Published in IET Communications
Received on 24th September 2009
Revised on 13th May 2010
doi: 10.1049/iet-com.2010.0149

ISSN 1751-8628

Exact symbol and bit error probabilities of linearly modulated signals with maximum ratio combining diversity in frequency non-selective Rician and Rayleigh fading channels

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Abstract: The exact symbol and bit error probabilities of linearly modulated signals in frequency non-selective Rician fading channels with Lth diversity branches are derived, where coherent detection with maximum ratio combining (MRC) is used at the receiver. For performance evaluation, the multiplicative distortion is combined with the additive Gaussian noise and form an additive noise is formed. This method allows computing the exact symbol and bit error probabilities of M-ary phase shift keying (M-PSK), M-ary quadrature amplitude modulation (M-QAM), M-ary amplitude modulation (M-AM) and M-ary amplitude modulation phase modulation (M-AMPM) for any arbitrary bit mappings. The results contain several versions of one simple integral. The error probabilities are also obtained for Rayleigh channel, which is a special case of Rice channel. Further, the results for Nakagami-m channel with the integer parameter are found. The results quite match with the previous ones while having simple forms. Also, for some unsolved cases, the exact error probabilities are presented. Simulations are carried out to verify the analytical evaluations.

1 Introduction

The problem of fading and its impact on the performance of wireless communication systems have long been of interest. Diversity techniques, which essentially amount to transmitting signals over independent fading channels, are used in practice to combat fading. Typical examples of diversity include time, frequency, polarisation and antenna (spatial) diversities. Maximum ratio combining (MRC), which maximises the output signal to noise ratio (SNR) is commonly used for coherent reception at the receiver. Rician fading channel is used to model the propagation paths consisting of one strong direct line of sight component and many random weaker components.

In [1], the authors derived the exact symbol and bit error probabilities of linearly modulated signals in single path Rician, Rayleigh and also AWGN channels. There have been several works on deriving the error probabilities in multipath Rician fading channels, with MRC at the receiver. In [2], the authors introduced a unified approach for error probability computation, which simplifies the performance evaluation over generalised fading channels [3]. In [4–6], closed-form expressions were derived for the average bit error probabilities of a class of modulation schemes in Rician fading channels. There have been also other works in the literature, for example [7–16] and references therein. However, to the best of author's knowledge, the mentioned papers used different methods for computing the error probabilities of different linear modulations with Gray coding and the error probabilities are in the forms of Marcum Q function, ${}_1F_1$ function, several summations, approximations or long complicated expressions.

In this paper, we consider linearly modulated signals with Lth order diversity in Rician fading channel, where coherent

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detection with MRC is used at the receiver. We combine the effect of fading channel, which is in the form of multiplicative distortion, with the additive Gaussian noise. The result will be presented in the form of an additive final noise. We will then obtain the probability density function (PDF) of the final noise. Consequently, the symbol error probabilities (SEPs) and bit error probabilities (BEPs) of different linearly modulated signals with any arbitrary bit mappings such as Gray or binary coding can be computed. The error probabilities are presented in the form of several versions of one simple integral, which can be easily solved by numerical methods. The obtained results quite match with the previously presented results while having simple forms. Further, for some unsolved cases, the exact error probabilities are computed. We also derive the error probabilities for Rayleigh channel, which is a special case of Rice channel. In this case, the results have analytical solutions. Considering that the Nakagami-m fading with the integer parameter m in the Lth order diversity channel is equivalent to the Rayleigh channel with the mLth order diversity [6], the error probabilities can be obtained for Nakagami channel as well. Further, we compare the performance of different modulations. We also provide simulation results to verify our analytical evaluations.

The rest of the paper is organised as follows. In Section 2, we consider the coherent detection of linearly modulated signals with MRC and combine the multiplicative distortion and additive Gaussian noise as an additive noise and then compute its PDF. In Section 3, we derive the exact SEPs and BEPs of different modulations. Section 4 presents numerical and simulation results. Finally, Section 5 concludes the paper.

2 System model

We assume a frequency flat slowly fading channel with L diversity receptions, each carrying the same information-bearing signal. The fading processes among the L diversity channels are assumed to be mutually statistically independent. The signal is also corrupted by an additive zero mean white Gaussian noise process. We consider coherent detection. For each channel, the sampled output of the matched filter, r_k, will be

$$r_k = g_k a_m + n_k, \quad k = 1, \ldots, L \tag{1}$$

where a_m represents one of the M-ary transmitted symbols. It is assumed that all symbols are equi-probable and have zero mean and unit average power, that is, $E(|a_m|^2) = 1$, $m = 1, \ldots, M$. $g_k = \beta_k e^{j\varphi_k}$ is the kth channel coefficient, which is a complex Gaussian process with non-zero mean $s_k = |E(g_k)|$ and variance σ_k^2 per real and imaginary components. As a result, β_k, the magnitude of g_k, will have Rice distribution [6] as

$$p_{\beta_k}(\beta) = \frac{\beta}{\sigma_k^2} \exp\left(-\frac{(s_k^2 + \beta^2)}{\sigma_k^2}\right) I_0\left(\frac{s_k \beta}{\sigma_k^2}\right),$$
$$\beta_k = |g_k|, \quad s_k = |E(g_k)| \tag{2}$$

where $I_0(x)$ is the modified Bessel function of order zero. $\{n_k, \quad k = 1, \ldots, L\}$ in (1) represent identically distributed (i.i.d.) uncorrelated complex Gaussian random processes with zero mean and power spectral density equal to N_0. Maximum ratio combiner multiplies each matched filter output (1) by the conjugate of the corresponding channel gain estimation $(\hat{g}_k^*)$ and then combines the results. We assume perfect channel estimations at the receiver, that is, $\hat{g}_k = g_k$, $k = 1, \ldots, L$. Hence, the output of MRC, that is, the random variable for decision is

$$D \triangleq a_m \sum_{k=1}^{L} \beta_k^2 + \sum_{k=1}^{L} \beta_k e^{-j\varphi_k} n_k, \quad \varphi_k = \arg g_k \tag{3}$$

Since $\sum_{k=1}^{L} \beta_k^2 > 0$, the above random decision variable will be equivalent to

$$D \equiv a_m + \frac{\sum_{k=1}^{L} \beta_k e^{-j\varphi_k} n_k}{\sum_{k=1}^{L} \beta_k^2} = a_m + Z,$$
$$Z \triangleq \frac{\sum_{k=1}^{L} \beta_k e^{-j\varphi_k} n_k}{\sum_{k=1}^{L} \beta_k^2} \tag{4}$$

We observe from (4) that the effects of multipath fading and additive Gaussian noise are combined and represented as the final additive noise Z. For error probability computation, we need to compute the PDF of Z. For this purpose, we first note that the random variable $X \triangleq \sum_{k=1}^{L} \beta_k e^{-j\varphi_k} n_k$ conditioned on β_k's, is a complex Gaussian noise with zero mean and variance equal to $N_0 \sum_{k=1}^{L} \beta_k^2$ per real and imaginary components [6]. Hence, the PDF of the complex random variable Z conditioned on $\sum_{k=1}^{L} \beta_k^2$ will be [6, 17]

$$p_{Z|\sum_{k=1}^{L} \beta_k^2}(z) = \frac{\sum_{k=1}^{L} \beta_k^2}{2\pi N_0} \exp\left(-\frac{\sum_{k=1}^{L} \beta_k^2}{2N_0} |z|^2\right) \tag{5}$$

We assume that all channels are identically distributed, that is, $\sigma_k^2 = \sigma^2$, $s_k = s$, $k = 1, \ldots, L$ (2). Therefore $Y \triangleq \sum_{k=1}^{L} \beta_k^2$ is a non-central Chi-square [6] random variable with $2L$ degrees of freedom and non-centrality parameter $\sum_{k=1}^{L} s_k^2 = Ls^2$. The Rice factor is defined as the ratio of the fixed scattered power to the random scattered powers, that is, $K_R \triangleq (s^2/2\sigma^2)$. Without loss of generality, assuming unit power for each β_k, $E(\beta_k^2) = 1$, then the PDF of Y in terms of Rice factor can be written [6] as

$$p_{Y=\sum_{k=1}^{L} \beta_k^2}(y) = (K_R + 1) e^{-LK_R} \left(\frac{(K_R + 1)y}{LK_R}\right)^{(L-1)/2} \times e^{-(K_R+1)y} I_{L-1}\left(2\sqrt{K_R(K_R + 1)Ly}\right) \tag{6}$$

where $I_{L-1}(x)$ is the modified Bessel function of order $L - 1$.

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We define the average SNR per channel as

$$\bar{\gamma}_c = \frac{E(\beta_k^2)}{2N_0} = \frac{1}{2N_0} \quad (7)$$

Lemma: The PDF of the final additive noise Z has circular symmetry, that is, it depends on $|Z|^2$.

Proof: In Appendix 1, from (5) to (7) we have derived the PDF of Z. It is

$$p_Z(z) = \frac{e^{-LK_R}}{\pi} \frac{L(\bar{\gamma}_c/(K_R+1))}{[1+|z|^2(\bar{\gamma}_c/(K_R+1))]^{(L+1)}} \times \left\{1 + \frac{K_R}{1+|z|^2(\bar{\gamma}_c/(K_R+1))}\right\} \times \exp\left\{\frac{LK_R}{1+|z|^2(\bar{\gamma}_c/(K_R+1))}\right\} \quad (8)$$

We observe that $p_Z(z)$ is circular symmetric. For error computations, we use polar coordinates [17] for PDF of Z which will be

$$p_{|z|,\arg z}(r,\theta) = \frac{e^{-LK_R}}{\pi} \frac{L(\bar{\gamma}_c/(K_R+1))r}{[1+r^2(\bar{\gamma}_c/(K_R+1))]^{(L+1)}} \times \left\{1 + \frac{K_R}{1+r^2(\bar{\gamma}_c/(K_R+1))}\right\} \times \exp\left\{\frac{LK_R}{1+r^2(\bar{\gamma}_c/(K_R+1))}\right\} \quad (9)$$

Thus

$$p_{|z|}(r) = \int_0^{2\pi} p_{|z|,\arg z}(r,\theta)\,d\theta = 2e^{-LK_R} \times \frac{L(\bar{\gamma}_c/(K_R+1))r}{[1+r^2(\bar{\gamma}_c/(K_R+1))]^{(L+1)}} \times \left\{1 + \frac{K_R}{1+r^2(\bar{\gamma}_c/(K_R+1))}\right\} \times \exp\left\{\frac{LK_R}{1+r^2(\bar{\gamma}_c/(K_R+1))}\right\} \quad (10)$$

Therefore from (9) and (10), we find that

$$p_{|z|,\arg z}(r,\theta) = \frac{p_{|z|}(r)}{2\pi}, \quad p_{\arg z}(\theta) = \frac{1}{2\pi} \quad (11)$$

which also arises from the circular symmetry. For Rayleigh channel, which is a special case of Rice channel, that is, $K_R = 0$, the PDF will be as

$$p_{|z|}(r) = (2L\bar{\gamma}_c r/[1+r^2\bar{\gamma}_c]^{(L+1)})$$

3 Performance evaluation

In this section, we compute the SEPs and BEPs of different linear modulations. The SEP is obtained as

$$\mathrm{SEP} = 1 - \sum_{m=1}^{M} p(a_m) \underbrace{\iint}_{R'_m} p_Z(z+a_m)\,dz \quad (12)$$

where R'_m is the decision region of the symbol a_m. For simplicity, we use polar coordinates and without loss of generality move the centre of coordinates to each a_m. So, considering (11), we have

$$\mathrm{SEP} = 1 - \frac{1}{M}\sum_{m=1}^{M} \underbrace{\iint}_{R_m} \frac{1}{2\pi} p_{|Z|}(r)\,dr\,d\theta \quad (13)$$

where R_m is the decision region R'_m moved by a_m. In order to compute the SEP, we calculate the first indefinite integral (Appendix 2), which is obtained as

$$S(r) \triangleq \int \frac{1}{2\pi} p_{|z|}(r)\,dr = -\frac{e^{-LK_R}}{2\pi} \frac{1}{[1+r^2(\bar{\gamma}_c/(K_R+1))]^L} \times \exp\left\{\frac{LK_R}{1+r^2(\bar{\gamma}_c/(K_R+1))}\right\} \quad (14)$$

So, if we can define the limits of the integrals for each decision region, in terms of $r_1^m(\theta)$, $r_2^m(\theta)$, θ_1^m and θ_2^m (which is shown for M-PSK in Fig. 1*b*), then noting (13) we have

$$\mathrm{SEP} = 1 - \frac{1}{M}\sum_{m=1}^{M}\int_{\theta_1^m}^{\theta_2^m} [S(r)]_{r_1^m(\theta)}^{r_2^m(\theta)}\,d\theta = 1 - \frac{1}{M}\sum_{m=1}^{M} I^m,$$

$$I^m \triangleq \int_{\theta_1^m}^{\theta_2^m} [S(r)]_{r_1^m(\theta)}^{r_2^m(\theta)}\,d\theta \quad (15)$$

In order to compute the BEP, we note that a single symbol error results in 1 bit to $\log_2 M$ bit errors. Hence, the BEP can be written as $\mathrm{BEP} = (1/\log_2 M)\sum_{j=1}^{\log_2 M} j\,\bar{P}(j)$ where j is the number of bits detected erroneously and $\bar{P}(j)$ is the probability that j-bit errors happen for each symbol i $(P_i(j))$, averaged over all symbols, that is, $\bar{P}(j) = (1/M)\sum_{i=1}^{M} P_i(j)$. Therefore

$$\mathrm{BEP} = \frac{1}{M\log_2 M}\sum_{j=1}^{\log_2 M}\sum_{i=1}^{M} jP_i(j) \quad (16)$$

3.1 M-PSK

3.1.1 Symbol error probability: In this case, all symbols have similar decision region (R) shown in Fig. 1*a*.

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IET Commun., 2011, Vol. 5, Iss. 1, pp. 12–26
doi: 10.1049/iet-com.2010.0149

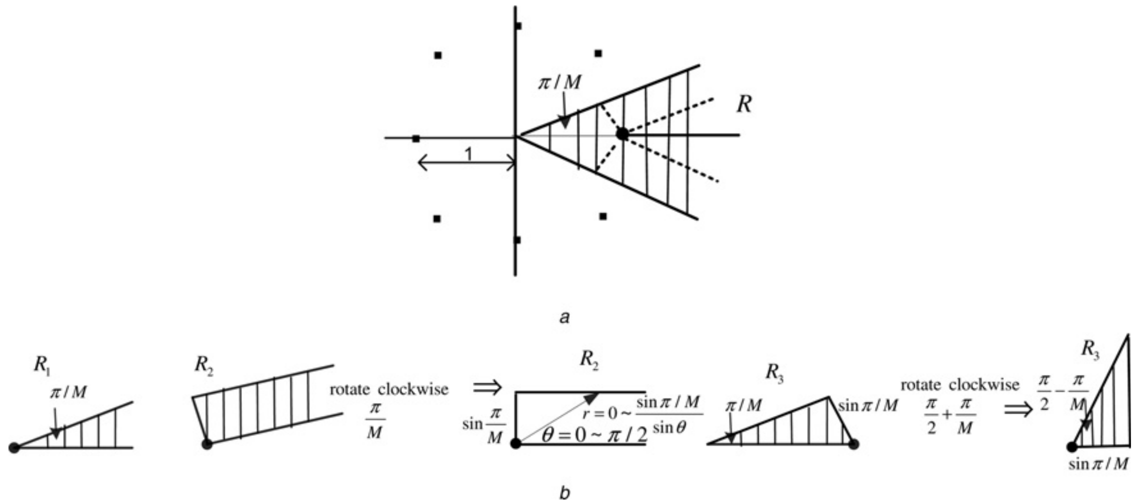

Figure 1 *Decision region of M-PSK and its division into three sub-regions, R = 2(R₁ + R₂ + R₃)*

a Decision region
b Three sub-regions

Considering Fig. 1*b*, in which the decision region is divided into three subregions, R_1, R_2 and R_3 as $R = 2(R_1 + R_2 + R_3)$, the SEP will be

$$\mathrm{SEP}_{\mathrm{MPSK}} = 1 - 2(I_1 + I_2 + I_3) \tag{17}$$

where I_1, I_2 and I_3 are the integrals of the PDF of Z over the sub-regions R1, R2 and R_3, respectively. Owing to the circular symmetry of $p_Z(\mathrm{z})$, we rotate R_2 and R_3 clockwise by the angles π/M and $\pi/2 + \pi/M$, respectively, as shown in Fig. 1*b*. In this way, we obtain

$$\begin{aligned} I_1 &= \int_0^{\pi/M} [S(r)]_0^{\infty}\, \mathrm{d}\theta = \frac{1}{2M} \\ I_2 &= \int_0^{\pi/2} [S(r)]_0^{((\sin \pi/M)/\sin\theta)}\, \mathrm{d}\theta \\ I_3 &= \int_0^{(\pi/2)-(\pi/M)} [S(r)]_0^{((\sin \pi/M)/\cos\theta)}\, \mathrm{d}\theta \\ &= \int_{\pi/2}^{\pi-(\pi/M)} [S(r)]_0^{((\sin \pi/M)/\sin\theta)}\, \mathrm{d}\theta \end{aligned} \tag{18}$$

From (14), (17), (18) and after some computations and simplifications, the SEP of M-PSK in Rician channel will be

$$\begin{aligned} \mathrm{SEP}_{\text{MPSK-Rice}} &= \frac{\mathrm{e}^{-LK_{\mathrm{R}}}}{\pi} \int_0^{\pi-(\pi/M)} \\ &\quad \times \frac{1}{[1 + ((\bar{\gamma}_{\mathrm{c}} \sin^2 \pi/M)/(K_{\mathrm{R}}+1)\sin^2\theta)]^L} \\ &\quad \times \exp\left\{\frac{LK_{\mathrm{R}}}{1 + ((\bar{\gamma}_{\mathrm{c}} \sin^2 \pi/M)/(K_{\mathrm{R}}+1)\sin^2\theta)}\right\} \mathrm{d}\theta \\ &= G\left(0, \pi - \frac{\pi}{M}, \bar{\gamma}_{\mathrm{c}}, L, K_{\mathrm{R}}, \sin\frac{\pi}{M}\right) \end{aligned} \tag{19}$$

where for simplicity, we have used $G(.)$. We define it as

$$\begin{aligned} G(\theta_1, \theta_2, \bar{\gamma}, L, K_{\mathrm{R}}, d) &\triangleq \frac{\mathrm{e}^{-LK_{\mathrm{R}}}}{\pi} \int_{\theta_1}^{\theta_2} \frac{1}{[1 + (\bar{\gamma} d^2/(K_{\mathrm{R}}+1)\sin^2\theta)]^L} \\ &\quad \times \exp\left\{\frac{LK_{\mathrm{R}}}{1 + (\bar{\gamma} d^2/(K_{\mathrm{R}}+1)\sin^2\theta)}\right\} \mathrm{d}\theta \end{aligned} \tag{20}$$

It is observed from (19) that the error probability has simple form. The integral in (19) can be easily solved using numerical methods. Further, our results show that equation (19) matches with [3, equation (9.15)] by substituting the momentum generating function of Chi-square distribution. For $L = 1$

$$\begin{aligned} &\mathrm{SEP}_{\text{MPSK-Rice},\, L=1} \\ &\quad = \frac{\mathrm{e}^{-K_{\mathrm{R}}}}{\pi} \int_0^{\pi-(\pi/M)} \frac{1}{[1 + ((\bar{\gamma}_{\mathrm{c}} \sin^2 \pi/M)/(K_{\mathrm{R}}+1)\sin^2\theta)]} \\ &\quad\quad \times \exp\left\{\frac{K_{\mathrm{R}}}{1 + ((\bar{\gamma}_{\mathrm{c}} \sin^2 \pi/M)/(K_{\mathrm{R}}+1)\sin^2\theta)}\right\} \mathrm{d}\theta \end{aligned} \tag{21}$$

It can be easily verified that the above equation is the same as that of the one obtained in [1].

For Rayleigh fading channel ($\mathrm{K_R} = 0$), the SEP will be

$$\begin{aligned} &\mathrm{SEP}_{\text{MPSK-Rayleigh}} \\ &\quad = \frac{1}{\pi} \int_0^{\pi-(\pi/M)} \frac{1}{[1 + ((\bar{\gamma}_{\mathrm{c}} \sin^2 \pi/M)/\sin^2\theta)]^L} \mathrm{d}\theta \\ &\quad = A\left(\pi - \frac{\pi}{M},\ \bar{\gamma}_{\mathrm{c}} \sin^2\frac{\pi}{M},\ L\right) \end{aligned} \tag{22}$$

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where $A(.)$ is [3, 18]

$$A(\varphi, c, L) \triangleq \frac{1}{\pi}\int_0^{\varphi} \frac{1}{[1+(c/\sin^2\theta)]^L} \mathrm{d}\theta$$
$$= \frac{\varphi}{\pi} - \frac{1}{\pi}\sqrt{\frac{c}{1+c}}\left\{\left(\frac{\pi}{2} + \mathrm{tg}^{-1}\left[-\sqrt{\frac{c}{1+c}}\cot\varphi\right]\right)\right.$$
$$\left.\times \sum_{k=0}^{L-1}\binom{2k}{k}\frac{1}{4^k(1+c)^k}\right\}$$
$$- \frac{1}{\pi}\sqrt{\frac{c}{1+c}}\sin\left(\mathrm{tg}^{-1}\left[-\sqrt{\frac{c}{1+c}}\cot\varphi\right]\right)$$
$$\times \sum_{k=0}^{L-1}\sum_{i=1}^{k}\frac{T_{ik}}{(1+c)^k}\left[\cos\left(\mathrm{tg}^{-1}\left[-\sqrt{\frac{c}{1+c}}\cot\varphi\right]\right)\right]^{2(k-i)+1} \quad (23)$$

where

$$T_{ik} = \frac{\binom{2k}{i}}{\binom{2(k-i)}{k-i}4^i[2(k-i)+1]}$$

We observe from (23) that the error probability in Rayleigh channel can be evaluated analytically.

For coherent BFSK, the results can be obtained from that of 2-PSK by replacing $\bar{\gamma}_c/2$ instead of $\bar{\gamma}_c$.

3.1.2 Bit error probability: In the following, we assume that Gray coding is used for bit mapping. The BEP for any bit constellations can be computed from (16).

4-PSK: The decision regions of 1-bit and 2-bit errors are demonstrated in Fig. 2 for the transmitted symbol 00. Owing to the symmetry, the other symbols have similar decision regions. Therefore considering (16), (20) and Fig. 2, the BEP is computed as

$$\mathrm{BEP}_{4\mathrm{PSK\text{-}Rice}} = G\left(0, \frac{\pi}{2}, \bar{\gamma}_b, L, K_R, 1\right) \quad (24)$$

$$\mathrm{BEP}_{4\mathrm{PSK\text{-}Rice}, L=1} = \frac{e^{-K_R}}{\pi}\int_0^{\pi/2}\frac{1}{[1+(\bar{\gamma}_b/(K_R+1)\sin^2\theta)]} \times \exp\left\{\frac{K_R}{1+(\bar{\gamma}_b/(K_R+1)\sin^2\theta)}\right\}\mathrm{d}\theta \quad (25)$$

where

$$\bar{\gamma}_b = \frac{\bar{\gamma}_c}{\log_2 M} \quad (26)$$

is the average SNR per bit of each channel. For Rayleigh channel, noting (23) we obtain

$$\mathrm{BEP}_{4\mathrm{PSK\text{-}Rayleigh}} = \frac{1}{2} - \frac{1}{2}\sqrt{\frac{\bar{\gamma}_b}{1+\bar{\gamma}_b}}\left\{\sum_{k=0}^{L-1}\binom{2k}{k}\frac{1}{4^k(1+\bar{\gamma}_b)^k}\right\} \quad (27)$$

It is easily verified that the BEP of 4-PSK is the same as that of 2-PSK for equal SNR per bit.

8-PSK: Gray coding and decision regions of 1, 2 and 3 bit errors for the transmitted symbol 000 are shown in Fig. 3. The BEP can be computed in the same manner as obtained for 4-PSK. After several calculations and some straightforward simplifications, the result is derived

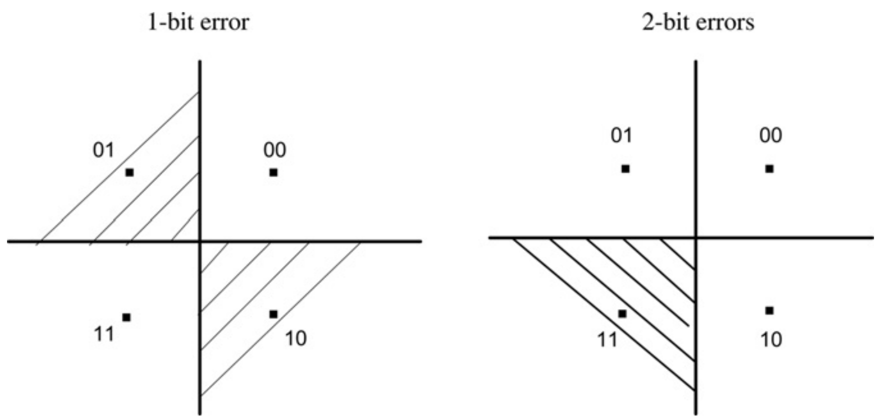

Figure 2 *Gray coding for 4-PSK and decision regions of 1-bit and 2-bit errors for the transmitted symbol 00*
Note that by symmetry, the other symbols have similar decision regions

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doi: 10.1049/iet-com.2010.0149

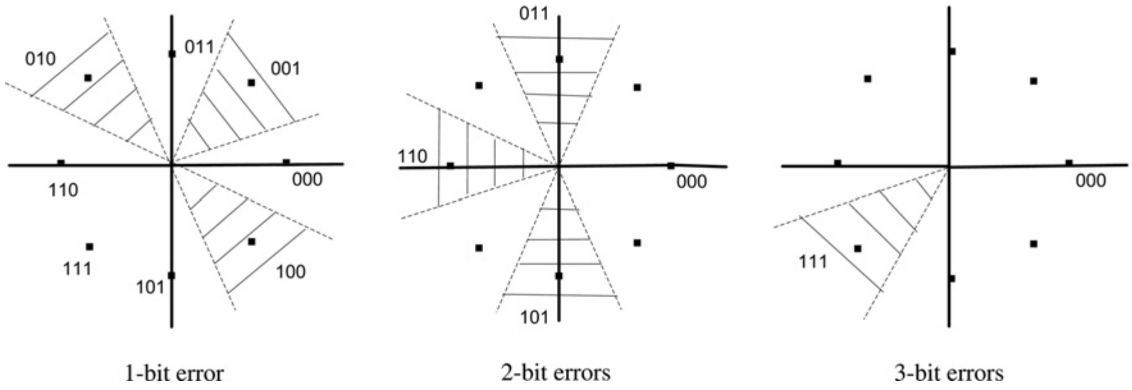

Figure 3 *Gray coding for 8-PSK and decision regions of 1-, 2- and 3-bit errors for the transmitted symbol 000*

as

$$\mathrm{BEP}_{\mathrm{8PSK\text{-}Rice}} = \frac{1}{3} G\left(0, \frac{7\pi}{8}, 3\bar{\gamma}_{\mathrm{b}}, L, K_{\mathrm{R}}, \sin \pi/8\right) + \frac{1}{3} G\left(0, \frac{5\pi}{8}, 3\bar{\gamma}_{\mathrm{b}}, L, K_{\mathrm{R}}, \sin 3\pi/8\right) \quad (28)$$

We observe that BEP has simple form and contains only two integrals. For $L = 1$, we obtain

$$\begin{aligned}\mathrm{BEP}_{\mathrm{8PSK\text{-}Rice},L=1} &= \frac{1}{3}\frac{\mathrm{e}^{-K_{\mathrm{R}}}}{\pi}\int_0^{7\pi/8} \frac{1}{[1 + ((3\bar{\gamma}_{\mathrm{b}} \sin^2 \pi/8)/(K_{\mathrm{R}}+1)\sin^2\theta)]} \\ &\quad \times \exp\left\{\frac{K_{\mathrm{R}}}{1 + ((3\bar{\gamma}_{\mathrm{b}} \sin^2 \pi/8)/(K_{\mathrm{R}}+1)\sin^2\theta)}\right\}\mathrm{d}\theta \\ &\quad + \frac{1}{3}\frac{\mathrm{e}^{-K_{\mathrm{R}}}}{\pi}\int_0^{5\pi/8} \frac{1}{[1 + ((3\bar{\gamma}_{\mathrm{b}} \sin^2 3\pi/8)/(K_{\mathrm{R}}+1)\sin^2\theta)]} \\ &\quad \times \exp\left\{\frac{K_{\mathrm{R}}}{1 + ((3\bar{\gamma}_{\mathrm{b}} \sin^2 3\pi/8)/(K_{\mathrm{R}}+1)\sin^2\theta)}\right\}\mathrm{d}\theta \end{aligned} \quad (29)$$

It can be easily verified that the above equation is the same as that of [1]. For Rayleigh channel

$$\mathrm{BEP}_{\mathrm{8PSK\text{-}Rayleigh}} = \frac{1}{3} A\left(\frac{7\pi}{8}, 3\bar{\gamma}_{\mathrm{b}} \sin^2 \pi/8, L\right) + \frac{1}{3} A\left(\frac{5\pi}{8}, 3\bar{\gamma}_{\mathrm{b}} \sin^2 3\pi/8, L\right) \quad (30)$$

where $A(.)$ has been defined in (23).

16-PSK: Using the same procedure applied in deriving BEP of 8-PSK and without referring to the details, the BEP of 16-PSK after long calculations is derived as

$$\begin{aligned}\mathrm{BEP}_{\mathrm{16PSK\text{-}Rice}} &= \frac{1}{4} G\left(0, \frac{15\pi}{16}, 4\bar{\gamma}_{\mathrm{b}}, L, K_{\mathrm{R}}, \sin \pi/16\right) \\ &\quad + \frac{1}{2} G\left(\frac{3\pi}{16}, \frac{\pi}{2}, 4\bar{\gamma}_{\mathrm{b}}, L, K_{\mathrm{R}}, \sin 3\pi/16\right) \\ &\quad + \frac{1}{4} G\left(0, \frac{7\pi}{16}, 4\bar{\gamma}_{\mathrm{b}}, L, K_{\mathrm{R}}, \sin 7\pi/16\right) \end{aligned} \quad (31)$$

$$\begin{aligned}\mathrm{BEP}_{\mathrm{16PSK\text{-}Rice},L=1} &= \frac{\mathrm{e}^{-K_{\mathrm{R}}}}{4\pi}\int_0^{15\pi/16} \frac{1}{[1 + ((4\bar{\gamma}_{\mathrm{b}} \sin^2 \pi/16)/(K_{\mathrm{R}}+1)\sin^2\theta)]} \\ &\quad \times \exp\left\{\frac{K_{\mathrm{R}}}{1 + ((4\bar{\gamma}_{\mathrm{b}} \sin^2 \pi/16)/(K_{\mathrm{R}}+1)\sin^2\theta)}\right\}\mathrm{d}\theta \\ &\quad + \frac{e^{-K_{\mathrm{R}}}}{2\pi}\int_{3\pi/16}^{\pi/2} \frac{1}{[1 + ((4\bar{\gamma}_{\mathrm{b}} \sin^2 3\pi/16)/(K_{\mathrm{R}}+1)\sin^2\theta)]} \\ &\quad \times \exp\left\{\frac{K_{\mathrm{R}}}{1 + (4\bar{\gamma}_{\mathrm{b}} \sin^2 3\pi/16/(K_{\mathrm{R}}+1)\sin^2\theta)}\right\}\mathrm{d}\theta \\ &\quad + \frac{\mathrm{e}^{-K_{\mathrm{R}}}}{4\pi}\int_0^{7\pi/16} \frac{1}{[1 + ((4\bar{\gamma}_{\mathrm{b}} \sin^2 7\pi/16)/(K_{\mathrm{R}}+1)\sin^2\theta)]} \\ &\quad \times \exp\left\{\frac{K_{\mathrm{R}}}{1 + ((4\bar{\gamma}_{\mathrm{b}} \sin^2 7\pi/16)/(K_{\mathrm{R}}+1)\sin^2\theta)}\right\}\mathrm{d}\theta \end{aligned} \quad (32)$$

We note that the BEP includes just three versions of one integral. For Rayleigh channel, we have

$$\mathrm{BEP}_{\mathrm{16PSK\text{-}Rayleigh}} = \frac{1}{4} A\left(\frac{15\pi}{16}, 4\bar{\gamma}_{\mathrm{b}} \sin^2 \pi/16, L\right) + \frac{1}{2} A\left(\frac{\pi}{2}, 4\bar{\gamma}_{\mathrm{b}} \sin^2 3\pi/16, L\right)$$

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$$-\frac{1}{2}A\left(\frac{3\pi}{16}, 4\bar{\gamma}_b \sin^2 3\pi/16, L\right)$$
$$+\frac{1}{4}A\left(\frac{7\pi}{16}, 4\bar{\gamma}_b \sin^2 7\pi/16, L\right) \qquad (33)$$

3.2 M-AM (M-ASK)

Fig. 4*a* demonstrates the decision regions. The SEP is derived as

$$\mathrm{SEP}_{\text{M-AM,Rice}} = 2\left(1-\frac{1}{M}\right)G\left(0, \frac{\pi}{2}, \bar{\gamma}_c, L, K_R, \sqrt{3/(M^2-1)}\right) \qquad (34)$$

$$\mathrm{SEP}_{\text{M-AM,Rice},L=1} = \left(1-\frac{1}{M}\right)\frac{2e^{-K_R}}{\pi}\int_0^{\pi/2}\frac{1}{[1+((3\bar{\gamma}_c/(M^2-1))/(K_R+1)\sin^2\theta)]} \times \exp\left\{\frac{K_R}{1+((3\bar{\gamma}_c/(M^2-1))/(K_R+1)\sin^2\theta)}\right\}\mathrm{d}\theta \qquad (35)$$

$$\mathrm{SEP}_{\text{M-AM,Rayleigh}} = \left(1-\frac{1}{M}\right)\left(1-\sqrt{\frac{3\bar{\gamma}_c}{3\bar{\gamma}_c+M^2-1}} \times \sum_{k=0}^{L-1}\binom{2k}{k}\frac{1}{4^k(1+3\bar{\gamma}_c/(M^2-1))^k}\right) \qquad (36)$$

3.3 M-QAM

In general, square M-QAM is composed of the quadrature combination of two $\sqrt{M}$-AM modulations, each with half of the total power. Therefore the SEP for M-QAM can be expressed as

$$\mathrm{SEP}_{\text{MQAM},\gamma} = 1-\left[1-\mathrm{SEP}_{\sqrt{M}\text{-AM},\gamma/2}\right]^2 \qquad (37)$$

Thus, using (34) and (36), the SEP can be calculated for square M-QAM in Rice and Rayleigh fading channels, respectively.

16-QAM: Since 16-QAM is commonly used, we directly obtain its SEP and BEP. Fig. 4*b* shows Gray coding and the decision regions. It can be verified that

$$\mathrm{SEP}_{\text{16QAM-Rice}} = 3G\left(0, \frac{\pi}{2}, \bar{\gamma}_c, L, K_R, \sqrt{0.1}\right) - \frac{9}{4}G\left(0, \frac{\pi}{4}, \bar{\gamma}_c, L, K_R, \sqrt{0.1}\right) \qquad (38)$$

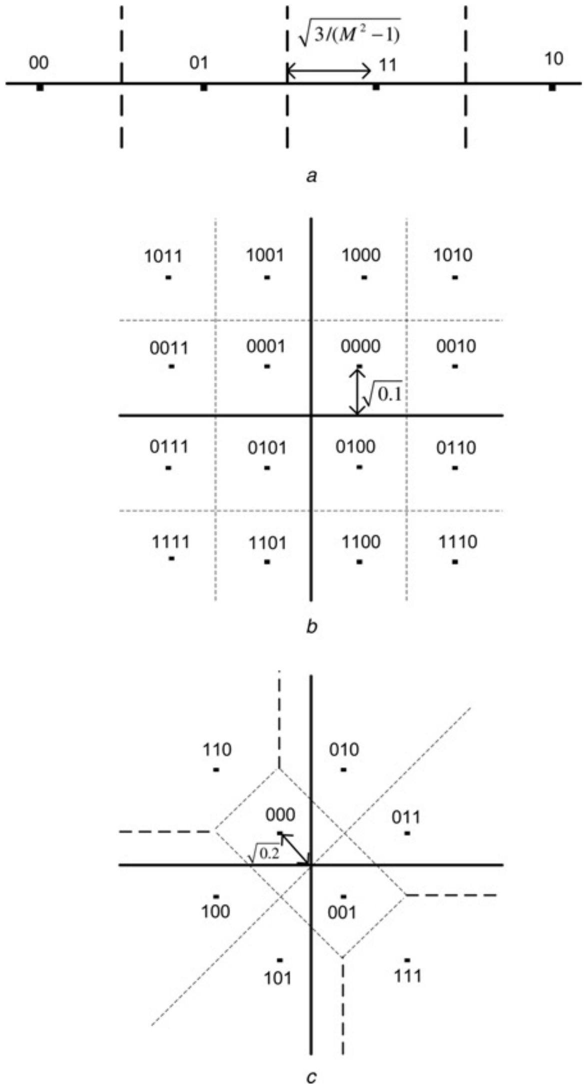

Figure 4 *Gray coding and decision regions for SEP evaluation of different modulations*

a M-AM (M-ASK)
b 16-QAM
c 8AM-PM

$$\mathrm{SEP}_{\text{16QAM-Rice},L=1} = \frac{3e^{-K_R}}{\pi}\int_0^{\pi/2}\frac{1}{[1+(0.1\bar{\gamma}_c/(K_R+1)\sin^2\theta)]} \times \exp\left\{\frac{K_R}{1+(0.1\bar{\gamma}_c/(K_R+1)\sin^2\theta)}\right\}\mathrm{d}\theta - \frac{9}{4}\frac{e^{-K_R}}{\pi}\int_0^{\pi/4}\frac{1}{[1+(0.1\bar{\gamma}_c/(K_R+1)\sin^2\theta)]} \times \exp\left\{\frac{K_R}{1+(0.1\bar{\gamma}_c/(K_R+1)\sin^2\theta)}\right\}\mathrm{d}\theta \qquad (39)$$

$$\mathrm{SEP}_{\text{16QAM-Rayleigh}} = 3A\left(\frac{\pi}{2}, 0.1\bar{\gamma}_c, L\right) - \frac{9}{4}A\left(\frac{\pi}{4}, 0.1\bar{\gamma}_c, L\right) \qquad (40)$$

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And after many computations and simplifications, we have computed the BEP as

$$\begin{aligned}\mathrm{BEP}_{16\mathrm{QAM\text{-}Rice}} = {} & \frac{1}{4}G\left(0, \frac{\pi}{2}, 4\bar{\gamma}_b, L, K_R, \sqrt{0.1}\right) \\ & + \frac{1}{4}G\left(0, \pi - \mathrm{tg}^{-1}\frac{1}{3}, 4\bar{\gamma}_b, L, K_R, \sqrt{0.1}\right) \\ & + \frac{1}{4}G\left(0, \frac{\pi}{2}, 4\bar{\gamma}_b, L, K_R, \sqrt{0.9}\right) \\ & + \frac{1}{4}G\left(0, \frac{\pi}{2} + \mathrm{tg}^{-1}\frac{1}{3}, 4\bar{\gamma}_b, L, K_R, \sqrt{0.9}\right) \\ & - \frac{1}{4}G\left(0, \frac{\pi}{2}, 4\bar{\gamma}_b, L, K_R, \sqrt{2.5}\right) \end{aligned} \quad (41)$$

$$\begin{aligned}&\mathrm{BEP}_{16\mathrm{QAM\text{-}Rice}, L=1} \\ &= \frac{e^{-K_R}}{4\pi}\int_0^{\pi/2} \frac{1}{[1 + (0.4\bar{\gamma}_b/(K_R+1)\sin^2\theta)]} \\ &\quad \times \exp\left\{\frac{K_R}{1 + (0.4\bar{\gamma}_b/(K_R+1)\sin^2\theta)}\right\} d\theta \\ &\quad + \frac{e^{-K_R}}{4\pi}\int_0^{\pi - \mathrm{tg}^{-1}(1/3)} \frac{1}{[1 + (0.4\bar{\gamma}_b/(K_R+1)\sin^2\theta)]} \\ &\quad \times \exp\left\{\frac{K_R}{1 + (0.4\bar{\gamma}_b/(K_R+1)\sin^2\theta)}\right\} d\theta \\ &\quad + \frac{e^{-K_R}}{4\pi}\int_0^{\pi/2} \frac{1}{[1 + (3.6\bar{\gamma}_b/(K_R+1)\sin^2\theta)]} \\ &\quad \times \exp\left\{\frac{K_R}{1 + (3.6\bar{\gamma}_b/(K_R+1)\sin^2\theta)}\right\} d\theta \\ &\quad + \frac{e^{-K_R}}{4\pi}\int_0^{(\pi/2)+\mathrm{tg}^{-1}(1/3)} \frac{1}{[1 + (3.6\bar{\gamma}_b/(K_R+1)\sin^2\theta)]} \\ &\quad \times \exp\left\{\frac{K_R}{1 + (3.6\bar{\gamma}_b/(K_R+1)\sin^2\theta)}\right\} d\theta \\ &\quad - \frac{e^{-K_R}}{4\pi}\int_0^{\pi/2} \frac{1}{[1 + (10\bar{\gamma}_b/(K_R+1)\sin^2\theta)]} \\ &\quad \times \exp\left\{\frac{K_R}{1 + (10\bar{\gamma}_b/(K_R+1)\sin^2\theta)}\right\} d\theta \end{aligned} \quad (42)$$

$$\begin{aligned}\mathrm{BEP}_{16\mathrm{QAM\text{-}Rayleigh}} = {} & \frac{1}{4}A\left(\frac{\pi}{2}, 0.4\bar{\gamma}_b, L\right) \\ & + \frac{1}{4}A\left(\pi - \mathrm{tg}^{-1}\frac{1}{3}, 0.4\bar{\gamma}_b, L\right) \\ & + \frac{1}{4}A\left(\frac{\pi}{2}, 3.6\bar{\gamma}_b, L\right) \\ & + \frac{1}{4}A\left(\frac{\pi}{2} + \mathrm{tg}^{-1}\frac{1}{3}, 3.6\bar{\gamma}_b, L\right) \\ & - \frac{1}{4}A\left(\frac{\pi}{2}, 10\bar{\gamma}_b, L\right) \end{aligned} \quad (43)$$

3.4 8AM-PM

Bit mapping for Gray coding and decision regions are shown in Fig. 4c. The SEP is derived as

$$\begin{aligned}&\mathrm{SEP}_{8\mathrm{AM\text{-}PM\text{-}Rice}} \\ &= \frac{1}{2}G\left(0, \frac{\pi}{2}, \bar{\gamma}_c, L, K_R, \sqrt{0.4}\right) \\ &\quad + \frac{9}{4}G\left(0, \frac{\pi}{2}, \bar{\gamma}_c, L, K_R, \sqrt{0.2}\right) \\ &\quad - 2G\left(0, \frac{\pi}{4}, \bar{\gamma}_c, L, K_R, \sqrt{0.2}\right) \end{aligned} \quad (44)$$

$$\begin{aligned}&\mathrm{SEP}_{8\mathrm{AM\text{-}PM\text{-}Rice}, L=1} \\ &= \frac{e^{-K_R}}{2\pi}\int_0^{\pi/2} \frac{1}{[1 + (0.4\bar{\gamma}_c/(K_R+1)\sin^2\theta)]} \\ &\quad \times \exp\left\{\frac{K_R}{1 + (0.4\bar{\gamma}_c/(K_R+1)\sin^2\theta)}\right\} d\theta \\ &\quad + \frac{9e^{-K_R}}{4\pi}\int_0^{\pi/2} \frac{1}{[1 + (0.2\bar{\gamma}_c/(K_R+1)\sin^2\theta)]} \\ &\quad \times \exp\left\{\frac{K_R}{1 + (0.2\bar{\gamma}_c/(K_R+1)\sin^2\theta)}\right\} d\theta \\ &\quad - \frac{2e^{-K_R}}{\pi}\int_0^{\pi/4} \frac{1}{[1 + (0.2\bar{\gamma}_c/(K_R+1)\sin^2\theta)]} \\ &\quad \times \exp\left\{\frac{K_R}{1 + (0.2\bar{\gamma}_c/(K_R+1)\sin^2\theta)}\right\} d\theta \end{aligned} \quad (45)$$

$$\begin{aligned}\mathrm{SEP}_{8\mathrm{AM\text{-}PM\text{-}Rayleigh}} = {} & \frac{1}{2}A\left(\frac{\pi}{2}, 0.4\bar{\gamma}_c, L\right) + \frac{9}{4}A\left(\frac{\pi}{2}, 0.2\bar{\gamma}_c, L\right) \\ & - 2A\left(\frac{\pi}{4}, 0.2\bar{\gamma}_c, L\right) \end{aligned} \quad (46)$$

The BEP contains 16 versions of the integral in (20), which is not presented here.

3.5 Nakagami fading

It has been shown [6] that Nakagmi-m fading with the integer parameter m in the Lth order diversity channel is equivalent to the Rayleigh channel with mLth order diversity. Therefore the error probabilities in Nakagami-m channel with the integer parameter can be obtained from those of Rayleigh channels.

4 Numerical results

In this section, we evaluate the performance of linearly modulated signals based on the analytical derivations provided in the previous section and also simulation results. Fig. 5 shows the SEPs of 2-PSK, 8-PSK and 16-QAM against the average SNR for different number of diversity branches in Rician channel with the Rice factor $K_R = 10$.

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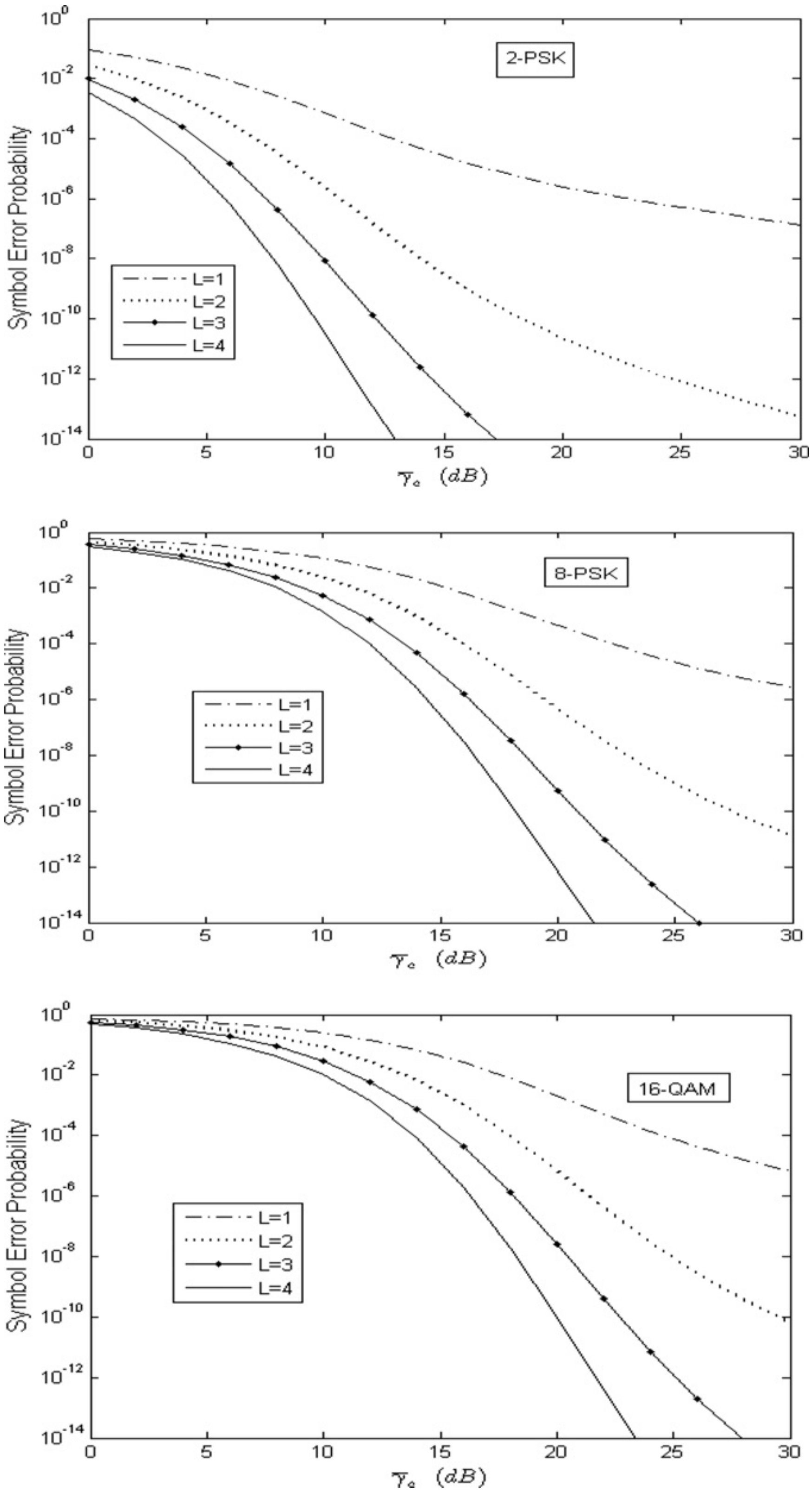

Figure 5 *SEP of different modulations against the average SNR for different number of diversity branches in Rician channel with* $K_R = 10$

It is observed that the error probability decreases with diversity order (L), as expected. As an example, the error probabilities of 2-PSK in Rician channel with $\bar{\gamma}_c = 10\,\text{dB}$ for $L = 1$, 2, 3 and four are 7×10^{-4}, 2.26×10^{-6}, 0.83×10^{-8} and 0.32×10^{-10}, respectively. The results for $L = 1$ quite match with those of [1].

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Figs. 6*a* and *b* compare the SEPs of different M-PSKs, 8AM-PM and 16-QAM for $L = 2$ in Rician ($K_R = 10$) and Rayleigh ($K_R = 0$) fading channels, respectively. It is observed that 16-QAM has better performance than 16-PSK. In addition, 8AM-PM works better than 8-PSK. We also observe that at large SNRs the error probabilities are exponentially reduced by the number of diversity branches L, SEP $\propto (1/(\bar{\gamma}_c)^L)$, that is, numerical results verify that the system achieves Lth order diversity for the case of independent branches. However, this exponential behaviour happens sooner in Rayleigh channel compared to the Rice channel. Our results for M-PSK quite match with those obtained in [3]. Comparing with [6], the same results are obtained while our expressions have simple

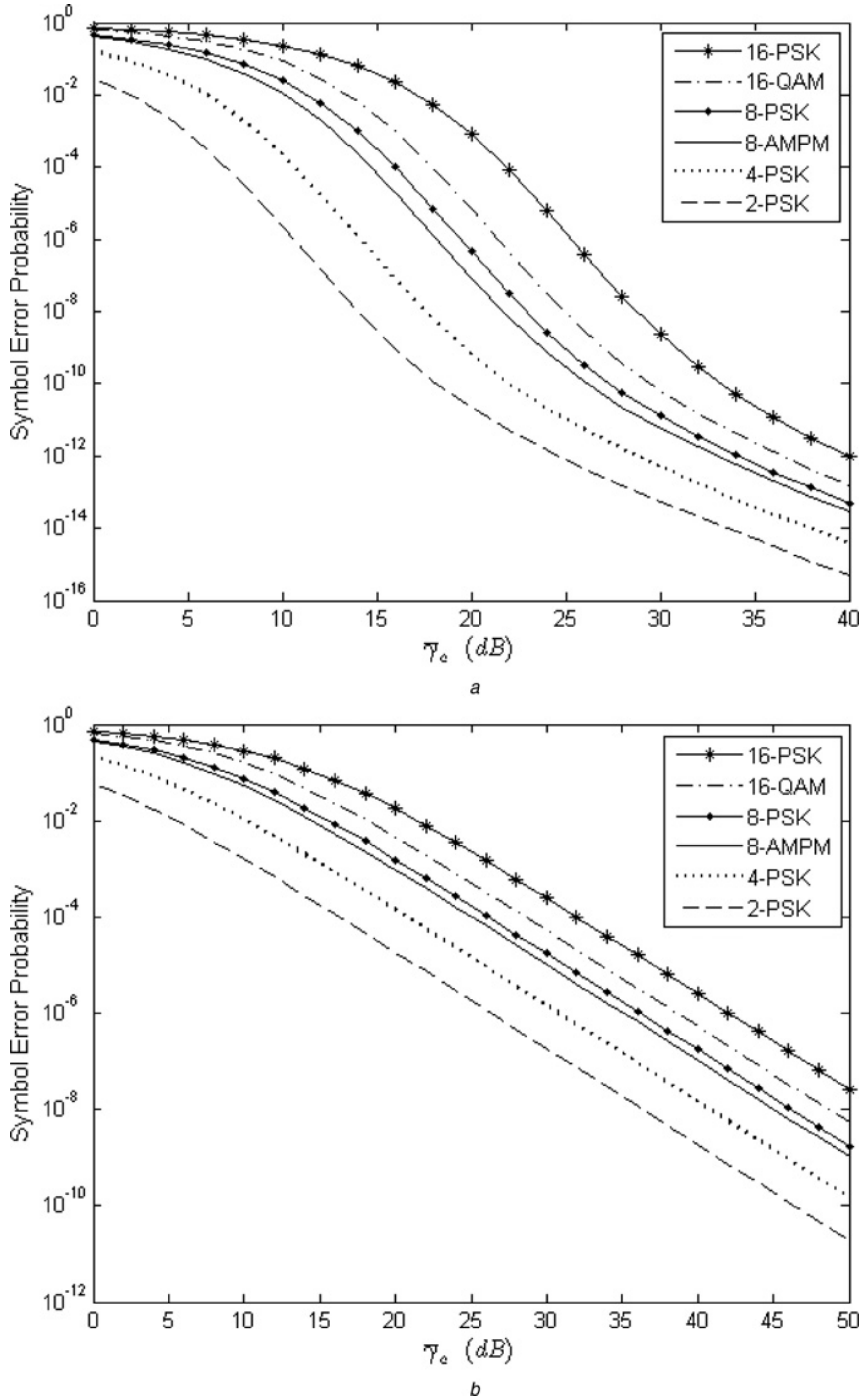

Figure 6 *SEPs of different modulations in Rice and Rayleigh channels for L = 2*

a Rice ($K_R = 10$)
b Rayleigh ($K_R = 0$)

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forms. Since the results are the same, they are shown as only one curve.

Figs. 7*a* and *b* show the BEPs of different *M*-PSKs and 16-QAM for $L = 2$ in Rice ($K_R = 10$) and Rayleigh ($K_R = 0$) channels, respectively. It is observed that the BEP of 16-QAM is very close to that of 8-PSK. We also observe that for high SNRs the error probabilities are proportional to the inverse of SNR powered by *L*, which indicates the *L*th order diversity.

Fig. 8 demonstrates the SEP of 4-PSK against the average SNR for different values of Rice factors (K_R) and $L = 2$. As expected, the performance improves by increasing the Rice factor.

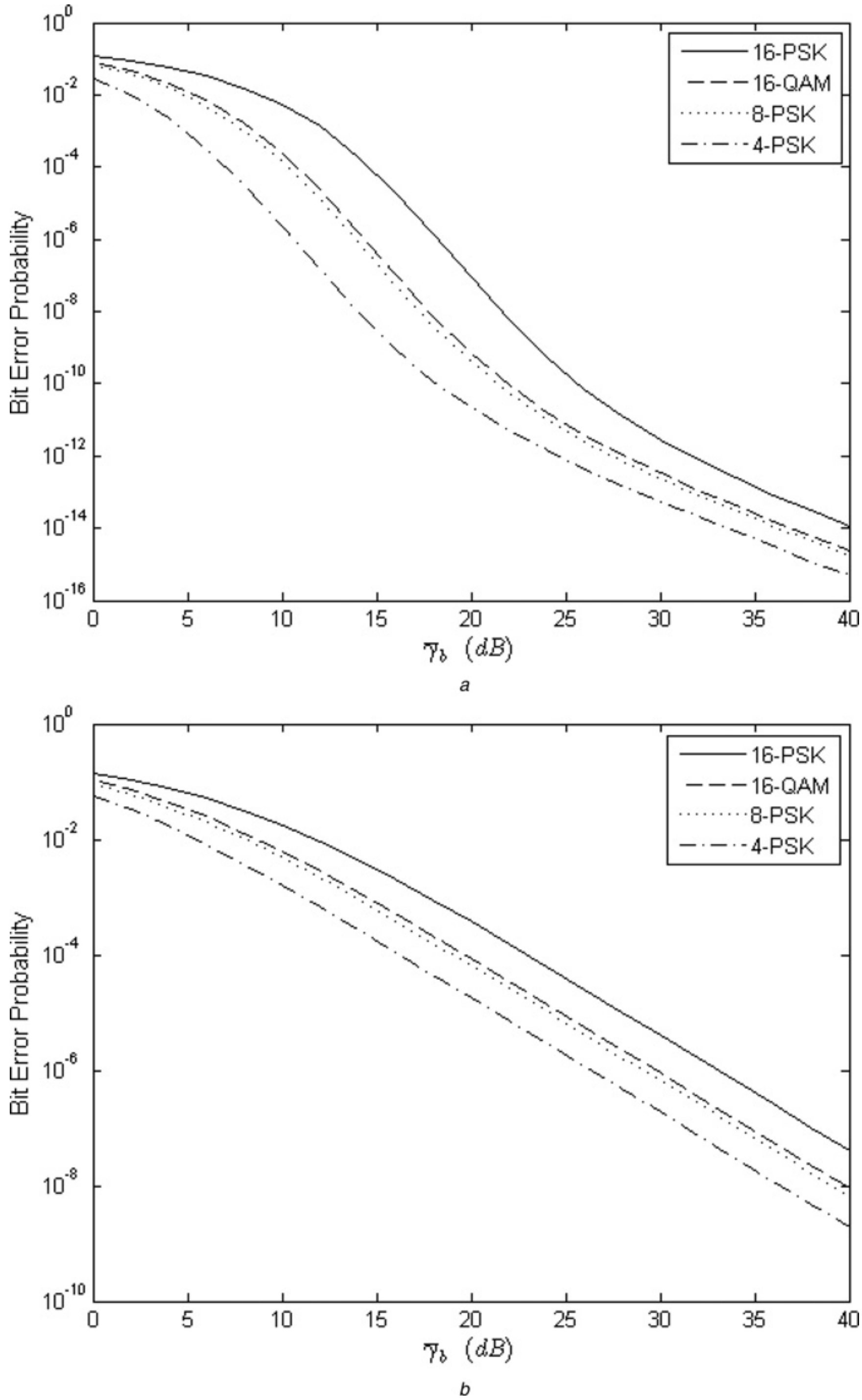

Figure 7 *BEP of different M-PSKs and 16-QAM in Rice and Rayleigh channels for L = 2*

a $K_R = 10$ (Rice)
b $K_R = 0$ (Rayleigh)

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doi: 10.1049/iet-com.2010.0149

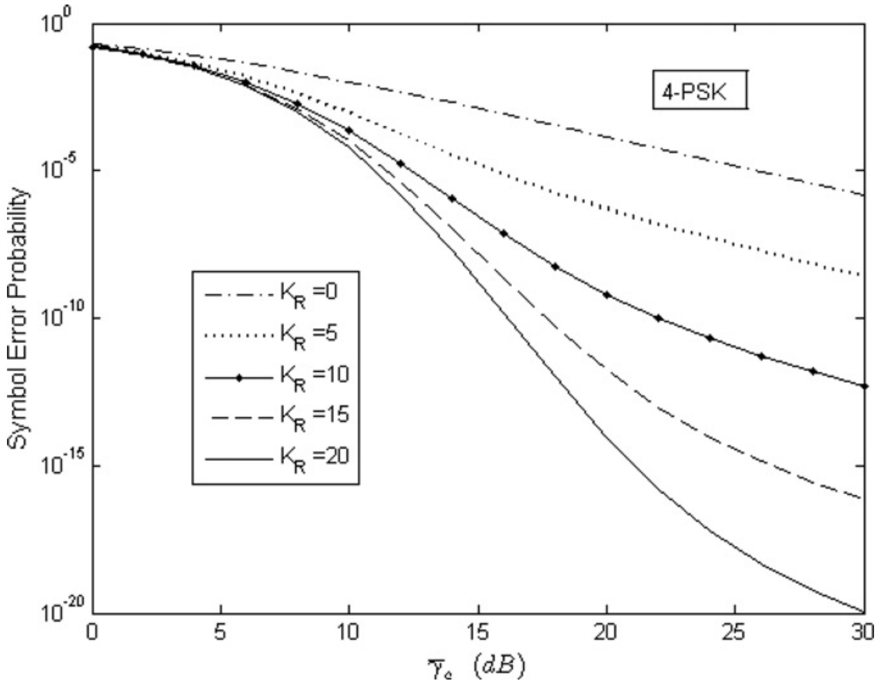

Figure 8 *SEP of 4-PSK against the average SNR for different Rice factors and L = 2*

In order to verify the analytical evaluations, we have provided simulations. Figs. 9*a* and *b* depict the analytical and simulation results for SEP and BEP of different modulations in Rician channel with $K_R = 10$ and diversity order $L = 2$. It is observed that the simulation results confirm the analytical evaluations. The results hold true for all cases.

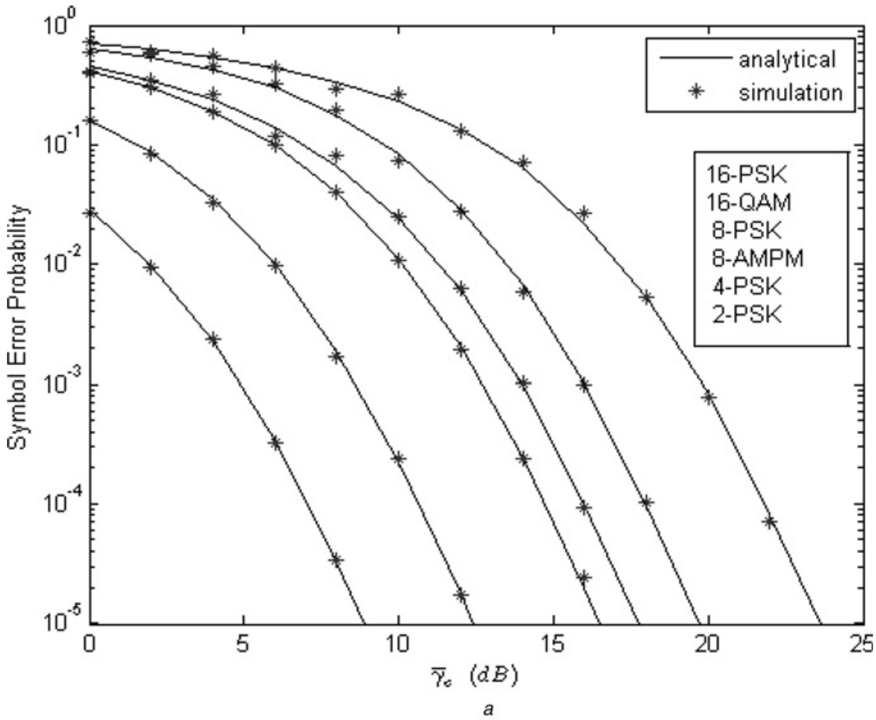

Figure 9 *Analytical and simulation results for different modulations against the average SNR in Rician channel with $K_R = 10$ and diversity order $L = 2$*

The legends are in the same order as the curves
a Symbol error probability
b Bit error probability

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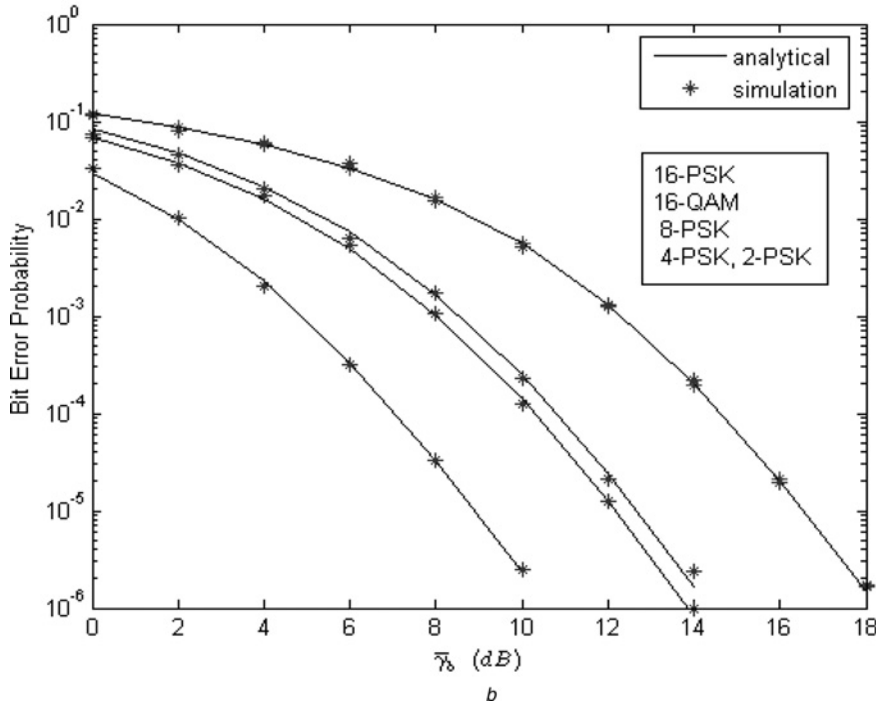

Figure 9 *Continued*

5 Conclusions

We obtained the exact SEPs and BEPs of linearly modulated signals in Rician frequency flat fading channels with diversity transmission and coherent detection with MRC at the receiver. The results were presented in the simple forms and contain several versions of a single integral, which can be easily solved numerically. By setting the Rice factor to zero, the results can be generalised to Rayleigh channel, which is the special case of Rice channel. In this case, the error probabilities have analytical solutions. Using the fact that Nakagmi-m fading with the integer parameter m in the Lth order diversity channel is equivalent to the Rayleigh channel with mLth order diversity, the error probabilities can be also derived for this channel. It was shown that at large SNRs, the error probability is proportional to the inverse of SNR powered by the number of diversity branches, which verifies that the system achieves Lth order diversity. The simulation results also confirm the analytical evaluations.

6 Acknowledgments

This work was supported by Iran National Science Foundation (INSF) under contract no. 11388.

The author would like to thank Prof. Masoumeh Nasiri-Kenari for her valuable comments. The author also would like to thank Prof. Jawad A. Salehi.

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8 Appendix 1: derivation of the PDF of the final additive noise *Z*

Here, we derive a closed form for the PDF of Z. We start from (6) and consider an alternative expression for $I_\alpha(y)$ [6, 17] as

$$I_\alpha(y) = \sum_{n=0}^{\infty} \frac{(y/2)^{\alpha+2n}}{n!\Gamma(\alpha+n+1)};$$
$$\Gamma(\alpha+n+1) = (\alpha+n)! \text{ integer } \alpha \tag{47}$$

Thus, substituting (47) in (6) gives

$$p_Y(y) = (K_\mathrm{R}+1)^L \mathrm{e}^{-LK_\mathrm{R}} y^{L-1} \mathrm{e}^{-(K_\mathrm{R}+1)y} \sum_{n=0}^{\infty} \frac{[LK_\mathrm{R}(K_\mathrm{R}+1)y]^n}{n!(L+n-1)!} \tag{48}$$

Therefore using (5), the PDF of the final additive noise Z is computed as

$$\begin{aligned} p_Z(z) &= \int_0^\infty p_{Z|Y}(z) p_Y(y)\,\mathrm{d}y \\ &= \frac{1}{2\pi N_0}(K_\mathrm{R}+1)^L \mathrm{e}^{-LK_\mathrm{R}} \int_0^\infty y \mathrm{e}^{-(y|z|^2/2N_0)} y^{L-1} \\ &\quad \times \mathrm{e}^{-(K_\mathrm{R}+1)y} \sum_{n=0}^{\infty} \frac{[LK_\mathrm{R}(K_\mathrm{R}+1)y]^n}{n!(L-1+n)!}\,\mathrm{d}y \\ &= \frac{1}{2\pi N_0}(K_\mathrm{R}+1)^L \mathrm{e}^{-LK_\mathrm{R}} \sum_{n=0}^{\infty} \frac{[LK_\mathrm{R}(K_\mathrm{R}+1)]^n}{n!(L-1+n)!} \\ &\quad \times \int_0^\infty \mathrm{e}^{-[(K_\mathrm{R}+1)+(|z|^2/2N_0)]y} y^{L+n}\,\mathrm{d}y \end{aligned} \tag{49}$$

where in the above, we have changed the place of integral and summation. Noting $\int_0^\infty \mathrm{e}^{-au} y^m\,\mathrm{d}y = (m!/a^{m+1})$ [18, 19], we obtain

$$\begin{aligned} p_Z(z) &= \frac{1}{2\pi N_0}(K_\mathrm{R}+1)^L \mathrm{e}^{-LK_\mathrm{R}} \sum_{n=0}^{\infty} \frac{[LK_\mathrm{R}(K_\mathrm{R}+1)]^n}{n!(L-1+n)!} \\ &\quad \times \frac{(L+n)!}{[(K_\mathrm{R}+1)+(|z|^2/2N_0)]^{(L+n+1)}} \\ &= \frac{1}{2\pi N_0}(K_\mathrm{R}+1)^L \mathrm{e}^{-LK_\mathrm{R}} \frac{1}{[(K_\mathrm{R}+1)+(|z|^2/2N_0)]^{(L+1)}} \\ &\quad \times \left\{ L \sum_{n=0}^{\infty} \frac{[LK_\mathrm{R}(K_\mathrm{R}+1)]^n}{[(K_\mathrm{R}+1)+(|z|^2/2N_0)]^n n!} \right. \\ &\quad \left. + \sum_{n=0}^{\infty} \frac{[LK_\mathrm{R}(K_\mathrm{R}+1)]^n}{[(K_\mathrm{R}+1)+(|z|^2/2N_0)]^n} \frac{n}{n!} \right\} \end{aligned} \tag{50}$$

Using $\mathrm{e}^x = \sum_{n=0}^{\infty}(x^n/n!)$, $\sum_{n=0}^{\infty}(nx^n/n!) = x\mathrm{e}^x$ [18, 19]

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and $\bar{\gamma}_c = (1/2N_0)(7)$, the PDF of Z can be written as follows

$$p_Z(z) = \frac{e^{-LK_R}}{\pi}\frac{L(\bar{\gamma}_c/(K_R+1))}{[1+|z|^2(\bar{\gamma}_c/(K_R+1))]^{(L+1)}} \times \left\{1+\frac{K_R}{1+|z|^2(\bar{\gamma}_c/(K_R+1))}\right\} \times \exp\left\{\frac{LK_R}{1+|z|^2(\bar{\gamma}_C/(K_R+1))}\right\} \quad (51)$$

It is observed that $p_Z(z)$ depends on $|z|^2$, that is, it has circular symmetry.

9 Appendix 2: calculation of the indefinite integral in (13)

In this Appendix, we compute the indefinite integral $S(r) \triangleq \int (1/2\pi) p_{|z|}(r)\,dr$, which is used for the error probability evaluations. Replacing $p_{|z|}(r)$ from (10) in the above yields

$$S(r) = \frac{e^{-LK_R}}{\pi}\int \frac{L(\bar{\gamma}_c/(K_R+1))r}{[1+r^2(\bar{\gamma}_c/(K_R+1))]^{(L+1)}} \times \left\{1+\frac{K_R}{1+r^2(\bar{\gamma}_c/(K_R+1))}\right\} \times \exp\left\{\frac{LK_R}{1+r^2(\bar{\gamma}_C/(K_R+1))}\right\} dr \quad (52)$$

By a change of variable $x = (1/(1+r^2(\bar{\gamma}_c/(K_R+1))))$ and using $\int x^m e^{ax} dx = \sum_{i=0}^{m} (-1)^i (m! x^{m-i}/(m-i)! a^{i+1})$ [18, 19], the result of the above integral will be in the following form

$$\begin{aligned} S(r) &= \frac{-Le^{-LK_R}}{2\pi}\int (x^{L-1} + K_R x^L)\exp(LK_R x)\,dx \\ &= -\frac{Le^{-LK_R}}{2\pi}\exp(LK_R x) \times \left\{\sum_{n=0}^{L-1}(-1)^n \frac{(L-1)!x^{L-1-n}}{(L-1-n)!(LK_R)^{n+1}} + K_R\sum_{n=0}^{L}(-1)^n \frac{L!x^{L-n}}{(L-n)!(LK_R)^{n+1}}\right\} \end{aligned} \quad (53)$$

Using $m = n+1$ in the first summation, writing the second summation into two parts and then replacing x yields

$$\begin{aligned} S(r) &= -\frac{Le^{-LK_R}}{2\pi}\exp(LK_R x)\left[\sum_{m=1}^{L}(-1)^{m-1} \times \frac{(L-1)!x^{L-m}}{(L-m)!(LK_R)^m} + K_R\sum_{n=0}^{L}(-1)^n\frac{L!x^{L-n}}{(L-n)!(LK_R)^{n+1}}\right] \\ &= -\frac{Le^{-LK_R}}{2\pi}\exp(LK_R x)\left[-\sum_{m=1}^{L}(-1)^m\frac{(L-1)!x^{L-m}}{(L-m)!(LK_R)^m} + K_R\sum_{n=1}^{L}(-1)^n\frac{L!x^{L-n}}{(L-n)!(LK_R)^{n+1}} + \frac{x^L}{L}\right] \\ &= -\frac{Le^{-LK_R}}{2\pi}\exp(LK_R x)\left[-\sum_{n=1}^{L}(-1)^n\frac{(L-1)!x^{L-n}}{(L-n)!(LK_R)^n} + \sum_{n=1}^{L}(-1)^n\frac{(L-1)!x^{L-n}}{(L-n)!(LK_R)^n} + \frac{x^L}{L}\right] \\ &= -\frac{e^{-LK_R}}{2\pi}\frac{1}{[1+r^2(\bar{\gamma}_c/(K_R+1))]^L} \times \exp\left\{\frac{LK_R}{1+r^2(\bar{\gamma}_c/(K_R+1))}\right\} \end{aligned} \quad (54)$$

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IET Commun., 2011, Vol. 5, Iss. 1, pp. 12–26
doi: 10.1049/iet-com.2010.0149