A Novel DFT-Based Method for Clipping Noise Suppression in OFDM Systems

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***Abstract*-- It is well known that clipping the OFDM signals in digital part of the transmitter is one of the simplest methods to reduce the peak to mean envelop power ratio. However, it suffers from additional clipping distortion, peak regrowth after digital to analog conversion, and out-of-band distortion. Recently, to combat the effect of in-band distortion and peak regrowth, it is proposed that before clipping, oversampling is performed by padding the modulating sequence with zeros. In this paper, we propose a robust DFT-based method (DBM) to reconstruct the clipped samples and mitigate the clipping distortion in the presence of channel noise at the expense of bandwidth expansion. We show through extensive simulations that by slightly increasing the bandwidth of the system, we can significantly improve the performance while limiting the maximum of the analog signal. Furthermore, we compare the performance of the DBM and the channel coding methods. It can be seen that for lower bandwidth expansions, the DBM outperforms the channel coding methods at moderate SNR values while for higher bandwidth expansions, the channel coding methods seem to be more efficient. Furthermore, we introduce a hybrid system which outperforms both the DBM and the channel coding methods at most SNR values**.

I. Introduction

Orthogonal frequency division multiplexing (OFDM) is one of the most promising approaches to achieving high-rate data transmission in wireless communication systems. However, OFDM signals exhibit a large Peak to Mean Envelope Power Ratio (PMEPR) which requires a linear amplifier; otherwise, the signal may suffer from significant spectral spreading and in-band distortion [1]-[2].

Several alternative solutions have been proposed to reduce the PMEPR of OFDM signals. Clipping the OFDM signal in digital part of the transmitter seems to be the simplest method. However, digital clipping suffers from three problems: in-band distortion, which degrades the bit-error-rate (BER) performance [1], out-of-band radiation, which reduces the spectral efficiency [2]-[3], and peak regrowth after digital to analog conversion, which results in an increase of PMEPR [1] and [4]-[7].

If clipping is performed directly, the resulting clipping noise will spread into in-band frequency and cannot be reduced by filtering. To address this aliasing problem, Li and Cimini [1] oversampled each block of OFDM by padding the original input with zeros and they showed that clipping of oversampled sequence, results in PMEPR improvement for sufficiently high values of clipping threshold (PMEPR of greater than 4) with negligible performance degradation. By further decreasing the threshold value, considerable power penalty must be paid. Furthermore, oversampled sequence clipping has also been proposed to reduce the peak regrowth after digital to analog conversion. The peak regrowth problem after digital to analog conversion has been analyzed analytically in [4], and [7] and also through simulations in [1] and [5]-[6] and oversampling rate of 4 is shown to be sufficient to bind the peak of the continuous signal.

In OFDM systems with oversampled sequence, one way to compensate the performance degradation is to reconstruct the clipped samples. This way was first proposed by Henkel in [8] and [9]. In this case, clipped samples are considered to be the lost ones that should be reconstructed by the other samples. There are different methods of reconstructing the lost samples of an oversampled signal that those are fully discussed in [10]-[14]. Meanwhile, some of the reconstructing methods, such as Reed Solomon (RS) decoding, used by Henkel in OFDM systems with oversampled sequence in [8] and [9], are very sensitive to additive noise [12] produced by channel and in-band distortion, which may result in instability of the reconstruction algorithm especially in OFDM wireless communication systems.

In this paper, we propose to use a robust DFT-base method (DBM) to compensate the SNR degradation due to digital clipping for low values of clipping threshold. In this method, we suggest the least square method to reconstruct the clipped samples which is a robust method against additive channel noise. But, our simulation results show that in a clipped OFDM system, because of considerable amount of in-band distortion, by completely removing the out-of-band components of the signal, even the LS method fails to reconstruct the lost samples and therefore we should not completely remove out-of-band components. On the other hand, results of extensive simulations also show that it is not necessary to save all the out-of-band radiation caused by clipping. It will also be shown that by increasing the system bandwidth from 1.25 to at most 2 times, one can improve the BER performance while lowering the clipping threshold value. The improvement becomes more significant for higher

SNR values when the major part of BER is due to clipping noise rather than the channel noise.

As it stated, in the DBM there is a tradeoff between bandwidth expansion (BW) and clipping distortion mitigation capability. It is well known that another method which addresses this trade-off is to employ conventional channel coding methods to improve performance degradation at the expense of bandwidth expansion (channel coding method) [15]. In this paper, we will also compare BER performance for the DBM and the channel coding methods. It can be seen that for lower bandwidth expansions, the DBM outperforms the channel coding method at moderate SNR values while for higher bandwidth expansions, the channel coding methods seem to be more efficient. Furthermore, we will introduce a hybrid system which outperforms both the DBM and the channel coding methods at most SNR values.

The paper is organized as follows; Section II introduces the OFDM signals and PMEPR factor to measure the amplitude fluctuation and then describes a typical system of interest. In Section III, the DBM is briefly discussed. Simulation results are presented in Section IV. In Section V, the comparison between the DBM and the channel coding method is considered, and finally, Section VI concludes the paper.

II. Definitions And System Description

The complex envelope of an OFDM signal at time t with N subcarriers can be expressed as follows,

$$S(t)=\sum_{n=0}^{N-1} c_n e^{j2\pi n f_0 t} \quad , \ 0 \le t \le T, \tag{1}$$

where f_0 is the subchannel spacing, T is the symbol period, and c_i is a complex representation of the subcarrier symbols produced form a chosen constellation set. Consequently, the transmitted OFDM signal will be the real part of that signal after up conversion to the carrier frequency f_c and can be written as,

$$G(t)=\mathrm{Re}\left\{\sum_{n=0}^{N-1} c_n e^{j2\pi(f_c+nf_0)t}\right\} \quad , \ 0 \le t \le T, \tag{2}$$

The amount of amplitude fluctuation of OFDM signals may be measured in terms of the ratio of the peak power of the envelope signal, to the average envelope power (PMEPR) of the signal. More specifically, the PMEPR of the signal is defined as [2],

$$PMEPR \equiv \underset{(c_0,\ldots,c_{N-1}),0\le t<T}{Max} \frac{|S(t)|^2}{P_{av}} \quad , \tag{3}$$

where P_{av} is equal to $E\{\|(c_0,c_1,\ldots,c_{N-1})\|^2\}$ which is a constant that depends on the constellation size and N (number of subcarriers).

In this paper, we consider a typical OFDM system with 32 subcarriers (N=32) and 16 QAM constellation in which oversampled sequence clipping is used to reduce PMEPR for oversampling rate of 4. As shown in Fig 1, oversampling is performed in the transmitter by padding the modulating vector, $C=(c_0,c_1,\ldots,c_{N-1})$, with $3N$ zeros, and then taking $4N$ point IFFT. The amplitude of base band oversampled signal is then clipped by a hard-limiter with characteristics as,

$$g\left(|x|e^{j\phi}\right)=\begin{cases}|x|e^{j\phi} & |x|\le A\\ Ae^{j\phi} & |x|>A.\end{cases} \tag{4}$$

As it was stated in section I, in order to reconstruct the clipped samples, we should save part of the out-of-band radiation caused by clipping. Therefore, we use an ideal filter to remove a portion of out-of-band components. The major part of out-of-band energy can be preserved with just 50% bandwidth expansion. Therefore, we consider a system with variable bandwidth expansions and we will discuss the effect of bandwidth expansion on the performance improvement of the DBM.

III. Structure Of The DBM

Oversampling and reconstructing a band-limited signal are equivalent to encoding and decoding in the channel coding methods [12] and [16]. It means that as long as the average rate of the samples of a signal is above the Nyquist rate, lost samples can be recovered based on the other samples. Below, we will have a brief discussion on the proposed DBM that we employ to reconstruct the lost samples of a signal in the presence of channel noise.

It is well known that taking FFT of an arbitrary vector X with length N is equivalent to multiplying the vector by a square matrix A whose elements are $a_{mn}= exp(-j2\pi mn/N)$. Thus we have,

$$\mathrm{FFT}(X)=AX \ . \tag{5}$$

To briefly describe the DBM, assume that arbitrary number of elements of FFT(X) say K, are zero. If K samples of X are lost and other samples are exactly available, we can recover the lost samples by solving a linear system of K equations and K unknowns. This idea is a basic principle for various methods of recovering the lost samples of a signal discussed in [10]-[13]. But it should be noted that in OFDM system, because channel noise and in-band distortion changes the values of all available samples, sensitivity to additive noise is a major issue [13].

Now suppose that the number of lost samples say P is less than the number of padded zeros ($P<K$). Methods discussed in [10]-[13] use a linear system with P equations and P unknowns for lost sample recovery and therefore, sensitivity to noise will be a major issue and the reconstruction process remains unstable for low SNRs.

To keep up with such a limitation, we try to incorporate all the padded zeros in calculating the lost samples by using the least square method. In this case, we have a system with K equations (number of padded zeros) but P unknowns (number of lost samples) where $K>P$. In order to achieve the optimum solution for this system, we consider the least square method (LS) for over-determined systems. In this method, instead of finding an exact solution for the equation system,

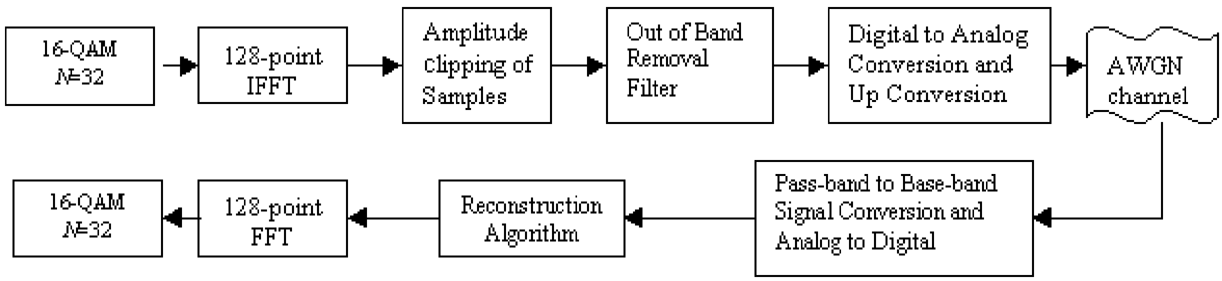

Fig. 1. OFDM transmitter and receiver architecture with digital clipping and DFT-Based Method

$$TX_L = b, \tag{6}$$

we try to solve the following optimization problem,

$$\hat{X}_L = \arg\left\{\underset{X}{Min}\left(\|TX - b\|^2\right)\right\}, \tag{7}$$

where b and T are $K\times 1$ and $K\times P$ matrices which can be found by rearranging the K equations that we have from our information about the zero padded elements in FFT*(X)*. In our simulation results, the LS method is implemented by pseudo-inverse algorithm in the MATLAB implementation.

According to our simulation results, using the LS method, the decoding process is much more stable. In this case, the more the number of padded zeros with respect to the number of lost samples is, the more stable the reconstruction process will become. Furthermore, as mentioned in Section II, in order to reconstruct the clipped samples, we should not omit all the out-of-band components of the clipped signal. In fact, if all the out-of-band part of the signal is removed, we will have only a low-pass version of the clipped signal and there will not be accurate non-clipped samples for recovering the clipped samples.

IV. Simulation Results

In order to present the performance of the DBM method, we over-sample the signal with the rate of 4, therefore up to 75% of sample loss can be tolerated. Moreover, as the PMEPR clipping threshold is at least 2, approximately 10% of the samples are clipped. Therefore, the number of remaining samples is sufficiently above the Nyquist rate and the decoding process will be robust against the additive noise. For example, we are able to reconstruct the clipped samples successfully even for the SNR values around 10db.

In this section, we present the simulation results for the system shown in Fig.1. In Fig.2, we use the PMEPR clipping threshold of 2 and BER performance for the system is shown for various SNR values. In order to study the effect of bandwidth expansion on BER improvement, the simulation is done for three bandwidth expansions of 25%, 50%, 100%, and also for the case without any reconstruction. Obviously, by increasing the SNR, we have more performance improvement. The reason is that in high SNR values, the major part of BER is due to clipping distortion rather than the channel noise. To gain a better insight, consider the system at BER of 10^{-3} for SNR of 16 dB. By using the reconstruction method for bandwidth expansion of %25, %50 and %100, we have 3.5, 4 and 5 dB improvement, respectively. For higher SNRs, this may increases to 7 dB and more by only % 25 bandwidth expansions. This improvement is also valid for clipping at PMEPR threshold of 3 as shown in Fig. 3. As it is shown, for BER of 10^{-5} (or SNR of 18 dB) by using the DBM for bandwidth expansion of %25, %50 and %100, we will have 2, 3 and 3.5 dB improvement, respectively.

It is worth mentioning that by eliminating the clipping distortion for large values of SNR, we have significant SNR improvements even for 25% bandwidth (BW) expansion as shown in Fig. 2. Also, by further decreasing the threshold value, we will have more out-of-band radiation and in order to save its major components, we need to have more bandwidth expansion, which is not attractive. Besides, as the number of clipped samples increases, the reconstruction process becomes more unstable.

Fig 4 shows the Complementary Cumulative Distribution Function (CCDF) of PMEPR (Pr{PMEPR>x}) for PMEPR clipping threshold of 2 and 3 and also for the case without clipping. Evidently, the peak regrowth is not a severe problem and the peak value, in the worst case, is less than twice of the PMEPR threshold. This is consistent with the results of [1],[6], and [7] for oversampling rate of 4.

V. Comparison Between the DBM and the Channel Coding Method

In previous sections, it was shown that in the DBM, there is a tradeoff between bandwidth expansion (BW) and clipping distortion mitigation capability. Another method which addresses this trade-off is to employ conventional channel coding methods to improve performance degradation at the expense of bandwidth expansion (channel coding method) [15]. In this section, we will compare BER performance for the DBM and the channel coding methods.

At the receiver, in the DBM, the reconstruction algorithm is performed before taking FFT. After 16-QAM demodulation, we will have our received bit stream. In the channel coding method, we do not have reconstruction procedure. Instead, after 16-QAM demodulation, the decoding is applied to the received encoded bits.

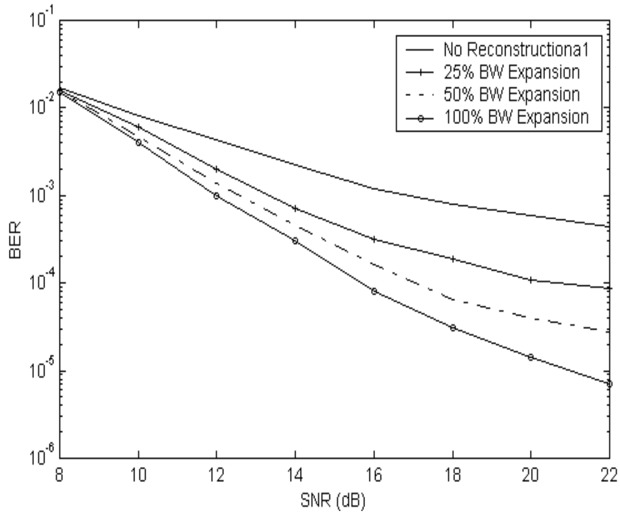

Fig. 2. BER versus SNR for PMEPR clipping threshold of 2 and for various BW expansions.

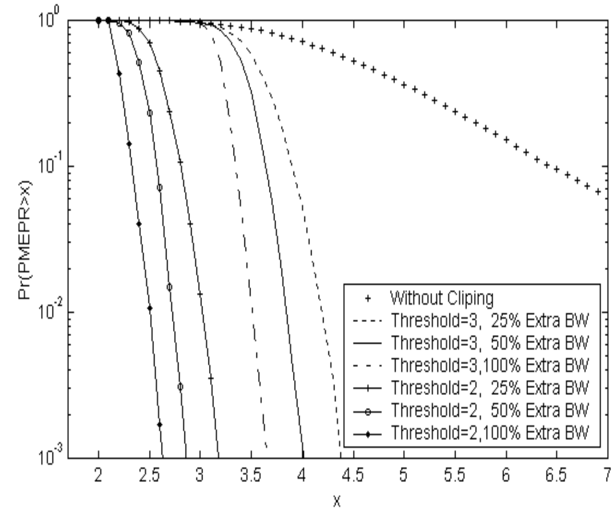

Fig. 4. The CCDF of PMEPR after digital clipping and without clipping for various BW expansions.

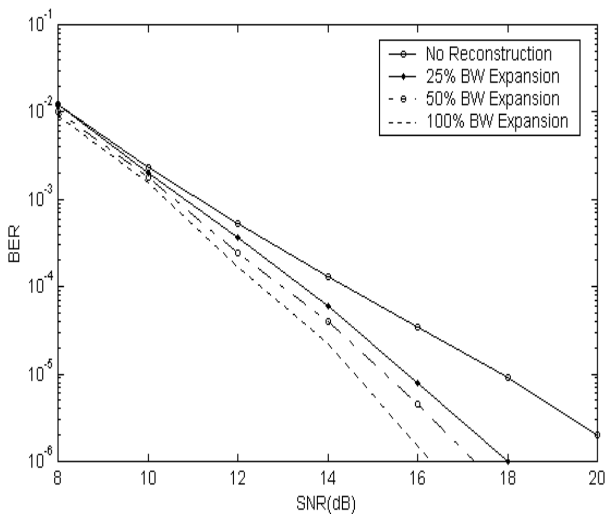

Fig. 3. BER versus SNR for PMEPR clipping threshold of 3 and for various BW expansions.

Table I. Normalized computational complexities for the DBM and the channel coding methods

Rate	DFT-Based Method	BCH	Convolutional
1/2	1	1.5	2
2/3	1	1.1	2
4/5	1	0.9	2

To compare, we present the simulation results for performance evaluation of the DBM and the channel coding method for different bandwidth expansions of 2, 1.5 and 1.25 (or different coding rates of 1/2, 2/3 and 4/5). In channel coding method, we use BCH codes with length 127 and message length of 64, 85 and 99. We use a 1/2 convolutional code with generator octal sequence of [15,17]. To generate 2/3 and 4/5 convolutional codes, we punctured the same 1/2 convolutional code [17]. It should be noted that these codes are chosen so that they have nearly equal encoding-decoding complexity with the DBM. The computational complexity, measured and normalized based on MATLAB simulation time, is shown in table 1.

In figures 6, 7 and 8, the performance results are shown for bandwidth expansion of 2, 1.5 and 1.25 respectively. It can be seen that for lower signal to noise ratio (SNR) values, the channel coding method has better performance. The reason is that at these values of SNR, the major BER is due to channel noise rather than clipping distortion. The DBM works only to suppress clipping noise while channel coding works for distortions caused by both channel noise and clipping. By increasing SNR, the DBM curve approaches coding curves and for higher SNR values, the DBM outperforms the channel-coding method.

The crossing points for the rate of 1/2 are 19 and 18.5 and for the rate of 2/3 are 16 and 12 dB with BCH and convolutional codes, respectively. At the rate of 4/5, these points move to 15 and 10dB. It can also be seen that for lower bandwidth expansions, the DBM outperforms the channel coding method at moderate SNR values while for higher bandwidth expansions, the channel coding methods seem to be more efficient.

In fig. 7, we have also shown the performance of a hybrid system in which we have combined both the channel coding and the DFT-Based method. In this method, we considered a system using the DBM while adding channel encoding and decoding block before 16-QAM modulation and after 16-QAM demodulation, respectively (Fig. 9). We used BCH

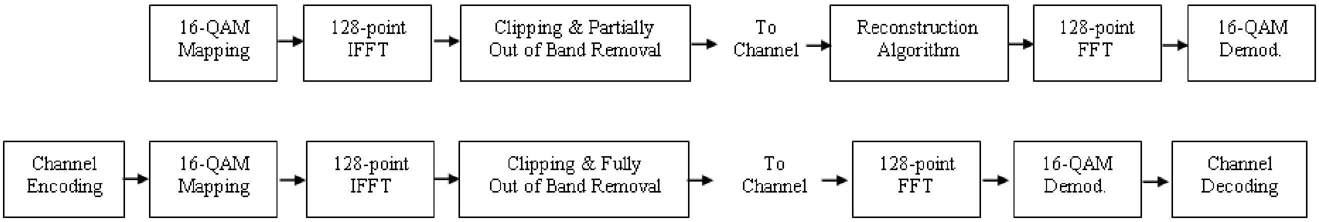

Fig. 5. Block diagrams of transmitter and receiver for the DFT-Based and the channel coding method

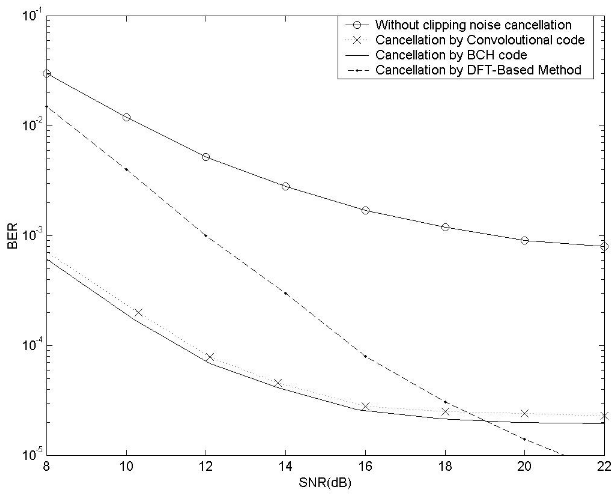

Fig. 6. BER versus SNR for the DFT-Based and the channel coding methods with BW expansion of 2 (or coding rate of 1/2)

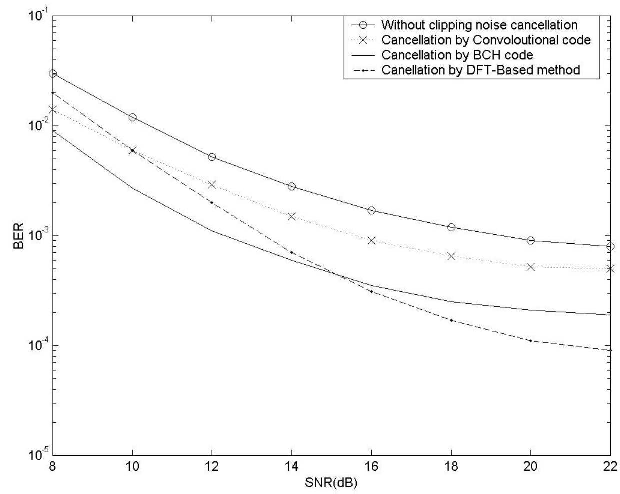

Fig. 8. BER versus SNR for the DFT-Based and the channel coding methods with BW expansion of 1.25 (or coding rate of 4/5)

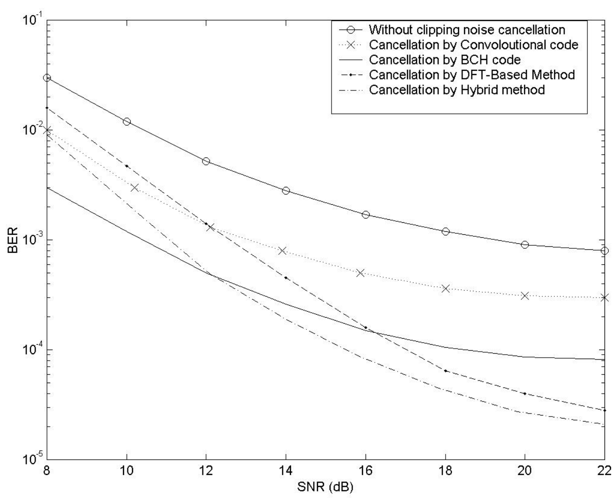

Fig.7. BER versus SNR for the DFT-Based and the channel coding methods with BW expansion of 1.5 (or coding rate of 2/3)

(127,106) code over the DBM system with 1.25 bandwidth expansion so that the total expansion of bandwidth is equal to 1.5. It can be seen that the hybrid system outperforms both the DBM and the channel coding methods at SNR values greater than 12dB. This is achieved at the expense of increasing the complexity.

V. Conclusions

In this paper, a robust DFT-based method (DBM) to reconstruct the clipped samples and mitigate the clipping distortion in the presence of channel noise at the expense of bandwidth expansion has been proposed. In the proposed method, by using the LS method for oversampled signal reconstruction, we removed the clipping distortion. Moreover, it has been shown that in the DBM, there is a tradeoff between bandwidth expansion and clipping distortion mitigation. It is shown that by only 25% extra bandwidth, we can significantly

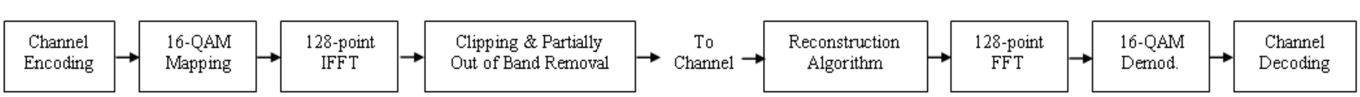

Fig. 9. Block diagrams of transmitter and receiver for the hybrid method

improve the BER performance of the system for clipping threshold of 2. It is also observed that the maximum of the peak regrowth is lower than twice of the clipping threshold.

Furthermore, the performances of the DBM and the channel coding method have been compared. It has been shown that for higher SNR values, the DBM has better performance. Finally we introduced the hybrid system which outperforms both the DBM and the Channel coding methods for SNR values greater than 12 db.

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