Quantized Ergodic Radio Resource Allocation in Cognitive Femto Networks with Controlled Collision and Power Outage Probabilities

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***Abstract*—A robust Ergodic Resource Allocation (ERA) scheme is proposed in this paper in the framework of an orthogonal frequency division multiple access (OFDMA) based underlay heterogeneous network in which the allocations are made so as to maximize the average network sum-rate while guaranteeing the macro network interference requirements with any desired high probability. In previously proposed ERA schemes, both in conventional and heterogeneous networks, the optimal solution is obtained assuming that average of constraints are satisfied. In a heterogeneous network, this is translated into the fact that instantaneous level of interference on macro users can not be guaranteed, i.e., there is an uncontrolled probability of collision which is not acceptable by macro network. In this paper, we reformulate the ERA problem by replacing the average based constraints, in our case, femto total power constraint and macro interference threshold constraint, with their probabilistic counterparts so that both constraints are satisfied instantaneously with any desired high probability. We consider both cases of continuous and quantized channel state information. The proposed problems are then solved based on three methods, namely, iterative, analytical, and hybrid approaches. The optimality of the proposed methods is also assessed and the iterative approach is shown to have a performance quite close to that of the optimal solution. Simulation results confirm the effectiveness of the proposed scheme to provide a robust instantaneous imposed interference on macro network and a robust femto total transmit power. We also investigate the convergence properties of the proposed iterative approach.**

***Index Terms*—Ergodic Resource Allocation, Cognitive Femto Networks, Collision Probability, Orthogonal Frequency Division Multiple Access (OFDMA), Channel Distribution Information (CDI).**

I. Introduction

EMBEDDING femto networks in cellular mobile systems has emerged as an appealing way to significantly improve the limited coverage of macro networks and to increase system capacity [1], [2], and [3]. If a femto network is allowed to simultaneously use the same spectrum as that of the marco network, it is referred to as a cognitive femto network [4] and [5]. The name is inspired by the idea of cognitive radio networks where spectrum sharing, as a promising access strategy for efficient use of the available spectrum is considered. In spectrum sharing, unlicensed (secondary) services are allowed to opportunistically access the under-utilized parts of the licensed spectrum [6], [7]. Underlay, overlay, and interweave spectrum sharing are different access strategies for the secondary network to access the primary spectrum [8].

Among different access strategies in spectrum sharing systems, we focus on underlay method similar to [9] where femto transmitter's access to the macro network spectrum is allowed provided that the interference inflicted to the macro network is kept below a predefined *interference threshold* [10], [11]. Dissatisfaction of this condition is referred to as a collision incident. In this paper, we focus on orthogonal division multiple access (OFDMA) based networks.

Generally, resource allocation (RA) plays a key role in implementing efficient multiuser communications over dynamic wireless environments (see, e.g., [12], [13], [14], [15], [16], [17], [18], [19], [20] and references therein). Two different types of such problems are in general considered in the literature, namely, instantaneous resource allocation (IRA) and ergodic resource allocation (ERA). In IRA, the instantaneous value of the utility function is optimized subject to constraints that have to also be satisfied instantaneously. To do so, for any new set of channel state information (CSI) values, the problem should be solved which, in a dynamic wireless environment, imposes a high computational complexity and excessive signaling overhead to the system.

On the other hand in ERA, the average of the utility function is optimized subject to constraints that should also be satisfied in average sense. ERA is accomplished in two phases, off-line phase which is carried before the communication starts where several parameters that are later used to allocate resources are computed based on long term channel conditions or more explicitly, the channel distribution information (CDI); and an online phase that takes place during communication where the transmitter uses the parameters computed during the off-line phase and the CSI to allocate resources. Consequently, since in ERA, allocations are made based on CDI which varies much less infrequently than CSI, ERA incurs much less computational complexity than IRA. This is very appealing form practical point of view.

Such appealing benefits of ERA come at a cost: as the short term channel fluctuations are not considered, the constraints can be only guaranteed in the long term average sense. In other words, there is a non-negligible probability that the constraints are not met instantaneously. Such a situation is observed in majority of prior works on ERA, in conventional [16], [21], [22], [23], [24], [25], [26], cognitive [27], [28], [29], and femto networks [9], which may not be acceptable from practical

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point of view. In particular, in case of spectrum sharing systems, i.e., cognitive and cognitive femto networks, this is translated into facing uncontrolled probability of collision. As an example, assume that the channel gain between femto base station (BS) and macro receiver on a ceratin subcarrier has a symmetric distribution. Satisfying the average-based interference threshold constraint implies a collision probability as high as one half which may not be acceptable by the primary service side. It is important to note that to avoid such a situation in cognitive networks, some works such as [30], [31], and [32], maximize the average sum-rate while using an instantaneous interference threshold constraint instead of an average based constraint. Although these works may be categorized into ergodic resource allocation, the corresponding problem should be solved for any new set of CSI values similar to an IRA problem. In other words, they do not bear anymore the main advantage of ERA over IRA problems, i.e., reduced computational complexity.

To deal with this important drawback of ERA, we propose a new scheme in which we replace the average based constraints with their probabilistic counterparts. In particular for the case of cognitive femto networks, the interference threshold constraint is replaced by the collision probability constraint. Consequently, the excessive interference caused by femto network on macro users can be avoided with any desired high probability. In other words, in contrast to the original problem, we gain control on the collision probability. Note that despite this modification, the allocations are still made based on CDI and not instantaneous CSI values.

Other average based constraints can also be treated the same way. For example, to meet the short term power limitations of femto base station, the average total power constraint is also replaced by the total power outage probability constraint. Therefore, our formulation can guarantee the short-term macro and femto network limitations with any desired probability. This provides a kind of robustness to the system. Therefore, we refer to the corresponding ERA problems as robust resource allocation (RERA).[1] Although in this paper we concentrate on a downlink of an OFDMA-based femto network, the proposed concept is valid for any other ERA framework.

The proposed RERA problem can not be directly solved due to the inclusion of the probabilistic constraints. To find the optimal solution, the problem is transformed into a convex problem and solved based on the dual decomposition method [15]. We also propose an iterative approach (IA) to solve the proposed problem without transformation. Simulation results verify that the solution obtained based on the IA approach is very close to the optimal solution. A less complex sub-optimal method, referred to as analytical approach (AA) is also proposed in which the probabilistic constraints are replaced by non-probabilistic sufficient constraints so that the problem can be readily solved based on the dual method. A combination of AA and IA, called the hybrid approach (HA), is also investigated which provides a trade-off between complexity and performance with respect to AA and IA.

[1]Note that robust resource allocation also refers to those problems where imperfect CSI is dealt with to provide a robust value of sum rate (or any other utility function), see for example [33], [34] and [35].

It is important to note that to reduce the system overhead signaling used to feedback CSI values between users and base station, channel quantization schemes have been proposed. In such schemes, also referred to as limited feed back systems, the CSI is quantized into a limited number of fading regions and instead of CSI values, the index of the fading region corresponding to that value is transmitted (see, e.g., [25], [27], [17], [23], [28], [30] and references therein). In this paper, we also consider the channel quantization and extend our analysis to this case by solving the quantized version of the proposed RERA problem abbreviated as QRERA.

The rest of the paper is organized as follows. In the next section, we describe the system model. In Section III, we propose the original ERA problem and replace the interference threshold constraint by the collision probability constraint and find the optimal solution. In Section IV, we further modify the ERA problem by substituting the total power constraint by the power outage probability constraint. Section V provides a similar analysis for the case QRERA and Section VI concentrates on complexity analysis of the proposed algorithms. In Section VII simulation results are presented and Section VII concludes the paper.

II. System Model

A. Network architecture

We consider a system model of a two-tier macro and femto networks. The considered networks consist of one central macro and F randomly distributed femto networks. The set of indices for femto BS's are denoted by $f \in \mathcal{F} = \{1, \dots, F\}$. The macro BS is indicated by 0 index. Thus, we index all BS's by $f \in \mathcal{F}_0 = \{0, 1, \dots, F\}$. The set of users at each fetmo network f is denoted by $\mathcal{K}_f = \{1, 2, \dots, K_f\}$ where K_f is the total number of users at femto network f. Our focus is mainly on the downlink of each femto network based on OFDMA technology.

The system bandwidth is B Hz which is licensed to the macro network. The spectrum is divided into N subcarriers each with $B_c = B/N$ Hz bandwidth. Subcarriers are indexed by n, where $n = 1, \dots, N$. It is further assumed that B_c is much smaller than that of the coherent bandwidth of the wireless channel. Therefore, the subcarriers undergo a flat fading. The femto networks do not have license to access the spectrum, however, opportunistic access is allowed utilizing dynamic spectrum access protocols. From the concept of cognitive networks, the femto networks play the role of secondary networks and macro network is considered as the primary network. Spectrum sharing is then assumed between macro and femto networks. On the other hand, no spectrum is shared between femto networks.

The set of allocated subcarriers to the femto network f is indicated by $\mathcal{N}_f$ where its cardinality is N_f. For the k_fth femto user, the quality of the associated channel with subcarrier n is modelled through $\alpha_{k_f,n}$, the channel to interference plus noise ratio (CINR) of subcarrier n, which is defined as:

$$\alpha_{k_f n} = \frac{T_{k_f n}}{N_0 B/N + V_{k_f n}}, \tag{1}$$

where $T_{k_f n}$ is the channel power gain between femto base station f and femto user k_f on subcarrier n. The received interference due to macro network activity at front-end of the k^fth femto user on subcarrier n is assumed to be a Gaussian variable whose variance is denoted by $V_{k_f n}$ [36], [37], [38], and [39].[2]

Throughout the paper, the channel state information within the femto network is abbreviated by F-CSI. The state information of the channel between femto transmitter and macro receiver, referred to as the interference channel, is abbreviated by I-CSI. The corresponding channel distribution information is denoted by I-CDI.

B. Statistical macro QoS provisioning

Throughout the paper, we refer to the interference requirements of macro network as the macro Quality of Servis (QoS) requirements. To meet the short-term macro QoS, we modify the average interference threshold constraint in conventional ERA problems to a one in which the macro QoS requirement is satisfied on the fast timescale with high probability although occasional outage may occur. To this end, the QoS of the k_0th macro user which transmits on subcarrier n is denoted by a pair of parameters, $(Q_{k_0 n}, \zeta^I_{k_0 n})$, where $Q_{k_0 n}$ is the interference threshold, and $\zeta^I_{k_0 n}$ is the maximum tolerable collision probability. For the k_0th macro user on subcarrier n, collision is experienced if the imposed interference by the each femto BS f on the k_0th macro user, $I^f_{k_0 n}(\boldsymbol{\alpha}^f_n, g^f_{k_0 n})$, is higher than $Q_{k_0 n}$. The collision probability constraint $\mathcal{CO}^f_{k_0 n}$ is then written as [41] and [42]

$$\mathcal{CO}^f_{k_0 n} = \Pr\left\{ I^f_{k_0 n}(\boldsymbol{\alpha}^f_n, g^f_{k_0 n}) > Q_{k_0 n} \right\} \leq \zeta^I_{k_0 n}, \quad (2)$$

where $g^f_{k_0 n}$ is the channel power gain between the k_0th macro user and femto BS f on subcarrier n and $K_f \times 1$ vector $\boldsymbol{\alpha}^f_n$ is the secondary power gain on subcarrier n across different femto users. The imposed interference, $I^f_{k_0 n}(\boldsymbol{\alpha}^f_n, g^f_{k_0 n})$, can be obtained as follows:

$$I^f_{k_0 n}(\boldsymbol{\alpha}^f_n, g^f_{k_0 n}) = \sum_{k_f \in \mathcal{K}_f} \rho_{k_f n} p_{k_f n}(\alpha_{k_f n}) g^f_{k_0 n}, \quad (3)$$

where $\rho_{k_f n} \in \{0,1\}$ is the assignment factor indicating whether subcarrier n is assigned to the k_f^{th} femto user and $p_{k_f n}$ is the allocated power for the k_f^{th} femto user on subcarrier n.

C. Statistical femto network limitations

Similar to the last subsection, the femto average total transmit power constraint in conventional ERA problems has to be modified to a one in which the instantaneous total transmit power constraint is below a desired threshold with arbitrarily low probability. Therefore, the transmission power constraint is considered by a pair of parameters, (P^T_f, ζ^p_f), where P^T_f is the maximum allowable transmission power, and ζ^p_f is the maximum tolerable power outage probability of each femto base station f. The power outage is experienced if the total allocated power for femto users by the femto BS f, $\sum_{k \in \mathcal{K}_f} \sum_{n \in \mathcal{N}_f} p_{k_f n}(\alpha_{k_f n}) \rho_{k_f n}$, is larger than P^T_f. The power outage probability constraint, $\mathcal{PO}_f$, is then defined as

$$\mathcal{PO}_f = \Pr\left\{ \sum_{k_f \in \mathcal{K}_f} \sum_{n \in \mathcal{N}_f} p_{k_f n}(\alpha_{k_f n}) \rho_{k_f n} > P^T_f \right\} \leq \zeta^p_f. \quad (4)$$

III. Partially Robust Ergodic Resource Allocation (PRERA)

In this section, we first start by reviewing the conventional ERA problem in which the two constraints (macro QoS and femto power constraints) are satisfied only in average sense. The ultimate goal of this paper is to alleviate the aforementioned drawback of the conventional ERA problem where both average interference threshold as well as total power constraints are replaced by their probabilistic counterparts. The resulting modified problem is called Fully Robust Ergodic Resource Allocation (FRERA). We have not been able to find the optimal solution for FRERA and therefore in Section IV, we propose an iterative approach (IA) to solve it. To be able to assess the optimality of the IA method, in this section we consider a simpler case where we replace the interference threshold constraint by the collision probability constraint but leave the other average-based constraint intact to form a new problem called Partially Robust Ergodic Resource Allocation (PRERA) problem. We then obtain the optimal solution of the proposed PRERA problem followed by a solution based on IA. Simulation results show the optimality of the proposed IA method. We then conjecture that the FRERA problem solution obtained based on IA should not also be far away from optimal. We also propose a less complex sub-optimal method called the analytical approach.

A. ERA and its solution

The allocated rate for femto user k_f on subcarrier n can be obtained as follows:

$$r_{k_f n} = \log(1 + \frac{p_{k_f n} \alpha_{k_f n}}{\Gamma_{k_f n}}), \quad (5)$$

where $\Gamma_{k_f n}$ is the SNR gap which is obtained for a given target bit error rate, $\text{BER}_{k_f n}$, and modulation type. Based on [43], Γ_{kn} can be written as

$$\Gamma_{k_f n} = c_1 \ln\left(\frac{c_2}{\text{BER}_{k_f n}}\right), \quad (6)$$

where c_1 and c_2 are constants which are related to the type of modulation and noise margins [37].

The RA problem is then defined as follows with the objective of maximizing the average sum rate subject to the average interference constraint as well as average total transmission power and subcarrier assignment constraints:

[2] We assume that the number of macro users is large. Since the transmitted signals from macro users can be considered to be independent, the resulting interference can be considered as a Gaussian random variable based on the central limit theorem. Now given the distribution is known by the femto service, the parameters of this distribution can be easily estimated based on the received signals from macro network using parametric density estimation methods [40].

Problem $\mathcal{O}^{ERA}$:

$$\max_{\boldsymbol{p}^f,\boldsymbol{\rho}^f} \quad \mathbf{E}_{\boldsymbol{\alpha}^f}\mathbf{E}_{\boldsymbol{g}^f}\left\{\sum_{n\in\mathcal{N}_f}\sum_{k_f\in\mathcal{K}_f}\rho_{k_fn}r_{k_fn}\right\}, \tag{7}$$

$$\text{s.t.} \quad \mathbf{E}_{\boldsymbol{\alpha}^f}\mathbf{E}_{\boldsymbol{g}^f}\left\{\sum_{n\in\mathcal{N}_f}\sum_{k_f\in\mathcal{K}_f}\rho_{k_fn}p_{k_fn}\right\}\leq P_f^T, \tag{8}$$

$$\mathbf{E}_{\boldsymbol{\alpha}^f}\mathbf{E}_{\boldsymbol{g}^f}\left\{\sum_{k_f\in\mathcal{K}_f}\rho_{k_fn}p_{k_fn}g^f_{k_0^*n}\right\}\leq Q_{k_0^*n},\forall n\in\mathcal{N}_f, \tag{9}$$

$$\sum_{k_f\in\mathcal{K}_f}\rho_{k_fn}=1,\forall n\in\mathcal{N}_f,\rho_{k_fn}\in\{0,1\},\ \forall k_f,n\in\mathcal{N}_f, \tag{10}$$

where (8) represents the average total transmission power on all fading realizations across all allocated subcarriers and femto users. Moreover, (9) denotes the the average imposed interference constraint on each subcarrier across all macro users and $k_0^*=\text{argmin}_{k_0\in\mathcal{K}_0}Q_{k_0n}$ [3]. Equation (10) guaranties that each subcarrier n is only assigned to one femto user.

This problem is non-convex due to constraint (10). In order to transform the problem into a convex problem, a sharing factor $\rho_{k_fn}\in[0,1]$ is introduced indicating the portion of time that subcarrier n is assigned to femto user k_f during each transmission frame. This time-sharing technique was first proposed in [12] and has been frequently used in the context of subcarrier assignment in OFDMA systems to convert a mixed integer programming problem into a convex optimization problem [44], [45]. In addition, we introduce a variable $s_{k_fn}=\rho_{k_fn}p_{k_fn}$ for all k_f and n. Clearly, s_{k_fn} becomes the actual amount of power allocated to user k_f on subcarrier n, whereas p_{k_fn} is the power as if subcarrier n is occupied by femto user k_f only. If $\rho_{k_fn}=0$, we always have $s_{k_fn}=0$ but p_{k_fn} is not necessarily equal to zero. Thus, the allocated rate for femto user k_f on subcarrier n is reformulated as follows

$$r_{k_fn}=\log(1+\frac{s_{k_fn}\alpha_{k_fn}}{\rho_{k_fn}\Gamma_{k_fn}}). \tag{11}$$

The original problem $\mathcal{O}^{ERA}$ is then transformed into:
Problem $\mathcal{O}^{ERA-convex}$:

$$\max_{\boldsymbol{s}^f,\boldsymbol{\rho}^f} \quad \mathbf{E}_{\boldsymbol{\alpha}^f}\mathbf{E}_{\boldsymbol{g}^f}\left\{\sum_{n\in\mathcal{N}_f}\sum_{k_f\in\mathcal{K}_f}\rho_{k_fn}r_{k_fn}\right\}, \tag{12}$$

$$\text{s.t.} \quad \mathbf{E}_{\boldsymbol{\alpha}^f}\mathbf{E}_{\boldsymbol{g}^f}\left\{\sum_{n\in\mathcal{N}_f}\sum_{k_f\in\mathcal{K}_f}s_{k_fn}\right\}\leq P_f^T, \tag{13}$$

$$\mathbf{E}_{\boldsymbol{\alpha}^f}\mathbf{E}_{\boldsymbol{g}^f}\left\{\sum_{k_f\in\mathcal{K}_f}s_{k_fn}g_{k_0^*n}\right\}\leq Q_{k_0^*n},\forall n\in\mathcal{N}_f, \tag{14}$$

$$\sum_{k_f\in\mathcal{K}_f}\rho_{k_fn}=1,\forall n\in\mathcal{N}_f,\rho_{k_fn}\in[0,1],\forall n\in\mathcal{N}_f,k_f\in\mathcal{K}_f. \tag{15}$$

This problem is a convex optimization problem and there exists a unique optimal solution, which can be obtained by dual decomposition method. The optimal power allocation for femto user k_f on subcarrier n is given by the following water-filling equation

$$p^*_{k_fn}=\frac{s^*_{k_fn}}{\rho_{k_fn}}=\left[\frac{1}{(\lambda+\mu_ng_{k_0^*n})\ln 2}-\frac{\Gamma_{k_fn}}{\alpha_{k_fn}}\right]^+, \tag{16}$$

where λ and $\mu_n,\ \forall n\in\mathcal{N}_f$ are lagrangian multipliers and $[x]^+\overset{\Delta}{=}\max\{x,0\}$. Subcarrier n is allocated to the femto user k_f i.e., $\rho_{k_fn}=1$ and $\rho_{k'n}=0,\forall k'\neq k$ if $\nu_n\leq H_{k_fn}$ otherwise $\rho_{k_fn}=0$ where $\nu_n,\ \forall n\in\mathcal{N}_f$ are Lagrangian multipliers related to subcarrier allocation constraints and $H_{k_fn}=\log(1+\frac{s_{k_fn}\alpha_{k_fn}}{\Gamma_{k_fn}})-\frac{s_{k_fn}\alpha_{k_fn}}{\ln 2(s_{k_fn}\alpha_{k_fn}+\Gamma_{k_fn})}$.

It has been shown in [16] that for ergodic OFDMA based problems, with probability very close to one, the optimal solution for $\mathcal{O}^{ERA-Convex}$ is also optimal for $\mathcal{O}^{ERA}$.

B. PRERA and its solution

Now we substitute the average interference threshold constraint by the collision probability constraint. The optimization problem is then reformulated as
Problem $\mathcal{O}^{PRERA}$:

$$\max_{\boldsymbol{p}^f,\boldsymbol{\rho}^f} \quad \mathbf{E}_{\boldsymbol{\alpha}^f}\left\{\sum_{n\in\mathcal{N}_f}\sum_{k_f\in\mathcal{K}_f}\rho_{k_fn}r_{k_fn}\right\}, \tag{17}$$

$$\text{s.t.} \quad \mathbf{E}_{\boldsymbol{\alpha}^f}\left\{\sum_{n\in\mathcal{N}_f}\sum_{k_f\in\mathcal{K}_f}\rho_{k_fn}p_{k_fn}\right\}\leq P_f^T, \tag{18}$$

$$\Pr\left\{\sum_{k_f\in\mathcal{K}_f}\rho_{k_fn}p_{k_fn}g_{k_0^*n}>Q_{k_0^*n}\right\}\leq\zeta^I_{k_0^*n},\forall n\in\mathcal{N}_f, \tag{19}$$

$$\sum_{k_f\in\mathcal{K}_f}\rho_{k_fn}=1,\forall n\in\mathcal{N}_f,\rho_{k_fn}\in\{0,1\},\forall k_f,n\in\mathcal{N}_f. \tag{20}$$

[3]The original constraint (9) is in fact $\mathbf{E}_{\boldsymbol{\alpha}}\mathbf{E}_{\boldsymbol{g}}\left\{\sum_{k_f\in\mathcal{K}_f}\rho_{kn}p_{kn}g_{k_0n}\right\}\leq Q_{k_0n},\forall n\in\mathcal{N}_f,k_0\in\mathcal{K}_0$ which necessitates to obtain N_fK_0+1 Lagrange multipliers. This imposes a high computational complexity. To reduce this complexity and make the problem more tractable, a new set of parameters $Q_{k_0^*n}$, $n\in\mathcal{N}_f$ are considered where k_0^* for each n is defined as $k_0^*=\text{argmin}_{k_0\in\mathcal{K}_0}Q_{k_0n}$. This results in a modified version of constraint (9) in which we only deal with $N+1$ Lagrange multipliers.

1) Optimal Approach (OA): The collision probability constraint, (19), can be simplified to

$$\Pr\left\{\sum_{k_f\in\mathcal{K}_f}\rho_{k_fn}p_{k_fn}g_{k_0^*n} > Q_{k_0^*n}\right\} \tag{21}$$

$$= \Pr\left\{g_{k_0^*n} > \frac{Q_{k_0^*n}}{\sum\limits_{k_f\in\mathcal{K}_f}\rho_{k_fn}p_{k_fn}}\right\} \tag{22}$$

$$= 1 - F_{g_{k_0^*n}}\left(\frac{Q_{k_0^*n}}{\sum\limits_{k_f\in\mathcal{K}_f}\rho_{k_fn}p_{k_fn}}\right) \le \zeta^I_{k_0^*n}. \tag{23}$$

Since the probability distribution function is monotonically increasing, for a given subcarrier n and macro user k_0^*, (21) can be substituted with

$$\sum_{k_f\in\mathcal{K}_f}\rho_{k_fn}p_{k_fn} \le \frac{Q_{k_0^*n}}{F^{-1}_{g_{k_0^*n}}\left(1-\zeta^I_{k_0^*n}\right)}. \tag{24}$$

Inequality (24) suggests a threshold on the allocated power to subcarrier n across all the femto users as a function of the collision probability constraint. Consequently, the original problem can be written as

Problem $\tilde{\mathcal{O}}^{PRERA-OA}$:

$$\max_{\boldsymbol{p}^f,\boldsymbol{\rho}^f} \quad \mathbf{E}_{\boldsymbol{\alpha}^f}\left\{\sum_{n\in\mathcal{N}_f}\sum_{k_f\in\mathcal{K}_f}\rho_{k_fn}r_{k_fn}\right\}, \tag{25}$$

$$\text{s.t.} \quad \mathbf{E}_{\boldsymbol{\alpha}^f}\left\{\sum_{n\in\mathcal{N}_f}\sum_{k_f\in\mathcal{K}_f}\rho_{k_fn}p_{k_fn}\right\} \le P_f^T, \tag{26}$$

$$\sum_{k_f\in\mathcal{K}_f}\rho_{k_fn}p_{k_fn} \le \frac{Q_{k_0^*n}}{F^{-1}_{g^f_{k_0^*n}}\left(1-\zeta^I_{k_0^*n}\right)}, \forall n\in\mathcal{N}_f, \tag{27}$$

$$\sum_{k_f\in\mathcal{K}_f}\rho_{k_fn} = 1, \forall n\in\mathcal{N}_f, \rho_{k_fn}\in\{0,1\}, \forall k_f, n\in\mathcal{N}_f. \tag{28}$$

Although we were able to transform the probabilistic constraint into a non-probabilistic one, the problem is still not convex. Similar to $\mathcal{O}^{ERA}$, we relax the subcarrier allocation indicator and introduce a variable $s_{k_fn} = \rho_{k_fn}p_{k_fn}$ for all k_f and n. Thus, the allocated rate for femto user k_f on subcarrier n is similar to (11). Finally, $\tilde{\mathcal{O}}^{PRERA-OA}$ is transformed into the following convex problem:

Problem $\hat{\mathcal{O}}^{PRERA-OA}$:

$$\max_{\boldsymbol{s}^f,\boldsymbol{\rho}^f} \quad \mathbf{E}_{\boldsymbol{\alpha}^f}\left\{\sum_{n\in\mathcal{N}_f}\sum_{k_f\in\mathcal{K}_f}\rho_{k_fn}r_{k_fn}\right\}, \tag{29}$$

$$\text{s.t.} \quad \mathbf{E}_{\boldsymbol{\alpha}^f}\left\{\sum_{n\in\mathcal{N}_f}\sum_{k_f\in\mathcal{K}_f}s_{k_fn}\right\} \le P_f^T, \tag{30}$$

$$\sum_{k_f\in\mathcal{K}_f}s_{k_fn} \le \frac{Q_{k_0^*n}}{F^{-1}_{g_{k_0^*n}}\left(1-\zeta^I_{k_0^*n}\right)}, \forall n\in\mathcal{N}_f, \tag{31}$$

$$\sum_{k_f\in\mathcal{K}_f}\rho_{k_fn} = 1, \forall n\in\mathcal{N}_f, \rho_{k_fn}\in[0,1], \quad \forall k, n\in\mathcal{N}_f. \tag{32}$$

The unique optimal solution of this problem can be obtained by the dual decomposition method. The optimal power allocation for femto user k_f on subcarrier n is given by the following water-filling equation

$$p^*_{k_fn} = \frac{s^*_{k_fn}}{\rho_{k_fn}} = \left[\frac{1}{(\lambda+\mu_n)\ln 2} - \frac{\Gamma_{k_fn}}{\alpha_{k_fn}}\right]^+, \tag{33}$$

Subcarrier n is allocated to the femto user k_f i.e., $\rho_{k_fn} = 1$ and $\rho_{k'_fn} = 0, \forall k'_f \neq k_f$ if $\nu_n \le H_{k_fn}$ otherwise $\rho_{k_fn} = 0$ where ν_n, $\forall n \in \mathcal{N}_f$ are Lagrangian multipliers related to subcarrier allocation constraints and $H_{k_fn} = \log(1 + \frac{s_{k_fn}\alpha_{k_fn}}{\Gamma_{k_fn}}) - \frac{s_{k_fn}\alpha_{k_fn}}{\ln 2(s_{k_fn}\alpha_{k_fn}+\Gamma_{k_fn})}$.

C. Iterative Approach (IA)

In this approach, instead of solving $\mathcal{O}^{PRERA}$ directly, a new RA problem is solved iteratively. To do so, the probabilistic constraint is eliminated and substituted by a non-probabilistic conventional constraint. The new optimization problem is solved iteratively until probability constraints are satisfied. In fact, IA adjusts the key parameters (maximum allowable average power of interference) such that after sufficient number of iterations, collision probability is satisfied. In other words in each iteration, the collision probability constraint is eliminated from $\mathcal{O}^{PRERA}$ and substituted by average interference threshold constraint for each subcarrier.

Let l be the iteration number and $\theta^n_{\mathcal{CO}} \in [0,1], \forall n \in \mathcal{N}_f$ be scaling parameters. The adjusted average maximum allowable interference constraint at the l^{th} iteration for subcarrier n is yielded as $\hat{Q}_n(l) = \theta^n_{\mathcal{CO}}\hat{Q}_n(l-1) \le \hat{Q}_n(l-1)$, $\forall n \in \mathcal{N}_f$. Thus, the new constraint is obtained as follows:

$$\sum_{k_f\in\mathcal{K}_f}\bar{g}^f_{k_0^*n}\mathbf{E}_{\alpha_{k_fn}}\left\{\rho_{k_fn}p_{k_fn}(\alpha_{k_fn})\right\} \le \hat{Q}_n(l), \forall n\in\mathcal{N}_f, \tag{34}$$

where $\bar{g}^f_{k_0n}$ is the average of $g^f_{k_0n}$.

IA can be implemented in four steps which is summarized in Table I.

For all femto transmitters, the corresponding process $\hat{Q}_n(l)$ is a decreasing of function l since $\theta^n_{\mathcal{CO}}(l) \in [0,1]$. Therefore, $\mathcal{CO}^f_{k_0^*n}(l)$ will be decreased by increasing l. It can be easily seen that as l tends to ∞, $\theta^n_{\mathcal{CO}}(l)$ tends to 1, implying the convergence of the algorithm.

Remark 1: In IA, we can control the speed of convergence via $\theta^n_{\mathcal{CO}}(l) = (1-\mathcal{D}^n_{\mathcal{CO}})^{\Theta}$ where $\Theta \ge 1$. In fact for $\Theta > 1$, the value of auxiliary parameters will be adjusted faster, making the algorithm converge quicker. However, this faster convergence comes at the cost of smaller femto ergodic sum rate. This is mainly because for $\Theta > 1$, the algorithm jumps over some feasible solutions that may satisfy the probabilistic constraints.

TABLE I
ALGORITHM I

Step1: Initialize the auxiliary variables, $\hat{Q}_n(l=1) = \pi_n^I \max_u Q_{k_0 n}$ where π_n^I is an arbitrary positive large number,
Step2: Generate the new optimization problem, $\mathcal{O}^{PRERA-IA}$, at the l^{th} iteration,
Step2.1: Similar to $\mathcal{O}^{ERA}$, relax the subcarrier allocation constraint and define a variable $s_{k_f n} = \rho_{k_f n} p_{k_f n}$,
Step2.2: Take the derivative of the corresponding Lagrangian function with respect to $s_{k_f n}$
Step2.3: Obtain the allocated power for femto user k_f on subcarrier n with some mathematical manipulations which is depicted in the second row of Table II.
Step3: Using the obtained power allocation and allocated subcarriers, compute the collision and power outage probabilities at the l^{th} iteration,
Step4: Compute $\mathcal{D}_{\mathcal{CO}}^n(l) = \lvert \mathcal{CO}_{k_0^* n}^f(l) - \zeta_{k_0^* n}^I \rvert$, $\forall n \in \mathcal{N}_f$,
Step4.1: If $\mathcal{D}_{\mathcal{CO}}^n(l) \leq \epsilon$,[4] $\forall n \in \mathcal{N}_f$, then go to ***Step5***,
Step4.2: Compute $\theta_{\mathcal{CO}}^n(l) = 1 - \mathcal{D}_{\mathcal{CO}}^n(l)$, $\forall n \in \mathcal{N}_f$, $l = l+1$ and adjust the auxiliary variables as $\hat{Q}_n(l) = \theta_{\mathcal{CO}}^n(l)\hat{Q}_n(l-1)$, then go to ***Step2***,
Step5: End.

Remark 2: Remark 1 shows that the designer can always make a compromise between the complexity of the RA problem and the performance of the femto network. A larger Θ, yields remarkably faster convergence at the expense of noticeably weaker performance, and vice versa.

Remark 3: In simulation results, we compare the sum-rate obtained based on the optimal approach and that of our proposed IA algorithm and show that the performance of IA is very close to optimal.

D. Analytical Approach (AA)

The required computational complexity of solving the optimization problem based on IA can be pretty high when we are dealing with probabilistic constraints. To tackle this issue, we propose the AA approach. In this approach, the considered probabilistic constraint is substituted by non-probabilistic using upper bounds as will be explained in the sequel.

1) Upper bound on collision probability based on Markov's inequality: We suggest an upper bound on the collision probability using the Markov's Inequality. For the k_0^{th} macro user on subcarrier n we have

$$\mathcal{CO}_{k_0 n}^f = \Pr\left\{ \sum_{k_f \in \mathcal{K}_f} p_{k_f n}(\alpha_{k_f n}) g_{k_0 n}^f > Q_{k_0 n} \right\} \tag{35}$$

$$= \mathbf{E}_{\boldsymbol{\alpha}} \left\{ \Pr\left\{ \sum_{k_f \in \mathcal{K}_f} p_{k_f n}(\alpha_{k_f n}) g_{k_0 n}^f > Q_{k_0 n} | \boldsymbol{\alpha}^f \right\} \right\} \tag{36}$$

$$\leq \mathbf{E}_{\boldsymbol{\alpha}^f} \left\{ \frac{\sum_{k_f \in \mathcal{K}_f} \bar{g}_{k_0 n}^f p_{k_f n}(\alpha_{k_f n})}{Q_{k_0 n}} \right\} \tag{37}$$

$$= \frac{\sum_{k_f \in \mathcal{K}_f} \bar{g}_{k_0 n}^f \mathbf{E}_{\alpha_{k_0 n}} \left\{ p_{k_f n}(\alpha_{k_f n}) \right\}}{Q_{k_0 n}}. \tag{38}$$

With some manipulations, (35) is modified to

$$\sum_{k_f \in \mathcal{K}_f} \bar{g}_{k_0 n} \mathbf{E}_{\alpha_{k_f n}} \left\{ p_{k_f n}(\alpha_{k_f n}) \right\} \leq \zeta_{k_0 n}^I Q_{k_0 n}. \tag{39}$$

Replacing (19) in $\mathcal{O}^{PRERA}$ with (39), the problem can be formulated into a tractable optimization problem called, $\mathcal{O}^{PRERA-AA-M}$. Taking the derivative of the corresponding Lagrangian function with respect to $s_{k_f n}$ and with some mathematical manipulations, the solution is obtained which is depicted in the third row of Table II.

2) Upper bound on collision probability based on Bernstein approximation: We can also use an upper bound on the collision probability using the Bernstein Approximation. Using this approximation, collision probability for k_0^{th} macro user on subcarrier n can be substituted with the following constraint [46], [47]

$$\inf_{\tau > 0} \mathbf{E}_{\boldsymbol{\alpha}^f} \left\{ \varpi_{k_0 n} + \tau \sum_{k_f \in \mathcal{K}_f} \Lambda(\tau^{-1} p_{k_f n}(\alpha_{k_f n})) \right\} \leq 0, \tag{40}$$

where

$$\varpi_{k_0 n} = -Q_{k_0 n} - \tau \log \zeta_{k_0 n}^I, \tag{41}$$

and

$$\Lambda(z) = \max_{f_g(x)} \left\{ \ln\left(\int e^{zx} df_g(x) \right) \right\}. \tag{42}$$

The tractability of the above modified constraint depends on efficiently computing $\Lambda(z)$. The following upper bound can be used to deal with this issue:

$$\Lambda(z_{k_0 n}^f) \leq \max\left\{ \mu_{k_0 n}^{f-} z_{k_0 n}^f, \mu_{k_0 n}^{f+} z_{f_0 n}^k \right\} + \frac{(\sigma_{k_0 n}^f z_{k_0 n}^f)^2}{2}, \tag{43}$$

where $\mu_{k_0 n}^{f-}$, $\mu_{k_0 n}^{f+}$ and $(\sigma_{k_0 n}^f)^2$ are fixed and can be determined based on the probability distribution. More details can be found in Table 1 of [47]. Substituting this upper bound in (40), we obtain

$$\mathbf{E}_{\boldsymbol{\alpha}^f} \left\{ \sum_{k_f \in \mathcal{K}_f} \max\left\{ \mu_{k_f n}^{k-} p_{k_f n}(\alpha_{k_f n}), \mu_{k_f n}^{k+} p_{k_f n}(\alpha_{k_f n}) \right\} \right. \tag{44}$$

$$\left. -Q_{k_0 n} + \sqrt{2 \log \frac{1}{\zeta_{k_0 n}^I}} \left(\sum_{k_f \in \mathcal{K}_f} (p_{k_f n}(\alpha_{k_f n}) \sigma_{k_0 n}^k)^2 \right)^{1/2} \right\} \leq 0.$$

We suppose that $g_{k_0 n}^f$ is bounded and belongs to $[a_{k_0 n}^f, b_{k_0 n}^f]$ and normalized to $[-1, 1]$. The normalized interference channel power is denoted by $\check{a}_{k_0 n}^f = \frac{b_{k_0 n}^f - a_{k_0 n}^f}{2}$ where $\check{b}_{k_0 n}^f = \frac{b_{k_0 n}^f + a_{k_0 n}^f}{2}$ and $\check{g}_{k_0 n}^f = \frac{g_{k_0 n}^f - \check{b}_{k_0 n}^f}{\check{a}_{k_0 n}^f}$. Thus, the normalized version of (44) can be obtained as follows [48]

TABLE II
ALLOCATED POWER FOR DIFFERENT APPROACHES IN PRERA

Approaches	Allocated power
IA	$\left[\frac{1}{(\lambda+\bar{g}^f_{k_0^* n}\mu_n)\ln 2}-\frac{\Gamma_{k_f n}}{\alpha_{k_f n}}\right]^+$
AA-B	$\left[\frac{1}{(\lambda+\Upsilon^f_{k_0^* n}\mu_n)\ln 2}-\frac{\Gamma_{k_f n}}{\alpha_{k_f n}}\right]^+$
AA-M	$\left[\frac{1}{(\lambda+\bar{g}^f_{k_0 n}\mu_n)\ln 2}-\frac{\Gamma_{k_f n}}{\alpha_{k_f n}}\right]^+$

$$\mathbf{E}_{\boldsymbol{\alpha}^f}\Bigg\{-Q_{k_0 n}+\sum_{k_f\in\mathcal{K}_f}(\check{b}^f_{k_0 n}+\check{a}^f_{k_0 n}\mu^{f+}_{k_0 n})p_{k_f n}(\alpha_{k_f n}) \quad (45)$$

$$+\sqrt{2\log\frac{1}{\zeta^I_{k_0 n}}}\Bigg(\sum_{k_f\in\mathcal{K}_f}(\check{a}^f_{k_0 n}p_{k_f n}(\alpha_{k_f n})\sigma^f_{k_0 n})^2\Bigg)^{1/2}\Bigg\}\le 0.$$

Using the fact that $\| x \|_2 \le \| x \|_1$ where $\| x \|_q$ denotes norm q vector of x, we can obtain a more tractable constraint as follows which substitutes the collision probability:

$$\mathbf{E}_{\boldsymbol{\alpha}^f}\Bigg\{-Q_{k_0 n}+\sum_{k_f\in\mathcal{K}_f}(\check{b}^f_{k_0 n}+\check{a}^f_{k_0 n}\mu^{f+}_{k_0 n})p_{k_f n}(\alpha_{k_f n}) \quad (46)$$

$$+\sqrt{2\log\frac{1}{\zeta^I_{k_0 n}}}\sum_{k_f\in\mathcal{K}_f}|\check{a}^f_{k_0 n}p_{k_f n}(\alpha_{k_f n})\sigma^f_{k_0 n}|\Bigg\}\le 0.$$

Thus, after some mathematical manipulation of (46), collision probability constraint (19) is substituted by the following average transmission power constraint:

$$\mathbf{E}_{\boldsymbol{\alpha}^f}\left\{\sum_{k_f\in\mathcal{K}_f}\Upsilon^f_{k_0 n}p_{k_f n}(\alpha_{k_f n})\right\}\le Q_{k_0 n}, \quad (47)$$

where $\Upsilon^f_{k_0 n}=(\check{b}^f_{k_0 n}+\check{a}^f_{k_0 n}\mu^{f+}_{k_0 n})+\sqrt{2\log\frac{1}{\zeta^I_{k_0 n}}}|\check{a}^f_{k_0 n}\sigma^f_{k_0 n}|$.

Using the modified constraint, $\mathcal{O}^{PRERA}$ can be formulated into a tractable optimization problem called $\mathcal{O}^{PRERA-AA-B}$.

Taking the derivative of the corresponding Lagrangian function with respect to $s_{k_f n}$ and with some mathematical manipulations, we obtain the allocated power for each femto user k_f on subcarrier n which is depicted in last row of Table II.

Remark 4: As can be seen from table II, the allocated powers for the case of IA and AA-M is only a function of the mean value of the interference channel gain. In other words, in contrast to the original ERA problem, we do not need the knowledge of I-CSI even in the online phase and only I-CDI is necessary to obtain the mean. This is another important advantage of the proposed scheme. This is not the case for AA-B method.

IV. FULLY ROBUST ERGODIC RESOURCE ALLOCATION (FRERA) AND ITS SOLUTION

In this section, in addition to the collision probability constraint, we replace the average power constraint with power outage probability constraint. We refer to the resulting problem as fully robust ergodic resource allocation (FRERA). In contrast to PRERA, we are not able to transform this problem into a convex one. Therefore, we are confined to use the proposed IA and AA algorithms. We also propose suboptimal algorithms referred to as hybrid approaches (HA). The FRERA problem is formulated as follows:

Problem $\mathcal{O}^{FRERA}$:

$$\max_{\boldsymbol{p}^f,\boldsymbol{\rho}^f}\sum_{k_f\in\mathcal{K}_f}\sum_{n_f\in\mathcal{N}_f}\mathbf{E}_{\alpha_{k_f n}}\Big\{\rho_{k_f n}r_{k_f n}(\alpha_{k_f n})\Big\}, \quad (48)$$

$$\text{s.t.} \quad (2),(4),(10),$$

A. Iterative Approach (IA)

In this approach we act similar to the case of PRERA. Instead of solving $\mathcal{O}^{FRERA}$ directly, a new RA problem is realized where the probabilistic constraints are eliminated and substituted by tractable constraints, i.e., the collision probability constraint is substituted by average interference threshold constraint and the power outage probability constraint is replaced by average transmission power constraint. The new optimization problem is solved iteratively until probability constraints are satisfied. In fact, IA adjusts the key parameters (maximum average power of interference and maximum average transmission power of femto base station) such that after sufficient number of iterations, collision and power outage probabilities are satisfied.

Let l be the iteration number and $\theta^n_{\mathcal{CO}}\in[0,1]$ and $\theta_{\mathcal{PO}_f}\in[0,1]$ be scaling parameters. The adjusted average maximum allowable interference constraint at the l^{th} iteration for subcarrier n and average maximum allowable transmission power of femto base station are yielded as $\hat{Q}_n(l)=\theta^n_{\mathcal{CO}}\hat{Q}_n(l-1)\le\hat{Q}_n(l-1)$, $\forall n\in\mathcal{N}_f$, and $\hat{P}^T_f(l)=\theta_{\mathcal{PO}_f}\hat{P}^T_f(l-1)\le\hat{P}^T_f(l-1)$, respectively. Thus, the new constraints are obtained as follows:

$$\sum_{k_f\in\mathcal{K}_f}\bar{g}^f_{k_f n}\mathbf{E}_{\alpha_{k_f n}}\Big\{\rho_{k_f n}p_{k_f n}(\alpha_{k_f n})\Big\}\le\hat{Q}_n(l), \quad \forall n\in\mathcal{N}_f, \quad (49)$$

$$\sum_{n\in\mathcal{N}_f}\sum_{k_f\in\mathcal{K}_f}\mathbf{E}_{\alpha_{k_f n}}\Big\{\rho_{k_f n}p_{k_f n}(\alpha_{k_f n})\Big\}\le\hat{P}^T_f(l), \quad (50)$$

IA can be implemented in four steps which is summarized in Table III.

B. Analytical Approach (AA)

Similar to the partially robust case, in order to reduce the computational complexity, we propose the AA approach. In this approach, probabilistic constraints are substituted by non-probabilistic ones using upper bounds as will be explained in the sequel.

TABLE III
ALGORITHM II

Step1: Initialize the auxiliary variables, $\hat{Q}_n(l = 1) = \pi_n^I \max_u Q_{k_0 n}$ and $\hat{P}_f^T(l = 1) = \pi^p P_f^T\ \pi_n^I$ where π_n^I and π^p are arbitrary positive large number, ***Step2:*** Generate the new optimization problem, $\mathcal{O}^{FRERA-IA}$, at the l^{th} iteration, ***Step2.1:*** Similar to $\mathcal{O}^{ERA}$, relax the subcarrier allocation constraint and define a variable $s_{k_f n} = \rho_{k_f n} p_{k_f n}$, ***Step2.2:*** Take the derivative of the corresponding Lagrangian function with respect to $s_{k_f n}$ ***Step2.3:*** Obtain the allocated power for femto user k_f on subcarrier n with some mathematical manipulations which is depicted in second row of Table IV. ***Step3:*** Using the obtained power allocation and allocated subcarriers, compute the collision and power outage probabilities at the l^{th} iteration, ***Step4***: Compute $\mathcal{D}^n_{\mathcal{CO}}(l) = \lvert\mathcal{CO}_{k_0^* n}(l) - \zeta^I_{k_0^* n}\rvert$, $\forall n$, and $\mathcal{D}_{\mathcal{PO}_f}(l) = \lvert\mathcal{PO}_f(l) - \zeta^p\rvert$, ***Step4.1***: If $\mathcal{D}^n_{\mathcal{CO}}(l) \leq \epsilon, \quad \forall n \in \mathcal{N}_f$, and $\mathcal{D}_{\mathcal{PO}_f}(l) \leq \epsilon$, then go to ***Step5***, ***Step4.2***: Compute $\theta^n_{\mathcal{CO}}(l) = 1 - \mathcal{D}^n_{\mathcal{CO}}(l)$, $\forall n \in \mathcal{N}_f$, $\theta_{\mathcal{PO}_f}(l) = 1 - \mathcal{D}_{\mathcal{PO}_f}(l)$, set $l = l + 1$ and adjust the auxiliary variables as $\hat{Q}_n(l) = \theta^n_{\mathcal{CO}}(l)\hat{Q}_n(l-1)$ and $\hat{P}_f^T(l) = \theta_{\mathcal{PO}_f}(l)\hat{P}_f^T(l-1)$, then go to ***Step2***, ***Step5:*** End.

1) Upper bound on power outage probability : We suggest an upper bound on the power outage probability as follows

$$\mathcal{PO} = \Pr\left\{\sum_{n\in\mathcal{N}_f}\sum_{k_f\in\mathcal{K}_f} p_{k_f n}(\alpha_{k_f n}) > P_f^T\right\} \tag{51}$$

$$\leq \Pr\left\{\max_{n,k_f}\left\{p_{k_f n}(\alpha_{k_f n})\right\} > \frac{P_f^T}{N_f}\right\} \tag{52}$$

$$= 1 - \Pr\left\{\max_{n,k}\left\{p_{k_f n}(\alpha_{k_f n})\right\} \leq \frac{P_f^T}{N_f}\right\} \tag{53}$$

$$= 1 - \prod_{n\in\mathcal{N}_f}\prod_{k_f\in\mathcal{K}_f}\Pr\left\{p_{k_f n}(\alpha_{k_f n}) \leq \frac{P_f^T}{N_f}\right\} \tag{54}$$

$$= 1 - \left[\Pr\left\{p_{k_f n}(\alpha_{k_f n}) \leq \frac{P_f^T}{N_f}\right\}\right]^{N_f K_f}. \tag{55}$$

Based on (51) and with some manipulations, (4) is modified to

$$\Pr\left\{p_{k_f n}(\alpha_{k_f n}) \leq \frac{P_f^T}{N_f}\right\} \geq \sqrt[N_f K_f]{1-\zeta_k^p}. \tag{56}$$

Using (56), we can replace the power outage probability constraint with the following sufficient constraint

$$p_{k_f n} \leq \frac{P_f^T F^{-1}_{\alpha_{k_f n}}(1 - \sqrt[N_f K_f]{1-\zeta_k^p})}{N\alpha_{k_f n}}, \tag{57}$$

TABLE IV
ALLOCATED POWER FOR DIFFERENT APPROACHES IN FRERA

Approaches	Allocated power
IA	$\left[\frac{1}{(\lambda+\bar{g}^f_{k_0^* n}\mu_n)\ln 2} - \frac{\Gamma_{k_f n}}{\alpha_{k_f n}}\right]^+$
AA-B	$\left[\frac{1}{\Upsilon^k_{k_0^* n}\mu_n \ln 2} - \frac{\Gamma_{k_f n}}{\alpha_{k_f n}}\right]^{\frac{P_f^T F^{-1}_{\alpha_{k_f n}}(1-\sqrt[N_f K_f]{(1-\zeta_k^p)})}{N\alpha_{k_f n}})}$
AA-M	$\left[\frac{1}{\bar{g}_{k_0 n}\mu_n \ln 2} - \frac{\Gamma_{k_f n}}{\alpha_{k_f n}}\right]^{\frac{P^T F^{-1}_{\alpha_{k_f n}}(1-\sqrt[N_f K_f]{(1-\zeta^p)})}{N_f\alpha_{k_f n}})}$
HA-IIAP-M	$\left[\frac{1}{\bar{g}^f_{k_0^* n}\mu_n \ln 2} - \frac{\Gamma_{k_f n}}{\alpha_{k_f n}}\right]^{\frac{P_f^T F^{-1}_{\alpha_{k_f n}}(1-\sqrt[N_f K_f]{(1-\zeta^p)})}{N_f\alpha_{k_f n}})}$
HA-AIIP-M	$\left[\frac{1}{(\lambda+\bar{g}^f_{k_0^* n}\mu_n)\ln 2} - \frac{\Gamma_{k_f n}}{\alpha_{k_f n}}\right]^+$

where $F^{-1}_{\alpha_{k_f n}}(\cdot)$ is the inverse cumulative density function (CDF) of random variable $\alpha_{k_f n}$.

2) Upper bound on collision probability: Similar to the partially robust case, we can obtain an upper bound on the collision probability using the Markov's Inequality and Bernestain approach.

If we use Markov's Inequality, the corresponding optimization problem, $\mathcal{O}^{FRERA}$, can be formulated into a tractable optimization problem called, $\mathcal{O}^{FRERA-AA-M}$, by replacing (2) with (39) and (4) with (57). Alternatively, if we adopt Bernestain approach, the corresponding optimization problem, $\mathcal{O}^{FRERA}$, can be formulated into a tractable optimization problem called, $\mathcal{O}^{FRERA-AA-B}$, by replacing (2) with (47) and (4) with (57). Taking the derivative of the corresponding Lagrangian function with respect to $s_{k_f n}$ and with some mathematical manipulations, the solution is obtained which is depicted in the third and fourth rows of Table IV.

C. The hybrid approach

As will be shown in simulation results, the performance of IA is considerably better than that of AA at the expense of larger computational complexity. To provide a trad-off between performance and complexity in RERA, we can treat one of the probabilistic constraints using AA and the other one based on IA. This method is called hybrid approach (HA) and depending on which method is applied to which constraint, we have several combinations. For example, if the collision probability constraint is treated based on IA and the power outage probability constraints is taken care of using AA based on Markov inequality, we demonstrate the resulting solution method by HA-IIAP-M where II denotes Iterative-Interference and AP-M denotes Analytical-Power-Markov. The resulting power allocation for HA-IIAP-M and HA-AI-IP-M has been reported in Table IV.

Remark 5: As can be seen from table IV, similar to the case of PRERA, the allocated powers for the case of IA and AA-M is only a function of the mean value of the interference channel gain. This also applies to HA case as long as we do not use the Bernstein method to treat any of the probabilistic constraints.

V. Quantized Robust Ergodic Resource Allocation (QRERA)

In this section, we formulate the quantized version of the RERA problem in which the domain of $\alpha_{k_f n}$ is divided into J *fading regions* indexed by $j = 0, 1, \ldots, J-1$ that are specified by their boundaries, $A_{k_f n}(j)$, where $A_{k_f n}(0) = 0$, $A_{k_f n}(J) = \infty$. In this system J-ary modulation is utilized in which $c(0) = 0$, $c(1) = 1$, and $c(j) = 2(j-1), j = 2, \ldots, J-1$ is the modulation rate for the j^{th} region.

The average transmission rate and the average transmission power for femto user k_f on subcarrier n are respectively obtained as the sum of the data rates and powers associated to each of the regions multiplied by the probability that CINR falls over that particular region [43]. In other words, we have

$$\mathbf{E}_{\alpha_{k_f n}}\left\{r_{k_f n}(\alpha_{k_f n}, \boldsymbol{A}_{k_f n})\right\} = \rho_{k_f n}\sum_{j=0}^{J-1} c(j)\int_{A_{k_f n}(j)}^{A_{k_f n}(j+1)} f_{\alpha_{k_f n}}(x)dx, \quad (58)$$

$$\mathbf{E}_{\alpha_{k_f n}}\left\{p_{k_f n}(\alpha_{k_f n}, \boldsymbol{A}_{k_f n})\right\} = \rho_{k_f n}\sum_{j=0}^{J-1} h_{k_f n}(j)\int_{A_{k_f n}(j)}^{A_{k_f n}(j+1)} \frac{f_{\alpha_{k_f n}}(x)}{x}dx, \quad (59)$$

where $f_{\alpha_{k_f n}}(x)$ is the probability density function (pdf) of $\alpha_{k_f n}$ and $h_{k_f n}(j)$ is expressed as:

$$h_{k_f n}(j) = \frac{2^{c(j)}-1}{c_1}\ln\frac{c_2}{\mathrm{BER}_{k_f n}}. \quad (60)$$

The RA problem is then obtained as follows:

Problem $\mathcal{O}^{QRERA}$:

$$\max_{\boldsymbol{A}^f, \boldsymbol{\rho}^f} \sum_{k_f\in\mathcal{K}_f}\sum_{n\in\mathcal{N}_f} \mathbf{E}_{\alpha_{k_f n}}\left\{r_{k_f n}(\alpha_{k_f n}, \boldsymbol{A}_{k_f n})\right\}, \quad (61)$$

$$\text{s.t.} \quad (2), (4), (10),$$

Similar to $\mathcal{O}^{RERA}$, probabilistic constraints prevent us to apply the dual method directly. Consequently, we propose three approaches, abbreviated by QIA, QAA and QHA to transform the proposed problem into one which can be solved using the dual method.

A. Quantized Iterative Approach (QIA)

We use an algorithm similar to Algorithm II where the tractable constraints which are defined in Step 2 of that algorithm are obtained as follows:

$$\sum_{k_f\in\mathcal{K}_f} \bar{g}_{k_0^* n}\mathbf{E}_{\alpha_{k_f n}}\left\{p_{k_f n}(\alpha_{k_f n}, \boldsymbol{A}_{k_f n})\right\} \le \hat{Q}_n(l), \forall n\in\mathcal{N}_f, \quad (62)$$

$$\sum_{n\in\mathcal{N}_f}\sum_{k_f\in\mathcal{K}_f} \mathbf{E}_{\alpha_{k_f n}}\left\{p_{k_f n}(\alpha_{k_f n}, \boldsymbol{A}_{k_f n})\right\} \le \hat{P}_f^T(l). \quad (63)$$

Taking the derivative of the corresponding Lagrangian function with respect to $A_{k_f n}(j)$ and with some mathematical manipulations, we can obtain the fading boundary at j^{th} region for femto user k_f on subcarrier n which is depicted in Table V. Using the obtained boundary regions and allocated subcarriers, the *Step 3* of Algorithm II is modified such that the collision probabilities at the l^{th} iteration $\mathcal{CO}^f_{k_0^* n}(l)$ are computed as follows:

$$\mathcal{CO}^f_{k_0^* n}(l) = \Pr\left\{\sum_{k_f\in\mathcal{K}_f} p_{k_f n}(\alpha_{k_f n})g_{k_0^* n} \ge Q_{k_0^* n}\right\} \quad (64)$$

$$= \Pr\left\{p_{k_f^* n}(\alpha_{k_f^* n})g_{k_0^* n} \ge Q_{k_0^* n}\right\} \quad (65)$$

$$= \mathbf{E}_{\alpha_{k_f^* n}}\left\{\Pr\left\{p_{k_f^* n}(\alpha_{k_f^* n})g_{k_0^* n} \ge Q_{k_0^* n}|\alpha_{k_f^* n}\right\}\right\} \quad (66)$$

$$= 1 - \mathbf{E}_{\alpha_{k_f^* n}}\left\{F_{g_{k_0^* n}}\left(\frac{Q_{k_0^* n}}{p_{k_f^* n}(\alpha_{k_f^* n})}\right)\right\} \quad (67)$$

$$= 1 - \sum_{j=0}^{J-1}\int_{A_{k^* n}(j)}^{A_{k^* n}(j+1)} F_{g_{k_0^* n}}\left(\frac{xQ_{k_0^* n}}{h_{k_f^* n}(j)}\right) f_{\alpha_{k_f^* n}}(x)dx. \quad (68)$$

The power outage probability can be obtained using power Probability Mass Function (PMF) of the femto BS. For a given boundary region and subcarrier assignment, we can use Probability Generating Function (PGF) to obtain the PMF of the allocated transmission power for each femto user. To do this, the allocated transmission power PGF of femto user k_f on subcarrier n is obtained as follows:

$$G_{p_{k_f n}(\alpha_{k_f n})}(Z) = \mathbf{E}_{\alpha_{k_f n}}\left\{Z^{p_{k_f n}(\alpha_{k_f n})}\right\} \quad (69)$$

$$= \sum_{j=0}^{J-1} Z^{\phi_{k_f n}(j)}\Pr\left\{p_{k_f n}(\alpha_{k_f n}) = \phi_{k_f n}(j)\right\},$$

where $\phi_{k_f n}(j) = \frac{h_{k_f n}(j)}{A_{k_f n}(j)}$. In this step, we know the set of subcarriers allocated to femto user k_f. Thus, the allocated femto user to subcarrier n is shown by $k_f(n)$. The PGF of total allocated transmission power can be obtained as follows where we assume femto channel coefficients are independent and identically distributed:

$$G_{\Upsilon(\boldsymbol{\alpha})}(Z) = \mathbf{E}_{\boldsymbol{\alpha}}\left\{Z^{\sum_{n\in\mathcal{N}_f} p_{k(n)n}(\alpha_{k_f(n)n})}\right\} \quad (70)$$

$$= \prod_{n\in\mathcal{N}_f}\mathbf{E}_{\alpha_{k_f(n)n}}\left\{Z^{p_{k_f(n)n}(\alpha_{k_f(n)n})}\right\} \quad (71)$$

$$= \prod_{n\in\mathcal{N}_f} G_{p_{k_f(n)n}(\alpha_{k_f(n)n})}(Z) = \left[G_{p_{k(n)n}(\alpha_{k(n)n})}(Z)\right] \quad (72)$$

$$= \left[\sum_{j=0}^{J-1} Z^{\phi_{k_f(n)n}(j)} \Pr\left\{ p_{k_f(n)n}(\alpha_{k_f(n)n}) = \phi_{k_f(n)n}(j) \right\} \right]. \quad (73)$$

where $\Upsilon(\boldsymbol{\alpha}^f) = \sum_{n\in\mathcal{N}_f} p_{k_f(n)n}(\alpha_{k_f(n)n})$.

Based on the Multinomial Theorem, (72) can be computed as follows:

$$G_{\Upsilon(\boldsymbol{\alpha}^f)}(Z) = \sum_{\substack{\sum_{j=0}^{J-1} d_j = N_f,\\ d_1,\ldots,d_J \geq 0}} \binom{N_f}{d_1,\ldots,d_J} \prod_{j=0}^{J-1} \left[Z^{\phi_{k_f(n)n}(j)} \right. \quad (74)$$
$$\left. \Pr\left\{ p_{k_f(n)n}(\alpha_{k_f(n)n}) = \phi_{k_f(n)n}(j) \right\} \right]^{d_j}.$$

Using (74), we can easily compute the power PMF of the femto BS. Thus, the power outage probability can be obtained as follows

$$\mathcal{PO}_f = \Pr\left\{ \Upsilon(\boldsymbol{\alpha}^f) > P_f^T \right\} = 1 - \sum_{x\in\Phi_f} \Pr\left\{ \Upsilon(\boldsymbol{\alpha}^f) = x \right\}, \quad (75)$$

where Φ_f is defined as

$$\Phi_f = \left\{ \sum_{n\in\mathcal{N}_f} \phi_{k_f(n)n}(j_n) | \sum_{n\in\mathcal{N}_f} \phi_{k_f(n)n}(j_n) \leq P_f^T, \quad (76)$$
$$j_n \in \left\{ 0,\ldots,J-1 \right\} \right\}.$$

B. Quantized Analytical Approach (QAA)

Similar to the continuous case, the probabilistic constraints are substituted by their corresponding upper bounds. With some manipulations of (51), the upper bound on the power outage probability for the quantized case can be obtained as follows:

$$\Pr\left\{ p_{k_f n}(\alpha_{k_f n}) \leq \frac{P_f^T}{N_f} \right\} = \Pr\left\{ \alpha_{k_f n} \geq \frac{N_f(2^{r_{k_f n}} - 1)}{P_f^T} \right\}. \quad (77)$$

Using (77) and (4), we can get the following inequality as the power outage probability constraint

$$r_{k_f n} \leq \log_2\left(1 + \frac{P_f^T}{N_f} F^{-1}_{\alpha_{k_f n}} (1 - \sqrt[N_f K_f]{1-\zeta^p}) \right). \quad (78)$$

Based on (78), the set of possible rate regions is obtained as follows

$$\Omega_{k_f n} = \left\{ c(j) | c(j) \leq \log_2\left(2 + \frac{P_f^T}{N_f} F^{-1}_{\alpha_{k_f n}} (1 - \sqrt[N_f K_f]{1-\zeta^p}) \right), \quad (79)$$
$$j = 0,\ldots,J-1 \right\}.$$

Based on (79), the total number of regions will be equal to the cardinality of $\Omega_{k_f n}$ and denoted by $L_{k_f n}$. The average transmission rate and average transmission power can be obtained as follows:

$$\mathbf{E}_{\alpha_{k_f n}}\left\{ r_{k_f n}(\alpha_{k_f n}, \boldsymbol{A}_{k_f n}) \right\} \quad (80)$$
$$= \rho_{k_f n} \sum_{j=0}^{L_{k_f n}-1} c(j) \int_{A_{k_f n}(j)}^{A_{k_f n}(j+1)} f_{\alpha_{k_f n}}(x) dx,$$

$$\mathbf{E}_{\alpha_{k_f n}}\left\{ p_{k_f n}(\alpha_{k_f n}, \boldsymbol{A}_{k_f n}) \right\} \quad (81)$$
$$= \rho_{k_f n} \sum_{j=0}^{L_{k_f n}-1} h_{k_f n}(j) \int_{A_{k_f n}(j)}^{A_{k_f n}(j+1)} \frac{f_{\alpha_{k_f n}}(x)}{x} dx,$$

If we use the upper bound on collision probability derived by Markov's inequality, the problem $\mathcal{O}^{QRERA}$ can be reformulated as follows

Problem $\mathcal{O}^{QFRERA-AA-M}$:

$$\max_{\boldsymbol{A}^f, \boldsymbol{\rho}^f} \sum_{k_f\in\mathcal{K}_f} \sum_{n\in\mathcal{N}_f} \mathbf{E}_{\alpha_{k_f n}}\left\{ r_{k_f n}(\alpha_{k_f n}, \boldsymbol{A}_{k_f n}) \right\}, \quad (82)$$

$$s.t. \sum_{k_f\in\mathcal{K}_f} \bar{g}_{k_0 n} \mathbf{E}_{\alpha_{k_f n}}\left\{ p_{k_f n}(\alpha_{k_f n}, \boldsymbol{A}_{k_f n}) \right\} \leq \zeta^I_{k_0 n} Q_{k_0 n},$$
$$\forall n \in \mathcal{N}_f, \quad (83)$$

$$\sum_{k_f\in\mathcal{K}_f} \rho_{k_f n} = 1, \forall n \in \mathcal{N}_f, \rho_{k_f n} \in \{0,1\}, \quad \forall k_f, n \in \mathcal{N}_f, \quad (84)$$

On the other hand, if we adopt the upper bound on collision probability using Bernestein approach, Equation (83) is replaced by

$$\sum_{k_f\in\mathcal{K}_f} \mathbf{E}_{\boldsymbol{\alpha}_{k_f n}} \left\{ \Upsilon^f_{k_0^* n} p_{k_f n}(\alpha_{k_f n}, \boldsymbol{A}_{k_f n}) \right\} \leq Q_{k_0^* n}, n \in \mathcal{N}_f, \quad (85)$$

to form $\mathcal{O}^{QFRERA-AA-B}$.

Taking the derivative of the corresponding Lagrangian function with respect to $A_{k_f n}(j)$ and with some mathematical manipulations, we can obtain the fading boundary at the j^{th} region for femto user k_f on subcarrier n which is depicted for both problems in Table V.

Similar to the continuous case, we can also consider Quantized Hybrid Approach (QHA) in which one of the probabilistic constraints is treated based on QIA and the other one is simplified based on QAA. This provides a trade-off between complexity and performance. The fading boundary regions corresponding to QHA-IIAP and QHA-AIIP are reported in Table V.

TABLE V
THE BOUNDARY FADING REGIONS OF FEMTO USER k_f ON SUBCARRIER n, $A_{k_f,n}(j)$

QIA	$(\lambda + \mu_n \bar{g}^f_{k_0^* n}) \frac{h_{k_f n}(j) - h_{k_f n}(j-1)}{c(j) - c(j-1)}, j = 1, \ldots, J-1$
QAA-M	$\mu_n \bar{g}^f_{k_0^* n} \frac{h_{k_f n}(j) - h_{k_f n}(j-1)}{c(j) - c(j-1)}, j = 1, \ldots, L_{k_f n} - 1$
QAA-B	$\mu_n \Upsilon^k_{k_0^* n} \frac{h_{k_f n}(j) - h_{k_f n}(j-1)}{c(j) - c(j-1)}, j = 1, \ldots, L_{k_f n}$
QHA-IIAP	$\mu_n \bar{g}^f_{k_0^* n} \frac{h_{k_f n}(j) - h_{k_f n}(j-1)}{c(j) - c(j-1)}, j = 1, \ldots, L_{k_f n} - 1$
QHA-AIIP	$\mu_n \bar{g}^f_{k_0^* n} \frac{h_{k_f n}(j) - h_{k_f n}(j-1)}{c(j) - c(j-1)}, j = 1, \ldots, J-1,$

VI. COMPUTATIONAL COMPLEXITY

In this part, we investigate the computational complexity for the proposed methods. As it was mentioned in previous sections, in the process of obtaining the near optimal solution using the dual decomposition approach, we adopt the ellipsoid method for updating Lagrange multipliers. For details of the ellipsoid method see [49].

Utilizing the results presented in [49], the number of iterations required to achieve δ-optimality, in a problem with X Lagrange multipliers is in order of $O\left(\frac{X^2}{\delta^2}\right)$. Moreover, in each iteration[5], it is required to obtain subcarrier allocation for all N sub-carriers. Therefore, in each iteration of the dual method, subcarrier allocation equation is obtained Y times where Y is the total number of assignment variables. Consequently, the total computational complexity is in order of $O\left(\frac{YX^2}{\delta^2}\right)$.

For example, in PRERA-AA the proposed optimization problem which is solved by dual decomposition method has $X = 2N_f + 1$ Lagrange multipliers. Moreover, the total number of assignment variables is $Y = N_f K_f$. Accordingly, the total computational complexity corresponding to dual decomposition solution is summed up to $O\left(\frac{K_f N_f (2N_f+1)^2}{\delta^2}\right)$. As an another example in PRERA-IA, it is necessary that the proposed optimization problem is solved $\Lambda_{PRERA-IA}(\theta)$ times using the dual method where $\Lambda_{PRERA-IA}(\theta)$ is the maximum number of iterations necessary for convergence. Therefore, the overall complexity is in order of $\Lambda_{PRERA-IA}(\theta)\mathcal{O}\left(\frac{K_f N_f (2N_f+1)^2}{\delta^2}\right)$. Note that the value of $\Lambda_{PRERA-IA}(\theta)$ depends on the considered system parameters and is a decreasing function of θ which is discussed in Section VII. The computational complexity for the remaining proposed methods can be evaluated similarly and are reported in Table VI.

TABLE VI
THE COMPUTATIONAL COMPLEXITY OF DIFFERENT METHODS

Scheme	Computational Complexity Scheme
ERA	$\mathcal{O}\left(\frac{K_f N_f (2N_f+1)^2}{\delta^2}\right)$
PRERA-AA-M	$\mathcal{O}\left(\frac{K_f N_f (2N_f+1)^2}{\delta^2}\right)$
PRERA-AA-B	$\mathcal{O}\left(\frac{K_f N_f (2N_f+1)^2}{\delta^2}\right)$
PRERA-IA	$\Lambda_{PRERA-IA}(\theta)\mathcal{O}\left(\frac{K_f N_f (2N_f+1)^2}{\delta^2}\right)$
FRERA-IA	$\Lambda_{FRERA-IA}(\theta)\mathcal{O}\left(\frac{K_f N_f (2N_f+1)^2}{\delta^2}\right)$
FRERA-AA-M	$\mathcal{O}\left(\frac{4K_f N_f^3}{\delta^2}\right)$
FRERA-HA-AIIP	$\Lambda_{FRERA-HA-AIIP}(\theta)\mathcal{O}\left(\frac{K_f N_f (2N_f+1)^2}{\delta^2}\right)$
QFRERA-AA-M	$\mathcal{O}\left(\frac{K_f N_f^3}{\delta^2}\right)$
FRERA-HA-IIAP	$\Lambda_{FRERA-HA-IIAP}(\theta)\mathcal{O}\left(\frac{K_f N_f^3}{\delta^2}\right)$
QFRERA-HA-IIAP	$\Lambda_{QFRERA-HA-IIAP}(\theta)\mathcal{O}\left(\frac{N_f^3 K_f}{\delta^2}\right)$
QFRERA-IA	$\Lambda_{QFRERA-IA}(\theta)\mathcal{O}\left(\frac{K_f N_f (N_f+1)^2}{\delta^2}\right)$
QFRERA-HA-AIIP	$\Lambda_{QFRERA-HA-AIIP}(\theta)\mathcal{O}\left(\frac{K_f N_f (N_f+1)^2}{\delta^2}\right)$

VII. NUMERICAL RESULTS

In this section, numerical results are presented. We consider an OFDMA-based macro network with $N = 1024$ subcarriers that includes a number of femto networks. We focus on the BS of one femto network which is allowed to use a subset of subcarriers, statically assigned to it, subject to macro network interference constraints. These subcarriers are concurrently used by the femto and macro networks but not other femto networks.[6] We investigate the convergence of the iterative algorithm as well as the effect the proposed robust ERA to stabilize the instantaneous values of femto BS consumed power as well as femto BS inflicted interference on the macro network. scenarios. We also show the impact of different system parameters, particularly collision and power outage probabilities, on the average sum rate of femto service.

In the considered framework, we set $N_f = 128$ unless otherwise stated. Femto users are assumed to be uniformly distributed in a cell of radius $R = 100$ m. We consider $K_0 = 15$ macro receivers which are in the vicinity of femto users with maximum distance $R_u = 1000$ m from the femto transmitters. The shadowing effect follows a log-normal distribution, and the small-scale fading is assumed to be Rayleigh distributed. Moreover, we set $W = 1$ Hz and $N_0 = 1$ Watts/Hz, and the capacity gap $\Gamma = \frac{-1.5}{\ln(\text{BER})} = 0.1937$ where the target BER is set to 10^{-4}. We also assume that $V_{k_f n}$ has a normal distribution with zero mean and variance 1, and the maximum transmit power of the femto base station is set to 1 Watt. The value of Q is set to $N_0 B$ Watts for all macro users over all sub-carriers unless otherwise stated. For the simulations where ζ^p and $\zeta^I_{k_0 n}$ are not variable, we set $\zeta^p = \zeta^I_{k_0 n} = 0.1$, and also $\zeta^I_{k_0 n} = \zeta^I$ for all k_f, u, and n. For the iterative algorithms, we set $\pi^I_n = 100$, $\pi^P = 100$, and $\epsilon = 10^{-4}$.

A. Robustness against instantaneous violation of the ergodic constraints

In this subsection, we investigate the effect of introducing $\mathcal{PO}^f$ and $\mathcal{CO}^f_{k_0 n}$ on the issue of satisfying the instantaneous transmit power constraints for the femto base station and the instantaneous interference constraints for the macro users. We first assume that at $t = 0$, the proposed problems are solved and the Lagrange multipliers are derived for the upcoming time instants. For each new instant of time, based on the Lagrange multipliers at $t = 0$ and new I-CSI and F-CSI values, new allocations are made for power and subcarriers.

[5] Not to be confused with the iteration in our propsed IA method.

[6] We acknowledge that this model can be improved by allowing dynamic subcarrier subset assignments to each femto BS as well as letting femto BS's to share subcarriers. However, this can be addressed in a future work.

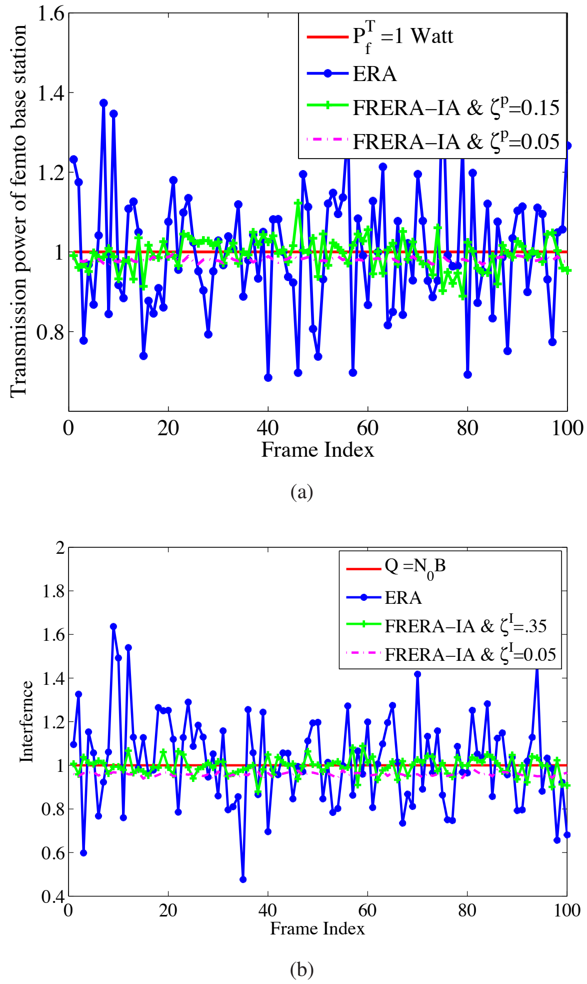

Fig. 1. (a) The impact of $\mathcal{PO}_f$ on the instantaneous femto total transmit power, (b) the impact of $\mathcal{CO}^f_{k_0n}$ on the on the instantaneous interference on macro service.

We start by comparing the instantaneous total transmit power of the femto base station in ERA and FRERA. In Fig. 1(a), FRERA has been solved based on IA for $\zeta^p = 0.15$ and $\zeta^p = 0.05$. As can be seen, in ERA, the transmit power of femto base station cannot be guaranteed instantaneously and shows large scale variations around P^T_f. In other words, with a high probability, the femto base station needs more transmit power than its limit, i.e., P^T_f. By using FRERA-IA, the variation range is reduced compared to that of ERA while the total power still exceeds its maximum with non-negligible probability. Such a probability can be controlled and kept under any desired value. In particular, we have set this value to 0.15 and 0.05. As can be seen, for $\zeta^p = 0.05$, the instantaneous total power is almost a straight line.

Similar results to the previous figure can be obtained for the instantaneous interference caused by femto network on macro network. In Fig. 1(b), we have shown the variations of interference on one subcarrier versus time for ERA and FRERA for $\zeta^I = 0.35$ and $\zeta^I = 0.05$, where FRERA is solved based on the IA method. As can be seen, the imposed interference by femto base station in ERA cannot be guaranteed instantaneously and has large scale variations around Q, meaning that with a high probability, femto base station imposes an interference higher than the threshold on

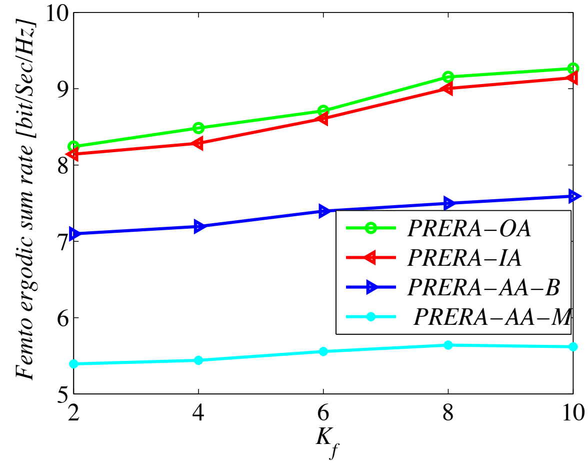

Fig. 2. Femto ergodic sum rate *vs.* K_f for PRERA and QPRERA with different levels of quantization.

macro users. This causes the macro users to experience severe performance degradation. Such a high probability is reduced to 5 and 35 percents in case of FRERA. As can be seen for $\zeta^p = 0.05$, the instantaneous interference is almost always below the desired threshold.

B. Performance comparison of PRERA and QPRERA

In Fig. 2, the performance of PRERA versus the total number of femto users K_F is compared. We have used the iterative approach due to its superior performance as well as the optimal approach as the benchmark. The performance of Bernsetin and Markov method is also demonstrated. It can be confirmed that the IA method results in average sum-rates very close to that of the optimal solution, implying that the proposed iterative approach has a close-to-optimum performance. It is also observed that AA-M has the worst performance and AA-B exhibits a performance worse than IA but better than AA-M. Note that IA and AA-M do not require the knowlge of I-CSI while AA-B needs this information in the transmission phase.

C. Effect of the value of Q on the system performance

In Fig. 3, for QFRERA, we show the femto ergodic sum rate versus different values of $\frac{Q}{N_0B}$. Clearly, by increasing the value of $\frac{Q}{N_0B}$, the femto ergodic sum rate is increased. When $\frac{Q}{N_0B} < 1$, increasing ζ^p from 0.1 to 0.2 has little effect on femto ergodic sum rate. It is because, in this case, the constraint corresponding to interference is the dominant constraint of the optimization problem, and the limit of transmit power of SUs does not affect the optimal value. However, in this case, the effect of increasing ζ^I is considerable.

When $\frac{Q}{N_0B} > 1$, each of $\mathcal{PO}_f$ or $\mathcal{CO}^f_{k_0n}$ can have a dominant effect. Therefore in this case, the best performance occurs when both ζ^p and ζ^I have large values, i.e., 0.2. Also, the worst performance is observed when $\zeta^p = 0.1$ and $\zeta^I = 0.1$.

D. Iterative algorithm and its convergence behavior

In Figs. 4(a) and 4(b), we study the effect of the value of Θ on the performance of iterative and hybrid approaches to

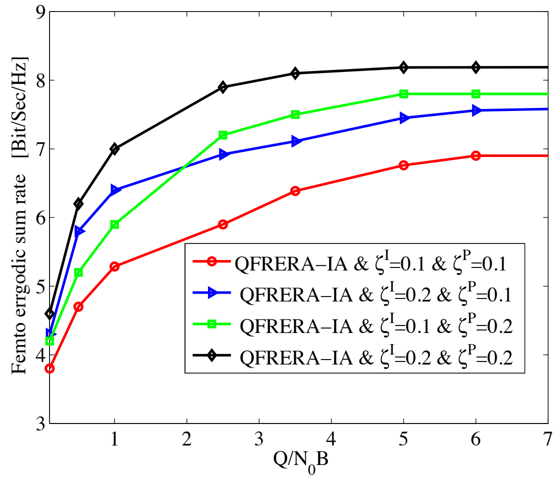

Fig. 3. Femto ergodic sum rate *vs.* $\frac{Q}{N_0 B}$ for QFRERA with different values of ζ^I and ζ^p.

solve different RERA problem. As can be seen, decreasing the value of Θ, increases the femto ergodic sum rate. This increase, however, is not cost free as will be explained later. Figs. 5(a) to 5(d) show the number of iterations required for IA and HA methods to converge when applied to different RERA problems. As can be seen in Figs. 5(a) to 5(d), decreasing the value of Θ, increases the convergence time of IA and HA methods. This is the cost we incur when we need larger sum rates.

These figures highlight the important role of Θ on the tradeoff between performance and convergence time for IA and HA. When a smaller convergence time to reach a solution is required (e.g., in highly dynamic situations such as fast moving users in femto network), larger values of Θ is appealing at the expense of reducing the femto ergodic sum rate.

In the last figure, we slightly modify the framework. We still assume that the macro network has 1024 subcarriers and femto networks can opportunistically use these subcarriers. However, instead of fixing $N_f = 128$ subcarriers for femto network, for a given number of femto BS's, F, we divided the set of 1024 subcarrers into F mutually exclusive subsets, each assigned statically to one femto BS. Then each femto BS can independently use the proposed RERA schemes to optimally allocate power and subcarriers within the assigned subcarrier subset. Therefore, the less the number of femto networks, the larger number of subcarrier out of 1024 can be deployed by each femto. We obtain the sum-rate based on the QIA scheme. As can be seen in Fig. 6, by increasing the number of femto networks, the sum-rate of each femto is decreased which is is not unexpectable.

VIII. Conclusion

In this paper we proposed a robust ergodic resource allocation scheme in the framework of an OFDMA-based underlay heterogenous network. In the proposed scheme, the average based constraints of conventional ERA problems are replaced by probabilistic constrains such that these constraints can be met instantaneously with any desired high probability. Due to the inclusion of the probabilistic constraints, the proposed ERA problems could not be solved directly based on the dual method. Therefore, we proposed an iterative approach to solve them. To reduce the complexity of the proposed iterative approach, we also proposed a suboptimal method called the analytical approach. A hybrid approach was also proposed that provides a trade-off between the good performance of the iterative approach and the low complexity of the analytical approach. We considered both cases of continuous and quantized F-CSI and the proposed solution methods were modified accordingly. In simulation results, we demonstrated the robustness resulting from the proposed scheme on the instantaneous femto total power and on femto interference on the macro service. We also investigated the convergence properties as well as optimality of the proposed iterative approach.

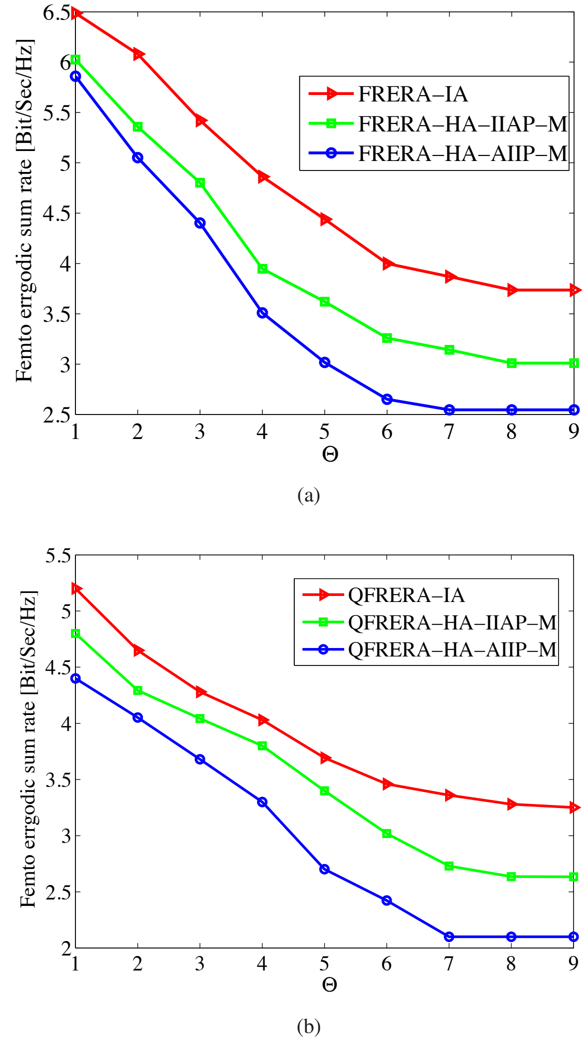

Fig. 4. (a) The impact of Θ on the performance of CRERA, (b) the impact of Θ on the performance of QRERA.

References

[1] V. Chandrasekhar, J. Andrews, and A. Gatherer, "Femtocell networks: a survey," *IEEE Commun. Mag.*, vol. 46, no. 9, pp. 59–67, 2008.

[2] K. Lee, O. Jo, and D.-H. Cho, "Cooperative resource allocation for guaranteeing intercell fairness in femtocell networks," *IEEE Commun. Lett.*, vol. 15, no. 2, pp. 214–216, 2011.

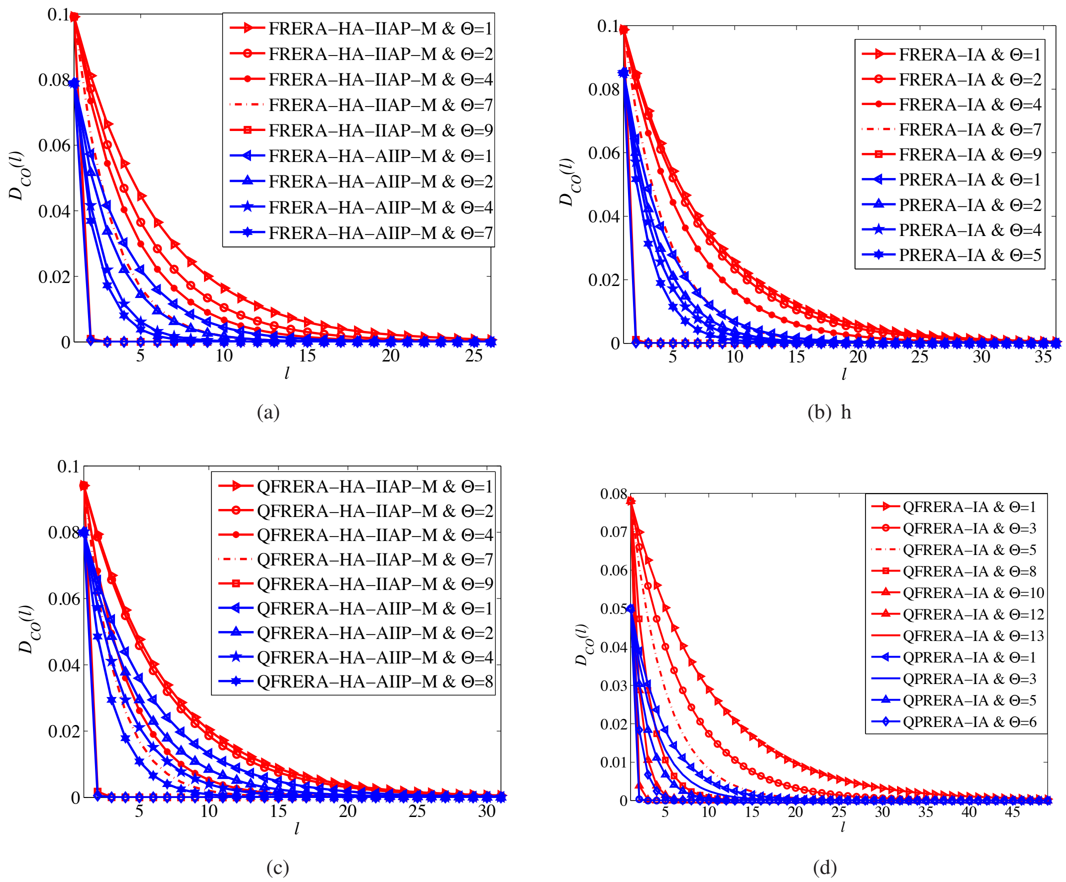

Fig. 5. The effect of the value of Θ on the convergence speed of hybrid and iterative algorithms.

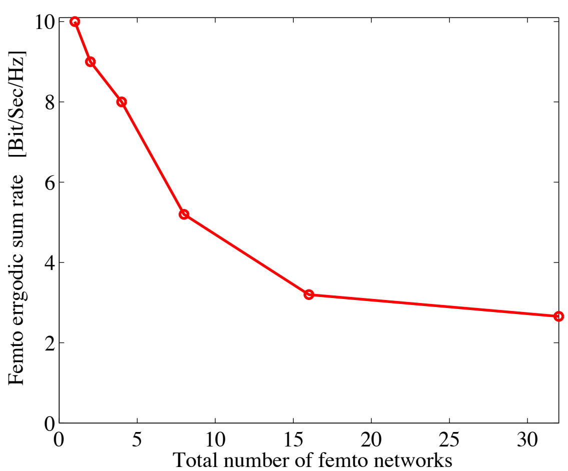

Fig. 6. Femto ergodic sum rate obtained based on QFRERA-IA *vs*. total number of femto networks.

[3] P. Xia, V. Chandrasekhar, and J. G. Andrews, "Open vs. closed access femtocells in the uplink," *IEEE Trans. Wireless Commun.*, vol. 9, no. 12, pp. 3798–3809, 2010.

[4] R. Urgaonkar and M. J. Neely, "Opportunistic cooperation in cognitive femtocell networks," *IEEE J. Sel. Areas Commun.*, vol. 30, no. 3, pp. 607–616, 2012.

[5] S. Wang, Z. H. Zhou, M. Ge, and C. Wang, "Resource allocation for heterogeneous cognitive radio networks with imperfect spectrum sensing," *IEEE J. Sel. Areas Commun.*, vol. 31, no. 3, pp. 464–475, November 2013.

[6] Q. Zhao and B. Sadler, "A survey of dynamic spectrum access: Signal processing, networking, and regulatory policy," *IEEE Signal Processing Mag.*, vol. 24, no. 3, pp. 79–89, May 2007.

[7] "Spectrum policy taskforce report," Federal Communications Commission, Tech. Rep., November 2002, [online] Available: http://hraunfoss.fcc.gov/edocs_public/attachmatch/DOC-228542A1.pdf.

[8] M. G. Khoshkholgh, K. Navaie, and H. Yanikomeroglu, "Access strategies for spectrum sharing in fading environment: Overlay, underlay and mixed," *IEEE Trans. Mobile Computing*, vol. 9, no. 12, pp. 1780–1793, December 2010.

[9] S. Han, B. H. Soong, and Q. D. La, "Power control based on subcarrier exclusion to mitigate downlink cross-tier interference in OFDMA tiered networks," *IEEE Wireless Commun. Lett.*, vol. 2, no. 2, pp. 179–182, April 2013.

[10] S. Haykin, "Cognitive radio: brain-empowered wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 23, no. 2, pp. 201–220, February 2005.

[11] M. Gastpar, "On capacity under receive and spatial spectrum-sharing constraints," *IEEE Trans. Inf. Theory*, vol. 53, no. 2, pp. 471–487, February 2007.

[12] Y. W. Cheong, R. S. Cheng, and K. B. Lataief, "Multiuser OFDM with adaptive subcarrier, bit, and power allocation," *IEEE J. Sel. Areas Commun.*, vol. 17, no. 10, 1999.

[13] R. Zhang, Y. c. Liang, and S. Cui, "Dynamic resource allocation in cognitive radio networks: A convex optimization perspective," *IEEE Signal Processing Mag.*, pp. 102–114, May 2010.

[14] W. W.-L. Li, Y. J. A. Zhang, A. M.-C. So, and M. Z. Win, "Slow adaptive OFDMA systems through chance constrained programming," *IEEE Trans. Signal Process.*, vol. 58, no. 7, pp. 3858–3869, July 2010.

[15] W. Yu and R. Lui, "Dual methods for nonconvex spectrum optimization of multicarrier systems," *IEEE Trans. Commun.*, vol. 54, no. 7, pp. 1310–1322, July 2006.

[16] I. C. Wong and B. L. Evans, "Optimal downlink OFDMA resource allocation with linear complexity to maximize ergodic capacity," *IEEE Trans. Wireless Commun.*, vol. 7, no. 3, pp. 962–971, March 2008.

[17] N. Mokari, P. Azmi, and H. Saeedi, "Quantized ergodic radio resource allocation in OFDMA-based cognitive DF relay-assisted networks,"

IEEE Trans. Wireless Commun., vol. 12, no. 10, pp. 5110–5123, October 2013.

[18] V. N. Ha and L. B. Le, “Fair resource allocation for OFDMA femtocell networks with macrocell protection,” *IEEE Trans. Veh. Technol.*, vol. 63, no. 3, pp. 1388–1401, March 2014.

[19] W. C. Cheung, T. Q. S. Quek, and M. Kountouris, “Throughput optimization, spectrum allocation, and access control in two-tier femtocell networks,” *IEEE J. Sel. Areas Commun.*, vol. 30, no. 2, pp. 561–574, April 2012.

[20] J.-H. Yun and K. G. Shin, “Adaptive interference management of OFDMA femtocells for co-channel deployment,” *IEEE J. Sel. Areas Commun.*, vol. 29, no. 6, pp. 1225–1241, Jun 2011.

[21] I. C. Wong and B. L. Evans, “Optimal resource allocation in the OFDMA downlink with imprefect channel knowledge,” *IEEE Trans. Commun.*, vol. 57, no. 1, pp. 232–241, January 2009.

[22] Z. Li and X. Wang, “Utility maximization over ergodic capacity regions of fading OFDMA channels,” *IEEE Trans. Wireless Commun.*, vol. 11, no. 7, pp. 2478–2485, July 2012.

[23] A. G. Marques, A. B. R. Gonzalez,IEEE Trans. R. Agrawal, J. L. R.-A. lvarez, J. Requena-Carrion, and J. Ramos, “Optimizing average performance of OFDM systems using limited-rate feedback,” *IEEE Trans. Wireless Commun.*, vol. 9, no. 10, pp. 3130–1279, October 2010.

[24] A. G. Marques, G. Giannakis, F. F. Digham, and F. J. Ramos, “Power-efficient wireless OFDMA using limited-rate feedback,” *IEEE Trans. Wireless Commun.*, vol. 7, no. 2, pp. 685–696, February 2008.

[25] D. J. Love, R. W. H. Jr, V. K. N. Lau, R. Berry, D. Gesbert, B. D. Rao, and M. Andrews, “An overview of limited feedback in wireless communication systems,” *IEEE J. Sel. Areas Commun.*, vol. 26, no. 8, pp. 1341–1365, March 2008.

[26] Y. Liu and W. Chen, “Limited-feedback-based adaptive power allocation and subcarrier pairing for OFDM DF relay networks with diversity,” *IEEE Trans. Veh. Technol.*, vol. 61, no. 6, pp. 2559–2571, July 2012.

[27] A. G. Marques, X. Wang, and G. B. Giannakis, “Dynamic resource management for cognitive radios using limited-rate feedback,” *IEEE Trans. Signal Process.*, vol. 57, no. 9, pp. 3651–3666, September 2009.

[28] Y. He and S. Dey, “Power allocation in spectrum sharing cognitive radio networks with quantized channel information,” *IEEE Trans. Commun.*, vol. 59, no. 6, pp. 1644–1656, June 2011.

[29] M. Abdallah, A. Salem, M. S. Alouini, and K. Qaraqe, “Discrete rate and variable power adaption for underlay cognitive networks,” *European Wireless Conference*, pp. 733–737, 2010.

[30] Y. He and S. Dey, “Throughput maximization in cognitive radio under peak interference constraints with limited feedback,” *IEEE Trans. Commun.*, vol. 59, no. 6, pp. 1644–1656, June 2011.

[31] R. Zhang, “On peak versus average interference power constraints for protecting primary users in cognitive radio networks,” *IEEE Trans. Wireless Commun.*, vol. 8, no. 4, pp. 2112–2120, April 2009.

[32] L. Musavian and S. Assa, “Fundamental capacity limits of cognitive radio in fading environments with imperfect channel information,” *IEEE Trans. Commun.*, vol. 57, no. 11, pp. 3472–3480, November 2009.

[33] R. H. Gohary and T. J. Willink, “Robust IWFA for open-spectrum communications,” *IEEE Trans. Signal Process.*, vol. 57, no. 12, pp. 4964 – 4970, December 2009.

[34] A. P. Iserte, D. P. Palomar, A. I. P. Neira, and M. A. Lagunas, “A robust MAXIMIN approach for MIMO communications with imperfect channel state information based on convex optimization,” *IEEE Trans. Signal Process.*, vol. 46, no. 1, pp. 346–360, January 2006.

[35] S. Parsaeefard and A. Sharafat, “Robust distributed power control in cognitive radio networks,” *IEEE Trans. Mobile Computing,*, vol. 12, no. 4, pp. 609–620, April 2013.

[36] R. Zhang, S. Cui, and Y. C. Liang, “On ergodic sum capacity of fading cognitive multiple-access and broadcast channels,” *IEEE Trans. Inf. Theory*, vol. 55, no. 11, pp. 5161–5178, November 2009.

[37] R. Zhang and Y.-C. Liang, “Exploiting multi-antennas for opportunistic spectrum sharing in cognitive radio networks,” *IEEE J. Sel. Topics Signal Process.*, vol. 2, no. 1, pp. 88–102, February 2008.

[38] A. Tajer and X. Wang, “Multiuser diversity gain in cognitive networks with distributed spectrum access,” *IEEE/ACM Trans. Netw.*, vol. 18, no. 6, pp. 1766–1779, December 2010.

[39] X. Gong, S. A. Vorobyov, and C. Tellambura, “Optimal bandwidth and power allocation for sum ergodic capacity under fading channels in cognitive radio networks,” *IEEE Trans. Signal Process.*, vol. 59, no. 4, pp. 1814–1826, April 2011.

[40] H. J. Lim, D. Y. Seol, and G. H. Im, “Joint sensing adaptation and resource allocation for cognitive radio with imperfect sensing,” *IEEE Trans. Commun.*, vol. 60, no. 4, pp. 1091–1100, April 2012.

[41] N. Mokari, K. Navaie, and M. G. Khoshkholgh, “Downlink radio resource allocation in OFDMA spectrum sharing environment with partial channel state information,” *IEEE Trans. Wireless Commun.*, vol. 10, no. 10, pp. 3482–3495, October 2011.

[42] M. G. Khoshkholgh, N. Mokari, and K. G. Shin, “Ergodic sum capacity of spectrum-sharing multiple access with collision metric,” *IEEE J. Sel. Areas Commun.*, vol. 31, no. 11, pp. 2528–2540, November 2013.

[43] A. J. Goldsmith, *Wireless Communications*. Cambridge University Press, 2005.

[44] M. Tao, Y.-C. Liang, and F. Zhang, “Resource allocation for delay differentiated traffic in multiuser OFDM systems,” *IEEE Trans. Wireless Commun.*, vol. 7, no. 6, pp. 2190–2201, June 2008.

[45] Z. Shen, J. G. Andrews, and B. L. Evans, “Adaptive resource allocation in multiuser OFDM systems with rate constraints,” *IEEE Trans. Wireless Commun.*, vol. 4, no. 6, pp. 2726–2737, Nov 2005.

[46] A. Nemirovski and A. Shapiro, “Convex approximations of chance constrained programs,” *SIAM J. Optim*, vol. 17, no. 4, pp. 969–996, 2006.

[47] A. Ben-Tal and A. Nemirovski, “Selected topics in robust convex optimization,” *Math. Prog., Ser. B,*, vol. 112, pp. 125–158, r 2008.

[48] N. Y. Soltani, S. J. Kim, and G. B. Giannakis, “Chance-constrained optimization of OFDMA cognitive radio uplinks,” *IEEE Trans. Wireless Commun.*, vol. 12, no. 3, March 2013.

[49] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.

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