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Performance Improvement of Level-Crossing A/D Converters

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Abstract—**Level Crossing (LC) Analog-to-Digital converters have been suggested as an asynchronous alternative to conventional schemes. It is our intention to improve the performance of these LC converters. In this paper, we also suggest alternative adaptive and multi-level adaptive LC schemes and use an iterative method to drastically improve the performance of LC converters. The impressive improvement of these schemes make LC converters a potential competitor to other conventional A/D converters such as Sigma Delta Modulators (SDM).**

Index Terms— ***Adaptive level-crossing, asynchronous A/D, iterative algorithm, non-uniform sampling***

I. INTRODUCTION

LC converters are asynchronous in nature and provide interesting properties such as low power dissipation, electromagnetic interference reduction and improved Figure-of-Merit compared to conventional A/D's [1]. In addition, the LC converters can match themselves to the varying dynamic range of the input analog signal without any loss of quality while conventional A/D's create more distortion if the dynamic range suddenly changes. Also for remote sensing signals that are almost constant and may vary just during limited moments, asynchronous A/Ds decrease the activity of the circuit in comparison with synchronous A/Ds and therefore lower power consumption [1].For non-bandlimited signals; Nyquist sampling cannot capture the characteristics of the input. LC converters can be more useful in these situations [2].

The architecture of Asynchronous A/Ds based on level-crossing scheme has been proposed in [1] and the performance of the system has been analyzed for a sinusoidal signal. In this paper, the time elapsed between samples is quantized which can cause quantization error propagation. A level-crossing scheme for A/Ds has been discussed in [3] where the non-uniform sampling sequence is transformed to uniform samples by polynomial interpolation, but the simulations are only done on sinusoidal signals. The LC scheme of non-bandlimited signal has been proposed in [2] and compared to uniform sampling the authors have shown that this type of sampling outperforms uniform sampling. In Section 2 we describe non-uniform sampling based on level-crossings [4] and then propose adaptive and multi-level adaptive level-crossing schemes for non-uniform sampling. Section 3 contains an analytical discussion on LC and non-uniform sampling theory [5]. In Section 4, we apply an iterative algorithm [6] to improve the performance of LC schemes. Simulation results and comparisons among proposed schemes are presented in Section 5. Section 6 concludes the paper.

II. LEVEL CROSSING (LC) SCHEMES

For a non-uniform sampling A/D converter, the conversion of samples takes place whenever a reference level is crossed by the continuous time signal. Hence, the amplitude of the sample is precise, but we need some kind of quantization in the time domain. This type of sampling is called Level Crossing (LC) and is shown in Fig. 1. When a reference level is crossed by a continuous time signal, the precise value of level amplitude a_0, and the time difference between two level crossings dt_0 should be transmitted. For digital transmission, dt_n must be quantized. To prevent the quantization error propagation, it is better to calculate time difference due to the quantized time of previous sample $Q(t_{n-1})$ instead of t_{n-1}. $Q(t_{n-1})$ is already known from the quantized values of t_{n-1}.

$$dt_n = t_n - Q(t_{n-1}) \tag{1}$$

The Effective Number of Bits (ENOB) is defined as the number of bits allocated to the quantized dt_n. The parameter ENOB is related to the Time Resolution (TR) to be defined later.

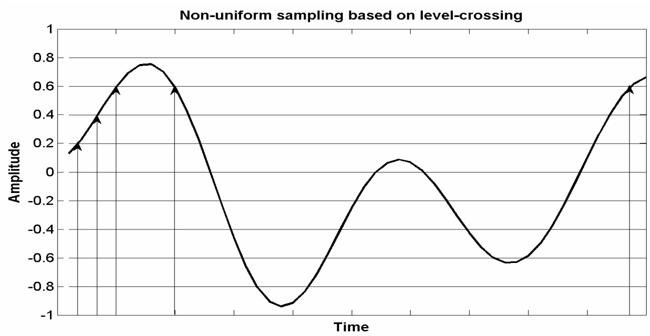

Fig. 1. Non-uniform sampling based on LC

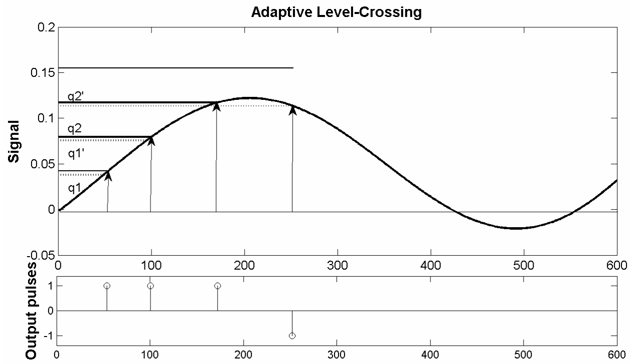

Fig. 2. Non-uniform sampling based on ALC

In LC, we need some information about the dynamic range of the input signal to adjust levels; to overcome this defect adaptive level crossing scheme has been proposed. Adaptive Level-Crossing (ALC) converter is similar to the LC, Fig 2.

This figure shows that there are two levels q_1 and q_2 which are adapted upward or downward depending on whether q_1 or q_2 intersects the signal first. If q_1 crosses the signal first, we send a positive pulse and the two levels are adapted upward and become $q_1^{'}$, $q_2^{'}$, on the other hand if q_2 crosses the signal first, we send a negative pulse and the two levels are moved downwards. The precise values of the levels can be decoded by these pulses, thus we just need to send pulses instead of level values for the LC case. For ALC, two parameters should be determined $d=q_2-q_1$ and $\delta=q_2-q'_1$. The value of d is a constant depending on the variance of the input signal and δ is a small number to assure that the adaptive levels remain in the dynamic range of the signal. To improve the SNR of reconstructed signal, it is better to adapt d with the signal slope. We propose a "Multi-Level Adaptive Level-Crossing" to achieve this. For example, for 2-level ALC, $d=d_1$ when the slope of the signal is high and $d=d_2<d_1$ when the slope is low. Fig. 3 compares the non-uniform samples of ALC and 2-level ALC. This figure shows that the 2-level ALC makes the non-uniform sample more uniform.

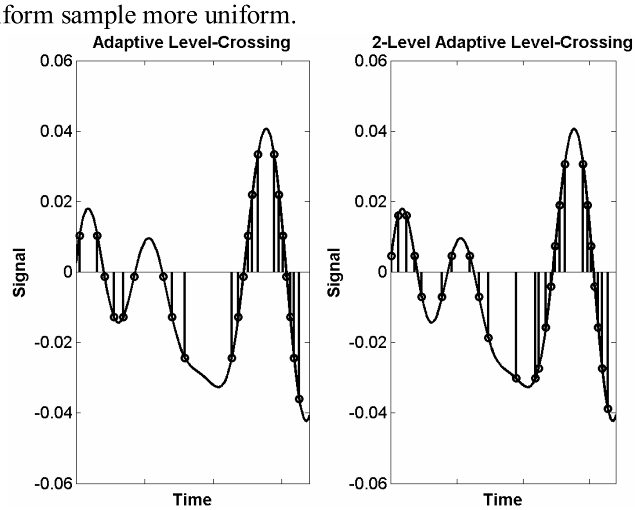

Fig. 3. Samples of ALC and 2-Level ALC

At the decoder (D/A), time indices are generated by accumulating time differences, and then the generated non-uniform samples are linearly interpolated and passed through a LP filter to yield an approximation of the original analog signal.

III. Analytical Discussion Based on Non-Uniform Sampling Theory

We would like to show a relation among the number of levels (N), distances in level-crossings (d), and Over-Sampling Ratio (OSR). To facilitate this analysis, we consider a band-limited Gaussian source with spectral density:

$$S(f) = \begin{cases} A & |f| \leq B \\ 0 & others \end{cases} \tag{2}$$

Since $x(t)$ is a zero-mean Gaussian process with variance σ_x^2 and spectral density $S(f)$, for the derivative $x'(t)$ we have

$$\sigma_{x'}^2 = \int_{-B}^{B} (2\pi f)^2 S(f) df = (2\pi B)^2 \frac{\sigma_x^2}{3} \tag{3}$$

Following the argument similar to Blake and Lindsey [7], for the mean number of level crossings of amplitude a in the interval of T, we have

$$v(a) = \frac{E\{n_a(T)\}}{T} = \int_{-\infty}^{\infty} |x'|\, p(a, x') dx' \tag{4}$$

For a zero-mean Gaussian process $x(t)$, we get

$$v(a) = \frac{2B}{\sqrt{3}} \exp(-\frac{a^2}{2\sigma_x^2}) \tag{5}$$

For $a=nd$ and n = 0, ±1, ±2…, ±N, the total number of crossings can be expressed as below:

$$r = \frac{2f_c}{\sqrt{3}} + \frac{4f_c}{\sqrt{3}} \sum_{n=1}^{N} \exp(-\frac{\alpha^2 n^2}{2}) \tag{6}$$

where α is d/σ. According to the non-uniform sampling theory, if the non-uniform samples satisfy the Nyquist condition on average [4]; the higher the OSR the better is quality of reconstructed signal in practice. Since OSR is the ratio of the sampling rate divided by the Nyquist rate, we have

$$OSR = \frac{1}{\sqrt{3}} (1 + 2\sum_{n=1}^{N} e^{-\frac{1}{2} n^2 \alpha^2}) \tag{7}$$

According to ([4], [5]), typical values of α is $0.01 \leq \alpha = d/\sigma_x \leq 0.5$. For large values of N, OSR can be approximated as:

$$OSR \cong \sqrt{\frac{2\pi}{3}} \frac{1}{\alpha} \tag{8}$$

Considering that $e^{-\frac{1}{2} n^2 \alpha^2} < e^{-\frac{1}{2} n\, \alpha^2}$, for small values of N we have:

$$OSR < \sqrt{\frac{4}{3}} \frac{1 - e^{-\frac{1}{2}(N+1)\alpha^2}}{1 - e^{-\frac{1}{2}\alpha^2}} - \frac{\sqrt{3}}{3} \quad N < \frac{1}{2\alpha} \tag{9}$$

The maximum OSR which is achievable for a specific number of levels N can be calculated from (7), i.e.

$$OSR_{\max} = \frac{1}{\sqrt{3}} (1 + 2N) \tag{10}$$

For N=1, 2 the relation between OSR and α from (7) is depicted in Fig. 4. The OSR, ENOB and TR are related by

$$OSR \geq \frac{TR^{-1}}{2B.2^{N}} \qquad (11)$$

The amplitude error which is caused by time quantization is discussed in [5], the normalized distortion (ND) is given by

$$ND = \frac{\sigma_e^2}{\sigma_x^2} = 0.40 - 20\log_{10}(\frac{TR^{-1}}{B}) \qquad dB \qquad (12)$$

It is evident that the SNR improves with increasing TR^{-1} for a given OSR, N and B. Fig. 4 is the experimental result to show the relation between SNR of the reconstructed signal versus TR^{-1} for the LC, ALC and 2-level ALC.

IV. ITERATIVE ALGORITHM

The iterative algorithm [4] is given by:

$$x_{k+1} = \lambda G\{x(t)\} + (I - \lambda G)\{x_k(t)\} \qquad (13)$$

where λ is the relaxation parameter that determines the convergence rate. $x_k(t)$ is the *kth* iteration and $x_0(t)$ can be any function of time. However, $x_0(t)=G\{x(t)\}$ can be a good choice to achieve faster convergence. In general, G can be either linear or non-linear operator. Defining the operators $\hat{G}=\lambda G$ and $E=I-\hat{G}$, we can rewrite (13) as

$$x_{k+1} = \hat{G}x + (I - \hat{G})x_k \qquad (14)$$

It is straightforward to show that (13) can be written as:

$$x_k(t)=(E^k+E^{k-1}+...+E+I)x_0(t) \qquad (15)$$

If G is a linear operator, we have:

$$E^k+E^{k-1}+...+E+I=\frac{I-E^{k+1}}{I-E} \qquad (16)$$

If the norm of operator E satisfies $||E||<1$, by increasing the number of iterations k, (13) approaches the inverse system $\hat{G}$; hence $x_k(t)$ converges to $x(t)$. In our application, the operator G

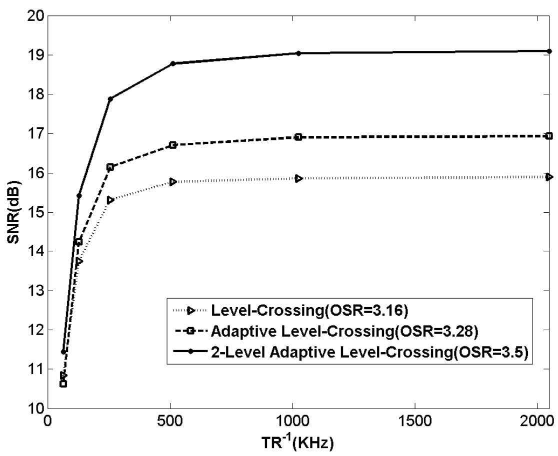

Fig.4. SNRs of the reconstructed signal vs. TR^{-1} for three samplings

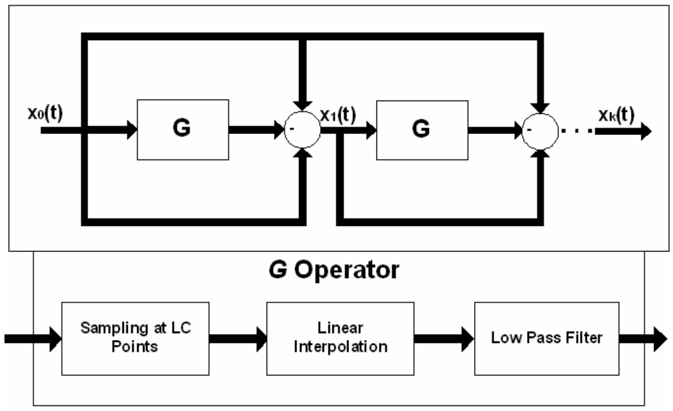

Fig.5. Iterative algorithm and G operator in LC A/D.

consists of sampling at the LC of the received signal and low-pass filtering as shown in Fig. 5.

V. LC-A/D DESIGN USING AN ITERATIVE ALGORITHM

In this section we show that if we use the iterative method depicted in Fig.5, we can get drastic improvement in the 3 schemes we discussed before (Fig. 1-3). In the Matlab simulations, we use zero-mean Gaussian source with a bandwidth of 4 KHz and a 3rd order IIR filter.

For a fair comparison, for the standard LC, the value of N is chosen to fully cover the dynamic range of the input signal and the parameter α is set to be 0.5. In case of ALC, the values of $d=0.5\sigma_x$ and $\delta=d/1000$ are chosen, and for 2-level ALC we set $d_1=0.25\sigma_x$, $d_2=0.5\sigma_x$, and $\delta=d_1/1000$. We observe that by increasing the TR^{-1}, the SNR saturates. The simulation results have been depicted for the three schemes at two TR^{-1} values in Fig. 6. As illustrated, for a given TR^{-1} the iterative algorithm cannot improve the SNR of the reconstructed signal beyond a certain threshold, and we should increase TR^{-1} to achieve higher SNR values. More details for different OSR's are shown in Table .1.

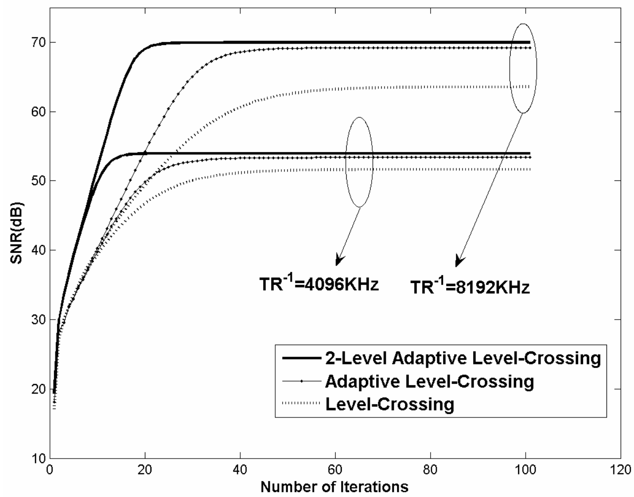

Fig. 6. SNR vs. number of iterations for TR^{-1}=4096, 8192 KHz, and λ=1.4.

TABLE 1: SNRS (DB) OF RECONSTRUCTED SIGNAL FOR DIFFERENT NUMBER OF ITERATIONS, TR^{-1}=8192 KHZ.

Iteration No.	LC		ALC		2-Level ALC	
	OSR=3.16 ENOB=12	OSR=3.97 ENOB=12	OSR=3.16 ENOB=12	OSR=3.97 ENOB=11	OSR=3.1 ENOB=12	OSR=3.97 ENOB=11
0	17.13 dB	18.75 dB	18.11 dB	19.99 dB	20.2 dB	22.5 dB
5	34.96 dB	53.59 dB	33.97 dB	59.81 dB	69.18 dB	69.76 dB
10	41.12 dB	62.49 dB	41.15 dB	69.91 dB	69.81 dB	70.25 dB
20	50.17 dB	68.84 dB	55.80 dB	70.20 dB	69.99 dB	70.25 dB
40	60.89 dB	69.39 dB	70.25 dB	70.20 dB	70.00 dB	70.25 dB
SATURATE	63.56 dB	69.36 dB	70.29 dB	70.20 dB	70.00 dB	70.25 dB

Table 1 shows that for a finite number of iterations, a fixed value for OSR, and finite precision for the TR^{-1}, the 2-Level ALC performs the best and the ALC is better than the LC converter. 2-Level ALC makes the level crossings more uniform hence converges more rapidly. Not surprisingly, the complexities of the encoders are inversely proportional to their performances. Simple LC, unlike the ALC and 2-Level ALC, needs to store the quantized amplitude of each sample and hence requires more internal bandwidth for its implementation.

VI. COMPARISON OF 2- LEVEL ALC WITH SDM A/D

Many commercial A/D converters use SDM. SDM performance can be improved with the iterative method. The difference in the case of SDM is that unlike the LC method, nonuniform samples are not known and hence the iterative method consists of the encoder and decoder to form the distortion operator [8]. This type of iteration is much more complex than the G operator of 2- level ALC. A comparison of the 2 techniques is shown in Table 2.

TABLE 2: COMPARISON OF SNR IN SDM AND 2-LEVEL ALC, CT=COMPUTER TIME PER ITERATION

Itr No.	SDM (OSR=32, q=1bit) CT=4.1sec	SDM (OSR=16, q=2bit) CT=4.19sec	2-Level ALC (OSR=2.48, ENOB=12) CT=0.05sec
0	27.02 dB	20.92 dB	14.50 dB
5	42.21 dB	35.91 dB	33.32 dB
10	43.74 dB	37.39 dB	43.51 dB
20	44.30 dB	37.47 dB	57.00 dB
40	44.60 dB	37.80 dB	63.16 dB

Table 2 shows the SNR values of SDM and 2-Leel ALC for different iteration steps. The Computer Time per iteration (CT) is also shown. It is clear that the 2-Level ALC outperforms both in terms of performance and computational complexity. In this table we assumed that the OSR multiplied by the number of quantizer bits are the same for SDM and 2-Level ALC.

VII. CONCLUSION

In this paper we have evaluated the performance of LC A/D and two proposed adaptive versions using an iterative method. We have shown through simulations that the 2-level LC outperforms the ALC and ALC outperforms the LC method. The simulations show that LC, ALC, and 2-level ALC are potential candidates for A/D converters and may be advantageous compared to SDM where much higher OSR's and complex noise shaping filters are needed. Reduced circuit power consumption and OSR are the potential advantages of the proposed methods. If we apply iterative methods for SDM, because we need to use an inverse system algorithm [8], the overall system becomes much more complex as verified by the computer time.

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