, X., Yang, Y.: 'Analysis of collision probability in vehicular ad hoc networks'. Proc. ACM/GEC'09, 2009, pp. 791–794
17 Javanmard, A., Ashtiani, F.: 'Analytical evaluation of average delay and maximum stable throughput along a typical two-way street for vehicular ad-hoc networks in sparse situations', *Comput. Commun.*, 2009, **32**, (16), pp. 1768–1780
18 Ma, X., Chen, X., Refai, H.H.: 'Performance and reliability of DSRC vehicular safety communication: a formal analysis', *EURASIP J. Wirel. Commun. Netw.*, 2009, pp. 1–13
19 Chao, X., Miyazawa, M., Pinedo, M.: 'Queueing networks' (John Wiley & Sons, 1999)
20 Kong, Z., Tsang, D.H.K., Bensaon, B., Gao, D.: 'Performance analysis of IEEE 802.11e contention-based channel access', *IEEE J. Sel. Areas Commun.*, 2004, **22**, (10), pp. 2095–2106
21 Xu, K., Gerla, M., Bae, S.: 'How effective is the IEEE 802.11 RTS/CTS handshake in ad hoc networks?'. Proc. Globecom'02, 2002, pp. 72–76
22 Kleinrock, L.: 'Queueing systems' (John Wiley & Sons, 1975), vol. 1

10 Appendix

10.1 Computation of $\rho^{(i),(t)}$ in (13) and (14)

In computing the steady state probability that a class-i vehicle has a packet at a slot seen by a class-t vehicle, we consider a simple two state Markov chain such that states 0, 1 denote the former vehicle is seen as empty and non-empty, respectively. Obviously, by considering $P_{u,v}^{(i),(t)}$ ($u, v = 0, 1$), that is, the transition probability from state u to state v of the Markov chain, we will have

$$\rho^{(i),(t)} = \frac{P_{01}^{(i),(t)}}{P_{01}^{(i),(t)} + P_{10}^{(i),(t)}}$$

In this respect $P_{10}^{(i),(t)}$ is computed as in the following

$$P_{10}^{(i),(t)} = P_{tx}^{(t)} \times (1 - \rho^{(t)}) \times (1 - \rho^{(i)}) + P_{\text{backoff}}^{(t)} \times P_{tx}^{(i)} \times (1 - \rho^{(i)}) \quad (36)$$

where $\rho^{(i)} = \Sigma_{e\in A_6}\rho_e^{(i)}$ denotes the traffic intensity of the class-i vehicle, that is, the steady state probability that the class-i vehicle is non-empty, and $P_{\text{backoff}}^{(t)}, P_{tx}^{(t)}$ denote the ratio of the number of backoff slots and successful transmission attempts, respectively, over total number of slots at which the class-t vehicle is non-empty. These probabilities are computed as in the following:

$$P_{tx}^{(i)} = \frac{\Sigma_{c\in A_7}(\rho_c^{(i)}/T_c^s)}{\Sigma_{d\in A_6}(\rho_d^{(i)}/T_d^s)} \quad (37)$$

$$P_{\text{backoff}}^{(i)} = \frac{\Sigma_{c\in A_8}(\rho_c^{(i)}/T_c^s)}{\Sigma_{d\in A_6}(\rho_d^{(i)}/T_d^s)}; \quad A_8 = \{B_1^{(i)}, \ldots, B_m^{(i)}\} \quad (38)$$

In writing (36), we have approximated arrival rate at each vehicle with a Poisson distribution and employ the fact that at an M/G/1 node, a departing customer sees the node in its steady state. For example, the first additive term in (36) denotes the case that in a slot corresponding to transmission phase, the class-t vehicle sees the class-i vehicle as non-empty (i.e. being at state 1) and at the end of the transmission, the former becomes empty itself. At some slots later (after a gap that the former does not see anything because it does not have any packet) regarding the assumption of Poisson arrival, the former vehicle sees the latter as empty with a probability equal to $1 - \rho^{(i)}$. The second additive term denotes that at current observation slot the class-t vehicle is in backoff and the class-i vehicle is transmitting its last packet (indicated by $P_{tx}^{(i)} \times (1 - \rho^{(i)})$).

On the other hand $P_{01}^{(i),(t)}$ is computed as in the following

$$\begin{aligned} P_{01}^{(i),(t)} = {} & P_{\text{backoff}}^{(t)} \times (1 - \exp(-\alpha_{0^{(i)}} \times T_{B_1^{(t)}}^s)) + P_{\text{RTS}}^{(t)} \\ & \times (1 - \exp(-\alpha_{0^{(i)}} \times T_{\text{RTS}_1^{(t)}}^s)) + P_{tx}^{(t)} \\ & \times (1 - \exp(-\alpha_{0^{(i)}} \times T_{T_1^{(t)}}^s)) + P_{tx}^{(t)} \times (1 - \rho^{(t)}) \times \rho^{(i)} \end{aligned} \quad (39)$$

$$P_{\text{RTS}}^{(i)} = \frac{\Sigma_{c\in A_9}(\rho_c^{(i)}/T_c^s)}{\Sigma_{d\in A_6}(\rho_d^{(i)}/T_d^s)}; \quad A_9 = \{\text{RTS}_1^{(i)}, \ldots, \text{RTS}_m^{(i)}\} \quad (40)$$

where different additive terms indicate different cases that at an observation slot, a class-t vehicle sees a class-i vehicle as empty (i.e. being at state 0 of the Markov chain) but at the next observation slot, it sees it as non-empty (i.e. transition to state 1 of the Markov chain). Since for this transition a packet should arrive at the class-i vehicle during the current observation slot (in the cases that the next observation slot is exactly the consecutive one, that is, the class-t vehicle does not become empty at the end of the current observation slot), we use the assumption of Poisson arrivals and the length of the current slot. The last additive term denotes the case that the next observation slot is not consecutive of the current slot because the class-t vehicle is transmitting its last packet (indicated by $P_{tx}^{(t)} \times (1 - \rho^{(t)})$). By solving the above Markov chain, we obtain $\rho^{(i),(t)}$. Such a Markov chain should be considered for $i, t = 1, \ldots, 6$.