Signal Activity Detection of Phase-Shift Keying Signals

Ali A. Tadaion, *Student Member, IEEE*, Mostafa Derakhtian, *Student Member, IEEE*, Saeed Gazor, *Senior Member, IEEE*, Mohammad M. Nayebi, *Senior Member, IEEE*, and Mohammad R. Aref

***Abstract*—We propose computationally inexpensive and efficient solutions for signal activity detection of phase-shift keying (PSK) signals in additive white Gaussian noise. We consider the complex amplitude of the signal, as well as the information sequence, as the unknown parameters. In addition, the noise variance is assumed unknown. We derive the generalized likelihood ratio test (GLRT) and suggest a computationally efficient implementation thereof. Furthermore, we develop a new inexpensive detector for binary PSK signals, which we will refer to as the generalized energy detector. To evaluate the performance of these detectors, we attempt to derive a uniformly most powerful invariant (UMPI) test as an optimal detector. It turns out that the UMPI test exists only if the signal-to-noise ratio is known. We use this UMPI test in order to obtain an upper-bound performance for the evaluation of invariant detectors, such as the GLRT. Simulation results illustrate and compare the performance and the efficiency of the proposed signal activity detectors.**

***Index Terms*—Additive Gaussian noise, binary detection problem, detection algorithms, generalized energy detector, generalized likelihood ratio (GLR), GLR detectors, invariances, invariant detectors, invariant hypothesis testing, matched filter detectors, maximal invariant statistic, maximum-likelihood (ML) detection, phase-shift keying (PSK), signal activity detection, signal classification, signal detection, uniformly most powerful invariant (UMPI) test.**

I. Introduction

SIGNAL activity detection (SAD) is a critical stage in the implementation of an effective communication system. For instance, detection of the presence of digitally modulated signals is an important problem within the context of wireless packet switching networks. A user may attempt to transmit signals, or is inactive. Particularly, in the presence of ambient noise, the reliability of the SAD scheme has a critical impact on the performance of these systems. Some approaches to address such a detection problem are attempted for a variety of other applications, including reconnaissance, surveillance, spectrum management and surveillance, signal confirmation, and some other intelligence-gathering activities, interference identification, modulation classification, and also in electronic warfare and threat analysis (e.g., see [1]–[20], [31], and the references therein).

For the presence detection of a phase-shift keying (PSK) signal, the traditional method of the energy detector or radiometer uses the energy level of the received signal as a criterion [2]. Such a method is susceptible to the *a priori* knowledge of the noise variance and interference [3]. Most of the existing signal-detection methods not only assume that the signal or some of its parameters are known, but also require the noise variance in order to determine the decision threshold [4]. Krasner [1] considered a model for the signal with unknown parameters (such as carrier frequency, carrier phase, and symbol sequence) as random variables, and developed the coherent and noncoherent detectors by averaging the detection criteria over the distribution of these parameters. If the probability density functions (pdfs) of the unknown parameters are provided, these detectors are able to detect the presence of a weak signal at the expense of high computational complexity (CC). Ramprashad and Parks proposed a locally most powerful invariant test to detect signals [8]. The most common assumption of the existing methods is that the noise variance is considered to be known, which in some practical situations may not hold. We assume neither any knowledge nor any distribution about the noise variance and the complex amplitude of the signal, and propose a suboptimal inexpensive detector for the SAD problem. In addition, we derive an optimal invariant test for the case of the known signal-to-noise ratio (SNR), and serve this optimal test in order to evaluate the efficiency of our detector, which turns out to perform very close to the optimal detector.

The remaining of the paper is organized as follows. In Section II, the SAD is discussed, and by substituting the maximum-likelihood (ML) estimates of the unknown parameters in the likelihood ratio (LR), a generalized LR (GLR) detector is derived in Section II-A. We suggest a simplified implementation with comparable performance to reduce the exponential CC due to the exhaustive search in the implementation of this GLR test (GLRT). In Section II-B, we present an intuitive detector based on matched subspace detectors for the case of binary PSK (BPSK)-modulated signals, which will be referred to as the generalized energy detector (GED) with substantially reduced CC at the expense of some small negligible performance loss. In Section II-C, we derive a uniformly most powerful invariant (UMPI) test in known SNR to evaluate the performance of the proposed detectors. Simulation results and comparison of the proposed methods are reported in Section III for the presented SAD algorithms. Section IV concludes the paper.

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A. A. Tadaion, M. Derakhtian, M. M. Nayebi, and M. R. Aref are with the Department of Electrical Engineering, Sharif University, Tehran, Iran (e-mail: tadaion@mehr.sharif.edu; derakhtian@mehr.sharif.edu; nayebi@sharif.edu; aref@sharif.edu).

S. Gazor is with the Department of Electrical and Computer Engineering, Queen's University, Kingston, ON K7L 3N6, Canada (e-mail: s.gazor@queensu.ca).

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II. Signal Activity Detection

In this section, the SAD of a PSK signal in additive white Gaussian noise (AWGN) is considered. Our signal detection is the following binary hypothesis test [8]:

$$\begin{cases} \mathcal{H}_0 : \mathbf{r} = \mathbf{n}, & \text{if PSK signal is absent} \\ \mathcal{H}_1 : \mathbf{r} = A\mathbf{c} + \mathbf{n}, & \text{if PSK signal is active} \end{cases} \tag{1}$$

where $\mathbf{r} = [r_0, \ldots, r_{N-1}]^T$ is the baseband representation of the received signal vector, $\mathbf{n} = [n_0, \ldots, n_{N-1}]^T$ is the complex zero-mean white Gaussian noise with unknown variance $\sigma^2 = E[|n_i|^2], \mathbf{c} = [c_0, c_1, \ldots, c_{N-1}]^T$ is the data vector, and $A = |A|e^{j\phi_c}$ is the unknown complex amplitude of the received signal, $|A|$ the magnitude and ϕ_c the carrier phase, which are assumed constant unknown during the period of observation [5]. The digitally modulated sequence c_n is selected from the set of PSK alphabets $\mathcal{C} = \{e^{j(2\pi k/M)}, k = 0, \ldots, M-1\}$, where M is the number of constellation points, e.g., in the BPSK scheme $c_n \in \mathcal{C} = \{+1, -1\}$. In this paper, we assume no intersymbol interference (ISI) and neglect synchronization inaccuracies in the sampling time and in the carrier frequency. In practice, the synchronization may be achieved using several methods, such as transmitting pilot tones for all users or non-data-aided algorithms [21].

A. GLR Detector

In this section, to obtain the GLR detector for (1), we substitute the ML estimates of the unknown parameters, A and σ^2, under each hypothesis in the pdfs of the observation and construct the LR [22]. The pdf of the observation $\mathbf{r}$ under the hypothesis $\mathcal{H}_1$ and $\mathcal{H}_0$ are as follows:

$$f(\mathbf{r}; \mathcal{H}_1) = \frac{1}{(\pi\sigma^2)^N} \exp\left\{-\frac{1}{\sigma^2}\|\mathbf{r} - A\mathbf{c}\|^2\right\} \tag{2a}$$

$$f(\mathbf{r}; \mathcal{H}_0) = \frac{1}{(\pi\sigma^2)^N} \exp\left\{-\frac{1}{\sigma^2}\|\mathbf{r}\|^2\right\}. \tag{2b}$$

Maximizing the above density functions, the ML estimates of A and σ^2 are

$$\begin{cases} \hat{A} = \frac{1}{N}\mathbf{c}^H\mathbf{r} \\ \widehat{\sigma_1^2} = \frac{1}{N}\|\mathbf{r} - \hat{A}\mathbf{c}\|^2 \\ \widehat{\sigma_0^2} = \frac{1}{N}\|\mathbf{r}\|^2. \end{cases} \tag{3}$$

Since $|c_k| = 1, k = 0, \ldots, N-1$, from (3), we have $\widehat{\sigma_1^2} = (1/N)\|\mathbf{r}\|^2 - (1/N^2)|\mathbf{c}^H\mathbf{r}|^2$. Substituting (3) in (2a) and (2b), respectively, the ratio of the LFs is

$$\frac{f(\mathbf{r}; \mathcal{H}_1)|_{(3)}}{f(\mathbf{r}; \mathcal{H}_0)|_{(3)}} = \frac{\left(\frac{1}{(\pi e)\widehat{\sigma_1^2}}\right)^N}{\left(\frac{1}{(\pi e)\widehat{\sigma_0^2}}\right)^N} = \left(\frac{\frac{1}{N}\|\mathbf{r}\|^2}{\frac{1}{N}\|\mathbf{r}\|^2 - \frac{1}{N^2}|\mathbf{c}^H\mathbf{r}|^2}\right)^N \tag{4}$$

where $a|_{(\cdot)}$ means substituting the results of equation numbered by $(\cdot)$ in the expression of a. Since x^N is an increasing function of x, comparing the above LR with the threshold T^N is equivalent to comparing $((1/N)\|\mathbf{r}\|^2/(1/N)\|\mathbf{r}\|^2 - (1/N^2)|\mathbf{c}^H\mathbf{r}|^2)$ with the threshold T. It is obvious that the comparison $((1/N)\|\mathbf{r}\|^2/(1/N)\|\mathbf{r}\|^2 - (1/N^2)|\mathbf{c}^H\mathbf{r}|^2) \lessgtr T$ is equivalent to $(1 - T/T) \lessgtr (|\mathbf{c}^H\mathbf{r}|^2/N\|\mathbf{r}\|^2)$. Therefore, the GLRT rejects $\mathcal{H}_0$ if

$$L_{\text{GLR}}(\mathbf{r}) = \frac{\max_{\mathbf{c}\in\mathcal{C}^N} |\mathbf{c}^H\mathbf{r}|^2}{N\|\mathbf{r}\|^2} > \eta_{\text{GLR}} \tag{5}$$

where $(\cdot)^H$ is the transpose conjugate, $\|\mathbf{r}\|^2 = \sum_{k=0}^{N-1} |r_k|^2$, and the threshold η_{GLR} is selected such that the probability of false alarm P_{fa} equals a predetermined value. The detection threshold is independent of the SNR and can be obtained by Monte Carlo simulation, assuming that signal is absent. Regardless of a phase ambiguity,[1] the data sequence is also detected in this method. In general, the CC of (5) is of the order of $\mathcal{O}(|\mathcal{C}|^{N-1})$, where $|\mathcal{C}|$ is the number of constellation points. If $|\mathcal{C}|$ is a large number (or if $|\mathcal{C}|$ is unknown), the CC of this detector could substantially be reduced by approximating $c_i \simeq e^{j\angle r_i}$ or by selecting the nearest neighbor of $e^{j\angle r_i}$ in the constellation set. In such a case, the decision variable becomes the ratio of L_1-norm to L_2-norm of the observed data.

We propose the following simplified implementation of this exhaustive search, which requires a CC of the order of $\mathcal{O}(N|\mathcal{C}|)$, and its performance is comparable to (5).

Step 1) Choose $\widehat{c}_0 \in \mathcal{C}$ arbitrarily and set $R_0 = \widehat{c}_0^H r_0$.

Step 2) For $k = 1, \ldots, N-1$, choose $\widehat{c}_k$ using the following:

$$\widehat{c}_k = \arg\max_{c_k\in\mathcal{C}} \left|R_{k-1} + c_k^H r_k\right|$$
$$R_k \triangleq R_{k-1} + \widehat{c}_k^H r_k. \tag{6}$$

Step 3) Compare $(|R_{N-1}|^2/N\|\mathbf{r}\|^2) \triangleq (\widehat{\max_{\mathbf{c}\in\mathcal{C}^N}} |\mathbf{c}^H\mathbf{r}|^2)$ $(/N\|\mathbf{r}\|^2)$ with the threshold η_{GLR}.

Simulation results illustrate that the performance of the above "simplified" implementation of GLR (S-GLR) is very close to the performance of the GLR implementation in (5).

B. GED for BPSK Signals

In this section, we propose an intuitive inexpensive detector with very low CC for the case of BPSK signals. Following (1), if we denote $t_k \triangleq r_k^2$, then under $\mathcal{H}_1$, we get $t_k = A^2 + n_k^2 + 2Ac_kn_k \triangleq A^2 + w_k$, since for a BPSK signal, we have $c_k^2 = 1$. Assuming that the noise variance is small compared with the signal power, we have $w_k \approx 2Ac_kn_k$; obviously, w_k is a zero-mean normal random variable with variance $4|A|^2\sigma^2$, where A is an unknown parameter. Thus, under $\mathcal{H}_1$, we have $E[t_k] \approx A^2$, while under $\mathcal{H}_0$, we have $E[t_k] \approx 0$. The test problem using the observed signal t_k is a particular case of the matched subspace

[1]For instance, in BPSK, this ambiguity is in the sign of vector $\mathbf{c}$. Such an ambiguity is resolvable, employing a differential modulation scheme.

detector (see, e.g., [23]–[25]) and the resulting detector rejects $\mathcal{H}_0$ if

$$T_{\rm GED} = \frac{\left|\sum_{k=0}^{N-1} t_k\right|^2}{\sum_{k=0}^{N-1}|t_k|^2} = \frac{\left|\sum_{k=0}^{N-1} r_k^2\right|^2}{\sum_{k=0}^{N-1}|r_k^2|^2} > \eta_{\rm GED} \quad (7)$$

where $\eta_{\rm GED}$ is chosen such that the false-alarm probability requirement is satisfied.

C. UMPI Detector

The uniformly most powerful (UMP) test does not necessarily exist for all composite hypothesis-testing problems in which either one or both of the hypotheses contain unknown parameters (e.g., see [26] and [27]). Specifically, for the PSK signal detection in AWGN (1), if the signal and noise parameters are unknown, the UMP test does not exist. We will show that the UMPI test does not exist either, except when the SNR is known. In invariant problems such as (1), the GLRT is invariant and asymptotically UMPI [23], [26]. Therefore, the resulting UMPI detector can be used as an optimal detector in order to evaluate the performance of the detectors derived in the previous subsections. A classical approach to derive a UMPI test is to express the problem invariance in terms of a transformation group, choose a maximal invariant,[2] derive the PDF of the maximal invariant under each hypothesis, and construct the LR (for examples, see [26], [28], and [29]). In the following, we find the groups of transformations under which the problem is invariant. Then, we derive the maximal invariant statistic for the groups and its PDFs under each hypothesis in Appendices I and II, respectively. Since the resulting LR depends on the SNR of the probable signal, the UMPI test does not exist. However, assuming that the SNR is known, the performance of the resulting UMPI detector can be used as an upper bound for the evaluation of any other invariant test which assumes unknown SNR, such as the GLRTs derived in the previous subsections.

We first show that the detection problem in (1) is invariant under the composition of the following transformation groups:

$$\begin{aligned} G_d &= \{g_d : g_d(\mathbf{r}) = d\,\mathbf{r} \quad \forall d \in \mathbb{C} - \{0\}\} \\ G_{\mathbf{e}} &= \{g_{\mathbf{e}} : g_{\mathbf{e}}(\mathbf{r}) = \mathbf{e} \odot \mathbf{r} \\ &\quad \mathbf{e} = [e_0, \ldots, e_{N-1}]^T \quad \forall e_n \in \mathcal{C}\} \end{aligned} \quad (8)$$

where $\odot$ denotes the element-wise multiplication. The above transformations, scale G_d, and the element-wise multiplication $G_{\mathbf{e}}$ are groups, since they are closed sets and associative, and contain the identity and inverse elements. In the following, we see that the distribution of the observations and the parameter spaces remains invariant under any composition of the transformation groups in (8) for each hypothesis.

1) Under $\mathcal{H}_1$, from $\mathbf{r} \sim \mathcal{N}(A\mathbf{c}, \sigma^2 I_N)$, we get $g_d(\mathbf{r}) = d\,\mathbf{r} \sim \mathcal{N}(dA\mathbf{c}, |d|^2\sigma^2 I_N)$, where I_N is the identity matrix. Since (A, σ^2) is unknown, the distribution family of the transformed signal is not changed. Under the null hypothesis, the proof is similar, except that $A = 0$.
2) For the second group $G_{\mathbf{e}}$ and under $\mathcal{H}_1$, we have $g_{\mathbf{e}}(\mathbf{r}) = A\mathbf{e}\odot\mathbf{c}+\mathbf{e}\odot\mathbf{n} \sim \mathcal{N}(A\mathbf{e}\odot\mathbf{c}, \sigma^2 I_N)$. Since $\mathbf{e}\odot\mathbf{c} \in \mathcal{C}^N$ and $\mathbf{c} \in \mathcal{C}^N$ are unknown, the distribution family generated by $\mathcal{N}(A\mathbf{c}, \sigma^2 I_N)$ for different values of unknown parameters is identical with those generated by $\mathcal{N}(A\mathbf{e} \odot \mathbf{c}, \sigma^2 I_N)$. Under $\mathcal{H}_0$, the proof is similar by substituting $A = 0$.

We find the maximal invariant statistic for the composite group G, consisting of the above subgroups G_d and $G_{\mathbf{e}}$. Then, using the densities of this maximal invariant statistic under each hypothesis, we form the LR for SAD. In Appendix I, the maximal invariant statistic for the composite group G is derived as follows:[3]

$$\begin{aligned} \mathbf{y} &= M_{\mathbf{h}}(M_d(\mathbf{r})) \\ &= \left[\sqrt[|\mathcal{C}|]{\left(\frac{r_0}{r_{N-1}}\right)^{|\mathcal{C}|}}, \ldots, \sqrt[|\mathcal{C}|]{\left(\frac{r_{N-2}}{r_{N-1}}\right)^{|\mathcal{C}|}}, 1\right]^T \end{aligned} \quad (9)$$

where $M_{\mathbf{h}}$ and M_d are defined in Appendix I. We must note that this maximal invariant is derived in a number of steps, where each step is associated with one of the subgroups.

If the LR does not depend on the unknown parameters, the UMPI test is to compare this LR with a threshold. In Appendix II, we derive the conditional distribution of the maximal invariant statistic $\mathbf{y}$ under each hypothesis, and construct the LR as follows:

$$\begin{aligned} L(\mathbf{r}) = \frac{e^{-N\rho}}{2^{N-1}} \sum_{j=1}^{|\mathcal{C}|^{N-1}} &\left[\exp\left(\rho \frac{\left|\sum_{k=0}^{N-1} \sqrt[|\mathcal{C}|]{r_k} u_{k,j}^H\right|^2}{\|\mathbf{r}\|^2}\right)\right. \\ &\left.\times \sum_{p=0}^{N-1} \binom{N-1}{p} \left(\rho \frac{\left|\sum_{k=0}^{N-1} \sqrt[|\mathcal{C}|]{r_k} u_{k,j}^H\right|^2}{\|\mathbf{r}\|^2}\right)^p \frac{1}{p!}\right] \end{aligned} \quad (10)$$

where the SNR is denoted by $\rho = (|A|^2/\sigma^2)$. The variable $u_{k,j} \in \mathcal{C}, k = 0, \ldots, N-1$ is the kth component of the jth possible sequence where $u_{N-1,j} \triangleq 1, j = 1, \ldots, |\mathcal{C}|^{N-1}$, and therefore, for $u_{0,j}, \ldots, u_{N-2,j}$, there are $|\mathcal{C}|^{N-1}$ different possible sequences to be considered. Since $\mathbf{c}$ is an N-tuple vector, there are $|\mathcal{C}|^{N-1}$ different possible sequences to be considered. The UMPI test, if it exits, rejects $\mathcal{H}_0$, if $L(\mathbf{r}) > \eta_{\rm UMPI}$, where the threshold is selected such that the probability of false-alarm requirement is satisfied. Since $L(\mathbf{r})$ in (10) depends on the SNR, the UMPI test can not be implemented if the SNR is unknown. In addition, provided the SNR value, this UMPI test allows us to numerically evaluate an upper bound for the performance of any invariant test, such as the GLR detector derived in the previous subsection.

III. Simulation Results

We evaluate and compare the performance of the proposed detectors by simulations. In all simulations, the complex amplitude of the signal, the variance of the noise, and the symbol sequence are unknown to the detectors, except in Fig. 1, where the SNR value is provided only to the UMPI detector. The SNR is defined as the ratio of the signal power to the noise variance, $\rho = (|A|^2/\sigma^2)$. In our simulations, we determined the threshold in each test experimentally, as follows. The decision statistics for 10^5 independent trials in the absence of signal were

[2]See [25] for the definition of the maximal invariant statistic.

[3]For a complex number z, the value $\sqrt[M]{z}$ denotes the unique solution of $x^M = z$ for which $\angle x \in (-\pi/M), (\pi/M)$.

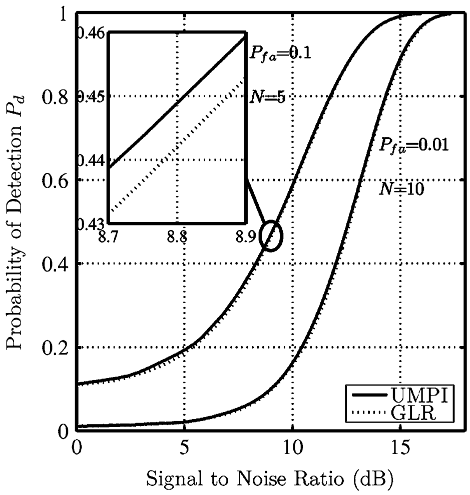

Fig. 1. Performance comparison of GLR and UMPI tests in terms of the probability of detection versus SNR for a BPSK signal with different numbers of samples $N = 5, 10$ and different $P_{\text{fa}} = 0.1, 0.01$.

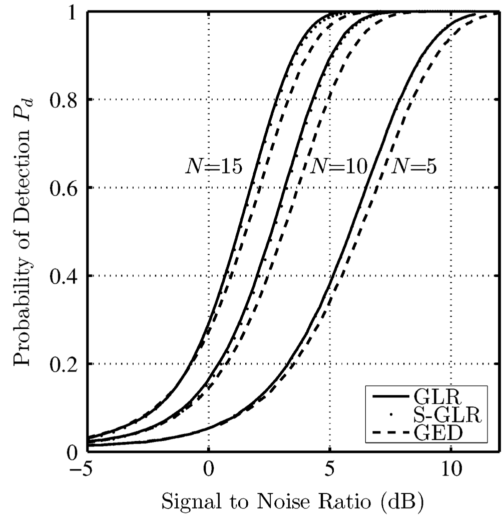

Fig. 2. Performance comparison of GLR, S-GLR, and GED for the SAD of a PSK signal. We considered a BPSK signal with different numbers of samples $N = 5, 10, 15$, and the probability of false alarm is $P_{\text{fa}} = 0.01$.

sorted in ascending order, and the threshold was chosen as the $\%100 \times P_{\text{fa}}$-percentile of the resulting data. For example, for $P_{\text{fa}} = 0.01$, the threshold is chosen as the $0.01 \times 10^5 = 10^3$th ordered data; i.e., such that $\%100 \times P_{\text{fa}}$ of the decision statistics are above the threshold.

Fig. 1 depicts the probability of detection versus SNR for the UMPI test (10) and GLRT (5), where $N = 5$ or $N = 10$, and $P_{\text{fa}} = 0.1$ or $P_{\text{fa}} = 0.01$ for BPSK signals. Simulations illustrate how close the GLR performance to UMPI performance is, even at small N, e.g., the UMPI detector outperforms the GLR detector by only 0.05 dB, while UMPI uses the SNR information for detection with high CC, and GLR detects the signal without this information. It is known that as $N \to \infty$, the performance of the GLRT (which is a suboptimal test) approaches the performance of the UMPI test (which uses the knowledge of SNR and is optimal). This means that the GLRT is asymptotically optimal. Although the optimality of the GLR criterion is unknown in the general binary test problem with unknown parameters, we observe the close-to-optimal performance of the GLRT even for small values of N. Such a close performance has also been observed in some other problems; for example, see [26]. The knowledge of SNR seems to result in no major performance improvement, and the close performance of the suboptimal GLRT to the optimal UMPI bound illustrates the efficiency of the GLRT.

Fig. 2 illustrates a performance comparison of the different proposed SAD algorithms for BPSK signaling. Different numbers of data symbols are considered for simulations, and the probability of false alarm is set to $P_{\text{fa}} = 0.01$. We observe that the performance improves as SNR or the number of samples increases. The performances of GLR (with exhaustive search) and S-GLR (simplified implementation) are indeed comparable (only about 0.07 dB apart), while the CC of the latter is substantially lower. For the number of samples $N = 10$ at a given probability of detection, the GLR results in 0.26 dB improvement gain in SNR compared with the GED. The price paid for these gains is their higher CC. The tradeoff between the number of multiplications performed in each detection algorithm (as the order of CC) and the performance loss (in decibels) is depicted in Table I, where the performance results are derived for $N = 10$. It is notable that GED is only applicable for BPSK signals. The low CC of S-GLR and GED in this comparison is observable.

The performance of the S-GLR is compared for BPSK and quaternary (Q)PSK at different sample numbers in Fig. 3, where $P_{\text{fa}} = 0.01$. As we expect, for the same number of samples, the S-GLR performs better for BPSK than for QPSK, since more unknown information is embedded in QPSK signals. In addition, increasing the number of samples improves the detection performance for both BPSK and QPSK signals.

IV. Conclusion

In this paper, we studied the presence detection of PSK signals with unknown parameters in AWGN. We developed a computationally efficient detector using the GLRT, and justified its performance by comparing against a UMPI test in smaller unknown parameter space. We proposed three detectors for the SAD. A GLR detector is derived, and a simplified implementation is proposed with reduced CC and comparable performance as an efficient detector. An intuitive and simple detector for BPSK signals is also proposed, and its performance is numerically evaluated to be close to the GLRT. In order to evaluate the performance of the proposed SAD algorithms, a UMPI test for known SNR is derived. We proved that the UMPI test does not exist if the SNR is unknown. Simulation results illustrate and compare the performances of the proposed detectors. The UMPI gives an upper performance bound. Since we observed that the new GLR solution performs close to this upper bound, we imply that knowledge of SNR does not affect the GLRT performance considerably in the considered circumstances. Such a close performance has been also observed between the GLRT and the UMPI bound in some other problems; for example, see [26]. However, the optimality of the GLR criterion is unknown, in general, and it is not easy to mathematically quantify the behavior of such a GLRT in the general case.

TABLE I
TRADEOFF BETWEEN CC AND PERFORMANCE: THE ORDER OF THE NUMBER OF MULTIPLICATIONS (A MEASURE OF CC) AND PERFORMANCE IN EACH OF THE PROPOSED DETECTORS FOR $N = 10$

Detector	Order of the Number of Complex Multiplications	Performance Loss in dB compared with UMPI
UMPI	$> N^2\|\mathcal{C}\|^N$	0
GLR	$N\|\mathcal{C}\|^{(N-1)}$	0.05
S-GLR	$N\|\mathcal{C}\|$	0.12
GED	N	0.3

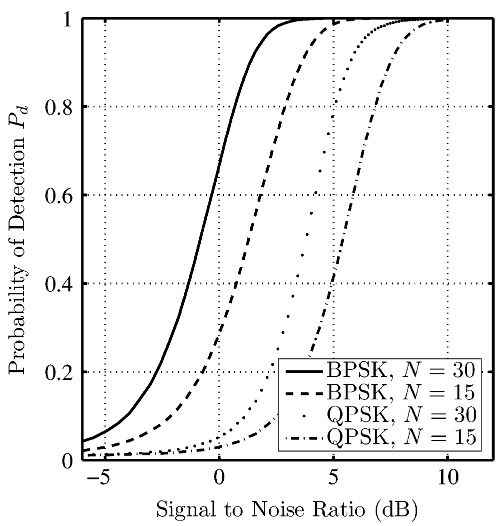

Fig. 3. Performance evaluation of S-GLR for the SAD of a PSK signal. We considered BPSK and QPSK signals with different numbers of samples $N = 15, 30$, and the probability of false alarm is $P_{\rm fa} = 0.01$.

APPENDIX I
MAXIMAL INVARIANT STATISTIC FOR COMPOSITE GROUP G

In this Appendix, using the following theorem [23], we obtain the maximal invariant statistic for the composite group G in two steps, each related to finding a maximal invariant statistic for one of the subgroups.

Theorem 1: Let G be a group of transformations, and let G_d and $G_{\mathbf{e}}$ be two subgroups generating G. Suppose that $\mathbf{x} = M_d(\mathbf{r})$ is a maximal invariant with respect to G_d, and for any $g_{\mathbf{e}} \in G_{\mathbf{e}}$, any $\mathbf{r}^{(1)}$, and any $\mathbf{r}^{(2)}$, we have

$$M_d\left(\mathbf{r}^{(1)}\right) = M_d\left(\mathbf{r}^{(2)}\right) \Rightarrow M_d\left(g_{\mathbf{e}}\left(\mathbf{r}^{(1)}\right)\right) = M_d\left(g_{\mathbf{e}}\left(\mathbf{r}^{(2)}\right)\right) \quad (11)$$

and there exists a maximal invariant statistic $\mathbf{y} \triangleq M_{\mathbf{h}}(\mathbf{x})$ under the group H of transformations $g_{\mathbf{h}}(\mathbf{x}) \triangleq M_d(g_{\mathbf{e}}(\mathbf{r}))$, then $\mathbf{y} = M_{\mathbf{h}}(M_d(\mathbf{r}))$ is maximal invariant with respect to G.

Proof: See [23, Ch. 6, Th. 2]. ■

Therefore, using the above theorem, a maximal invariant for the scale group G_d is given by

$$\mathbf{x} = M_d(\mathbf{r}) = \left[\frac{r_0}{r_{N-1}}, \dots, \frac{r_{N-1}}{r_{N-1}}\right]^T. \quad (12)$$

This statistic is invariant, since

$$M_d(g_d(\mathbf{r})) = \left[\frac{d\,r_0}{d\,r_{N-1}}, \dots, \frac{d\,r_{N-1}}{d\,r_{N-1}}\right]^T = \left[\frac{r_0}{r_{N-1}}, \dots, \frac{r_{N-1}}{r_{N-1}}\right]^T = M_d(\mathbf{r}). \quad (13)$$

It is also maximal-invariant, since for any given $\mathbf{r}^{(1)}$ and $\mathbf{r}^{(2)}$, from $M_d(\mathbf{r}^{(1)}) = M_d(\mathbf{r}^{(2)})$, we get $(r_n^{(1)}/r_{N-1}^{(1)}) = (r_n^{(2)}/r_{N-1}^{(2)})$ for $n = 0, \dots, N-2$, i.e., taking $d = (r_{N-1}^{(1)}/r_{N-1}^{(2)})$, we have $r_n^{(1)} = (r_{N-1}^{(1)}/r_{N-1}^{(2)})r_n^{(2)} = d\,r_n^{(2)}$ for $n = 0, \dots, N-2$.

Following *Theorem 1*, in order to derive a maximal invariant for the second group, assuming $M_d(\mathbf{r}^{(1)}) = M_d(\mathbf{r}^{(2)})$, we get

$$\begin{aligned} M_d\left(g_{\mathbf{e}}(\mathbf{r}^{(1)})\right) &= M_d(\mathbf{e} \odot \mathbf{r}^{(1)}) \\ &= \left[\frac{e_0 r_0^{(1)}}{e_{N-1} r_{N-1}^{(1)}}, \dots, \frac{e_{N-2} r_{N-2}^{(1)}}{e_{N-1} r_{N-1}^{(1)}}\right]^T \\ &= \left[\frac{e_0 r_0^{(2)}}{e_{N-1} r_{N-1}^{(2)}}, \dots, \frac{e_{N-2} r_{N-2}^{(2)}}{e_{N-1} r_{N-1}^{(2)}}\right]^T \\ &= M_d\left(\mathbf{e} \odot \mathbf{r}^{(2)}\right) \\ &= M_d\left(g_{\mathbf{e}}(\mathbf{r}^{(2)})\right). \end{aligned} \quad (14)$$

Therefore, the condition (11) is satisfied. Now, we should find a group $G_{\mathbf{h}}$ which acts on $\mathbf{x}$ and then a maximal invariant under that group. According to *Theorem 1*, we have $g_{\mathbf{h}}(\mathbf{x}) = M_d(g_{\mathbf{e}}(\mathbf{r}))$, therefore $g_{\mathbf{h}}(\mathbf{x}) = [h_0 x_0, \dots, h_{N-2} x_{N-2}]^T$, where $h_n \triangleq (e_n/e_{N-1}) \in \mathcal{C}$ and a maximal invariant under $G_{\mathbf{h}}$ is

$$\mathbf{y} \triangleq M_{\mathbf{h}}(\mathbf{x}) = \left[\sqrt[|\mathcal{C}|]{x_0^{|\mathcal{C}|}}, \dots, \sqrt[|\mathcal{C}|]{x_{N-2}^{|\mathcal{C}|}}\right]^T \quad (15)$$

because from $h_n \in \mathcal{C}$, we get $\sqrt[|\mathcal{C}|]{h_n^{|\mathcal{C}|}} = 1$, therefore

$$M_{\mathbf{h}}(g_{\mathbf{h}}(\mathbf{x})) = \left[\sqrt[|\mathcal{C}|]{h_0^{|\mathcal{C}|} x_0^{|\mathcal{C}|}} \cdots, \sqrt[|\mathcal{C}|]{h_{N-2}^{|\mathcal{C}|} x_{N-2}^{|\mathcal{C}|}}\right]^T = M_{\mathbf{h}}(\mathbf{x}).$$

Furthermore, $\forall n = 1, \dots, N-2$, from $M_{\mathbf{h}}(\mathbf{x}^{(1)}) = M_{\mathbf{h}}(\mathbf{x}^{(2)})$, we get $(x_n^{(1)})^{|\mathcal{C}|} = (x_n^{(2)})^{|\mathcal{C}|}$; that means $x_n^{(1)} = h_n x_n^{(2)}$, where $h_n = e^{(j2\pi k_n/|\mathcal{C}|)} \in \mathcal{C}$, for some integer k_n. Therefore,

$g_{\mathbf{h}}(\mathbf{x}^{(2)}) = \mathbf{x}^{(1)}$, for some $g_{\mathbf{h}} \in G_{\mathbf{h}}$. Hence, $M_{\mathbf{h}}$ is the maximal invariant under the group $G_{\mathbf{h}}$. The maximal invariant under the composite group G is given by

$$\begin{aligned}\mathbf{y} &= M_{\mathbf{h}}(M_d(\mathbf{r})) \\ &= \left[\sqrt[|\mathcal{C}|]{\left(\frac{r_0}{r_{N-1}}\right)^{|\mathcal{C}|}}, \ldots, \sqrt[|\mathcal{C}|]{\left(\frac{r_{N-2}}{r_{N-1}}\right)^{|\mathcal{C}|}}, 1 \right]^T \end{aligned} \quad (16)$$

where $M_{\mathbf{h}}$ and M_d are as defined above.

Appendix II
Derivation of pdf of the Maximal Invariant

In this Appendix, we derive the pdf of the maximal invariant $\mathbf{y}$ under each hypothesis. It will be done in steps. The density of the observation $\mathbf{r}$ under $\mathcal{H}_1$ is

$$\begin{aligned} f(\mathbf{r};\mathcal{H}_1) &= \frac{1}{(\pi\sigma^2)^N} \exp\left\{-\frac{1}{\sigma^2}\|\mathbf{r} - A\mathbf{c}\|^2\right\} \\ &= \frac{e^{-N\rho}}{(\pi\sigma^2)^N} \exp\left\{-\frac{1}{\sigma^2}\|\mathbf{r}\|^2 + \frac{2}{\sigma^2}\mathrm{Re}(A\mathbf{r}^H\mathbf{c})\right\} \end{aligned} \quad (17)$$

where $\rho = (|A|^2/\sigma^2)$. The distribution under $\mathcal{H}_0$ is obtained by substituting $A = 0$ in the above. In order to calculate the pdf of $\mathbf{x}$, we consider the auxiliary variable $\alpha \triangleq r_{N-1}$ and use the simple relationship between $\mathbf{x}$ and $\mathbf{r}$, i.e., $r_n = \alpha x_n, n = 0, \ldots, N-2$. Therefore, the Jacobian determinant is $|\alpha|^{2(N-1)}$ and the pdf of $[\mathbf{x}, \alpha]$ is the following:

$$\begin{aligned} f(\mathbf{x},\alpha;\mathcal{H}_1) &= \frac{e^{-N\rho}}{(\pi\sigma^2)^N} \\ &\times e^{-\frac{|\alpha|^2(\|\mathbf{x}\|^2+1) - 2\mathrm{Re}(A\alpha^H(\mathbf{x}^H\mathbf{c}+c_{N-1}))}{\sigma^2}} |\alpha|^{2(N-1)}. \end{aligned} \quad (18)$$

The pdf of $\mathbf{x}$ can be obtained by integrating (18) over the auxiliary random variable α, i.e.,

$$\begin{aligned} f(\mathbf{x};\mathcal{H}_1) &= \int_{\alpha\in\mathbb{C}} f(\mathbf{x},\alpha;\mathcal{H}_1) d\alpha \\ &= \frac{e^{-N\rho}}{(\pi\sigma^2)^N} \int_{\alpha\in\mathbb{C}} |\alpha|^{2(N-1)} \\ &\times e^{-\frac{|\alpha|^2\left(\|\mathbf{x}\|^2+1\right) - 2\mathrm{Re}\left(A\alpha^H\left(\mathbf{x}^H\mathbf{c}+c_{N-1}\right)\right)}{\sigma^2}} d\alpha. \end{aligned} \quad (19)$$

Substituting the polar representation of α gives

$$\begin{aligned} f(\mathbf{x};\mathcal{H}_1) &= \frac{e^{-N\rho}}{(\pi\sigma^2)^N} \int_0^\infty \int_0^{2\pi} |\alpha|^{2N-1} e^{-\frac{1}{\sigma^2}|\alpha|^2(\|\mathbf{x}\|^2+1)} \\ &\times e^{\frac{2}{\sigma^2}\mathrm{Re}(A|\alpha|e^{-j\angle\alpha}(\mathbf{x}^H\mathbf{c}+c_{N-1}))} d|\alpha| d\angle\alpha. \end{aligned} \quad (20)$$

By virtue of the integral definition of the zero-order Bessel function of the first kind [30], we get

$$\begin{aligned} &\int_0^{2\pi} \exp\left\{\frac{2}{\sigma^2}\mathrm{Re}(A|\alpha|e^{-j\angle\alpha}(\mathbf{x}^H\mathbf{c}+c_{N-1}))\right\} d\angle\alpha \\ &\qquad = 2\pi I_0\left(\frac{2}{\sigma^2}|A\alpha(\mathbf{x}^H\mathbf{c}+c_{N-1})|\right). \end{aligned} \quad (21)$$

Using the above, we simplify (20) and obtain the following pdf of $\mathbf{x}$ under $\mathcal{H}_1$ [30]:

$$\begin{aligned} f(\mathbf{x};\mathcal{H}_1) &= \frac{2\pi e^{-N\rho}}{(\pi\sigma^2)^N} \int_0^\infty |\alpha|^{2N-1} e^{-\frac{|\alpha|^2(\|\mathbf{x}\|^2+1)}{\sigma^2}} \\ &\times I_0\left(\frac{2}{\sigma^2}|\alpha||A(\mathbf{x}^H\mathbf{c}+c_{N-1})|\right) d|\alpha| \\ &= \frac{(N-1)! e^{-N\rho}}{\pi^{N-1}} \frac{e^{\frac{|A(\mathbf{x}^H\mathbf{c}+c_{N-1})|^2}{\sigma^2(\|\mathbf{x}\|^2+1)}}}{(\mathbf{x}\|^2+1)^N} \\ &\times \sum_{p=0}^{N-1} \binom{N-1}{p} \left(\frac{|A(\mathbf{x}^H\mathbf{c}+c_{N-1})|^2}{\sigma^2(\|\mathbf{x}\|^2+1)}\right)^p \frac{1}{p!}. \end{aligned} \quad (22)$$

The pdf of $\mathbf{x}$ under $\mathcal{H}_0$ is obtained by replacing $A = 0$ in (22) as follows:

$$f(\mathbf{x};\mathcal{H}_0) = \frac{(N-1)!}{\pi^{N-1}} \frac{1}{(\|\mathbf{x}\|^2+1)^N}. \quad (23)$$

The pdf of $\mathbf{y}$ in (15) is

$$f(\mathbf{y};\mathcal{H}_1) = \begin{cases} 0, & \text{if } \exists\angle y_i \notin \left(-\frac{2\pi}{|\mathcal{C}|}, \frac{2\pi}{|\mathcal{C}|}\right] \\ \sum_{j=1}^{|\mathcal{C}|^{N-1}} f_{\mathbf{x}}(\mathbf{t}_j \odot \mathbf{y};\mathcal{H}_1), & \text{if } \forall\angle y_i \in \left(-\frac{2\pi}{|\mathcal{C}|}, \frac{2\pi}{|\mathcal{C}|}\right] \end{cases}$$

where $\mathbf{t}_j = [t_{0,j}, \ldots, t_{N-1,j}]^T$, and $t_{k,j} \in \mathcal{C}$ is the kth component of the jth possible sequence. Since $\mathbf{t}$ is an N-tuple vector, there are $|\mathcal{C}|^{N-1}$ different possible sequences to be considered $(t_{N-1,j} \triangleq 1, j = 1, \ldots, |\mathcal{C}|^{N-1})$. Therefore, we have

$$\begin{aligned} f(\mathbf{y};\mathcal{H}_1) &= \frac{(N-1)! e^{-N\rho}}{\pi^{N-1}\|\mathbf{y}\|^{2N}} \sum_{j=1}^{|\mathcal{C}|^{N-1}} e^{\rho\frac{\left|\sum_{k=0}^{N-1} y_k^H u_{k,j}\right|^2}{\|\mathbf{y}\|^2}} \\ &\times \sum_{p=0}^{N-1} \binom{N-1}{p} \left(\rho\frac{\left|\sum_{k=0}^{N-1} y_k^H u_{k,j}\right|^2}{\|\mathbf{y}\|^2}\right)^p \frac{1}{p!} \end{aligned} \quad (24)$$

where $u_{k,j} = t_{k,j}c_k \in \mathcal{C}$ is the kth component of the jth possible sequence, and similarly, there are $|\mathcal{C}|^{N-1}$ different possible such sequences to be considered $(u_{N-1,j} \triangleq 1, j = 1, \ldots, |\mathcal{C}|^{N-1})$. By replacing $\rho = 0$ in (24), the pdf of $\mathbf{y}$ under $\mathcal{H}_0$ is

$$f(\mathbf{y};\mathcal{H}_0) = \frac{(N-1)! 2^{N-1}}{\pi^{N-1}} \sum_{j=1}^{|\mathcal{C}|^{N-1}} \frac{1}{\|\mathbf{y}\|^2}. \quad (25)$$

We obtain (10) by constructing $L(\mathbf{r}) = (f(\mathbf{y};\mathcal{H}_1)/f(\mathbf{y};\mathcal{H}_0))$ and some manipulation from (24), (25) and (9). Since the LR in (10) depends on the SNR of the probable signal, the UMPI test exists only if the SNR is known.

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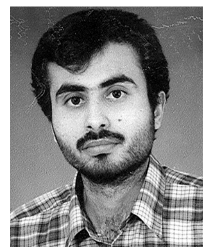
Ali Akbar Tadaion (S’05) was born in Iran in 1976. He received the B.Sc. degree in electronics and the M.Sc. degree in communication systems in 1998 and 2000, respectively, from Sharif University of Technology, Tehran, Iran, where he is currently working toward the Ph.D. degree.

From August 2004 to June 2005, he was with the Department of Electrical and Computer Engineering, Queen’s University, Kingston, ON, Canada, as a Visiting Researcher.

His main research interests are statistical signal processing, detection theory, and array signal processing.



Mostafa Derakhtian (S’05) was born in Iran in 1974. He received the B.Sc. and the M.Sc. degrees in communication systems in 1996 and 1998, respectively, from Sharif University of Technology, Tehran, Iran, where he is currently working toward the Ph.D. degree.

His main research interests are statistical signal processing, detection theory, and array signal processing.

Saeed Gazor (S’94–M’95–SM’98) received the B.Sc. degree in electronics and the M.Sc. degree in communication systems from Isfahan University of Technology, Isfahan, Iran, in 1987 and 1989, respectively, and the Ph.D. degree in signal and image processing from the Département Signal, École Nationale Supérieure des Télécommunications, Paris, France (Telecom Paris), in 1994 (all with highest honors).

From 1995 to 1998, he was with the Department of Electrical and Computer Engineering, Isfahan University of Technology. From January 1999 to July 1999, he was with the Department of Electrical and Computer Engineering, University of Toronto, Toronto, ON, Canada. Since July 1999, he has been with the Department of Electrical and Computer Engineering, Queen’s University, Kingston, ON, Canada, where now he holds the position of Associate Professor with tenure. He is also cross-appointed to the Department of Mathematics and Statistics at Queen’s University. His main research interests are array signal processing, statistical and adaptive signal processing, speech processing, MIMO communication systems, networking, analog adaptive circuits, channel modeling, and information theory.

Dr. Gazor received the Premier’s Research Excellence Award of the Province of Ontario in 2004.

Mohammad M. Nayebi (S’89–M’94–SM’05) was born in Iran in 1967. He received the B.S.E.E. and M.S.E.E. degrees in 1988 and 1990, respectively, both from Sharif University of Technology, Tehran, Iran, with the honor of first rank. He received the Ph.D. degree in electrical engineering in 1994 from Tarbiat Modarres University, Tehran, Iran, with the same honor.

He joined the Sharif University of Technology faculty in 1994, and became an Associate Professor in 1998, and Professor in 2003. His main research interests are radar signal processing and detection theory. Also, he has published more than 70 technical papers in international and national journals and conferences.

Mohammad R. Aref was born in Iran in 1951. He received the B.Sc. degree in 1974 from Tehran University, Tehran, Iran, and the M.Sc. and Ph.D. degrees in 1975 and 1979, respectively, from Stanford University, Stanford, CA, all in electrical engineering.

From 1994 to 1997, he was the Chancellor of Tehran University. He is now a Professor of Electrical Engineering with Sharif University of Technology, Tehran, Iran, and has published numerous technical papers on communication theory in international journals and conference proceedings.