

Blind Iterative Nonlinear Distortion Compensation Based on Thresholding

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Abstract—The sampling process in electrical devices includes nonlinear distortion that needs to be compensated to boost up the system efficiency. In this brief, a blind method is suggested for nonlinear distortion compensation. The core idea is to leverage the sparsity of the signal to cope with the ill-posedness of the distortion compensation task. The proposed scheme is an iterative method based on out of support energy minimization, in which the support information is not available. An adaptive thresholding operator is used to give a rough approximation of the support according to the estimated signal at each iteration. Various simulation scenarios have validated the capability of the suggested scheme in compensating the miscellaneous distorting functions and have confirmed its superiority over the other techniques.

Index Terms—Blind distortion compensation, iterative thresholding, nonlinear distortion compensation, sparsity.

I. INTRODUCTION

THE SAMPLING process in most of the applications includes some sort of nonlinear distortion that is due to the saturation stages of the electrical devices. The nonlinear distortions occur in satellite communications [1], amplifying systems [2], audio signal processing [3], electro-optical transducers [4], endoscopic video images [5], and electro-chemical transducers [6]. Compensating the nonlinear distortions would boost up the efficiency of the systems.

In supervised distortion compensation techniques [7], the distorting function is known or can be estimated with the aid of some training inputs. The distorting function should be identified using a system identification technique [2]. Then, an inversion scheme should be applied to recover the nondistorted signal. In some of the applications, the distorting system is not available or the training inputs cannot be used due to the high costs or the time consuming process. In those cases, the blind distortion compensation techniques shall be used. The blind compensation schemes aim to recover the nondistorted signal from its distorted version. Hence, the distortion compensation

problem would be ill-posed and some prior information is required to solve it.

In [8], the bandlimitedness of the original signal is used as the prior information. The out of band energy of the signal is minimized to compensate for the nonlinear distortion. The drawback of this scheme is that it assumes the bandwidth of the signal as known. Although this side information may be available in some communicational applications, it is an unrealistic assumption in some others. In [9], the application of the nonlinear distortion compensation in source separation of postnonlinear models has been investigated. In [10], the sparsity of the signal has been considered as a prior information. The sparsity solution of the ill-posed nonlinear distortion compensation problem is considered as the recovered signal.

The sparsity property refers to the case where most of the signal entries are zero in some transform domain [11], [12]. The main focus of the recent researches has been drawn on the effect of sparsity over ill-posed linear distorting operators [13], [14]. Many solutions have been suggested in this scope for the linear sparse recovery problem and different theoretical aspects of it have been investigated [15]–[18]. The nonlinearity in sparse recovery problems is an issue that has not yet been thoroughly looked into and many aspects of it still remain untouched.

What we suggest here is another sparsity-based blind nonlinear distortion compensation technique. The idea is to minimize the out of support energy of the signal, while the signal support is not available beforehand. However, what we have as a prior knowledge is that the signal is sparse in some domain such as discrete Fourier transform (DFT) or discrete cosine transform (DCT). To exploit this information, we apply an adaptive thresholding operator on the signal transform in the sparsity domain. We suggest to start from an initially large support for the signal that would be decreased by increasing the threshold value, iteratively. At each iteration, a part of the distortion is compensated by minimizing the energy out of the obtained support for that iteration. In this way, the outcome of each iteration is less-distorted compared to that of the previous one and the algorithm converges to the nondistorted signal after a number of iterations.

The simulation results confirm that our suggested scheme outperforms the state-of-art methods in the quality of the recovered signal. The proposed method is capable of compensating different kinds of distorting functions applied on signals with various sparsity rates, while its counterparts fail in compensating some special kinds of nonlinearities. Moreover,

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the proposed method has shown acceptable robustness against additive noise.

The rest of this brief is organized as follows. Section II gives an overview of the problem modeling. The proposed method is illustrated in Section III. The simulation results are reported in Section IV. Section V gives conclusive remarks on the work.

II. PROBLEM MODELING

Suppose that we have the continuous signal $x_c(t)$ that is distorted with the memoryless nonlinear function $f(\cdot)$. Hence, the distorted signal $y_c(t)$ can be represented as

$$y_c(t) = f(x_c(t)). \quad (1)$$

Since the function $f(\cdot)$ is nonlinear, there would be nonzero coefficients in its power series expansion that is obtained as

$$f(x) = \sum_{i=1}^{\infty} f_i x^i. \quad (2)$$

As the function $f(\cdot)$ becomes more nonlinear, the more number of f_i 's would be nonzero. Combining (1) and (2), we would have

$$y_c(t) = \sum_{i=1}^{\infty} f_i x_c(t)^i. \quad (3)$$

The above relation in the frequency domain would be translated to

$$Y_c(f) = f_1 X_c(f) + f_2 X_c(f) * X_c(f) + \dots \quad (4)$$

By convolving two signals, their supports would be added. Hence, (4) indicates the fact that the nonlinear distortion increases the support of the distorted signal. Sampling the distorted function, we would have the discrete signal $y(\cdot)$ as

$$y(k) = y_c(t) \Big|_{t=\frac{k}{F_s}} \quad (5)$$

where F_s is the sampling frequency. The power series expansion of the compensating function, $g(\cdot) = f^{-1}(\cdot)$, would be

$$g(y) = \sum_{i=1}^P w_i y^i. \quad (6)$$

The compensated discrete signal $s(k)$ can be obtained as

$$s(k) = g(y(k)) = y(k) + \sum_{i=2}^P y(k)^i w_i(k). \quad (7)$$

The goal of distortion compensation task is to obtain the w_i coefficients from the distorted signal $y(k)$ and reconstruct the compensated signal $s(k)$.

III. PROPOSED ITERATIVE THRESHOLDING METHOD FOR OUT OF SUPPORT ENERGY MINIMIZATION

In this section, the proposed iterative thresholding method for out of support energy minimization (ITOSEM) is illustrated for the unsupervised compensation of the nonlinear distortion. In this method, the sparsity property of the underlying signal is exploited to address the blind nonlinear distortion compensation problem. As mentioned earlier, the nonlinear distortion

increases the number of nonzero entries of the distorted signal. Hence, the support of the signal is extended. Being aware of this fact, we minimize the out of support energy of the signal to obtain the recovered signal. However, the support of the signal is not known beforehand. The prior information that we want to take advantage of is the sparsity of the signal that is imposed on the recovered signal with the aid of an adaptive thresholding operator. Therefore, we start from a sufficiently large initial support that is decreased iteratively by lowering the threshold value. Minimizing the energy out of the initial support, we obtain an approximation of the original signal that is still distorted. In the next iteration, we aim to gradually reduce the distortion by repeating the same process for the approximated signal with decreased support. Iteration by iteration, the distortion is decreased and the recovered signal approaches the original one.

Now, we want to describe the energy minimization task conducted at each iteration of our proposed algorithm. Suppose that we want to obtain the N -dimensional compensated signal vector \mathbf{c} from the distorted signal vector \mathbf{d} , where the signal support is assumed to be Λ . The energy minimization task is conducted as follows [10]: The compensated signal vector in the transform domain is obtained as

$$\mathbf{v} = \Psi \mathbf{c} \quad (8)$$

where

$$\mathbf{c} = [c(1), c(2), \dots, c(N)]^T \quad (9)$$

and the matrix Ψ indicates for the transform matrix of the domain in which the nondistorted signal is sparse, e.g., DFT or DCT. The vector \mathbf{v} is consisted of two disjoint parts: 1) \mathbf{v}_1 , which contains the entries of \mathbf{v} inside the support set Λ and 2) \mathbf{v}_2 , which includes the entries of \mathbf{v} out of that support. Thus, we have

$$\begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \end{bmatrix} = \begin{bmatrix} \Psi_1 \\ \Psi_2 \end{bmatrix} \mathbf{c} \quad (10)$$

where Ψ_1 includes the rows of Ψ that correspond to the entries in Λ and Ψ_2 contains the rest. In other words, for the $N \times N$ matrix Ψ and the set Λ with $|\Lambda| = m_2$, we define the projection operator $P_\Lambda : C^{N \times N} \rightarrow C^{m_2 \times N}$ in a way that $P_\Lambda(\Psi)$ removes the rows of Ψ corresponding to the indices in set Λ and keeps the rest. In this way, we would have: $\Psi_2 = P_\Lambda(\Psi)$. Moreover, rewriting (7) over N time slots, we can obtain the compensated signal vector, \mathbf{c} , as

$$\mathbf{c} = \mathbf{d} + \mathbf{U}\mathbf{w} \quad (11)$$

where

$$\mathbf{U} = \begin{bmatrix} d(1)^2 & \dots & d(1)^P \\ d(2)^2 & \dots & d(2)^P \\ \dots & \dots & \dots \\ d(N)^2 & \dots & d(N)^P \end{bmatrix} \quad (12)$$

and

$$\mathbf{d} = [d(1), d(2), \dots, d(N)]^T \quad (13)$$

and

$$\mathbf{w} = [w_2, w_3, \dots, w_P]^T. \quad (14)$$

It should be noted that the distorting function (and its inverse) is assumed not to change over the N time slots; therefore, the vector \mathbf{w} would be the same for all N time slots. The aim is to estimate \mathbf{w} from the vector \mathbf{d} . To this end, we would minimize the out of support energy of the compensated vector. The out of support signal \mathbf{v}_2 can be written as

$$\mathbf{v}_2 = \Psi_2 \mathbf{c} = \Psi_2 (\mathbf{d} + \mathbf{U}\mathbf{w}). \quad (15)$$

Minimizing the energy of \mathbf{v}_2 with least squares minimization, we obtain

$$\mathbf{w} = -((\Psi_2 \mathbf{U})^T \Psi_2 \mathbf{U})^{-1} (\mathbf{d}^T \Psi_2^T \Psi_2 \mathbf{U}). \quad (16)$$

According to (11), the compensated signal can be obtained from the estimated \mathbf{w} . The above procedure is repeated at each iteration of our proposed scheme. Hence, at iteration i of the proposed algorithm, we replace: $\mathbf{c} = \mathbf{s}^i$, $\mathbf{d} = \mathbf{s}^{i-1}$, $\Lambda = \Omega^i$, $\Psi_2 = \Gamma^i$ (defined in Algorithm 1, line 13), and $\mathbf{U} = \mathbf{U}^{i-1}$. The initial value for the compensated signal is $\mathbf{s}^0 = \mathbf{y}$. As mentioned earlier, a large initial support set is considered at the first iteration. Then, the support set is decreased gradually by iteration. The definition of the support sets is based on the sparsity property of the original signal in the transform domain. The support sets of different iterations Ω^i are obtained by a thresholding operator applied on the transform coefficients where the threshold value increases iteration by iteration. Hence, Ω^i is the set of indices of the transform coefficients ($\Psi \mathbf{s}^{i-1}$) that are absolutely greater than $\theta(i)$, which is increasing with the iteration number. The steps of the proposed ITOSEM method are illustrated in Algorithm 1, where the thresholding operator, $T(x, \theta)$, is defined as

$$T(x, \theta) = \begin{cases} 0 & \text{if } |x| < \theta \\ x & \text{if } |x| \geq \theta. \end{cases} \quad (17)$$

At each iteration of this algorithm, the support set Ω^i is obtained through a thresholding operation in the transform domain (line 12). Then, the out of support entries are selected by the projection operator $P_{\Omega^i}(\cdot)$ (line 13). The vector \mathbf{w} is estimated according to the out of support part of the signal (line 14). A regularization parameter μ has been added to ensure the invertibility of the matrix. The approximation of the compensated signal is updated (line 15). In order to increase the robustness of the method against noise, a thresholding operator is applied at the end of each iteration (lines 16 and 17) which eliminates a small portion of negligible entries in Ψ domain. The denoising parameter α could be set to any value less than one based on the estimated noise power. After a number of iterations, the compensated signal is obtained.

IV. SIMULATION RESULTS

In this section, the simulation results are reported.

A. Initialization

The random data set is generated as follows. For each of the sparsity rates, a set of 100 random sparse signals of size $N = 1000$ are generated. The nonzero locations are selected uniformly at random and the magnitudes of those entries are

Algorithm 1 ITOSEM Method

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1: input:
2: An initial support set  $\Omega^0$ .
3: A distorted signal  $\mathbf{y} \in \mathbb{R}^N$ .
4: The maximum number of iterations  $iter_{max}$ .
5: The denoising parameter  $\alpha$  ( $\alpha < 1$ ).
6: The regularization parameter  $\mu$ .
7: output:
8: The compensated signal  $\hat{\mathbf{s}} \in \mathbb{R}^n$ .
9: procedure ITOSEM( $\mathbf{y}, \mathbf{s}$ )
10:  $\mathbf{s}^0 \leftarrow \mathbf{y}$ 
11: for  $i=1:iter_{max}$  do
12:    $\Omega^i \leftarrow \{j|v = \Psi \mathbf{s}^{i-1}, |v_j| > \theta(i)\}$ 
13:    $\Gamma^i \leftarrow P_{\Omega^i}(\Psi)$ 
14:    $\mathbf{w}^i \leftarrow -(\mu \mathbf{I} + (\Gamma^i \mathbf{U}^{i-1})^T \Gamma^i \mathbf{U}^{i-1})^{-1} (\Gamma^i \mathbf{s}^{i-1})^T \Gamma^i \mathbf{U}^{i-1}$ 
15:    $\mathbf{s}^i \leftarrow \mathbf{U}^{i-1} \mathbf{w}^i + \mathbf{s}^{i-1}$ 
16:    $\mathbf{d}^i \leftarrow T(\Psi \mathbf{s}^i, \alpha \theta(i))$ 
17:    $\mathbf{s}^i \leftarrow \Psi^T \mathbf{d}^i$ 
18: end for
19: return  $\hat{\mathbf{s}} \leftarrow \mathbf{s}^{iter_{max}}$ 
20: end procedure

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TABLE I
DISTORTING FUNCTIONS

$F_1(x)$	$\arctan(5x)$	$F_4(x)$	$\tanh(3x)$
$F_2(x)$	$\begin{cases} 2x + 0.5 & x < -0.5 \\ 2x - 0.5 & x > 0.5 \\ x & \text{otherwise} \end{cases}$	$F_5(x)$	$\ln(1 + x/2)$
$F_3(x)$	$\frac{3}{1 + 2\exp(-5x)} - 1$	$F_6(x)$	$\begin{cases} \tanh(3x) & x > 0 \\ \operatorname{atan}(5x) & x < 0 \end{cases}$

obtained according to a random Gaussian distribution of zero mean and unit variance. The inverse DCT of the generated sparse vectors are produced as the signal vectors. Hence, the signals of our synthetic dataset are sparse in DCT domain ($\Psi = \text{DCT}$ matrix). In all of our simulations, we have set the regularization parameter $\mu = 0.1$. The maximum number of iterations, $iter_{max}$, is set to 40. The threshold values, $\theta(i)$, are chosen in the following manner. At first iteration, 90% of the signal coefficients pass the threshold. Then, at each iteration, this value is decreased by 1%. In a way that after 40 iterations (the last iteration), 50% of the coefficients remain above the corresponding threshold value. In other words, over 40 iterations, m_2 is changed from $0.9N$ to $0.5N$ with a step of $0.01N$. The proposed nonlinear distortion compensation technique is compared to the method in [10] as a benchmark. The parameter σ in [10] has been set to 0.01. The input signals are distorted by the five distorting functions that are given in Table I. The listed distorting functions in the table are depicted in Fig. 1 to elucidate their different nonlinearities. The functions include the odd-symmetric ones such as $F_1(x)$, $F_2(x)$, and $F_4(x)$, and the nonsymmetric ones such as $F_3(x)$, $F_5(x)$, and $F_6(x)$. What we require from the distorting function is its reversibility that can be guaranteed by strictly increasing or strictly decreasing property. For the functions simulated in this brief, without loss of generality, we have considered

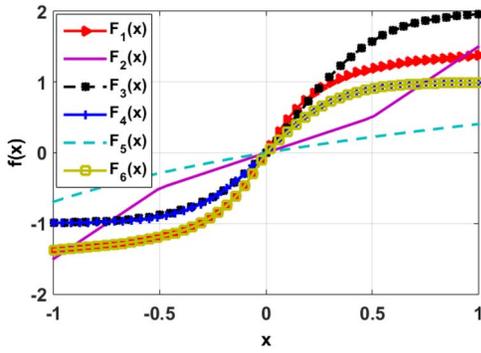


Fig. 1. Distorting functions.

TABLE II
MEAN SDR FOR $P = 5$

function	method	sparsity rate				
		0.1	0.2	0.3	0.4	0.5
$F_1(x)$	ITOSEM	33.48	33.33	33.30	33.26	30.73
	[10]	27.48	27.66	27.60	27.43	27.66
$F_2(x)$	ITOSEM	31.37	31.40	31.37	31.40	31.45
	[10]	18.05	18.27	18.22	18.12	18.37
$F_3(x)$	ITOSEM	30.54	30.68	31.08	30.38	30.78
	[10]	10.62	10.41	9.71	8.64	6.77
$F_4(x)$	ITOSEM	32.53	32.76	32.29	32.00	31.54
	[10]	22.11	22.35	22.28	22.10	22.39
$F_5(x)$	ITOSEM	82.64	83.60	81.27	81.16	77.20
	[10]	17.49	17.72	17.64	17.60	17.78
$F_6(x)$	ITOSEM	21.03	21.14	20.90	20.60	19.64
	[10]	10.82	10.96	10.91	10.86	10.99

the strictly increasing case. The performance measure that we mainly use is the signal-to-distortion-ratio (SDR) that is defined as

$$\text{SDR} = 10 \log \left(\frac{\mathbf{s}^T \mathbf{s}}{(\mathbf{s} - \widehat{\mathbf{s}}^*)^T (\mathbf{s} - \widehat{\mathbf{s}}^*)} \right) \quad (18)$$

where $\widehat{\mathbf{s}}^*$ indicates for the normalized compensated signal. The normalization is conducted as: $\widehat{\mathbf{s}}^* = K \widehat{\mathbf{s}}$ where $K = \text{argmin}_k (\mathbf{s} - k \widehat{\mathbf{s}}^*)^T (\mathbf{s} - k \widehat{\mathbf{s}}^*)$.

B. Random Signal

The mean SDR of the compensated signal over the set of 100 random signals for the six distorting functions and different sparsity rates are given in Table II. In this table, we depict the results for the maximum power of $P = 5$ in Taylor series expansion in (6). According to this table, for all of the distorting functions and sparsity rates, the mean SDR of our method is considerably more than that of the method in [10]. In the case of $F_1(x)$, $F_2(x)$, and $F_4(x)$, our method manifests 6, 13, and 10 dB improvement over its counterpart. Moreover, the method in [10] fails in compensating the distortions caused by $F_3(x)$ and $F_6(x)$ by offering very low mean SDR of around 10 dB. This observation shows that the proposed method has superior performance compared to the method in [10] in the case of odd-symmetrical functions. The reconstructions of $F_5(x)$ show higher mean SDR compared to those of the other distorting functions since this function, as depicted in Fig. 1,

TABLE III
MEAN SDR OF THE NOISY DATA WITH SPARSITY RATE OF 0.2 FOR $P = 5$

function	method	SNR (dB)					
		5	15	25	35	45	55
$F_1(x)$	ITOSEM	5.10	8.48	18.46	27.18	31.70	32.71
	[10]	5.24	10.99	18.47	25.16	27.31	27.63
$F_2(x)$	ITOSEM	5.62	15.03	24.64	30.44	31.81	32
	[10]	2.89	9.81	16.05	17.99	18.26	18.27
$F_3(x)$	ITOSEM	4.76	10.42	19.82	27.81	31.88	32.89
	[10]	5.12	9.46	10.54	10.66	10.68	10.68
$F_4(x)$	ITOSEM	5.20	9.36	18.44	26.93	32.67	34.15
	[10]	5.49	11.63	18.19	21.67	22.28	22.34
$F_5(x)$	ITOSEM	5.62	14.96	25.48	35.69	45.52	55.11
	[10]	5.95	13.30	16.98	17.64	17.71	17.72
$F_6(x)$	ITOSEM	4.84	8.44	17.07	21.58	22.34	22.41
	[10]	5.09	9.38	10.75	10.94	10.96	10.96

is much closer to a linear distorting function with roughly fixed slope.

C. Performance in the Presence of Measurement Noise

In this part, we study the noise issue and investigate how the two distortion compensation schemes behave in the presence of noise. For this test, we add the additive white Gaussian noise to the distorted signal as

$$\mathbf{y} = f(\mathbf{x}) + \mathbf{n} \quad (19)$$

and then we apply the compensation schemes for the signal reconstruction. In this scenario, the set of 100 random signals with sparsity rate of 0.2 is simulated. The mean SDRs of the compensated signal for various values of input signal to noise ratio (SNR) are given in Table III. For the low SNR values ($\text{SNR} < 20$), we set $\alpha = 0.9$ and for the higher SNRs ($\text{SNR} > 20$), we set $\alpha = 0.5$. This is because in the case of low SNR, the noise power is large and we need to have a strong denoising scheme that leads to a larger thresholding parameter. According to this table, the proposed method offers higher values of mean SDR in almost all of the cases. This scenario confirms the efficiency of the suggested scheme even in the presence of noise.

D. Speech Signal

A sparse speech signal was simulated by sparsifying a speech utterance sourced from the TIMIT speech corpus. The speech signal, sampled at 16 kHz, is first normalized to increase its amplitude and then thresholded in the DCT domain to limit its sparsity rate to 0.4, i.e., 40% of its largest magnitude DCT coefficients are kept and the rest are set to zero. The sparse speech signal is distorted by the nonlinear distorting functions of Table I. The SDR curves of the proposed method and the method in [10] for $P = 5$ and $T = 20$ are depicted in Fig. 2. According to this figure, the proposed method outperforms its counterpart in the case of speech distortion compensation. In order to have a subjective evaluation of the efficiency of the proposed method, the distorted and compensated signals together with the original signal are depicted in Fig. 3. The same speech signal has been distorted

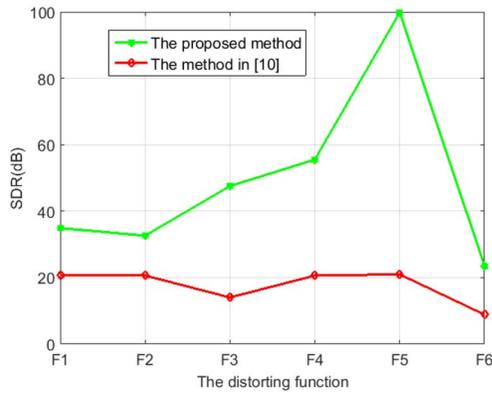


Fig. 2. SDR comparison of the sparse speech signal with sparsity rate of 0.4 for different distorting functions.

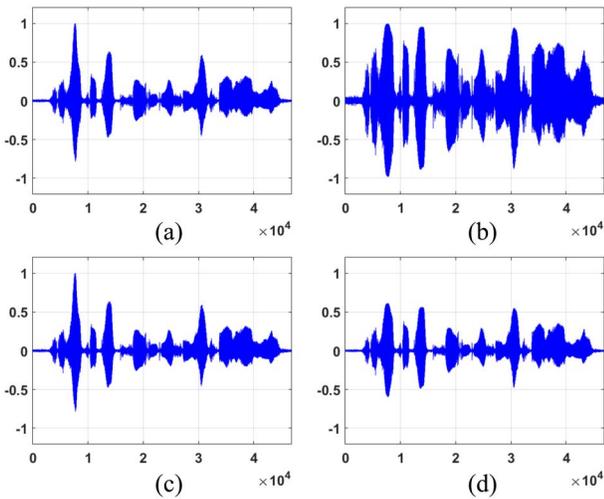


Fig. 3. (a) Original signal. (b) Signal distorted by $F_4(x)$. (c) Signal compensated by our method. (d) Signal compensated by the method in [10].

by the function $F_4(x)$ and recovered with the proposed algorithm and the benchmark. As depicted in the figure, we observe that the suggested method has obviously better compensation compared to its counterpart. The recovered signal using the proposed method is subjectively similar to the original one.

E. Effect of Residual Elements

In this section, we investigate the performance of the suggested method in the case of compressible signals, which have very small entries that are not necessarily zero. In the dataset of this scenario, the negligible nonzero entries (residuals) are chosen according to the Gaussian distribution with zero mean and variance of α . The simulation results for different values of α and various distorting functions are depicted in Table IV. According to this table, in most of the cases, the ITOSEM method exhibits better performance in the compensation of compressible signals with negligible entries.

V. CONCLUSION

A nonlinear distortion compensation technique called ITOSEM is suggested in this brief. The proposed method attempts to enforce the sparsity constraint on the signal by

TABLE IV
MEAN SDR OF THE COMPRESSIBLE SIGNAL WITH SPARSITY RATE OF 0.4 FOR $P = 5$

α	method	function					
		$F_1(x)$	$F_2(x)$	$F_3(x)$	$F_4(x)$	$F_5(x)$	$F_6(x)$
0.01	ITOSEM	30.82	31.26	30.62	31.29	47.92	20.48
	[10]	27.43	18.12	10.64	22.09	17.6	10.85
0.02	ITOSEM	27.49	30.42	28.65	28.93	39.28	20.24
	[10]	27.39	18.1	10.64	22.05	17.57	10.85
0.03	ITOSEM	23.34	28.75	25.71	25.34	34.05	17.91
	[10]	27.31	18.06	10.61	22	17.55	10.84

minimizing its out of support energy. Different simulation scenarios indicate that the suggested scheme is superior to the other state-of-the-art method in almost all of the cases.

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