A Novel Preamble-Based Frame Timing Estimator for OFDM Systems

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***Abstract*—This study presents a novel method for frame timing estimation in OFDM systems. The new method is based on the generation and utilization of a new sequence from the two available preambles in the practical OFDM systems (such as WiMAX and WLAN) for correlation, and is capable of having an extended correlation length. The superior performance of the new method is demonstrated in terms of mean square error (MSE) in comparison with the previous methods.**

***Index Terms*—OFDM, preamble, timing, synchronization.**

I. Introduction

ORTHOGONAL frequency division multiplexing (OFDM) as a technique that has been used in wireless systems such as WLAN and WiMAX, is sensitive to timing and frequency offsets. In [1], Schmidl proposed a method for timing synchronization that uses the correlation between the two identical parts of a preamble in the time domain. The timing metric of this method has a plateau that decreases its accuracy. The authors in [2] and [3] tried to reduce this plateau by changing the structure of the Schmidl's preamble. These methods still had either large variances or poor performance in fading channels. The method proposed in [4] estimates the timing offset independent of the preamble structure by making a sequence from the preamble. This scheme is highly vulnerable to the carrier frequency offset (CFO). None of the methods discussed above are capable of using the two available preambles in practical OFDM systems (such as WiMAX and WLAN). In addition, these schemes are unable to use the whole correlation products for timing estimation. In this paper, we propose a method that not only utilizes the two preambles, but also is capable of taking advantage of the whole correlation products between the two preambles. It is shown that the new method considerably improves the performance in comparison with the previous methods.

II. System Description

In the considered OFDM system, two OFDM symbols are sent for synchronization. The overall available bandwidth B is divided into N subchannels each with bandwidth B_{sub}, i.e. $B = NB_{sub}$, and the cyclic prefix (CP) has a length of G sampling periods. The transmitted information symbol in the nth subchannel in the frequency domain for the mth ($m \geq 1$) OFDM symbol is represented by $X_m(n)$, and the time domain signal $x_m(k)$ can be written as

$$x_m(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_m(n) e^{j\frac{2\pi}{N}kn}, \quad m \geq 1 \tag{1}$$

At the receiver, the received signal in the time domain has the form

$$r_m(k) = e^{j\frac{2\pi}{N}k\varepsilon} \sum_{l=0}^{L_h-1} h(l) x_m(k-l-\theta) + z_m(k), \ 0 \leq k \leq N-1 \tag{2}$$

where θ and ε are the timing offset and normalized CFO, respectively, $z_m(k)$ is additive white Gaussian noise (AWGN) with zero-mean and variance σ_z^2, and $h(l),\ l = 0, 1, \cdots, L_h - 1$ represents the channel impulse response with the length L_h.

III. Proposed Method

The proposed method is motivated by two goals: the utilization of both preambles that are available in communications systems such as WiMAX and WLAN for timing synchronization; and most importantly, effective mitigation of noise.

Let $\mathbf{r_1^d} = [r_1^d(0), r_1^d(1), \cdots, r_1^d(N-1)]$ be the first received vector at the time instant d, where $r_1^d(k)$ denotes the kth element of $\mathbf{r_1^d}$, and $\mathbf{r_2^d} = [r_2^d(0), r_2^d(1), \cdots, r_2^d(N-1)]$ be the second received vector with a distance of G samples from the first received vector, respectively. Thus, we have

$$\mathbf{r_1^d} = [r(d), r(d+1), \cdots, r(d+N-1)], \tag{3}$$

$$\mathbf{r_2^d} = [r(d+N+G), r(d+N+G+1), \cdots, r(d+2N+G-1)]. \tag{4}$$

Consider the previous methods such as [1] that use the following correlation function

$$P_{SC}(d) = \sum_{n=0}^{(N/2)-1} \overline{r}(d+k) r(d+k+(N/2)), \tag{5}$$

where $\overline{r}(k)$ is the complex conjugate of $r(k)$. It is observed that methods such as [1], use only the following $N/2$ products from the first received vector for correlation: $\{\overline{r}(d)r(d + (N/2)), \cdots, \overline{r}(d+(N/2)-1)r(d+N-1)\}$.

In the absence of the preamble, i.e. when the first received vector contains only noise samples, the method in [1] uses the correlation between $N/2$ uncorrelated noise samples. When the preamble (which consists of two identical halves in the time domain) is present, [1] obtains the correlation between $N/2$ correlated samples of the received preamble.

In this work, we propose that instead of utilizing $N/2$ products from one preamble for correlation, we use the entire

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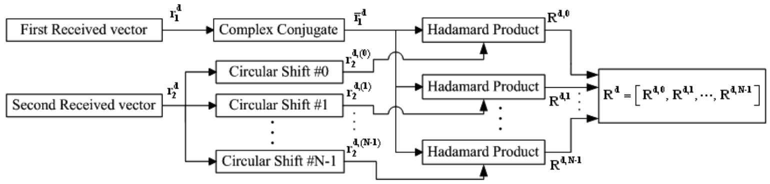

Fig. 1. The block diagram of generation of $\mathbf{R^d}$.

$$\mathbf{R^d} = \Big[\underbrace{\{ \overline{r}_1^d(0) r_2^d(0), \overline{r}_1^d(1) r_2^d(1), \cdots, \overline{r}_1^d(N-1) r_2^d(N-1) \}}_{\mathbf{R^{d,0}}}, \underbrace{\{ \overline{r}_1^d(0) r_2^d(N-1), \overline{r}_1^d(1) r_2^d(0), \cdots, \overline{r}_1^d(N-1) r_2^d(N-2) \}}_{\mathbf{R^{d,1}}}, , \cdots, \underbrace{\{ \overline{r}_1^d(0) r_2^d(1), \overline{r}_1^d(1) r_2^d(2), \cdots, \overline{r}_1^d(N-2) r_2^d(N-1), \overline{r}_1^d(N-1) r_2^d(0) \}}_{\mathbf{R^{d,N-1}}} \Big] \tag{7}$$

products that two preambles provide for correlation. These products are expressed as

$$\begin{aligned} \mathbf{R^d} = \{ & \overline{r}_1^d(0) r_2^d(0), \overline{r}_1^d(0) r_2^d(1), \cdots, \overline{r}_1^d(0) r_2^d(N-1), \\ & \overline{r}_1^d(1) r_2^d(0), \overline{r}_1^d(1) r_2^d(1), \cdots, \overline{r}_1^d(1) r_2^d(N-1), \\ & \overline{r}_1^d(N-1) r_2^d(0), \overline{r}_1^d(N-1) r_2^d(1), \cdots, \\ & \overline{r}_1^d(N-1) r_2^d(N-1) \} \end{aligned} \tag{6}$$

It is observed that the number of these products is N^2, which is far greater than the number of products used in the conventional methods $(N/2)$. The correlation between these products (6) and the corresponding products made from the pure preamble is proposed for timing estimation in this paper. Hence, in the absence of the preambles, the correlation between the N^2 uncorrelated noise samples and the samples of the pure preambles can be obtained and in the presence of the two preambles, the correlation between the N^2 highly correlated samples of the received preamble vectors and the pure preambles are computed. Since the samples of noise and pure preambles are uncorrelated, the difference between the correlation values of N^2 samples in the presence and absence of the two preambles will be considerably greater than the difference between the correlation values of $(N/2)$ samples in the presence and absence of one preamble. Hence, increasing the correlation length can significantly mitigate the noise and improve the performance. In the following, we demonstrate how we can use the products $\mathbf{R^d}$ in (6) for correlation.

The products in (6) can be equivalently written as (7) (for simplicity in formulation and complexity evaluation as demonstrated in section IV). According to (7), $\mathbf{R^d}$ can be expressed in terms of N subvectors as

$$\mathbf{R^d} = [\mathbf{R^{d,0}}, \mathbf{R^{d,1}}, \cdots, \mathbf{R^{d,N-1}}], \tag{8}$$

where $\mathbf{R^{d,n}} = \mathbf{\overline{r}_1^d} \circ \mathbf{r_2^{d,(n)}}$, $n = 0, 1, \cdots, N-1$, $\mathbf{\overline{r}_1^d} = [\overline{r}_1^d(0), \overline{r}_1^d(1), \cdots, \overline{r}_1^d(N-1)]$ and $\mathbf{r_2^{d,(n)}}$ is the vector obtained by circularly shifting of $\mathbf{r_2^d}$ by an amount equal to n,

$$\mathbf{r_2^{d,(n)}} = [r_2^d(N-n), \cdots, r_2^d(N-1), r_2^d(0), \cdots, r_2^d(N-n-1)]. \tag{9}$$

Furthermore, $\circ$ denotes the Hadamard product (element by element multiplication of the two vectors). The block diagram of generation of $\mathbf{R^d}$ is depicted in Fig. 1.

A sequence corresponding to (7) and (8) can be also generated from the pure preamble vectors as follows

$$\mathbf{C} = [\mathbf{C^0}, \mathbf{C^1}, \cdots, \mathbf{C^{N-1}}], \tag{10}$$

where $\mathbf{C^n} = \mathbf{\overline{s}_1} \circ \mathbf{s_2^{(n)}}$, $n = 0, 1, \cdots, N-1$, and $\mathbf{s_m} = [s_m(0), s_m(1), \cdots, s_m(N-1)]$, $m = 1, 2$ is the mth preamble vector in the time domain. Furthermore, $\mathbf{s_2^{(n)}}$ is obtained by circularly shifting of $\mathbf{s_2}$ by an amount equal to n. In the absence of CFO, the correlation between (8) and (10), i.e.

$$P(d) = \left| \sum_{k=0}^{\lambda N-1} R^d(k) \overline{C}(k) \right|, \quad 1 \le \lambda \le N \tag{11}$$

can be used for timing estimation, where λ is the number of subvectors of $\mathbf{R^d}$ that is used for correlation and $L = \lambda N$ is the total length of the utilized correlation.

In the presence of CFO, (11) is severely affected by the frequency offset. To show the effect of the CFO on (11), we consider the samples of the mth received preamble vector in an AWGN channel

$$r_m(k) = e^{j\frac{2\pi}{N}k\varepsilon} s_m(k-\theta) + z_m(k), \; 0 \le k \le N-1, \; m = 1, 2. \tag{12}$$

When the preambles are received $(d = \theta)$, ignoring the noisy terms (since CFO does not change the properties of the noise), the sequence derived from the received signal (7), can be written as (13). It is observed that except the first subvector of $\mathbf{R^d}$, all subvectors of $\mathbf{R^d}$ consist of two parts. Each of these parts and each of the subvectors as indicated in (13) are affected differently by the CFO (they are multiplied by different exponential factors). Hence, the CFO can cause the elements $R^d(k)\overline{C}(k)$, $k = 0, 1, \cdots, L-1$ to add destructively and destroy the peak of $P(d)$ at the start of the frame.

As a solution to this problem, we divide $\mathbf{R^d}$, into several groups (as shown in (13)). The elements of each group have the characteristic of being multiplied by the same exponential factor. After multiplication by the corresponding elements of $\mathbf{C}$, the elements of each group are added together and their magnitudes are obtained. This procedure prevents $R^d(k)\overline{C}(k)$, $k = 0, 1, \cdots, L-1$ from adding destructively. Next, the magnitudes of different groups are added together.

$$\mathbf{R^d} = \mathbf{R}^{\theta} = e^{j\frac{2\pi}{N}G\varepsilon}\Big[\{\underbrace{\overline{s}_1(0)s_2(0), \overline{s}_1(1)s_2(1), \cdots, \overline{s}_1(N-1)s_2(N-1)}_{\mathbf{R^{d,0}}}\},$$
$$\{\underbrace{e^{j\frac{2\pi}{N}(N-1)\varepsilon}\overline{s}_1(0)s_2(N-1)}_{R^{d,1}(0)}, \underbrace{e^{-j\frac{2\pi}{N}\varepsilon}\overline{s}_1(1)s_2(0), e^{-j\frac{2\pi}{N}\varepsilon}\overline{s}_1(2)s_2(1), \cdots, e^{-j\frac{2\pi}{N}\varepsilon}\overline{s}_1(N-1)s_2(N-2)}_{R^{d,1}(1), R^{d,1}(2), \cdots, R^{d,1}(N-1)}\}, \quad (13)$$
$$, \cdots, \{\underbrace{e^{j\frac{2\pi}{N}\varepsilon}\overline{s}_1(0)s_2(1), e^{j\frac{2\pi}{N}\varepsilon}\overline{s}_1(1)s_2(2), \cdots, e^{j\frac{2\pi}{N}\varepsilon}\overline{s}_1(N-2)s_2(N-1)}_{R^{d,N-1}(0), R^{d,N-1}(1), \cdots, R^{d,N-1}(N-2)}, \underbrace{e^{-j\frac{2\pi}{N}(N-1)\varepsilon}\overline{s}_1(N-1)s_2(0)}_{R^{d,N-1}(N-1)}\}\Big]$$

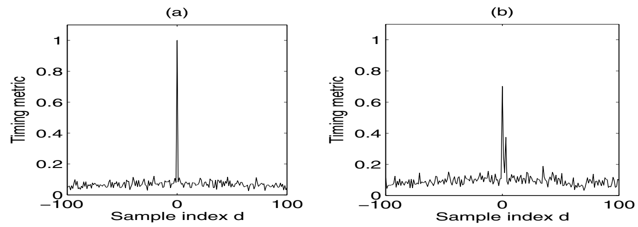

Fig. 2. Proposed timing metric ($N = 256$, $G = 16$, and $L = 4N$), a) under no noise and channel distortion conditions, b) in a 5-tap Rayleigh fading channel

Therefore, the proposed correlation function is obtained as

$$P(d) = \sum_{n=0}^{\lambda-1}\left|\sum_{k=0}^{N-n-1} R^d(k+nN+n)\overline{C}(k+nN+n)\right| + \sum_{n=1}^{\lambda-1}\left|\sum_{k=0}^{n-1} R^d(k+nN)\overline{C}(k+nN)\right|, \quad (14)$$

Finally, the proposed timing metric is expressed as follows

$$M(d) = \frac{P(d)}{R(d)} \quad (15)$$

where

$$R(d) = \sum_{k=0}^{\lambda N-1}\left|R^d(k)\right|^2, \quad 1 \leq \lambda \leq N. \quad (16)$$

In Figs. 2 a and b, we have depicted this metric in two cases of ideal condition (no noise and no channel distortion) and in a multipath fading channel, respectively. Here, the first and second preambles are composed of four and two identical parts in the time domain, respectively. It is obvious that the new metric has an impulse-like shape.

Although, for simplicity, (14) is designed based on the order $\mathbf{R^{d,0}}, \mathbf{R^{d,1}}, \cdots, \mathbf{R^{d,N-1}}$, in (8), in general, any arbitrary order can be utilized. For example, for a correlation length of $L = 2N$, we can use $\mathbf{R^{d,2}}$ and $\mathbf{R^{d,3}}$, instead of $\mathbf{R^{d,0}}$ and $\mathbf{R^{d,1}}$, respectively. In this case, (14) should be changed in the way that the elements with the same exponential factor are added together and then the magnitude is obtained.

IV. Computational Complexity

The complexity of the new method is evaluated by considering its correlation function (14). Without loss of generality, we consider the nth $1 \leq n \leq N-1$ subvcetor of $\mathbf{R^d}$ at the time instant d that is given in (17).

$$\mathbf{R^{d,n}} = \big[\overline{r}_1(d)r_2(d-n+2N+G), \cdots, \overline{r}_1(d+n-1)r_2(d+2N+G-1), \overline{r}_1(d+n)r_2(d+N+G), \cdots, \overline{r}_1(d+N-1)r_2(d-n+2N+G-1)\big] \quad (17)$$

At the time instant $d+1$, $\mathbf{R^{d+1,n}}$ is obtained as (18).

$$\mathbf{R^{d+1,n}} = \big[\overline{r}_1(d+1)r_2(d-n+2N+G+1), \cdots, \overline{r}_1(d+n)r_2(d+2N+G), \overline{r}_1(d+n+1)r_2(d+N+G+1), \cdots, \overline{r}_1(d+N)r_2(d-n+2N+G)\big] \quad (18)$$

It is observed that $\mathbf{R^{d+1,n}}$ can be obtained in terms of $\mathbf{R^{d,n}}$ as shown in (19).

$$\mathbf{R^{d+1,n}} = \big[R^{d,n}(1), \cdots, R^{d,n}(n-1), \overline{r}_1(d+n)r_2(d+2N+G), R^{d,n}(d+n+1), \cdots, R^{d,n}(d+N-1), \overline{r}_1(d+N)r_2(d-n+2N+G)\big] \quad (19)$$

Hence, for calculation of $\mathbf{R^{d,n}}$ at each timing instant, we need two multiplications (with the exception of the $n = 0$ subvector of $\mathbf{R^d}$ that needs only one multiplication). Furthermore, we need N multiplications for multiplying the elements of $\mathbf{R^{d,n}}$ by $\mathbf{C^n}$. Thus, we approximately need $\lambda N + 2\lambda$ multiplications and $\lambda N - 1$ additions for computation of $P(d)$ at each timing instant d, where λ is the number of subvectors of $\mathbf{R^d}$ that is used for correlation.

Reduced Complexity Correlator (RCC)

It is noticed that $P(d)$ has a high computational complexity. To reduce the complexity, we propose that $\mathbf{C}$ in (14) be replaced with $\mathbf{u} + j\mathbf{v}$, where the elements of $\mathbf{u}$ and $\mathbf{v}$ are the sign values of the real and imaginary parts of the elements of $\mathbf{C}$, i.e. $\mathbf{u} = \text{sgn}\left(\text{Re}\{\mathbf{C}\}\right)$ and $\mathbf{v} = \text{sgn}\left(\text{Im}\{\mathbf{C}\}\right)$.

The timing metric using RCC is proposed as

$$M_{RCC}(d) = \frac{P_{RCC}(d)}{R(d)}, \quad (20)$$

where $P_{RCC}(d)$ and $R(d)$ are given in (21) and (16), respectively. In this case, we only need 2λ, $1 \leq \lambda \leq N$ multiplications and $\lambda N - 1$ additions to compute $P(d)$ at each timing instant d.

In Table I, we have demonstrated the complexity of the proposed methods in comparison with the previous methods. It is obvious that RCC has an acceptable complexity that can be determined based on the performance that we expect from the estimator. As will be shown in the next section, the performance of the new estimator is directly related to the total correlation length $L = \lambda N$. Hence, depending on the expected performance, the new estimator can be designed.

$$P_{RCC}(d) = \sum_{n=0}^{\lambda-1} \left| \sum_{k=0}^{N-n-1} R^d(k+nN+n) \left(\overline{u}(k+nN+n) + j\overline{v}(k+nN+n) \right) \right| + \sum_{n=1}^{\lambda-1} \left| \sum_{k=0}^{n-1} R^d(k+nN) \left(\overline{u}(k+nN) + j\overline{v}(k+nN) \right) \right|, \ 1 \le \lambda \le N, \quad (21)$$

TABLE I
COMPUTATIONAL COMPLEXITY OF DIFFERENT ESTIMATORS

Method	**Multiplication**	**Addition**
Schmidl [1]	2	2
Ren [3]	1	$N-1$
Kang [4]	2	$N-1$
Proposed (14)	$\lambda N + 2\lambda,\ 1 \le \lambda \le N$	$\lambda N - 1$
Proposed RCC (21)	$2\lambda,\ 1 \le \lambda \le N$	$\lambda N - 1$

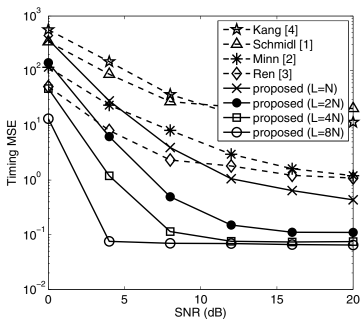

Fig. 3. Timing MSE of the proposed estimator (15) along with the previous estimators in SUI-1 channel ($N = 64$ and $G = 8$).

V. RESULTS

We use computer simulations to assess the performance of the new method. An OFDM system with $N = 64$ subcarriers, the CP length of $G = 8$ samples, and two preambles is considered. The first and second preambles are composed of four and two identical parts in the time domain, respectively. Furthermore, the normalized CFO is set to $\varepsilon = 10.4$. Stanford university interim (SUI) channel modeling [5] has been used to generate a frequency selective channel. The channel is SUI-1 with the sampling rate of 3 MHz.

In Fig. 3, we have depicted the mean square error (MSE) of the proposed timing metric (15) in comparison with the MSEs of previous methods. It is obvious that the proposed method has a significantly better performance than the previous methods and as the correlation length L increases, the performance improves.

Fig. 4 demonstrates the performance of the proposed RCC metric (20) along with those of the previous estimators. Again, it is noticed that the proposed method has a considerably lower MSE in comparison with the previous methods. Furthermore, it is concluded from the comparison of Figs. 3 and 4 that for a fixed correlation length, the proposed metric in (15) has a lower MSE than the RCC in (20).

In Fig. 5, the MSE of different estimators are illustrated in a multipath Rayleigh fading channel with $L_h = 5$ paths, path delays $l = 0, 1, \cdots, L_h - 1$ and the channel delay profile $e^{-(l/L_h)}$. Here, we have $N = 64$ and $G = 12$. It is evident that the proposed method has a remarkably better performance. Note that the performance of the new method might be worse than the previous methods for the same complexity. However, by sacrificing the complexity a great improvement is obtained.

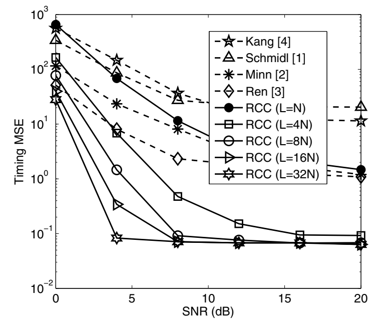

Fig. 4. Timing MSE of the proposed RCC (20) along with the previous estimators in SUI-1 channel ($N = 64$ and $G = 8$).

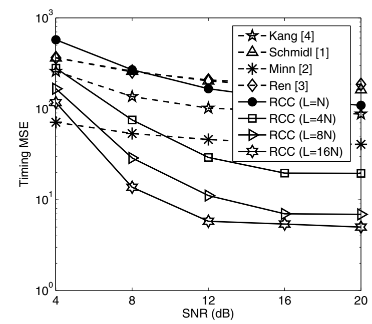

Fig. 5. Timing MSE of the proposed RCC (20) along with the previous estimators in a 5-tap Rayleigh fading channel ($N = 64$ and $G = 12$).

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