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Coding advantage decomposition inequality for the space–frequency block codes

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Abstract: In this study, the authors present a new criterion which can be used to decompose the coding advantage (CA) of space–frequency block codes (SFBCs). The proposed decomposition separates the CA into two parts, called precoder-CA and channel-CA. The precoder-CA represents the effects of the precoder and the signal constellation, and can be used in the design of the precoder of SFBCs. The channel-CA, on the other hand, gives an indication of the effects of the channel characteristics, and can be utilised for optimising the permutation parameter. The authors have shown that the proposed decomposition method leads to a lower computational complexity for the optimisation procedure compared with the general form of the CA. As an application of the proposed method, the authors used it to optimise the channel-CA of a high-rate SFBC. Simulation results confirmed better performance for the modified high-rate SFBC compared with recently proposed SFBCs in the literature.

1 Introduction

Fading is one of the most repellent defects of nowadays wireless communication, which causes a significant performance degradation compared with the additive white Gaussian noise channels. Space–time coding as a sort of multi-input multi-output (MIMO) systems is a promising modern channel coding technique to overcome the fading effect by introducing diversity.

To combat intersymbol interference in the frequency-selective channels, orthogonal frequency-division multiplexing (OFDM) technique could be used.

MIMO–OFDM takes advantage of both OFDM and MIMO for multipath and fading channels, respectively. Space–frequency block codes (SFBCs) are one of the techniques for implementing MIMO–OFDM systems. The performance criteria for SFBCs are discussed in [1, 2].

Space–frequency coding scheme exploits both the multipath and the space diversities. As proved in [2, 3], the maximum diversity advantage of SFBCs is equal to LM_TM_R, where M_T is the number of transmit antennas, M_R is the number of receive antennas and L is the number of independent delay paths. Numerous SFBCs have been proposed to date [2–6]. In [2], the authors show that using a simple mapping the structure of every full-diversity space–time-block code could be used to construct full-diversity SFBCs, but comes at the cost of losing the symbol-rate when the number of delay paths increases. In [3], a systematic SFBC is proposed, which could achieve full diversity and rate 1 for any number of transmit antennas.

Space–time–frequency block codes (STFBCs) have been developed by introducing the time dimension to SFBCs that can, at most, achieve the diversity advantage of LM_TM_Rr, where r is the rank of the temporal correlation matrix [7]. The design criteria for STFBCs are discussed in [7–9]. STFBCs and SFBCs proposed in [10–14] are just some examples of research study currently being pursued in this area. In [12], linear transform based full-diversity SFBCs and STFBCs are proposed which feature the best performance to the best of our knowledge.

Coding advantage (CA) of SFBCs is associated with channel parameters such as delay and power profiles (DPPs), which cause different optimisations for various channels. More importantly, a high complexity computation is required for CA calculation, which makes the procedure of optimising the SFBCs very hard and in some cases impossible.

In [3], the authors constructed a SFBC based on the design criteria which they were proposed earlier in [2]. This particular SFBC was wisely designed to have separable CA, that is, the CA of this code was decomposed into two parts as: extrinsic and intrinsic diversity products. This property of the SFBC in [3] is very important because the optimisation of this SFBC can be done with a very lower complexity calculation using extrinsic and intrinsic diversity products instead of CA itself. Therefore it could be very helpful for designing and optimising a SFBC if its CA can be decompose. However, in [3], neither a criterion nor a condition was presented for SFBCs with separable CA. We considered that it could be exploited further by developing a method that would formalise this property so as to be applicable to other SFBCs. In this paper, we explore a lower bound for the CA of SFBCs, which separates the effects of the precoder and the permutation on the performance of the coding method. This is advantageous because for any arbitrary channel, obtaining an optimum permutation is much faster and simpler through calculations than through the use of the CA. Also designing of the precoder can be done separately. As an application of the proposed method, the new criterion is used to improve the performance of the high-rate SFBCs in [1].

The rest of this paper is organised as follows: in the next section the system model is introduced and is followed by Section 3, which presents the proposed methods in detail. Section 4 defines the design parameters and explains the structure of a high-rate SFBC. The discussion then moves to an analysis of the approach aimed at improving performance of a high-rate SFBC algorithm using the proposed method. Section 5 assesses the simulation results and makes comparisons against other SFBCs. The paper concludes in Section 6 by highlighting its contribution towards SFBC improvement and suggesting further research work.

1.1 Notations

Capital boldface letters stands for matrices, and boldface lower case alphabet with a bar represents vectors. The element on the ith row and the jth column of matrix $\boldsymbol{A}$ is denoted by $\boldsymbol{A}(i, j)$. The ith element of vector $\bar{\boldsymbol{a}}$ is indicated by $\bar{\boldsymbol{a}}(i)$. Superscripts $(\cdot)^{\mathrm{T}}$, $(\cdot)^{\mathrm{H}}$, and $(\cdot)^*$ indicate transpose, Hermitian and complex conjugations, respectively. By $\circ$ and $\otimes$, we mean the Hadamard and the Tensor products, respectively. Notation $\mathbf{1}_{a\times b}$ stands for an $a \times b$ matrix of ones, $\boldsymbol{I}_a$ denotes an identity matrix of size $a \times a$ and $\mathbb{C}$ stands for the complex field. diag($a_1, a_2, \ldots, a_n$) represents a diagonal $n \times n$ matrix whose diagonal entries are $a_1, a_2, \ldots, a_n$.

2 System model

In a MIMO–OFDM system with N_c subcarriers and M_t transmit antennas, each codeword of a SFBC could be written as an $N_c \times M_t$ matrix, thus

$$\boldsymbol{C} = \begin{bmatrix} c_1(0) & c_2(0) & \cdots & c_{M_t}(0) \\ c_1(1) & c_2(1) & \cdots & c_{M_t}(1) \\ \vdots & \vdots & \ddots & \vdots \\ c_1(N_c-1) & c_2(N_c-1) & \cdots & c_{M_t}(N_c-1) \end{bmatrix} \in \mathbb{C}^{N_c \times M_t} \tag{1}$$

where $c_i(n)$'s are data transmitted by the ith transmit antenna at the nth subcarrier. After applying an N_c-point inverse fast Fourier transform to each column of $\boldsymbol{C}$ and appending cyclic prefix, the transmitter simultaneously sends the OFDM symbols over M_t transmit antennas.

For a receiver with M_r receive antennas, in a frequency-selective channel between each pair of the transmit and the receive antennas, we assume that there are L independent delay-paths with the same DPPs. Channel impulse response from the transmit antenna i to the receive antenna j could be modelled as

$$h_{i,j}(\tau) = \sum_{l=0}^{L-1} \alpha_{i,j}(l)\delta(\tau - \tau_l) \tag{2}$$

In (2), τ_l's are delays, $\alpha_{i,\ j}(l)$'s are zero-mean complex Gaussian random variables with variance σ_l^2 indicating the complex amplitude corresponding to the lth path of the ith transmit and the jth receive antennas and $\sum_{l=0}^{L-1} \sigma_l^2 = 1$ for normalisation purposes.

At the jth receive antenna, first the appended cyclic prefix is removed from the received signal. Then, a fast Fourier transform at the nth frequency subcarrier is performed. The resulting signal can now be expressed as

$$r_j(n) = \sum_{i=1}^{M_T} c_i(n) H_{i,j}(n) + \mathcal{N}_j(n), \quad n = 0, 1, \ldots, N_c - 1 \tag{3}$$

In (3)

$$H_{i,j}(n) = \sum_{l=0}^{L-1} \alpha_{i,j}(l) w^{n\tau_l}, \quad n = 0, 1, \ldots, N_c - 1 \tag{4}$$

where $w = \mathrm{e}^{-\mathrm{j}2\pi\Delta f}$, $\Delta f = \mathrm{BW}/N_c$, BW is the total bandwidth of the system, and $\mathcal{N}_j(n)$ denotes the additive white complex Gaussian noise

3 Proposed CA decomposition inequality

In this section, we aim to derive a decomposition expression for the CA of the SFBCs. The CA of SFBCs depends on the code structure and the channel characteristics. Therefore optimising the CA is complex and involves a great deal of computations. However, using the proposed decomposition method, one could optimise the code structure and, for any arbitrary channel, finding an optimum permutation parameter does not require a complex calculation.

It has already been proved that in an L-ray channel a SFBC could achieve the diversity order of LM_t. Consequently, to achieve maximum available diversity, at least each LM_t rows of the matrix $\boldsymbol{C}$ must be constructed together and contain the information of the same symbols. To minimise complexity of the receiver while attempting to attain maximum diversity, most existing SFBCs are designed by using sub-blocks of LM_t rows of the code, which are joined together to construct a certain type of SFBCs. Therefore, the construction of a given SFBC could be described based on the design of the sub-blocks of size $\Gamma\ M_t \times M_t$ matrices $\boldsymbol{G}_p$, for $p = 1, 2, \ldots, N_c/\Gamma M_t$, where $1 \le \Gamma \le L$, and N_c being a multiple of $\Gamma\ M_t$. Note that this condition does not cause any concern since zero padding or the use of a smaller code blocks could resolve this problem.

Let us assume each codeword of an SFBC as follows

$$\boldsymbol{C} = \begin{bmatrix} \boldsymbol{G}_1^{\mathrm{T}} & \boldsymbol{G}_2^{\mathrm{T}} & \cdots & \boldsymbol{G}_{N_c/LM_t}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \in \mathbb{C}^{N_c \times M_t} \tag{5}$$

Defining $\boldsymbol{G}_p$ as the primary codeword, the design and analysis of the SFBC splits into its $\Gamma M_t \times M_t$ submatrices $\boldsymbol{G}_p$.

The pairwise error probability between two distinct codewords $\boldsymbol{C}$ and $\tilde{\boldsymbol{C}}$ is shown to be upper bounded as [2]

$$P(\boldsymbol{C} \to \tilde{\boldsymbol{C}}) \le \binom{2vM_r - 1}{vM_r} \left(\prod_{i=1}^{v} \lambda_i\right)^{-M_r} \left(\frac{\rho}{M_t}\right)^{-vM_r} \tag{6}$$

where λ_i's are the non-zero eigenvalues and v is the minimum rank of $\boldsymbol{\Delta}\circ\boldsymbol{R}$, with $\boldsymbol{\Delta} \triangleq (\boldsymbol{C} - \tilde{\boldsymbol{C}})(\boldsymbol{C} - \tilde{\boldsymbol{C}})^{\mathrm{H}}$ and $\boldsymbol{R}$ denoting the frequency correlation matrix.

Now, we could simply use the inequality (6) for $\boldsymbol{G}_p$ as

$$P(\boldsymbol{G} \to \tilde{\boldsymbol{G}}) \leq \binom{2\varsigma M_r - 1}{\varsigma M_r} \left(\prod_{i=1}^{\varsigma} \gamma_i\right)^{-M_r} \left(\frac{\rho}{M_t}\right)^{-\varsigma M_r} \tag{7}$$

where γ_i's are the non-zero eigenvalues of $\hat{\boldsymbol{\Delta}} \circ \hat{\boldsymbol{R}}$, with $\hat{\boldsymbol{\Delta}} \triangleq (\boldsymbol{G} - \tilde{\boldsymbol{G}})(\boldsymbol{G} - \tilde{\boldsymbol{G}})^{\mathrm{H}}$, $\hat{\boldsymbol{R}}$ is a principal submatrix of $\boldsymbol{R}$ with the same indexing which $\hat{\boldsymbol{\Delta}}$ lies in the matrix $\boldsymbol{\Delta}$. Matrices $\boldsymbol{G}$ and $\hat{\boldsymbol{G}}$ are two distinct primary codewords of size $\Gamma M_t \times M_t$.

The diversity order of the SFBC is equal to v and the minimum value of $\prod_{i=1}^{v} \lambda_i$ is known as the CA.

Using a similar structure for $\boldsymbol{G}_p$'s and taking two distinct codewords $\boldsymbol{C}$ and $\tilde{\boldsymbol{C}}$, there is at least one index $p_0 (1 \leq p_0 \leq N_c/LM_t)$ such that $\boldsymbol{G}_{p_0}$ and $\tilde{\boldsymbol{G}}_{p_0}$ are different. We may further assume $\boldsymbol{G}_p = \tilde{\boldsymbol{G}}_p$ for any $p \neq p_0$ since the rank of $\boldsymbol{\Delta} \circ \boldsymbol{R}$ does not decrease if $\boldsymbol{G}_p \neq \tilde{\boldsymbol{G}}_p$ for some $p \neq p_0$. Thus, the rank and the CA of a SFBC is equal to the primary codeword described in (7), that is, $v = \varsigma$ and

$$\min\left(\prod_{i=1}^{v} \lambda_i\right) = \min\left(\prod_{i=1}^{\varsigma} \gamma_i\right)$$

For full-diversity SFBCs with $\Gamma = L$, the matrix $\hat{\boldsymbol{\Delta}} \circ \hat{\boldsymbol{R}}$ is full-rank. Hence, all the eigenvalues of $\hat{\boldsymbol{\Delta}} \circ \hat{\boldsymbol{R}}$ are non-zero, and we could calculate the CA by utilising the following determinant

$$\mathrm{CA} = \prod_{i=1}^{v} \gamma_i = \det\left(\hat{\boldsymbol{\Delta}} \circ \hat{\boldsymbol{R}}\right), \quad v = LM_t \tag{8}$$

Clearly, the CA of SFBCs is related to $\boldsymbol{R}$ which represents the correlation of the channel impulse response. This, therefore, results in different optimum structures of the codes in various channels. To the best of the authors' knowledge, only the SFBCs proposed in [3] have a separable CA, that is, the CA of this code can be decomposed into two parts as: extrinsic and intrinsic diversity products. However, neither a structure nor a condition has been proposed for SFBCs with separable CA. Hence, the following theorem is proposed for decomposing the CA of any SFBC. In Appendix 3, we discuss how the CA of SFBC in [3] can be decomposed as the product of two parts, and argue why this guideline is not applicable for other SFBCs.

Let us now introduce a lower bound for the CA of full-diversity SFBCs, which decomposes the CA into two parts: precoder-CA (P_{CA}) and channel-CA (C_{CA}). The precoder-CA and the channel-CA represent the effects of the precoder and the channel characteristics on the performance of the coding, respectively. The precoder-CA can be used for precoder design and the permutation parameter of the SFBC and be optimised with channel-CA with much lower complexity calculation than using the CA.

To extract the effect of channel DPPs from the CA in (8), the expression $\det(\hat{\boldsymbol{\Delta}} \circ \hat{\boldsymbol{R}})$ needs to be changed to $\det(\tilde{\boldsymbol{\Delta}} \circ \hat{\boldsymbol{\Xi}} \circ \hat{\boldsymbol{R}})$, where $\hat{\boldsymbol{\Delta}} = \tilde{\boldsymbol{\Delta}} \circ \hat{\boldsymbol{\Xi}}$. The matrix $\hat{\boldsymbol{\Xi}}$ consists of constants within the code and therefore, $\tilde{\boldsymbol{\Delta}}$ indicates the remaining parts of the code. Now the following theorem for decomposition of the CA of SFBCs is presented.

Theorem: Coding advantage decomposition inequality (CADI): for any full-diversity SFBC, if we have

$$\hat{\boldsymbol{\Delta}} = \hat{\boldsymbol{\Delta}} \circ \hat{\boldsymbol{\Xi}} \tag{9}$$

where $\tilde{\boldsymbol{\Delta}}$ is a positive semi-definite (PSD) matrix with non-zero diagonal entries and $\hat{\boldsymbol{\Xi}}$ is a PSD matrix, then the CA of SFBCs has a lower bound as follows

$$\mathrm{CA} \geq P_{\mathrm{CA}}.C_{\mathrm{CA}} \tag{10}$$

where $P_{CA} \triangleq \prod_{i=1}^{v} \Delta(i, j)$ and $C_{\mathrm{CA}} \triangleq \det\left(\hat{\boldsymbol{\Xi}} \circ \hat{\boldsymbol{R}}\right)$.

Proof: Substituting the assumption $\tilde{\boldsymbol{\Delta}} = \tilde{\boldsymbol{\Delta}} \circ \hat{\boldsymbol{\Xi}}$ in (8) gives

$$\mathrm{CA} = \det\left(\tilde{\boldsymbol{\Delta}} \circ \hat{\boldsymbol{\Xi}} \circ \hat{\boldsymbol{R}}\right) \tag{11}$$

The frequency correlation matrix is PSD because from [2], we have

$$\boldsymbol{R} = \boldsymbol{W}\boldsymbol{\Lambda}\boldsymbol{W}^{\mathrm{H}} \tag{12}$$

where

$$\boldsymbol{W} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \omega^{\tau_0} & \omega^{\tau_1} & \cdots & \omega^{\tau_{L-1}} \\ \vdots & \vdots & \ddots & \vdots \\ \omega^{(N_c-1)\tau_0} & \omega^{(N_c-1)\tau_1} & \cdots & \omega^{(N_c-1)\tau_{L-1}} \end{bmatrix}_{N_c \times L} \tag{13}$$

$\omega = \mathrm{e}^{-\mathrm{j}2\pi\Delta f}$, and

$$\boldsymbol{\Lambda} = \mathrm{diag}\left(\sigma_0^2, \sigma_1^2, \ldots, \sigma_{L-1}^2\right) \tag{14}$$

We could rewrite (12) as

$$\boldsymbol{R} = \boldsymbol{W}\boldsymbol{\Lambda}^{(1/2)}\boldsymbol{\Lambda}^{(1/2)}\boldsymbol{W}^{\mathrm{H}} \tag{15}$$

Since $\boldsymbol{\Lambda}^{1/2}$ is a diagonal real matrix, we have: $\boldsymbol{\Lambda}^{1/2} = \boldsymbol{\Lambda}^{1/2\mathrm{H}}$. Thus

$$\boldsymbol{R} = \left(\boldsymbol{W}\boldsymbol{\Lambda}^{1/2}\right)\left(\boldsymbol{W}\boldsymbol{\Lambda}^{1/2}\right)^{\mathrm{H}} \tag{16}$$

which proves that $\boldsymbol{R}$ is a PSD matrix. The matrix $\hat{\boldsymbol{R}}$ is also PSD because any principal submatrix of a PSD matrix is also PSD [15, p. 397]. Since the matrices $\hat{\boldsymbol{\Xi}}$ and $\hat{\boldsymbol{R}}$ are PSD, the matrix $\hat{\boldsymbol{\Xi}} \circ \hat{\boldsymbol{R}}$ is therefore PSD [15, p. 458]. Now, using the Oppenheim inequality [15, p. 480] for the Hadamard product, we could write

$$\det\left(\tilde{\boldsymbol{\Delta}} \circ \hat{\boldsymbol{\Xi}} \circ \hat{\boldsymbol{R}}\right) \geq \prod_{i=1}^{v} \tilde{\boldsymbol{\Delta}}(i, i) \times \det\left(\hat{\boldsymbol{\Xi}} \circ \hat{\boldsymbol{R}}\right) \tag{17}$$

Defining $P_{\mathrm{CA}} \triangleq \prod_{i=1}^{v} \tilde{\boldsymbol{\Delta}}(i, i)$ and $C_{\mathrm{CA}} \triangleq \det\left(\hat{\boldsymbol{\Xi}} \circ \hat{\boldsymbol{R}}\right)$, we arrive at (10). □

Remark 1: To optimise each parameter of a full-diversity SFBC using the CA, the value of CA needs to be calculated. This requires finding the minimum value of M^{N_s} determinants of $LM_t \times LM_t$ matrices, where M is the constellation size and N_s is the number of symbols used in each sub-block $\boldsymbol{G}_p$. For example, for the high-rate SFBC in [1], $N_s = LM_t^2$. This decomposition is valuable because in situations where the exact value of the CA is not needed, the lower bound of the CA can be used which requires less complex calculations. For instance, in order to obtain the optimum permutation for a full-diversity SFBC from N_γ possible permutations, using the CA as the objective function means $N_\gamma M^{N_s}$ determinants of $LM_t \times LM_t$ matrices must be calculated, whereas with the C_{CA}, it only requires calculating N_γ determinants. In other words, the computation complexity of the order of $\mathcal{O}(C \times M^{N_s})$ decreases to $\mathcal{O}(C)$, where $C = N_\gamma (LM_t)^3$. Since the reduction in the complexity is significant, it clearly results in a very swift and low-cost optimisation process in the MIMO–OFDM systems.

The proposed decomposition process may at first seem hard to perform but, as will be shown below, the CADI theorem can be easily applied to some of the best SFBCs published recently. Some examples are discussed below.

Some examples of the CADI for the SFBCs:

Example 1: In [12], we see that $\hat{\boldsymbol{\Delta}}_{\text{BCDD}}$ can be easily decomposed as $\hat{\boldsymbol{\Delta}}_{\text{BCDD}} = \tilde{\boldsymbol{\Delta}}_{\text{BCDD}} \circ \hat{\boldsymbol{\Xi}}_{\text{BCDD}}$, where $\hat{\boldsymbol{\Xi}}_{\text{BCDD}}$ is a matrix with entries of zeros and ones. To do this, we define $\hat{\boldsymbol{\Xi}}_{\text{BCDD}}$ as

$$\hat{\boldsymbol{\Xi}}_{\text{BCDD}}(i, j) = \begin{cases} 1, & \text{if } \hat{\boldsymbol{\Delta}}_{\text{BCDD}}(i, j) \neq 0 \\ 0, & \text{if } \hat{\boldsymbol{\Delta}}_{\text{BCDD}}(i, j) = 0 \end{cases} \tag{18}$$

which means $\tilde{\boldsymbol{\Delta}}_{\text{BCDD}} = \hat{\boldsymbol{\Delta}}_{\text{BCDD}}$.

Example 2: For the quasi-orthogonal space–frequency block codes (QOSFBCs) [4], we can use the same technique, $\hat{\boldsymbol{\Delta}}_{\text{QOSF}} = \tilde{\boldsymbol{\Delta}}_{\text{QOSF}} \circ \hat{\boldsymbol{\Xi}}_{\text{QOSF}}$, where $\tilde{\boldsymbol{\Delta}}_{\text{QOSF}} = \hat{\boldsymbol{\Delta}}_{\text{QOSF}}$, and

$$\hat{\boldsymbol{\Xi}}_{\text{QOSF}}(i, j) = \begin{cases} 1, & \text{if } \hat{\boldsymbol{\Delta}}_{\text{QOSF}}(i, j) \neq 0 \\ 0, & \text{if } \hat{\boldsymbol{\Delta}}_{\text{QOSF}}(i, j) = 0 \end{cases}$$

Example 3: For the SFBCs proposed in [3], we can write $\hat{\boldsymbol{\Delta}}_{[3]} = \hat{\boldsymbol{\Delta}}_{[3]} \circ \hat{\boldsymbol{\Xi}}_{[3]}$, where $\tilde{\boldsymbol{\Delta}}_{[3]} = \hat{\boldsymbol{\Delta}}_{[3]}$ and

$$\hat{\boldsymbol{\Xi}}_{[3]}(i, j) = \begin{cases} 1, & \text{if } \hat{\boldsymbol{\Delta}}_{[3]}(i, j) \neq 0 \\ 0, & \text{if } \hat{\boldsymbol{\Delta}}_{[3]}(i, j) = 0 \end{cases}$$

The proofs that matrices $\hat{\boldsymbol{\Xi}}_{\text{QOSF}}$, $\hat{\boldsymbol{\Xi}}_{\text{BCDD}}$ and $\hat{\boldsymbol{\Xi}}_{[3]}$ are PSD, are provided in Appendix 1.

Since for all the mentioned codes, $\tilde{\boldsymbol{\Delta}} = \hat{\boldsymbol{\Delta}}$ and $\hat{\boldsymbol{\Delta}} = (\boldsymbol{G} - \tilde{\boldsymbol{G}})(\boldsymbol{G} - \tilde{\boldsymbol{G}})^{\text{H}}$, $\tilde{\boldsymbol{\Delta}}$ is PSD for all of these SFBCs. Here, we obtained PSD matrices $\tilde{\boldsymbol{\Delta}}$ and $\hat{\boldsymbol{\Xi}}$ for codes [3, 4, 12], which means CADI is valid for these codes. It is obvious that for these SFBCs we do not have to look for special matrices $\hat{\boldsymbol{\Xi}}$ and $\tilde{\boldsymbol{\Delta}}$.

4 SFBCs improvement based on the CADI

In [1], a systematic design of high-rate full-diversity SFBCs is presented for MIMO–OFDM systems in the frequency-selective channels. In what follows, the structure of the SFBCs in [1] is investigated in brief.

Suppose $N_L = 2^{\lceil \log_2 L \rceil}$, $N_q = 2^{\lceil \log_2 M_t \rceil}$ and $K = N_L N_q$. Then, the proposed codes in [1] can be written as follows

$$\boldsymbol{C} = \begin{bmatrix} \boldsymbol{B}_1^{\text{T}} & \boldsymbol{B}_2^{\text{T}} & \cdots & \boldsymbol{B}_J^{\text{T}} \end{bmatrix} \in \mathbb{C}^{N_C \times M_T} \tag{19}$$

where $J = N_C/K$ and $\boldsymbol{B}_i$ for $i = 1, 2,\ldots, J$ is expressed as

$$\boldsymbol{B}_i = \begin{bmatrix} \boldsymbol{X}_1^{\text{T}} & \boldsymbol{X}_2^{\text{T}} & \cdots & \boldsymbol{X}_{N_L}^{\text{T}} \end{bmatrix}^{\text{T}} \tag{20}$$

where the matrix $\boldsymbol{X}_m$ is of size $N_q \times M_T (N_q \geq M_T)$ for $m = 1,\ldots, N_L$ and is given by (see (21))

where $k_m = (m - 1)M_T$ and

$$\bar{\boldsymbol{x}}_n^{\text{T}} = \boldsymbol{V}\bar{\boldsymbol{s}}_n^{\text{T}} \tag{22}$$

In (22), $\bar{\boldsymbol{s}}_n$ denotes a vector of $N_q N_L$ symbols taken from the constellation points, for $n = 1, 2,\ldots, N_q$ and $\phi = \text{e}^{\text{j}\pi/32}$ is another code parameter. Also $\boldsymbol{V} \in \mathbb{C}^{N_q N_L \times N_q N_L}$ stands for the Vandermonde matrix whose parameters are selected in the same way as those in [3].

At this point, we perform CADI for the recently proposed high-rate SFBCs in [1], as described above.

Let us consider a MIMO–OFDM system with two transmit antennas. The sub-block $\boldsymbol{G}_p$ of the codeword in (19) can be written as

$$\boldsymbol{G}_{\text{p-HRC}} = \begin{bmatrix} \boldsymbol{X}_1^{\text{T}} & \boldsymbol{X}_2^{\text{T}} & \cdots & \boldsymbol{X}_{N_L}^{\text{T}} \end{bmatrix}^{\text{T}} \in \mathbb{C}^{2N_L \times 2} \tag{23}$$

where

$$\boldsymbol{X}_m = \begin{bmatrix} \bar{\boldsymbol{x}}_1(k_m + 1) & \phi\bar{\boldsymbol{x}}_2(k_m + 1) \\ \phi\bar{\boldsymbol{x}}_2(k_m + 2) & \bar{\boldsymbol{x}}_1(k_m + 2) \end{bmatrix} \in \mathbb{C}^{2\times 2} \tag{24}$$

Here, we can rewrite $\hat{\boldsymbol{\Delta}}_{\text{HRC}} \circ \hat{\boldsymbol{R}}$ as

$$\hat{\boldsymbol{\Delta}}_{\text{HRC}} \circ \hat{\boldsymbol{R}} = \tilde{\boldsymbol{\Delta}}_{\text{HRC}} \circ \hat{\boldsymbol{\Xi}}_{\text{HRC}} \circ \hat{\boldsymbol{R}} \tag{25}$$

where $\hat{\boldsymbol{\Xi}}_{\text{HRC}}$ has the following structure for $M_T = 2$

$$\hat{\boldsymbol{\Xi}}_{\text{HRC}} = \boldsymbol{1}_{L\times L} \otimes \begin{bmatrix} 2 & \phi^* + \phi \\ \phi^* + \phi & 2 \end{bmatrix} \tag{26}$$

It has already been proved that this code can achieve full diversity and that the matrix $\hat{\boldsymbol{\Xi}}_{\text{HRC}}$ is PSD because it could be written as: $\hat{\boldsymbol{\Xi}}_{\text{HRC}} = \boldsymbol{D}\boldsymbol{D}^{\text{H}}$ where $\boldsymbol{D} = \boldsymbol{1}_{L\times 1} \otimes \begin{bmatrix} 1 & \phi \\ \phi & 1 \end{bmatrix}$.

For practical constellations such as binary phase shift keying (BPSK), quadrature phase shift keying (QPSK), 4-QAM, 16-QAM and 64-QAM, numerical results confirm that for any two distinct codewords $\boldsymbol{C}$ and $\tilde{\boldsymbol{C}}$, the matrix $\tilde{\boldsymbol{\Delta}}_{\text{HRC}}$ is PSD.

$$\boldsymbol{X}_m = \begin{bmatrix} \bar{\boldsymbol{x}}_1(k_m + 1) & \phi\bar{\boldsymbol{x}}_2(k_m + 1) & \cdots & \phi^{M_T - 1}\bar{\boldsymbol{x}}_{M_T}(k_m + 1) \\ \phi^{N_q - 1}\bar{\boldsymbol{x}}_{N_q}\left(k_m + \left\lfloor \dfrac{M_T}{N_q} + 1 \right\rfloor\right) & \bar{\boldsymbol{x}}_1(k_m + 2) & \cdots & \phi^{M_T - 2}\bar{\boldsymbol{x}}_{M_T - 1}(k_m + 2) \\ \vdots & \vdots & \ddots & \vdots \\ \phi\bar{\boldsymbol{x}}_2(k_m + M_T) & \phi^2\bar{\boldsymbol{x}}_3(k_m + M_t) & \cdots & \phi^{(1 - \lfloor (M_T/N_q) \rfloor)M_T}\bar{\boldsymbol{x}}_{(1 - \lfloor (M_T/N_q) \rfloor)M_T + 1}(k_m + M_T) \end{bmatrix} \tag{21}$$

The above two conditions allow us to use the inequality (22)

$$\mathrm{CA_{HRC}} \geq \prod_{i=1}^{v} \tilde{\boldsymbol{\Delta}}_{\mathrm{HRC}}(i, i) \times \det\left(\hat{\boldsymbol{\Xi}}_{\mathrm{HRC}} \circ \hat{\boldsymbol{R}}\right) \quad (27)$$

Now, in order to improve the performance of this SFBC, we could maximise its CA and this is done by maximising the lower bound of (27).

In Appendix 2, P_{CA} of this code has been proved to be non-zero. Therefore we can maximise its C_{CA} of the code by introducing a permutation parameter in the code.

In what follows, we describe a permuted version of the code in [1], namely permuted high-rate code (P-HRC).

Let us proceed to generate $\boldsymbol{\Psi}_k$'s as follows

$$\boldsymbol{\Psi}_k = \left[\boldsymbol{P} \otimes \boldsymbol{I}_2\right]\boldsymbol{\Phi}_k \in \mathbb{C}^{N_L\gamma_{\mathrm{SD}}\times 2},\ k = 1, 2, \ldots, \left\lfloor \frac{N_c}{N_L\gamma_{\mathrm{SD}}} \right\rfloor \quad (28)$$

where γ_{SD} is an even integer which represents the permutation parameter and could be optimised using the C_{CA}. The matrices $\boldsymbol{\Phi}_k$'s of size $N_L\gamma_{\mathrm{SD}} \times 2$ are constructed by sticking $(\gamma_{\mathrm{SD}}/2)$ distinct $\boldsymbol{G}_{p\text{-P-HRC}}$ together and $\boldsymbol{P} = \left(\boldsymbol{P}_1^{\mathrm{T}} \ \ \boldsymbol{P}_2^{\mathrm{T}} \ \ \cdots \ \ \boldsymbol{P}_{N_L}^{\mathrm{T}}\right)^{\mathrm{T}} \in \mathbb{C}^{N_L(\gamma_{\mathrm{SD}}/2)\times N_L(\gamma_{\mathrm{SD}}/2)}$ is the permutation matrix, where

$$\boldsymbol{P}_i = \left(\bar{\boldsymbol{e}}_i^{\mathrm{T}} \ \ \bar{\boldsymbol{e}}_{N_L+i}^{\mathrm{T}} \ \ \cdots \ \ \bar{\boldsymbol{e}}_{((\gamma_{\mathrm{SD}}/2)-1)N_L+i}^{\mathrm{T}}\right)^{\mathrm{T}} \in \mathbb{C}^{(\gamma_{\mathrm{SD}}/2)\times N_L(\gamma_{\mathrm{SD}}/2)} \quad (29)$$

for $i = 1, 2, \ldots, N_L$, and $\bar{\boldsymbol{e}}_i \in \mathbb{C}^{1\times N_L(\gamma_{\mathrm{SD}}/2)}$ is a vector whose components are all zeros except for the ith element, that is, 1.

Now, we can formulate the proposed SFBCs as shown below

$$\boldsymbol{C}_{\mathrm{new}} = \left[\boldsymbol{\Psi}_1^{\mathrm{T}} \ \ \boldsymbol{\Psi}_2^{\mathrm{T}} \ \ \cdots \ \ \boldsymbol{\Psi}_{(N_c/N_L\gamma_{\mathrm{SD}})}^{\mathrm{T}} \ \ \boldsymbol{Z}^{\mathrm{T}}\right]^{\mathrm{T}} \in \mathbb{C}^{N_c\times 2} \quad (30)$$

where $\boldsymbol{Z}$ is a zero matrix of size $\left(N_c - \gamma_{\mathrm{SD}}N_L\left\lfloor \frac{N_c}{\gamma_{\mathrm{SD}}N_L} \right\rfloor\right) \times 2$.

This modification results in an additional parameter, γ_{SD}, and causes C_{CA} to relate to this new parameter. Thus, we could optimise this parameter for different channels.

Now, by performing the same operations as in (23)–(27), for the proposed SFBCs, we finally obtain

$$\mathrm{CA_{P-HRC}} \geq \prod_{i=1}^{v} \tilde{\boldsymbol{\Delta}}_{\mathrm{P-HRC}}(i, i) \times \det\left(\hat{\boldsymbol{\Xi}}_{\mathrm{P-HRC}} \circ \hat{\boldsymbol{R}}(\gamma_{\mathrm{SD}})\right) \quad (31)$$

where $\hat{\boldsymbol{R}}(\gamma_{\mathrm{SD}})$ is a submatrix of matrix $\boldsymbol{R}$ with the same indices, which $\hat{\boldsymbol{\Delta}}_{\mathrm{P-HRC}}$ lies in the matrix $\boldsymbol{\Delta}_{\mathrm{P\text{-}HRC}}$. Finally, we could formulate an expression to find the optimum positive integer $\gamma_{\mathrm{SD}}^{\mathrm{OP}}$ to maximise the C_{CA} of P-HRC as

$$\gamma_{\mathrm{SD}}^{\mathrm{OP}} = \arg \max_{1<\gamma_{\mathrm{SD}}<(N/\Gamma)} \det\left(\hat{\boldsymbol{\Xi}}_{\mathrm{P-HRC}} \circ \hat{\boldsymbol{R}}(\gamma_{\mathrm{SD}})\right) \quad (32)$$

To optimise the permutation parameter $\gamma_{\mathrm{SD}}^{\mathrm{OP}}$ using (32), the correlation matrix $\boldsymbol{R}$ is needed which can be constructed by DPPs. There are two possible scenarios at the transmitter of a MIMO–OFDM system, DPPs are either available or absent. When the DPPs are accessible, simply the matrix $\boldsymbol{R}$ can be constructed and then using (32), $\gamma_{\mathrm{SD}}^{\mathrm{OP}}$ can be calculated. If there are no prior knowledge of the DPPs at the transmitter, the artificial DPPs (ADPPs) proposed in [14] are used to construct the correlation matrix and then calculating $\gamma_{\mathrm{SD}}^{\mathrm{OP}}$ by (32). In the next section, we presented simulation results for both scenarios.

It seems important to describe how the improved SFBC affects the system at the receiver end. Since only one new parameter, namely $\gamma_{\mathrm{SD}}^{\mathrm{OP}}$, is introduced to the conventional structure of the SFBC in [1], the only extra calculation at the receiver is the multiplication of the permutation matrix to the codeword before the ML decoder. Consequently, the complexity of the ML decoder does not change. Also the only extra information which is needed at the receiver is the value of $\gamma_{\mathrm{SD}}^{\mathrm{OP}}$, which can be accessible using the same way that the other parameters of the code such as ϕ, N_q or N_L are reported to the receiver.

It is worth mentioning that, by following the same guidelines decomposition can be easily carried out for any number of the transmit antennas.

Remark 2: As discussed in Section 3, the CADI achieves optimisation at low complexity. To illustrate this point, we consider an example of a practical MIMO–OFDM system.

For a MIMO–OFDM transmitter with two transmit antennas and $N_C = 128$ subcarriers in the 6-ray TU channel model, optimisation of the γ_{SD} parameter of the P-HRCs, using the CA for 4-QAM constellation, calls for a lengthy calculation of $N_\gamma M^{N_s} = 10{*}4^{4*6} \simeq 10^{15}$ determinants of 12×12 matrices, that is, a complexity order of $\mathcal{O}(10^{18})$. In contrast, if the CADI is applied, then, to maximise the C_{CA} of P-HRC only $N_\gamma = 10$ determinants of 12×12 matrices needs to be computed, that is, a complexity of the order of $\mathcal{O}(10^4)$.

5 Simulation results

At the transmitter, there can be two possible scenarios: either we have DPPs or we do not. Here, we simulated both of these scenarios and then examined the results as described below.

5.1 Presence of DPPs

We considered a MIMO–OFDM system with two transmit antennas, $N_C = 128$ subcarriers, BW = 1 MHz and a cyclic prefix of length 15 μs and a 2-ray equal power frequency-selective channel with a 5 μs delay spread. The parameters of the proposed optimum SFBC in [12] were calculated as: $\theta = 2$, $\gamma_{\mathrm{op}} = 32$. For the P-HRC, the optimum value of γ_{SD} was equal to 12, using the objective function (32).

To achieve the same spectral efficiencies, 4-QAM and 16-QAM constellations were used for P-HRC and SFBC in [12], respectively. Fig. 1 shows the simulation results for both the P-HRC and optimum SFBC in [12]. As can be seen the former SFBC outperforms the latter in [12].

5.2 Absence of DPPs

In the absence of channel state information at the transmitter, two methods have been proposed to date: the random permutation [3] and the permutation based on the ADPPs [14]. In [14], we saw that using the ADPPs almost always results in an improved performance. Considering of [14] optimisation of γ_{SD} is therefore possible using the ADPPs and the objective function (32).

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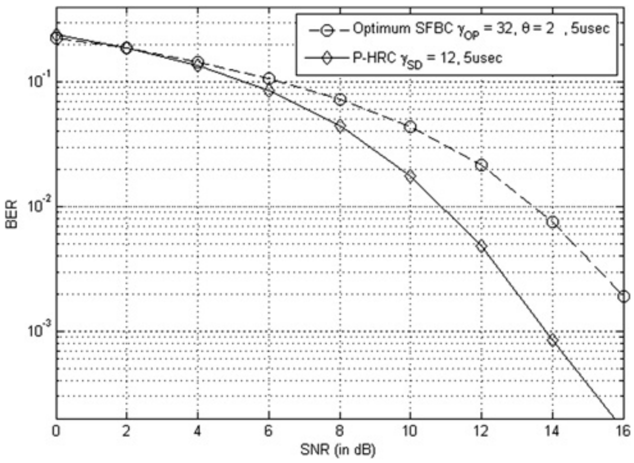

Fig. 1 *BER performance, N = 128, delay spread 5 µs, 16-QAM for optimum code in [12] and 4-QAM for P-HRC*

Presence of DPPs

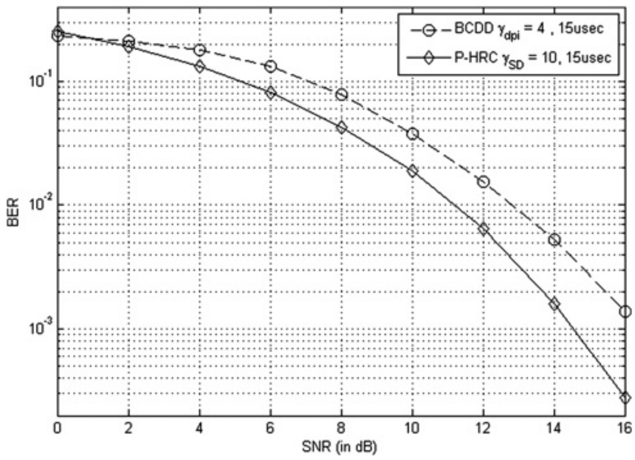

Fig. 2 *BER performance, N = 128, delay spread 15 µs, 16-QAM for the BCDD code and 4-QAM for the P-HRC*

Absence of DPPs

Let us explain what ADPPs are. An artificial delay profile is constructed based on the following equations

$$\tau_{A_\ell} = \begin{cases} 0, & \ell = 0 \\ \dfrac{\tau_{\max}}{d^{L-\ell}}, & \ell = 1, 2, \ldots, L-1 \end{cases} \tag{33}$$

where $d = 3$ for $2 \le L \le 8$, $d = 1.5$ for $L \ge 9$ and $\tau_{\max}$ is the length of cyclic prefix. Also artificial power profile is constructed based on the artificial delay profile as expressed below

$$\vartheta^2_{A_\ell} = \frac{e^{-0.26\tau_{A_\ell}}}{\sum_{k=0}^{L-1} e^{-0.26\tau_{A_k}}}, \quad \ell = 0, 1, \ldots, L-1 \tag{34}$$

And thus ADPPs refers to the combination of $\left\{\tau_{A_\ell}\right\}_{\ell=0}^{L-1}$ and $\left\{\vartheta^2_{A_\ell}\right\}_{\ell=0}^{L-1}$.

Now using ADPPs, we can construct an artificial channel covariance matrix and in this case optimisation of γ_{SD} is simply the same as when the DPPs are known at the transmitter.

So, we considered a MIMO–OFDM system with two transmit antennas, $N_C = 128$ subcarriers, BW = 1 MHz and a cyclic prefix of length 20 µs. Also, to obtain the same spectral efficiency, 16-QAM and 4-QAM were used for the BCDD and P-HRC, respectively. For a 2-ray equal power channel with a 15 µs delay spread, $\gamma_{dpi} = 4$ for the BCDD code. To calculate γ_{SD} for P-HRC, first, an artificial channel covariance matrix is produced using ADPPs and then the proper value of γ_{SD} can be easily calculated from (32). Fig. 2 shows the average bit-error-rate (BER) against the average signal-to-noise ratio of both the BCDD and the P-HRC in the case of absence of DPPs. It is worth mentioning that this comparison is fair since we used the same parameters of BCDD as in [12] and then discovered that they are the best possible parameters of this code for equal power, two ray, frequency-selective channel, with delay spread of 15 µs.

6 Conclusion and future works

In this paper, we presented a new criterion which enabled us to separate the CA of the SFBCs into two key parts namely, precoder-CA and channel-CA parts. The precoder-CA was indicative of the effect of the precoder, the signal constellation and the code structure. The channel-CA, represented the effects of the channel DPPs of the frequency-selective channels as well as the applied permutation. The proposed decomposition was derived from the lower bound of the CA that was formulated in this paper. The advantage of this approach was shown to be in its ability to reduce the complexity of optimising the SFBCs parameters. As an instance, the proposed decomposition was performed on a high-rate SFBC to optimise its channel-CA. Simulation results also verified the modified SFBC's superior performance over other popular SFBCs. Encouraged by the results so far, future work aims to build on this criterion by extending the innovative solution to space–time–frequency codes and exploring the possibility of designing new codes that are based on this scheme.

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8 Appendix 1

In this appendix, we show that the matrices $\hat{\boldsymbol{\Xi}}_{\text{QOSF}}$, $\hat{\boldsymbol{\Xi}}_{\text{BCDD}}$ and $\hat{\boldsymbol{\Xi}}_{[3]}$ introduced in Section 3 are PSD. For any matrix $\boldsymbol{A}$, where $\boldsymbol{A} = \boldsymbol{D}\boldsymbol{D}^{\text{H}}$, and $\boldsymbol{D}$ is an arbitrary matrix, $\boldsymbol{A}$ is a Hermitian PSD matrix. Thus, we can write $\hat{\boldsymbol{\Xi}} = \boldsymbol{D}\boldsymbol{D}^{\text{H}}$ for the mentioned codes and therefore, $\hat{\boldsymbol{\Xi}}$ has been proved to be a Hermitian positive semi-definite matrix.

For the BCDD codes with $M_t = 2$, we have: $\hat{\boldsymbol{\Xi}}_{\text{BCDD}} = \boldsymbol{1}_{L\times L} \otimes \boldsymbol{I}_{2\times 2}$. It is easy to see that by defining $\boldsymbol{D}_{\text{BCDD}} = (1/2)(\boldsymbol{1}_{L\times 1} \otimes \boldsymbol{I}_{2\times 2})$, we could write

$$\hat{\boldsymbol{\Xi}}_{\text{BCDD}} = \boldsymbol{D}_{\text{BCDD}}\boldsymbol{D}_{\text{BCDD}}^{\text{H}} \tag{35}$$

Proving that $\hat{\boldsymbol{\Xi}}_{\text{BCDD}}$ is PSD. Also for QOSFBC with $M_t = 2$, $\hat{\boldsymbol{\Xi}}_{\text{QOSFBC}}$ has the same structure as $\hat{\boldsymbol{\Xi}}_{\text{BCDD}}$ and therefore, it is PSD. For the proposed code in [3], $\hat{\boldsymbol{\Xi}}_{[3]} = \boldsymbol{I}_{M_T\times M_T} \otimes \boldsymbol{1}_{L\times L}$, $\hat{\boldsymbol{\Xi}}_{[3]} = \boldsymbol{D}_{[3]}\boldsymbol{D}_{[3]}^{\text{H}}$, $\boldsymbol{D}_{[3]} = \boldsymbol{I}_{M_T\times M_T} \otimes \boldsymbol{1}_{L\times 1}$. This implies that $\hat{\boldsymbol{\Xi}}_{[3]}$ is PSD.

9 Appendix 2

In this appendix, we show that $\prod_{i=1}^{2L}\tilde{\boldsymbol{\Delta}}_{\text{HRC}}(i,i)$ is non-zero. Using (19)–(25), we can expand this product as

$$\prod_{i=1}^{2L}\tilde{\boldsymbol{\Delta}}_{\text{HRC}}(i,i) = \prod_{i=1}^{2L}\left(\left(\boldsymbol{X}_i^1 - \hat{\boldsymbol{X}}_i^1\right)^2 + \left(\boldsymbol{X}_i^2 - \hat{\boldsymbol{X}}_i^2\right)^2\right) \tag{36}$$

Since the precoder of this code is a Vandermonde matrix [2], $\prod_{i=1}^{2L}\left(\boldsymbol{X}_i^1 - \hat{\boldsymbol{X}}_i^1\right)^2$ is non-zero and, similarly, so is $\prod_{i=1}^{2L}\left(\boldsymbol{X}_i^2 - \hat{\boldsymbol{X}}_i^2\right)^2$. Thus, $\left(\boldsymbol{X}_i^1 - \hat{\boldsymbol{X}}_i^1\right)^2$ and $\left(\boldsymbol{X}_i^2 - \hat{\boldsymbol{X}}_i^2\right)^2$ are non-zero for any i. Therefore it can be concluded that $\prod_{i=1}^{2L}\tilde{\boldsymbol{\Delta}}_{\text{HRC}}(i,i)$ is also non-zero.

10 Appendix 3

In this appendix, we explain how the CA of SFBC in [3] can be decomposed as product of two parts, and argue why this guideline is not applicable for other SFBCs.

In [3], at each subcarrier only one of the transmit antennas is sending a symbol, with the other antennas being switched off (zeros are placed in the codeword for other antennas). This more or less expresses the main concept behind the decomposition of the CA in [3]. Now let us explain why these zeros in the codeword are essential for the purpose of decomposing the CA.

In an L-ray channel with M_t transmit antennas, the sub-block G of SFBC in [3] has the following structure

$$\boldsymbol{G}_{[3]} = \text{diag}\left(\boldsymbol{X}_1, \boldsymbol{X}_2, \ldots, \boldsymbol{X}_{M_t}\right) \tag{37}$$

where $\boldsymbol{X}_i = [x_{(i-1)L+1}, x_{(i-1)L+2}, \ldots, x_{iL}]^{\text{T}}$, with $[x_{(i-1)L+1}, x_{(i-1)L+2}, \ldots, x_{iL}]^{\text{T}} = \boldsymbol{V}[s_{(i-1)L+1}, s_{(i-1)L+2}, \ldots, s_{iL}]$, where s_i is taken from the used constellation and $\boldsymbol{V}$ is the Vandermonde matrix with parameters presented in [3].

The matrix $\hat{\boldsymbol{\Delta}}_{[3]}$ could simply be written as (see (38))

$$\hat{\boldsymbol{\Delta}}_{[3]} = \begin{bmatrix}
\delta_1\bar{\delta}_1 & \delta_1\bar{\delta}_2 & \ldots & \delta_1\bar{\delta}_L & 0 & 0 & 0 & 0 & \ldots & 0 & \ldots & 0 & 0 \\
\delta_2\bar{\delta}_1 & \delta_2\bar{\delta}_2 & \ldots & \delta_2\bar{\delta}_L & 0 & 0 & 0 & 0 & \ldots & 0 & \ldots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & 0 & 0 & 0 & 0 & \ldots & 0 & \ddots & 0 & 0 \\
\delta_L\bar{\delta}_1 & \delta_L\bar{\delta}_2 & \ldots & \delta_L\bar{\delta}_L & 0 & 0 & 0 & 0 & \ldots & 0 & \ldots & 0 & 0 \\
0 & 0 & \ldots & 0 & \delta_{L+1}\overline{\delta_{L+1}} & \delta_{L+1}\overline{\delta_{L+2}} & \ldots & \delta_{L+1}\overline{\delta_{2L}} & \ldots & 0 & \ldots & 0 & 0 \\
0 & 0 & \ldots & 0 & \delta_{L+2}\overline{\delta_{L+1}} & \delta_{L+2}\overline{\delta_{L+2}} & \ldots & \delta_{L+2}\overline{\delta_{2L}} & \ldots & 0 & \ldots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \ldots & 0 & \ddots & 0 & 0 \\
0 & 0 & \ldots & 0 & \delta_{2L}\overline{\delta_{L+1}} & \delta_{2L}\overline{\delta_{L+2}} & \ldots & \delta_{2L}\overline{\delta_{2L}} & \ldots & 0 & \ldots & 0 & 0 \\
0 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & \ldots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & \ldots & 0 & 0 \\
0 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 & \ldots & \delta_{(M_t-1)L}\overline{\delta_{(M_t-1)L}} & \ldots & \delta_{(M_t-1)L}\overline{\delta_{(M_t-1)L}} & \delta_{(M_t-1)L}\overline{\delta_{(M_t-1)L}} \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 & \ldots & \delta_{M_tL-1}\overline{\delta_{(M_t-1)L}} & \ldots & \delta_{M_tL-1}\overline{\delta_{M_tL-1}} & \delta_{M_tL-1}\overline{\delta_{M_tL}} \\
0 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 & \ldots & \delta_{M_tL}\overline{\delta_{(M_t-1)L}} & \ldots & \delta_{M_tL}\overline{\delta_{M_tL-1}} & \delta_{M_tL}\overline{\delta_{M_tL}}
\end{bmatrix} \tag{38}$$

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where $\left[\delta_1, \delta_2, \ldots, \delta_{M_tL}\right]^{\mathrm{T}} = \boldsymbol{V}\left(\left[s_1, s_2, \ldots, s_{M_tL}\right]^{\mathrm{T}} - \left[\breve{s}_1, \breve{s}_2, \ldots, \breve{s}_{M_tL}\right]^{\mathrm{T}}\right)$, with $\left\{s_1, s_2, \ldots, s_{M_tL}\right\}$ and $\left\{\breve{s}_1, \breve{s}_2, \ldots, \breve{s}_{M_tL}\right\}$ being any two distinct strings of symbols ($\bar{\delta}$ indicates the complex conjugate of δ).

Here, we show that the penalty for losing the freedom of transmission via more than one antenna at each subcarrier is well worth it.

Using the n-linearity property of determinant function we can write the CA as

$$\det\left(\hat{\boldsymbol{\Delta}}_{[3]} \circ \hat{\boldsymbol{R}}\right) = \prod_{i=1}^{M_tL} \left|\delta_i\right|^2 * \det\left(\hat{\boldsymbol{\Delta}}_{\mathrm{res}[3]} \circ \hat{\boldsymbol{R}}\right) \tag{39}$$

The matrix $\hat{\boldsymbol{\Delta}}_{\mathrm{res}[3]}$ is a binary matrix with the following form

$$\hat{\boldsymbol{\Delta}}_{\mathrm{res}[3]} = \boldsymbol{I}_{M_T \times M_T} \otimes \mathbf{1}_{L \times L} \tag{40}$$

In [3], the normalised values of $\prod_{i=1}^{M_tL} \left|\delta_i\right|^2$ and $\det\left(\hat{\boldsymbol{\Delta}}_{\mathrm{res}[3]} \circ \hat{\boldsymbol{R}}\right)$ are defined as intrinsic and extrinsic diversity products, respectively.

This decomposition would be impossible to achieve if it was not for the zeros placed judiciously in the code structure. On the other hand, inserting zeros would result in the loss of the freedom to use all the available antennas at the transmitter. Given the inefficiency of the above scheme, we were motivated to exploit the idea and derive an improved criterion which could be applicable to more complex SFBCs.

It is worth mentioning that if the proposed method, that is, CADI is used for SFBC in [3], we have

$$\begin{aligned} \det\left(\hat{\boldsymbol{\Delta}}_{[3]} \circ \hat{\boldsymbol{R}}\right) &= \det\left(\hat{\boldsymbol{\Delta}}_{[3]} \circ \hat{\boldsymbol{\Delta}}_{\mathrm{res}[3]} \circ \hat{\boldsymbol{R}}\right) \\ &\geq \prod_{i=1}^{v} \hat{\boldsymbol{\Delta}}_{[3]} \det\left(\hat{\boldsymbol{\Delta}}_{\mathrm{res}[3]} \circ \hat{\boldsymbol{R}}\right) \end{aligned} \tag{41}$$