Optimisation of binary integrators for decentralised detection

Y. Norouzi, F. Gini and M.M. Nayebi

Abstract: Binary integrators are an important part of the receiver in many operating radar systems. The optimisation of a binary integrator is not a simple task, because it requires the solution of a $(k \times n)$-dimensional nonlinear optimisation problem, where n is the number of integrated bits (or the number of sensors in a distributed radar or sensor network) and k is the number of the design parameters of the single-pulse detector. An algorithm that converts the multi-dimensional optimisation problem into a one-dimensional problem, so reducing considerably the computational complexity, is developed. This reduction in computational complexity makes the real-time optimisation possible and practical, so it is very helpful for mobile sites in which the optimisation should be performed continually. The proposed algorithm can be applied when either the 'AND' or the 'OR' integration rule is adopted. The results are illustrated by means of two study cases. In the first case, the binary integrator of a constant false alarm rate radar detector is optimised; in the second one a decentralised detection system composed by n similar sensors is considered and the decision rules are jointly optimised according to the Neyman–Pearson criterion.

1 Introduction

In a single-sensor detection system, the decision about the presence or absence of a target in the cell under test (CUT) is often based on single-pulse processing. In many cases, the performance of this simple single-pulse detector is not satisfactory. To improve performance, radar systems commonly employ binary integration [1–3]. In this case, the receiver declares that a target is present in the CUT if there are at least k detections out of n successive single-pulse binary decisions; otherwise, the target is declared to be absent. Such a binary integration is quite common in pulse-Doppler airborne radars. In these systems, the output of Doppler processor (MTD) is binary integrated to improve the detection performance. '3 out of 8' and '4 out of 8' schemes are commonly used in such systems.

A k-out-n integration rule is quite often implemented in decentralised detection also [4–7]. In fact, in modern systems some type of diversity is used to obtain more reliability and survivability. A simple method to obtain this diversity is to use n cooperating receiving sensors in place of one. In an ideal system the signals of all these sensors are sent to a fusion centre, which decides about the presence or absence of the target in the CUT. In a real scenario these sensors may be placed far from each other and far from the fusion centre, and as a result of limited bandwidth and of some other practical issues, it may be better that each local sensor takes a preliminary decision and transmits only its binary decision to the fusion centre. The fusion centre takes the final decision about the presence of the target based on these n binary decisions, that is by processing these n bits [4]. Therefore again we encounter the concept of binary integration.

In many cases the performance of these sensors is not the same, even if they are of the same kind. For example these sensors may be radars of the same kind located at different geographical areas, as shown in Fig. 1. They collect samples of signals backscattered by the same target; the distance of the target is different for different radars and as a result the radars will receive echo signals having different signal-to-noise power ratios (SNRs). Moreover, different grazing angles may also cause different radars to observe different clutter statistics. According to the Neyman–Pearson criterion, the decision thresholds of different local sensors may be different and they should be jointly optimised to achieve the highest detection probability for a fixed probability of false alarm at the fusion centre [1].

We encounter a similar problem in a single-sensor radar system employing a binary integrator. For example if the sensor is surveillance radar with a mechanically rotating antenna, as the antenna rotates the echo pulses received during the time on target (ToT) will be modulated in amplitude according to the antenna beam pattern; therefore they will have different amplitudes, that is different SNRs.

In this work, we describe an efficient approach to the global optimisation of the detection thresholds (The thresholds could be different from one another to take into account the different SNR values for each pulse. Even if the sensors are all equal, sometimes better performance can be achieved by selecting different thresholds.). These findings are derived under the assumption that the binary integrator adopts the 'AND' or the 'OR' fusion rule.

Such a complexity-reduced optimisation algorithm is very helpful for mobile sites. In this site, the environment

doi:10.1049/iet-rsn:20060087

Paper first received 24th June 2006 and in revised form 18th June 2007

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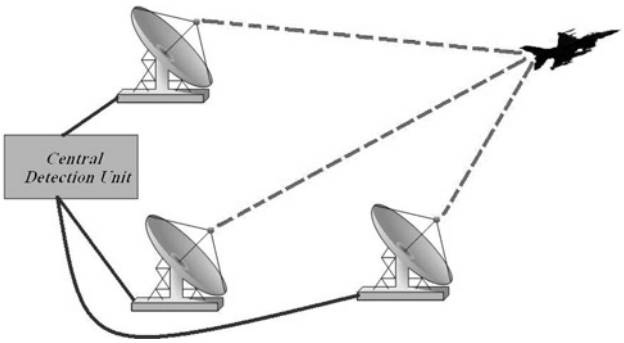

Fig. 1 *Decentralised detection system (here all the radars are the same, but because of different distance from target, their detection performance is different)*

surrounding the sensors changes continually. So the optimisation of the detector should be carried in an almost real-time manner. Such an optimisation is possible only if the algorithms are not complicated.

Previous works: Binary integration was first introduced in 1950s and 1960s [2, 3, 8]. At that time the most important feature of this method was its simplicity. Further investigations showed that binary integration is also robust to heavy-tailed noise [9]. A very similar optimisation problem shows up in the field of decentralised detection. Tenney and Sandell investigated the optimum Bayes detector for the case of two sensors [4]. Sadjadi extended this work to the case of n sensors and M hypotheses [10]. Chair and Varshney developed the optimum structure for the central detector, under the assumption of known local detectors' structure [11]. In all the above-mentioned works, it was assumed that the signals received by the sensors are mutually independent. The more complicated case of correlated signals is discussed by Drakapoulos and Lee in [12], Lauer and Sandell in [13] and by Lin *et al.* [14]. Kazakos *et al.* found some bound on the error probability of decentralised detectors [15]. Liu *et al.* developed some methods based on a genetic algorithm to find the optimum solution for designation of a decentralised detector [16]. Sometimes it is possible for each sensor to send more than one bit for each observation; the problem of optimum quantisation has been addressed by Duman and Salehi in [17].

In most of these works it has been assumed that the communication link between sensors and central unit is ideal, but in a real scenario we have to take into account the non-ideality of communication channels. The problem of limited bandwidth of the communication channel in a decentralised system has been addressed by Chamberland and Veeravalli [18] and by Gini *et al.* [19]. Appadwedula *et al.* investigated the case where the sensors are powered by some batteries, therefore the total energy is constrained [20].

In the field of radar decentralised detection, Thomopoulos *et al.* [21] showed that in order to achieve optimal detection performance the local detectors and the fusion centre should be jointly optimised under the Neyman–Pearson criteria. Barkat and Varshney [22, 23], Longo and Lops [24] and Nguyen *et al.* [5] analysed some decentralised constant false alarm rate (CFAR) detectors. Gini *et al.* [6, 7, 25] developed some methods to find the optimum solution for decentralised detection systems; their method reduces a $(k \times n)$-dimensional optimisation problem to some n-dimensional optimisation problem, so reducing considerably the complexity of problem.

We should mention that in addition to radar systems, the binary integration is of the same importance in sensor network communication systems. Lots of papers are also published in this field. Some new contributions in this field can be found in [26, 27].

Consider a decentralised detection system that is composed of three sensors characterised by the receiver operating characteristics (ROC) shown in Fig. 2. At the fusion centre, a final decision is taken based on the AND fusion rule. Assuming that the probability of false alarm (P_{fa}) at the fusion centre is equal to 5×10^{-2}, the achievable probability of detection (P_{d}) as a function of the local P_{fa} of the first and second sensors is shown in Fig. 3 (the local P_{fa} of third sensor is deterministically obtained by these two since the total P_{fa} is fixed.) The surface in Fig. 3 has three local maxima, but only one of them is the global maximum. If we want to find this optimal operating point by means of some gradient-type method, it may be trapped in one of the local maxima. To avoid such an event we should first use a sufficiently fine grid search, to find some point near the global optimal point and then we can use a gradient-type method. Obviously, the amount of operations needed for such a search increases exponentially with the number n of sensors. This makes the optimisation process very time-consuming, even for a moderately high number of sensors.

In this paper, we show that for the special cases of binary integrator employing an 'AND' or 'OR' fusion rule the multi-dimensional optimisation problem can be converted to a 1D problem. The proposed algorithm can be used to optimise the binary integrator of a CFAR detection system, that is to find the optimal number of bits (the window size of the binary integrator) and the optimal local threshold values. Besides, the algorithm can also be used for the multi-radar scenario with similar sensors as well as any other type of sensor networks.

Some authors have proved that even if the sensors are similar, the optimum solution can be achieved by selecting different threshold values [28, 29]. We will show when it is better to use different thresholds and when the optimal solution is achieved by using equal local thresholds.

The rest of this paper is organised as follows. In Section 2 we introduce the assumptions and the notation that are used throughout the paper. In Section 3 we derive some useful equations for the AND fusion rule. Then, we develop an algorithm to find the optimal operating points. In Section 4 we consider the OR fusion rule. The case of $(n-1)$-out-of-n fusion rule is treated in Section 5. We show that in this case the problem cannot be reverted to a 1D one, as in the case of the AND and the OR fusion rules. However, in this case we

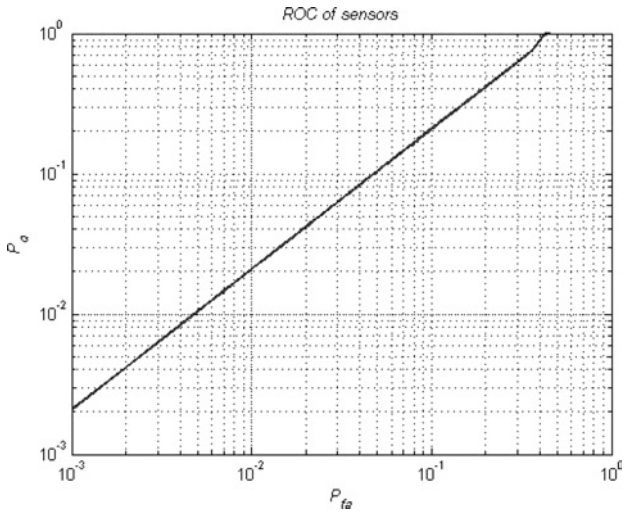

Fig. 2 *ROC of the single detectors which are used in Fig. 3*

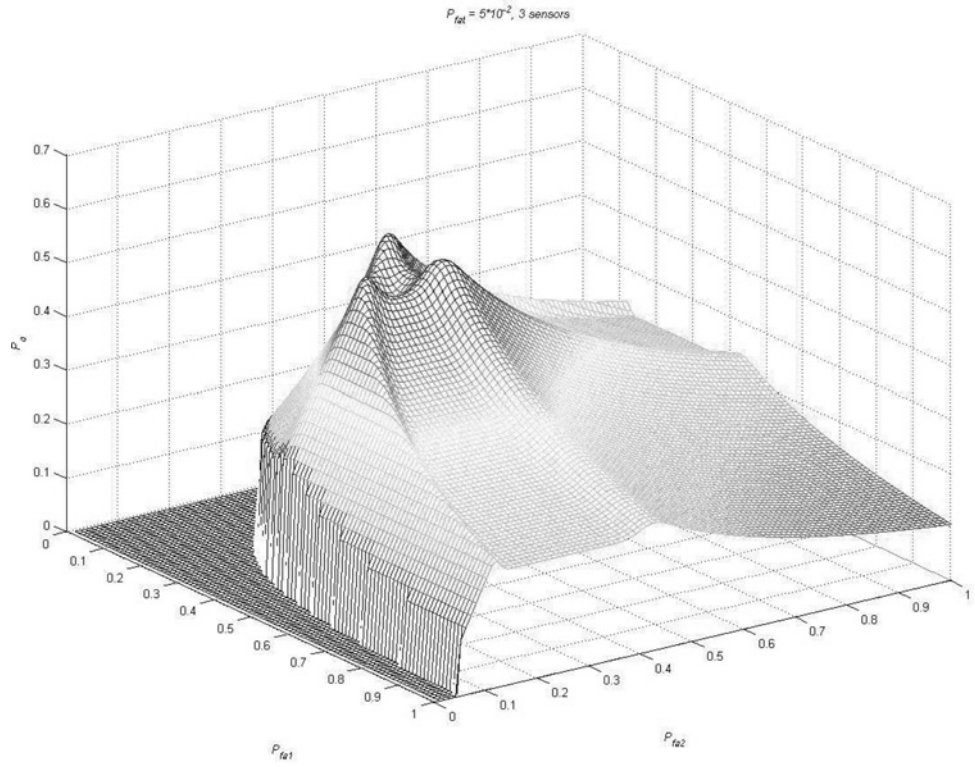

Fig. 3 *Total P_d of a distributed system composed by three sensors; AND binary integration at the fusion centre, total $P_{fa} = 5 \times 10^{-2}$*

can reformulate the problem as a 3D optimisation problem. In Section 6 we use the proposed method to optimise the well-known log-*t* CFAR detector. In Section 7 we consider the case of a decentralised sensor system composed by similar sensors. Finally, in Section 8, some conclusions and some hints for future investigation are reported.

2 Basic assumptions

The definitions that are commonly used in decentralised detection are now introduced and their relations with binary integration are described. The probability of detection of *i*th sensor is denoted by $P_d(i)$ and its probability of false alarm by $P_{fa}(i)$. When the specification of the local detection rule requires one to fix only the detection threshold, for each fixed SNR we have a one-to-one relationship between $P_d(i)$ and $P_{fa}(i)$. In some cases, specification of the local detection rule requires to fix k multiple parameters. For example in an ordered statistics (OS) CFAR detector, it is needed to fix the rank of the selected sample and the detection threshold. In this case, for every $P_{fa}(i)$ value, there exist several sets of thresholds and therefore several values of $P_d(i)$, and it is not possible to define uniquely $P_d(i)$ as a function of $P_{fa}(i)$. However, in [6] (see also [25]), it is shown that in order to optimise detection performance in the Neyman–Pearson sense, for every value of $P_{fa}(i)$ we should select the $k-1$ additional parameters (additional with respect to the local detection threshold) in order to achieve the highest value of $P_d(i)$. The 'optimised' $P_d(i)$ is unique for every value of the local detection threshold, so it is unique for every value of $P_{fa}(i)$. Therefore it is possible to state the following relationships

$$\begin{aligned} &P_{fa}(i) = x_i, \quad 0 \le x_i \le 1, \quad i = 1, 2, \ldots, n \\ &P_d(i) = f_i(x_i) \end{aligned} \tag{1}$$

where $f_i(\cdot)$ represents the relation between the 'optimised' $P_d(i)$ and the $P_{fa}(i)$, that is the 'optimised' ROC curve of the *i*th sensor. When the sensors are similar, the $f_i(\cdot)$ functions are all the same. However, as mentioned before, even in the case of similar sensors the best performance may be achieved by choosing different local detection thresholds, that is different x_is.

In the case of a single-sensor system employing a binary integrator, $P_d(i)$ and $P_{fa}(i)$ represent, respectively, the detection and false-alarm probabilities for the *i*th pulse in the stream of n successive pulses. Again, if the SNRs are different for different pulses, then the $f_i(\cdot)$ are different; otherwise they are all the same.

3 AND decision rule

In some scenarios, the detector should declare the presence of a target only if in all single observations the target is detected. For example assume that in a cognitive radio network the sensors want to decide whether a frequency band is free or it is used by a primary licensed user. In such a scenario, it is reasonable to declare a frequency band as a free band, whenever all the sensors have declared it as a free band. In this example if we define the declaration of a free band as detection, then the final detection should be announced if and only if all sensors have declared detection. Such a method is the AND decision rule. Under the assumption that the local decisions are conditionally independent, the probability of false alarm at the fusion centre (P_{fa}) is given by

$$P_{fa} = \prod_{i=1}^{n} P_{fa}(i) \tag{2}$$

Using the notation of (1), we have

$$P_{\text{fa}} = \prod_{i=1}^{n} x_i$$
$$P_{\text{d}} = \prod_{i=1}^{n} f_i(x_i) \tag{3}$$

where P_{d} is the probability of detection at the fusion centre. We should select the x_{i}s to achieve a P_{fa} equal to a given value, say α, and at the same time to obtain the highest possible value of P_{d}. A well-known method to carry out this optimisation is the Lagrange multipliers method [6]. It requires calculation of the derivatives of the following functional Q with respect to each x_{i} as well as λ

$$Q(x_1, x_2, \ldots, x_n, \lambda) = \prod_{i=1}^{n} f_i(x_i) - \lambda\left(\prod_{i=1}^{n} x_i - \alpha\right) \tag{4}$$

Then, these derivatives are set equal to zero. Carrying out these derivations, we obtain the following set of equations that must be jointly solved

$$f_k'(x_k)\prod_{\substack{i=1\\ i\neq k}}^{n} f_i(x_i) - \lambda\prod_{\substack{i=1\\ i\neq k}}^{n} x_i = 0, \quad k = 1, 2, \ldots, n$$
$$\prod_{i=1}^{n} x_i - \alpha = 0 \tag{5}$$

where $f_i'(\cdot)$ is the derivative of $f_i(\cdot)$. The cost function in (5) can be simplified to as follows

$$\frac{f_k'(x_k)}{f_k(x_k)}\prod_{i=1}^{n} f_i(x_i) - \frac{\lambda}{x_k}\prod_{i=1}^{n} x_i = 0, \quad k = 1, 2, \ldots, n \tag{6}$$

and then it can be expressed as

$$\frac{x_k f_k'(x_k)}{f_k(x_k)} = \frac{\lambda\prod_{i=1}^{n} x_i}{\prod_{i=1}^{n} f_i(x_i)} = \frac{\lambda P_{\text{fa}}}{P_{\text{d}}}, \quad k = 1, 2, \ldots, n \tag{7}$$

The right part of above equation is the same for all n equations. Therefore by setting $L_k(x_k) = (x_k f_k'(x_k))/f_k(x_k)$ and $w = (\lambda P_{\text{fa}})/P_{\text{d}}$, we obtain the following set of equations

$$L_1(x_1) = L_2(x_2) = \cdots = L_n(x_n) = w \tag{8}$$

If we denote by $\Gamma_k(\cdot)$ the inverse function of $L_k(\cdot)$, the local probabilities of false alarm are given by $x_k = \Gamma_k(w)$; once the optimal x_k has been found, the local thresholds are then obtained from the local ROC. The optimisation problem is basically reduced to the problem of finding w as the solution of the following equation

$$g(w) = \prod_{k=1}^{n} \Gamma_k(w) - \alpha = 0 \tag{9}$$

Unfortunately, quite often $\Gamma_k(\cdot)$ cannot be expressed in closed form. In such a case we should solve (9) numerically, for example by means of the Newton's method (Newton's method is one of the most widely used methods to find numerically the solution of an equation in the form $g(w) = 0$). It uses a local linear approximation of the function g and it finds the solution by solving the following recursive equation [30]

$$w^{(k+1)} = w^{(k)} - \frac{g(w^{(k)})}{g'(w^{(k)})} \tag{10}$$

where $g'(w)$ denotes the first derivative of $g(w)$. Newton's method can be used to solve (9) through the following steps:

1. An initial estimation for the value of w is needed; to obtain this we can assume that the sensors have the same probability of false alarm, that is

$$x_i = \sqrt[n]{\alpha}, \quad i = 1, 2, \ldots, n \tag{11}$$

Then we assume that the true value of x_{i} is between $1/10$ and 10 times the above calculated value. For all these values we calculate the corresponding w using (8); we select the initial value of w as the minimum or maximum of these values; this initial value is named $w^{(0)}$.
2. For any $w^{(k)}$, the following equations should be solved with respect to x_{i}, using some numerical method

$$L_i(x_i) = w^{(k)}, \quad i = 1, 2, \ldots, n \tag{12}$$

3. The value of $w^{(k+1)}$ is calculated from $w^{(k)}$ and x_{i}s of (7) by solving the following equation

$$w^{(k+1)} = w^{(k)} - \beta\frac{1 - \alpha(\prod_{i=1}^{n} x_i)^{-1}}{\sum_{i=1}^{n}(x_i(\mathrm{d}L_i(x_i)/\mathrm{d}x_i))^{-1}} \tag{13}$$

This equation is derived from (10) by calculating the derivative of $g(w)$ which is equal to

$$g'(w) = \prod_{i=1}^{n}\Gamma_i(w)\cdot\sum_{i=1}^{n}\frac{\mathrm{d}\Gamma_i(w)}{\mathrm{d}w} = \prod_{i=1}^{n}\Gamma_i(w)\cdot\sum_{i=1}^{n}\left(x_i\frac{\mathrm{d}L_i(x_i)}{\mathrm{d}x_i}\right)^{-1} \tag{14}$$

Now if we simplify (10) we obtain (13).
Parameter β should be selected between zero and one. In Newton's method it is equal to one, but using this value sometimes we do not have convergence; therefore it may be necessary to select a value of β lower than one.
4. Knowing the next value of w (i.e. $w^{(k+1)}$), the next estimations of x_{i}s should be calculated by solving (12). The next value of P_{fa} is calculated inserting these x_{i}s in (3). Now if this value of P_{fa} is close enough to the desired value, then the algorithm terminates; otherwise it returns to Step 3.

In each step in the algorithm, two main tasks should be performed. First, we calculate the solution of (12); therefore the amount of operations needed by this part increases linearly with the number of sensors. Secondly, (13) should be calculated in each step. The amount of calculations needed by this equation is linearly dependent on the number of sensors. Therefore the amount of operations required to implement the above algorithm is a linear function of the number n of sensors, whereas in all other methods, such as the one suggested in [6], the number of operations increases exponentially with n. Another distinguishing fact of the method proposed here is that it converts the information about the optimum point from n-dimensional space to a 1D space. We will use this important fact in Sections 6 and 7. In some cases it is not possible to find a closed equation for the ROC of the detector. In this case one can fit the experimental ROC curve of the detector (e.g. obtained by Monte Carlo simulation) by a closed-form equation and then use the above algorithm for the optimisation.

4 OR decision rule

The OR detector accepts the existence of a target if at least one sensor declares the existence of a target. P_{fa} and P_d are related as follows

$$P_{fa} = 1 - \prod_{i=1}^{n} (1 - x_i) \tag{15}$$

$$P_d = 1 - \prod_{i=1}^{n} (1 - f_i(x_i)) \tag{16}$$

As in the previous section, x_i and $f_i(x_i)$ denote the P_{fa} and P_d of ith local sensor, that is $P_{fa}(i)$ and $P_d(i)$. In this case, optimisation via the Lagrange method produces the following functional Q to be maximised

$$Q(x_1, x_2, \ldots, x_n, \lambda) = 1 - \prod_{i=1}^{n} (1 - f_i(x_i)) - \lambda\left(1 - \prod_{i=1}^{n} (1 - x_i) - \alpha\right) \tag{17}$$

After the usual derivations, similar to the AND detector case, we come up with the following equations to be solved

$$\frac{(1 - x_1) f_1'(x_1)}{(1 - f_1(x_1))} = \frac{(1 - x_2) f_2'(x_2)}{(1 - f_2(x_2))} = \cdots = \frac{(1 - x_n) f_n'(x_n)}{(1 - f_n(x_n))} = w \tag{18}$$

that is $L_1(x_1) = L_2(x_2) = \cdots = L_n(x_n) = w$, where we define

$$L_k(x_k) = \frac{(1 - x_k) f_k'(x_k)}{(1 - f_k(x_k))} \tag{19}$$

These equations are quite similar to that of the AND detector. Again, by defining $\Gamma_k(\cdot)$ as the inverse of $L_k(\cdot)$, we come up with the following equation to be solved with respect to w

$$G(w) = 1 - \prod_{i=1}^{n} (1 - \Gamma_k(w)) - \alpha = 0 \tag{20}$$

Therefore the steps for finding the optimum operating point (i.e. the optimal thresholds) for the OR detector are similar to those for the AND detector. If we use Newton's method, it is necessary to solve following recursive equation

$$w^{(k+1)} = w^{(k)} + \beta \frac{1 - (1 - \alpha)/\prod (1 - x_i)}{\sum_{i=1}^{n} ((1 - x_i)(\mathrm{d}L_i(x_i)/\mathrm{d}x_i))^{-1}} \tag{21}$$

Equation (21) is derived similarly to (13). Following the steps of the algorithm described for the AND detector, just replacing the equations of the AND detector with those of OR detector, we finally come up with the solution.

5 '$(n-1)$-out-of-n' detector

The issue now is whether or not we can find similar simplifying equations for the general 'k-out-of-n' binary integration rule. Unfortunately, we found that this is not possible. However, it is possible to reduce the dimension of the optimisation problem in the case of '$(n-1)$-out-of-n' rule. According to this rule, a detection is declared either if all sensors detect the signal or when one does not detect but all the other $n-1$ sensors declare a detection. Since these two events are mutually exclusive, the probability of detection by '$(n-1)$-out-of-n' rule is the sum of these two probabilities

$$P_d = \prod_{i=1}^{n} P_d(i) + \sum_{i=1}^{n} (1 - P_d(i)) \prod_{\substack{j=1 \\ j \neq i}}^{n} P_d(j) = \left(1 - n + \sum_{i=1}^{n} \frac{1}{P_d(i)}\right) \prod_{i=1}^{n} P_d(i) \tag{22}$$

The equation for P_{fa} is obtained by replacing $P_d(i)$ with $P_{fa}(i)$ in (22). Again we can use Lagrange multipliers method to find the optimum solution minimising the following functional

$$Q(x_1, x_2, \ldots, x_n, \lambda) = \left(1 - n + \sum_{i=1}^{n} \frac{1}{f_i(x_i)}\right) \prod_{i=1}^{n} f_i(x_i) - \lambda\left(\left(1 - n + \sum_{i=1}^{n} \frac{1}{x_i}\right) \prod_{i=1}^{n} x_i - \alpha\right) \tag{23}$$

The derivative of Q with respect to x_k is

$$\frac{\partial Q}{\partial x_k} = -\frac{f_k'(x_k)}{f_k^2(x_k)} \prod_{i=1}^{n} f_i(x_i) + \left(1 - n + \sum_{i=1}^{n} \frac{1}{f_i(x_i)}\right) \frac{f_k'(x_k)}{f_k(x_k)} \prod_{i=1}^{n} f_i(x_i) - \lambda\left(-\frac{1}{x_k^2} \prod_{i=1}^{n} x_i + \left(1 - n + \sum_{i=1}^{n} \frac{1}{x_i}\right) \frac{1}{x_k} \prod_{i=1}^{n} x_i\right) \tag{24}$$

Equating the derivative to zero, we obtain the following equations

$$\frac{x_k f_k'(x_k)}{f_k(x_k)} \cdot \frac{(1 - n + \sum_{i=1}^{n} (1/f_i(x_i)) - (1/f_k(x_k)))}{(1 - n + \sum_{i=1}^{n} (1/x_i) - (1/x_k))} = \frac{\lambda \prod_{i=1}^{n} x_i}{\prod_{i=1}^{n} f_i(x_i)}, \quad k = 1, 2, \ldots, n \tag{25}$$

In the above equation three terms are in common for all values of k; defining these three terms as follows

$$w = \frac{\lambda \prod_{i=1}^{n} x_i}{\prod_{i=1}^{n} f_i(x_i)}, \quad \psi = 1 - n + \sum_{i=1}^{n} \frac{1}{f_i(x_i)}$$

$$\xi = 1 + n - \sum_{i=1}^{n} \frac{1}{x_i} \tag{26}$$

we can simplify (25) as

$$\frac{x_k^2 f_k'(x_k)}{f_k^2(x_k)} \cdot \frac{(f_k(x_k) - 1/\psi)}{(x_k - 1/\xi)} = \frac{w\xi}{\psi}, \quad k = 1, 2, \ldots, n \tag{27}$$

To find the solution for above equation, we should first begin with an initial guess of x_is. Then, w, ψ and ξ can be calculated using (26). The next values of x_is are calculated using (27). Doing these steps iteratively, we can find the optimum values of x_is. Of course, the solution is not as easy as that of the AND or the OR rule.

For any other 'k-out-of-n' integration rule, we obtain a similar equation which is not possible to be changed to a function with single variable; therefore it is not possible

to obtain the same complexity reduction as in the AND and OR cases. Yet, solving the optimisation problem in the 3D space is simpler than solving the problem in the original n-dimensional space.

6 Application to CFAR detection optimisation

In many radar systems, several pulses are received that have been backscattered from target. The amplitude of these echoes change in accordance to the radar antenna beam pattern. The samples of the received signal are then sent to a CFAR detector and its outputs, which are binary digits, are processed by a binary integrator. If we name the reference samples of the CFAR processor 'v_i' and the CUT data 'u', then many CFAR processors decide about the presence or absence of the target in the CUT by implementing the following rule (e.g. [31])

$$u \underset{H_1}{\overset{H_1}{\gtrless}} \gamma \cdot A(v_1, v_2, \ldots, v_n) + B(v_1, v_2, \ldots, v_n) \tag{28}$$

Here, A and B are some statistics calculated from reference samples, and γ is a constant set in order to provide the desired value of P_{fa}. In conventional CFAR detectors the same value of γ is used in successive pulses, but as was previously mentioned it may be better to use different values for different successive pulses to obtain the highest P_{d}, while still maintaining the desired P_{fa}. Now we show how the results of the previous sections can be applied to the optimisation of the CFAR detector.

We want to optimise a system composed by the cascade of a CFAR detector and a binary integrator which adopts the AND rule. The CFAR algorithm is of log-t type, that can maintain a constant P_{fa} in heavy-tailed clutter, such as Weibull, Lognormal or K-distributed clutter [31]. The maximum SCR of the target is 7 dB, but due to the amplitude modulation impressed by antenna beam pattern, the echo of this target is received in $n = 13$ successive pulses whose SCR values are for example 1, 2, . . . , 6, 7, 6, . . . , and 1 dB. This situation, for example, can represent a radar system whose antenna 3 dB beamwidth is equal to 1°, its pulse repetition frequency is 350 Hz and its rotation speed is 8.3 rpm. The underlying clutter is assumed to be lognormal with parameter $\sigma = 0.5$. The simulated log-t CFAR has 16 reference cells, the binary integrator adopts the AND rule. The detection performance of this system is reported in Fig. 4. The solid line represents the overall performance of detector when the same threshold is used for all samples. The dashed line represents the performance achieved after optimisation. It is almost obvious that the difference between the optimised and non-optimised detectors may be considerable.

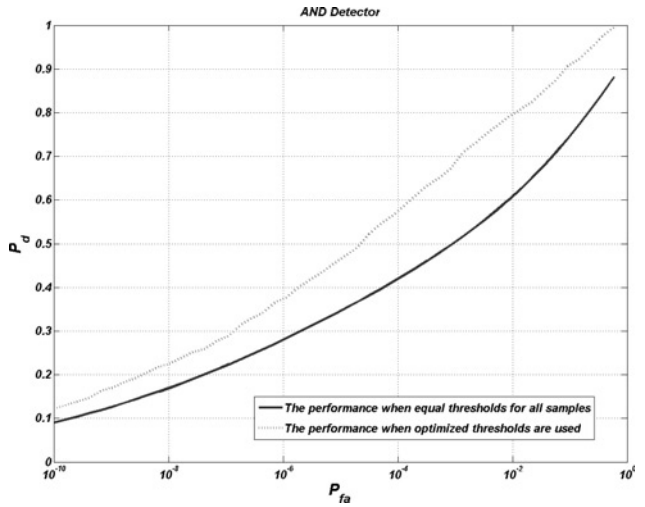

Fig. 4 *Detection performance of the system composed by a log-t CFAR detector and a binary integrator*

The target echo is received in several successive pulses, but in the first and last pulses the power of the received echo is low, and it may be better not to use these samples because when we increase the number of samples in the binary integrator both the P_{fa} and P_{d} will change. In order to maintain the previous P_{fa} we should change the threshold, but this change in threshold may reduce the P_{d}; this is especially true if the newly added sample has a low SCR. Therefore there exist an optimum number of bits used which should be used for binary integration.

This optimum number can be calculated easily by the optimisation method mentioned in Section 3. Referring to Fig. 4, if for example we require the system to operate with $P_{\text{fa}} = 10^{-3}$ and $P_{\text{d}} = 0.7$, we obtain that the 'local' P_{fa} relative to the samples with SCR less than 3 dB is almost equal to 1, which means these samples should not be used to take the final decision. The reason for this behaviour can be understood from Fig. 5; here the function $L(\cdot)$ is plotted for some values of SCR. As we see the minimum value of this function for each sensor is different from others, therefore when the line of constant w goes down, after this minimum (the minimum of the L-function), it has not any intersection with some curves; it means that we cannot find any point in which the gradient of the total detection probability function (P_{dt}) is equal to zero and also this point satisfies the constraint on total P_{fa}. Referring to calculus, in this case the optimum solution is located on the boundaries that is $P_{\text{fa}} = 1$. Note that if the P_{fa} of a sensor is equal to 1, it means that this sensor should not be used in the decision process. Using this method we can find the optimum number of processed samples.

7 Distributed detection with similar sensors

As it is mentioned in [28, 29], even if all the sensors in a distributed system are equal, the optimum solution may be achieved via different local decision thresholds, that is different values of $P_{\text{fa}}(i)$. We first consider the case of the

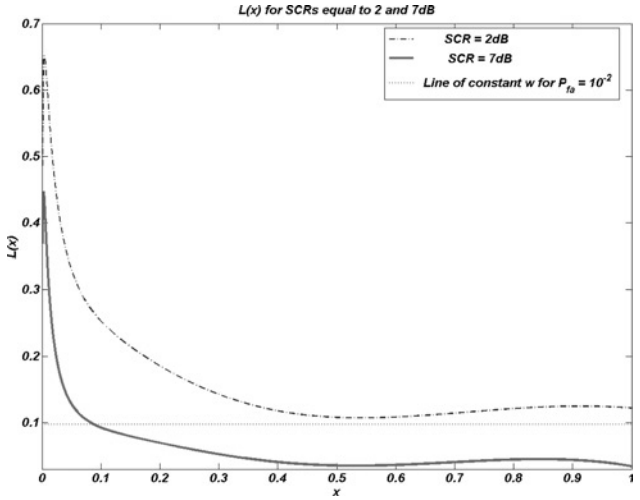

Fig. 5 *L(x) for AND detector and SCR = 2 and 7 dB (the line of constant w has not any intersection with the graph of SCR = 2 dB, which means that for this detector the optimum point is on the boundary)*

AND detector and we will show when we should select the same $P_{fa}(i)$s and when we should search for different $P_{fa}(i)$s. As it was shown in Section 3, the optimum operating point is obtained by jointly solving (8) and (9). This approach can be used both for similar sensors and for non-similar sensors.

Fig. 6 shows $f(x)$ as a function of x for a cell-averaged CFAR (CA-CFAR); the target signal and the noise are assumed exponentially distributed with different powers. In this figure the SNR is set equal to 10 dB and the number n of reference cells is 10. The $L(\cdot)$ function of this detector for the AND integration strategy is reported in Fig. 7; it is a one-to-one function, which proves that the optimum point for a combination of n sensors of this type will be achieved through selection of the same $P_{fa}(i)$s because otherwise the $L(\cdot)$ function of different sensors will be obtained for different values, which is in contradiction with optimality regarding Lagrange method [30]. The only other possible optimal point is to select the $P_{fa}(i)$ of some sensors on the border, that is 0 or 1, and select the same value for other sensors. Since we are considering the AND detector, it is not possible to select $P_{fa}(i)$ of any sensor equal to zero (Otherwise the final results will be equal to zero all the time.). Therefore the only possibility is to select some $P_{fa}(i)$s equal to 1, which means that we do not use the data from these sensors in the decision process.

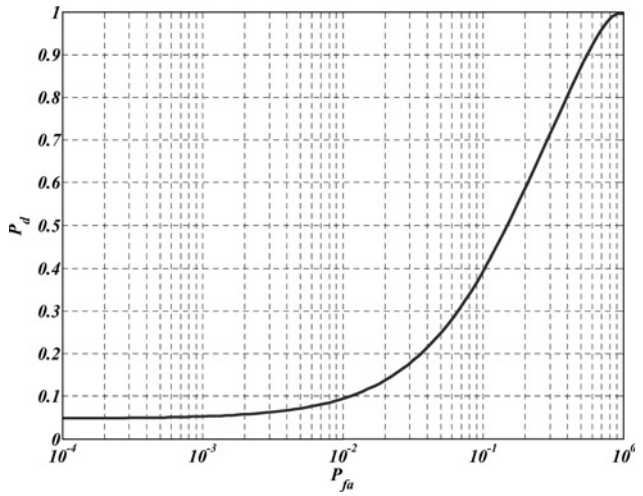

Fig. 6 *ROC of a CA-CFAR with 10 reference cells (noise and signal samples are exponentially distributed)*

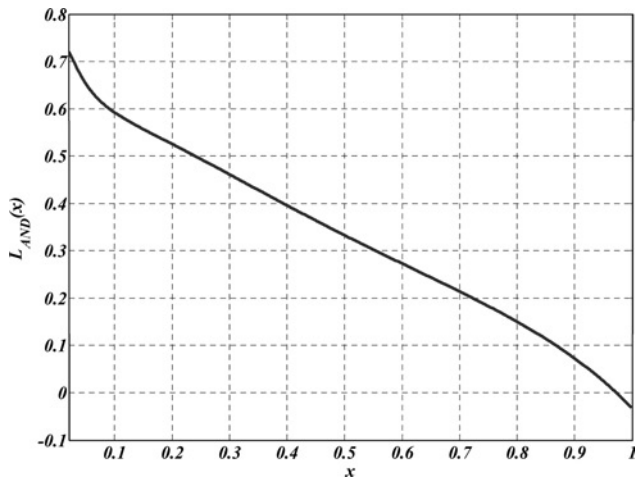

Fig. 7 *L-function of the ROC shown in Fig. 6*

So we can state that: for the AND binary integration rule with similar sensors, if the $L(\cdot)$ function is one-to-one, the optimum solution is to select equal $P_{fa}(i)$s for some sensors and not to use the others.

Sometimes $L(\cdot)$ is not one-to-one. Consider the following ROC

$$f(x)=\begin{cases}1, & a+d<x\le 1\\ x^{-b/d(d+a)}\mathrm{e}^{b/d(x-1)}, & a<x\le a+d\\ a^{-(a/d+2b)}x^{b-a((b-1)/d)} & a-d<x\le a\\ \quad\times\mathrm{e}^{1/d(2ab-(b+1)x-b)}, & \\ a^{-(a/d+2b)}(a-d)^{b-1-a((b-1)/d)} & 0\le x\le a-d\\ \quad\times\mathrm{e}^{1/d(ab-a-b)}x, & \end{cases} \tag{29}$$

Here a is some value between 0 and 1, b is greater than 1 and d is a positive value less than a and $1-a$. This function is shown in Fig. 8 for a, b and d equal to 0.5, 5 and 0.05, respectively. $L(\cdot)$ has the following simple form

$$L(x)=\begin{cases}0, & a+d<x\le 1\\ -\dfrac{b}{d}(x-a)+b, & a<x\le a+d\\ \dfrac{b-1}{d}(x-a)+b, & a-d<x\le a\\ 1, & 0\le x\le a-d\end{cases} \tag{30}$$

This function is shown in Fig. 9. For some values of w, the equation $L(x)=w$ has multiple solutions; therefore it is possible to find the optimum point where the $P_{fa}(i)$s are not the equal. For values of w between 1 and b, the equation $L(x)=w$ has two solutions which are

$$x_1=\frac{d}{b}(b-w)+a,\quad x_2=\frac{d}{b-1}(w-b)+a \tag{31}$$

Therefore if there are n of these sensors, the optimum solution is to select the $P_{fa}(i)$s of k of them equal to x_1 and $n-k$ others equal to x_2. In this case it is not easy to find a closed solution for optimum value of k. All possible values should be examined and the optimum value should be selected.

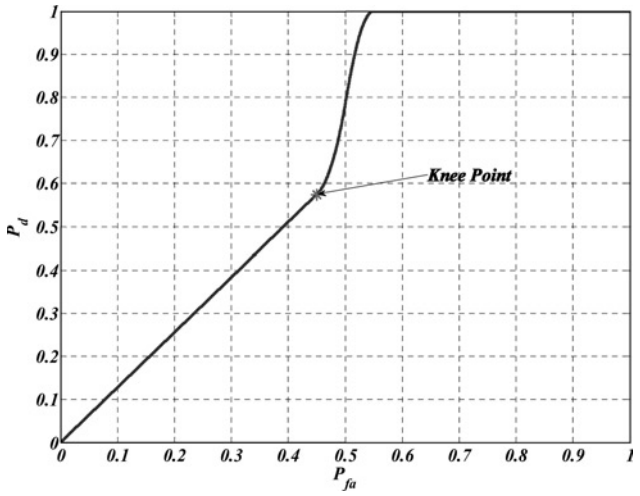

Fig. 8 *ROC for the function of (29) for $a=0.5$, $b=5$ and $d=0.05$*

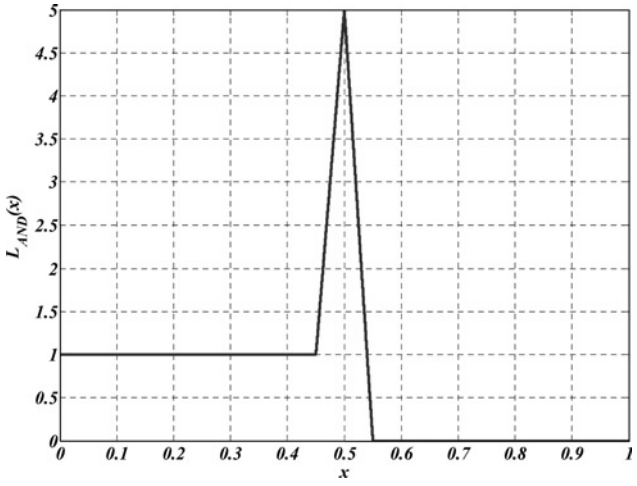

Fig. 9 *L-function of (30) for $a = 0.5$, $b = 5$ and $d = 0.05$*

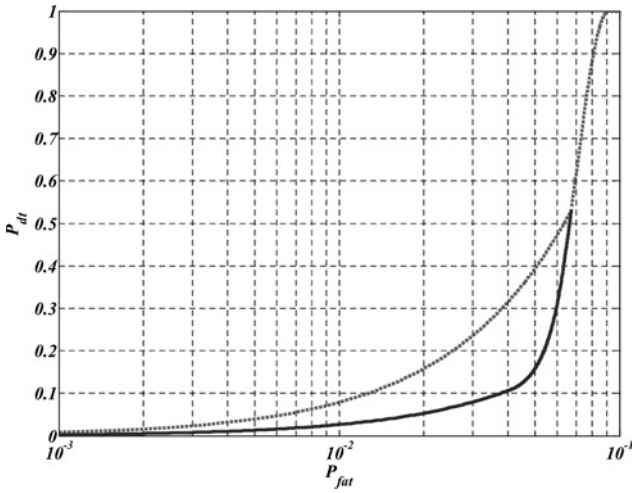

Fig. 10 *P_d as a function of P_{fa} for AND combination of four similar sensors whose ROC is shown in Fig. 8 (solid line represents the case of the same $P_{fa}(i)$s and dashed line shows the optimal case)*

In Fig. 10, P_{d} is shown as a function of P_{fa}; the number of identical sensors is $n = 4$; the ROC of each sensor is plotted in Fig. 8. In Fig. 10 the solid line shows the case where the $P_{\mathrm{fa}}(i)$s of all sensors identical; the dashed line shows the case where they are optimised without the constraint of being all the same.

Now consider the general case of 'k-out-of-n' detectors with n similar sensors. If again we refer to Figs. 8 and 9, in regions in which $L(x)$ is not one-to-one, $f(x)$ has an abrupt increase. For the points near this zone (e.g. the knee point in Fig. 8), if we slightly increase x, P_{d} will increase significantly; but if we slightly decrease x, P_{d} will decrease only a little. Therefore if by selecting the same $P_{\mathrm{fa}}(i)$s for all sensors, the local P_{fa} occurs near this knee point, then we can increase the $P_{\mathrm{fa}}(i)$s of some sensors, while decreasing those of the others in order to maintain the predefined value of P_{fa} while increasing significantly the global P_{d}. Therefore roughly speaking, for all 'k-out-of-n' detectors with n similar sensors we can say that: if the local ROCs have a region of abrupt increase then there is a possibility that the optimum solution is achieved by selecting different $P_{\mathrm{fa}}(i)$s.

8 Conclusions

The subject of this paper is the optimal design of the binary integration rule within a (possibly distributed) radar detection system or generally in a sensor network. We derived an efficient method that converts the classical n-dimensional optimisation problem into a 1D problem, so reducing considerably the computational complexity. Such a complexity reduction is quite beneficial for mobile sites, because the new algorithm lets the detectors to optimise their operation continually whenever their condition is changed.

The proposed algorithm can be applied when either the 'AND' or the 'OR' binary integration rule is implemented in the detector. The results have been illustrated by means of two study cases: (1) the optimisation of the binary integration rule in a CFAR detector; (2) the optimisation of the local decision rules in a distributed sensor system employing the 'AND' or the 'OR' fusion rule at the fusion centre. Additionally, we investigated when and which a collection of n similar sensors in AND and OR architecture should be treated differently. We failed to find similar powerful complexity reduction for the general 'k-out-of-n' binary integration rule. Therefore this subject remains for further investigation. Besides, through the paper we used the assumption of independent sensors which is sometimes violated; this case could be the subject of further research.

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