Interference Suppression for Space-Time Coded CDMA via Decision-Feedback Equalization

Mohammad H. Taghavi N. and Babak H. Khalaj

***Abstract*—A single-user receiver structure is proposed for space-time coded code-division multiple-access (CDMA) downlink in a multiuser frequency-selective channel. This structure is a two-dimensional (2-D) decision-feedback equalizer (2D-DFE) whose filters are optimized based on the MMSE criterion to mitigate noise, intersymbol interference (ISI), and multiuser interference (MUI) with a moderate complexity. By modeling the spreading codes of the interfering users as random sequences, system performance was evaluated using the Gaussian approximation. Two models for the desired user's spreading sequence have been considered and compared. Our numerical results show that in both cases the 2D-DFE exhibits significant performance improvement over the standard space-time coded RAKE, especially in interference-limited conditions. It is also observed that the gain obtained by using DFE in a MISO channel is less that in a SISO channel and this problem can be solved by providing diversity at the receiver.**

***Index Terms*—Decision-feedback equalization, multiuser interference (MUI), RAKE, transmit diversity, wideband code-division multiple-access (CDMA).**

I. Introduction

A MAJOR factor limiting the capacity of wireless channels is the unreliability due to slow fading, which is generally treated by increasing the degrees of freedom in the system, especially by means of multiple antenna diversity. In code-division multiple-access (CDMA) systems, the large signal bandwidth facilitates the employment of other types of diversity. However, in third-generation (3G) CDMA systems, the processing gain at high data rates can be as low as four, making spatial diversity very effective. Consequently, various spatial diversity methods, including Alamouti's space-time block code [1], have been proposed and adopted for 3G CDMA systems [2], especially for high data rate, indoor applications.

In CDMA systems, multiuser interference (MUI) and intersymbol interference (ISI) caused by multipath dispersion are other major problems, which cannot be well suppressed by conventional receivers such as RAKE. Most known space-time coding techniques were originally designed for point-to-point links, without considering the multiuser characteristics of the channel. Hence, for achieving the benefits of spatial diversity in multiuser channels, either new diversity schemes should be developed [3] or the existing methods should be efficiently combined with interference cancellation techniques. In the latter case, it will generally be inevitable to lose some of the features of the single-user schemes in order to make a balance between fading and MUI.

The generalization of multiuser detection (MUD) algorithms for space-time coded systems has been studied, e.g., in [4] and [5]. However, MUD schemes suffer from high computational complexity that makes them infeasible for many applications, especially at mobile units. As lower complexity solutions, several variations of chip equalization [6] are generalized for transmit diversity, e.g., in [7], most of them applicable only on multiple-antenna receivers. Among other works, in [8], a linear adaptive receiver has been proposed that filters the RAKE fingers before space-time decoding. Also, in [9], the performance of an MMSE structure along with a MAP channel decoder for forward error correction is compared for different space-time codes in CDMA downlink.

Many equalization techniques have also been proposed for space-time coding over TDMA or other systems where MUI is not a major concern [10]. Among these works, Lindskog *et al.* proposed an extension of Alamouti's code that achieves the full path and space diversity [11]. However, in order to implement this scheme in CDMA, time-reversal should be performed in the chip-level, which is infeasible, because reversing the spreading code and applying the reversed channel impulse response at the receiver can break its orthogonality to other codes, unless all the users use space-time coding and operate at equal rates. Furthermore, the spreading code changes from one symbol to another due to scrambling, which destroys the structure exploited in [11].

In this paper, we propose and analyze a two-dimensional MMSE decision-feedback equalizer for space-time coded WCDMA. In this structure, as a generalization of the 1D-DFE proposed in [12], the received signal is first passed through a spreading code matched filter and then sampled at multiples of chip rate before entering the DFE. The feed-forward filter minimizes the effects of noise, MUI, and ISI, while the feedback filter cancels the remainder of ISI. The necessity of the decision-feedback is inspired by the fact that at low processing gains, where transmit diversity is generally used, ISI can be comparable to MUI, hence complete cancellation of the ISI can result in a reasonable performance improvement.

The advantage of the proposed receiver over similar two-dimensional equalizers proposed for space-time coding is that it compromises between diversity and interference cancellation, and it can be implemented in both synchronous and asynchronous systems with either single- or multiple-antenna receivers. As a single-user receiver, the 2D-DFE does not require prior knowledge of the spreading codes of other users, and can

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be easily applied in multirate systems. Two cases have been considered for implementing the 2D-DFE. In the first case, the desired user's spreading code is explicitly used for deriving the equalizer parameters, so the equalizer should be updated as the spreading code changes over different symbols. In the second case, spreading codes of all users are considered as random sequences, thus reducing the computational complexity. As will be shown, both structures provide the highest gains in high data rates with a moderate number of users, which is typical in many indoor applications.

The rest of the paper is organized as follows. Section II describes the problem model. In Section III, the proposed 2-D decision feedback equalizer architecture is explained. Performance of 2D-DFE is evaluated and compared with that of 2D-RAKE and 1D-DFE in Section IV. Finally, Section V concludes the paper.

II. System Model

We consider a wideband CDMA downlink channel with transmit diversity, in which, for simplicity of equations, two transmit antennas at the base station and one receive antenna at the mobile unit are considered. We assume that all the K active users within the cell of interest are using the transmit diversity and the inter-cell interference is negligible, but extending the problem to the general case only requires minor modifications to the statistics of MUI. Also, we will first proceed with the assumption that all users have equal symbol rates, and in Appendix we will show how the solution can be applied to the multi-rate scenario.

Let $s_k^{(i)}(n), n = 0 \ldots, N-1$, be the kth user's space-time coded data sequence to be transmitted from antenna i within a specific time frame. The transmitted signal of user k from antenna i will then have the form

$$x_k^{(i)}(t) = \sqrt{\frac{E_k}{2}} \sum_{n=0}^{N-1} s_k^{(i)}(n) a_{k,n}(t - nT_s) \qquad (1)$$

where T_s is the symbol duration, E_k is the total energy per symbol for user k, and $a_{k,n}(t)$ is the spreading waveform of user k at the nth symbol period. We can write $a_{k,n}(t)$ as

$$a_{k,n}(t) = \frac{1}{\sqrt{G_p}} \sum_{m=0}^{G_p-1} c_{k,n}(m) p(t - mT_c) \qquad (2)$$

where G_p is the processing gain, T_c is the chip duration, $p(t)$ is the chip pulse shape signal, and $c_{k,n}(m)$ is the kth user's spreading sequence which may vary over different symbols.

The channels from the two base station antennas to the receiver are modeled as quasi-stationary multipath fading channels with impulse responses $h^{(1)}(t)$ and $h^{(2)}(t)$ given by

$$h^{(i)}(t) = \sum_{l=0}^{L-1} h_l^{(i)} \delta(t - \tau_l), \quad i = 1, 2 \qquad (3)$$

where L is the number of paths, $h_l^{(i)}$ is a complex coefficient modeling the amplitude and phase variations for the lth path, and τ_l is the delay related to the lth path. The number of paths and their delay times are equal for the two channels, since the distance between the transmitting antennas is of the order of ten wavelengths or less. Consequently, the received signal from all K users is represented after demodulation as

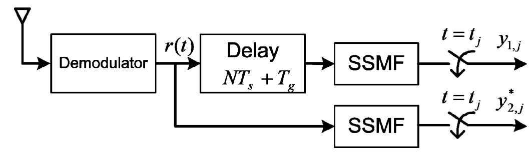

Fig. 1. Preprocessing at the receiver.

$$r(t) = \sum_{i=1}^{2} \sum_{l=0}^{L-1} \sum_{k=0}^{K-1} h_l^{(i)} x_k^{(i)}(t - \tau_l) + n(t) \qquad (4)$$

where $n(t)$ is white Gaussian noise with variance $N_0/2$ per complex dimension.

For transmit diversity, we employ Alamouti's space-time block code for two transmitter and one receiver antennas [1]. Space-time encoding can be performed using either a symbol-level [1], [2], or a block-level [11] method. With a symbol-level encoding, space-time coded transmission is performed on every two subsequent symbols $b_k(2n)$ and $b_k(2n+1)$. Thus, as a result of inter-symbol interference, the received signal at each instant will be a mixture of the information symbols and their complex conjugates, and widely linear filtering must be used for effective MMSE detection [13]. On the other hand, in a block-level encoding scheme, space-time coding is performed over two sub-streams of odd and even symbols, and the encoded sub-streams are transmitted during two subsequent periods separated by a guard time T_g greater than the channel memory. In other words, with an original block of length 2 N, we will have

$$\begin{cases} s_k^{(1)}(n) = b_k(2n) \\ s_k^{(2)}(n) = b_k(2n+1) \end{cases}, \quad \begin{cases} s_k^{(1)}(n+N) = -b_k^*(2n+1) \\ s_k^{(2)}(n+N) = b_k^*(2n) \end{cases} \qquad (5)$$

for $n = 0, \ldots, N-1$. Now if we simply conjugate the samples received at the second period, the complex conjugates of the symbols will not appear anymore, resolving the need for widely linear filtering. While widely linear filtering does not change the order of complexity of the receiver, in this paper we use the block-level encoding scheme for simplicity of explanation, but extending the analysis to the symbol-level case is straightforward. Note, however, that the time-reversal proposed in [11] is not performed at the transmitter.

III. Receiver Structure

Let's assume that $b_0(2n)$ and $b_0(2n+1)$ are the desired symbols for detection. As shown in Fig. 1, $r(t)$ is first passed through two parallel filters matched to the corresponding spreading waveforms of the user, and then sampled at times $t_j, j = 0, \ldots, M-1$. The output sequence of the two samplers, $y_{1,j}$ and $y_{2,j}$, can be written as

$$y_{1,j} = \int_0^{T_s} a_{0,n}^*(\tau) r(\tau + nT_s + t_j)\, d\tau \qquad (6)$$

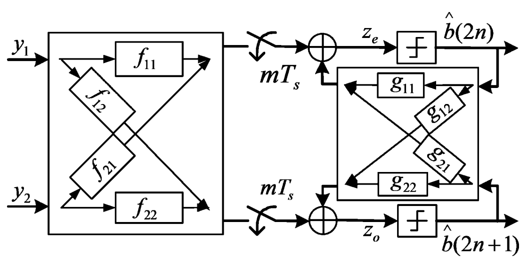

Fig. 2. Space-time coded DFE structure for two transmit antennas.

$$y_{2,j} = \left[\int_0^{T_s} a^*_{0,n+N}(\tau) r(\tau + (n+N)T_s + T_g + t_j)\, d\tau \right]^*. \tag{7}$$

Assuming ideal correlation properties for the spreading waveforms, there will be no ISI and MUI in these samples. In this case, using a 2D-RAKE structure is optimal [14]. However, in practice, the nonzero auto- and cross-correlations of the shifted spreading waveforms introduce ISI and MUI, making the RAKE receiver work poorly in the high interference regime. As an alternative, Abdulrahman *et al.* suggested to use a fractionally spaced MMSE-DFE for single transmit antenna multiuser CDMA system [12].

In order to achieve the performance improvement brought by DFE in a multiple transmit antenna scenario, the structure shown in Fig. 2 for the two-antenna case is introduced. The feed-forward filters, $f_{ij}(n), i, j \in \{1, 2\}$, each have N_f taps and the sampling is performed at m times the chip-rate, i.e., $t_j = jT_c/m, j = 0, \ldots, N_f - 1$ in (6) and (7). The feedback filters, $g_{ij}(n), i, j \in \{1, 2\}$, operate at the symbol rate and each have N_b taps. The input to the first decision unit for detecting $b_0(2n)$ is equal to

$$\begin{aligned} z_e = & \sum_{l=0}^{N_f-1} [f_{11}(N_f - 1 - l) y_{1,l} + f_{21}(N_f - 1 - l) y_{2,l}] \\ & + \sum_{i=1}^{N_b} [g_{11}(N_b - i) \hat{b}_0(2n - 2i) \\ & + g_{21}(N_b - i) \hat{b}_0(2n - 2i + 1)]. \end{aligned} \tag{8}$$

Assuming no decision error, we can rewrite (8) in matrix form

$$z_e = \underline{F}_e^H \underline{X} \tag{9}$$

where

$$\begin{aligned} \underline{F}_e^H = & [f_{11}(N_f - 1), \ldots, f_{11}(0), f_{21}(N_f - 1), \ldots, f_{21}(0) \\ & g_{11}(N_b - 1), \ldots, g_{11}(0), g_{21}(N_b - 1), \ldots, g_{21}(0)] \end{aligned} \tag{10}$$

$$\begin{aligned} \underline{X} = & [y_{1,0}, \ldots, y_{1,N_f-1}, y_{2,0}, \ldots, y_{2,N_f-1}, b_0(2n-2) \\ & \ldots, b_0(2n - 2N_b), b_0(2n-1) \ldots \\ & b_0(2n - 2N_b + 1)]^T. \end{aligned} \tag{11}$$

The weight vector should be chosen to minimize the mean square error, $J = E\{|z - b_0(2n)|^2\}$. The solution to this MMSE problem is

$$\underline{F}_{e,\text{opt}} = A^{-1} \underline{P} \tag{12}$$

where

$$A = E\{\underline{X}\,\underline{X}^H\} \tag{13}$$

and

$$\underline{P} = E\{\underline{X} b_0^*(2n)\}. \tag{14}$$

Matrix $\mathbf{A}$ and vector $\underline{P}$ can be derived knowing the channel and assuming that the interfering users' spreading codes and all the information symbols are independent random sequences. With respect to the desired user's spreading code, we will consider the following two cases. In the first case, which we call the deterministic case, we employ the exact values of the spreading code and its autocorrelation samples to derive the equalizer taps. In the second case, which we call the random case, we model the desired user's code as a random sequence and only use its second-order statistics. Details of the derivation of the filter coefficients for both cases are given in the Appendix.

In most commercial systems, the spreading code varies over different symbols due to the long PN code used for scrambling. Therefore, in the deterministic case, the equalizer parameters should be updated for each pair of symbols, while in the random case, the same equalizer can be used for all symbols as long as the channel remains unchanged, hence resulting in a significant decrease in the system complexity. Furthermore, in a multicode system, where the receiver demodulates more than one code at a time, the same equalizer taps can be used for all codes. On the other hand, since in the random case, unlike the deterministic case, ISI is not calculated and mitigated accurately, we expect that the latter outperforms the former, especially when noise and MUI are low and ISI is the major concern.

From a complexity point of view, it can be seen that the most time-consuming parts of the algorithm are the computation and inversion of matrix $\boldsymbol{A}$. In the non-adaptive standard implementation, the number of arithmetic operations for computing each of the 4 N_f^2 element of the four $N_f \times N_f$ sub-matrices $E\{\underline{Y}_i \underline{Y}_j^H\}$, similar to (30), is at most proportional to L^2. Also, since generally $N_b < N_f$, the inversion of $\boldsymbol{A}$ by Gaussian elimination has complexity of $O(N_f^3)$. Therefore, the computational complexity of the algorithm is $O(N_f^2 L^2 + N_f^3)$.

IV. Performance Analysis

The minimum mean-square error for the general system described by (9) and (12) can be written as [12]

$$J_{\min} = 1 - \underline{P}^H A^{-1} \underline{P}. \tag{15}$$

Also, using the Gaussian approximation, the overall signal-to-noise ratio per symbol at the decision unit can be estimated from the MMSE as [15]

$$\gamma_s = (1 - J_{\min})/J_{\min}. \tag{16}$$

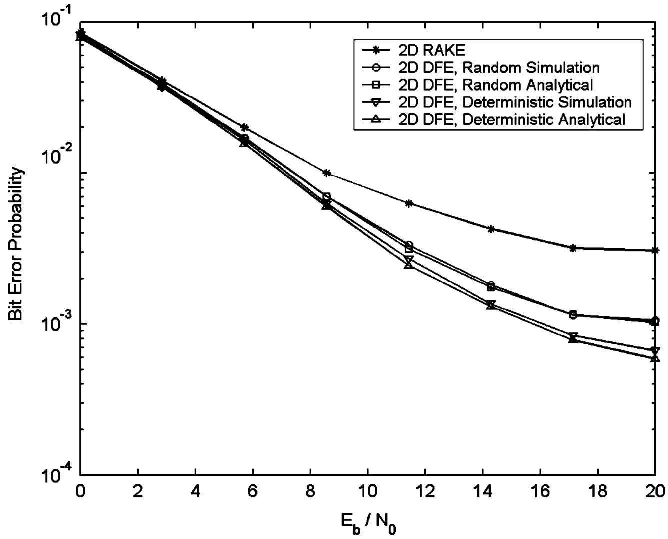

Fig. 3. Receiver BEP versus E_b/N_0 ($L = 3$ paths and $K = 8$ users).

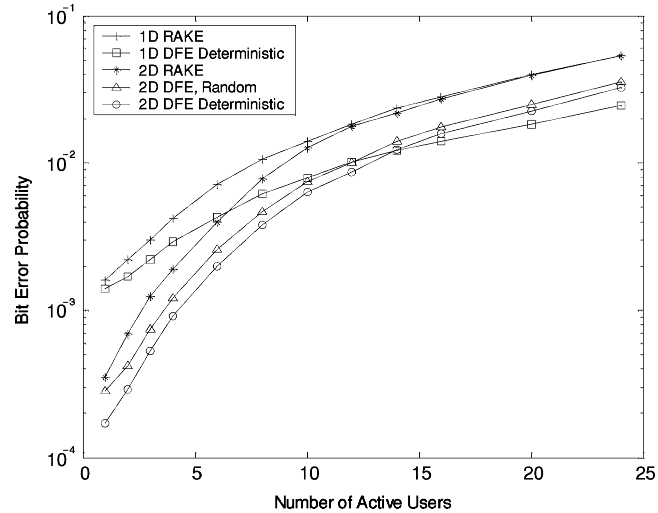

Fig. 4. BEP versus the number of active users at $E_b/N_0 = 10$ dB ($L = 3$ paths).

Thus, the bit error probability (BEP) with QPSK modulation can be approximated by

$$P_{e,\text{bit}} = Q(\sqrt{\gamma_s}) = Q(\sqrt{\underline{P}^H A^{-1} \underline{P}/(1 - \underline{P}^H A^{-1} \underline{P})}). \quad (17)$$

Using this result, we study the performance of a chip-rate 2D-DFE in a WCDMA forward link with QPSK modulation and spreading factor of 32. Since the BEP given by (17) depends on the channel coefficients and the desired user's spreading code (in the deterministic case), the expected value of $P_{e,\text{bit}}$ is calculated by numerically averaging over several channel realizations. We have considered chip-synchronous Rayleigh fading channels with $L < 32$ paths. The channel coefficients are complex Gaussian random variables with variances exponentially decreasing with the path delay, such that the variance of the Lth coefficient is 10 dB less than that of the first path. Also, the composite spreading code for each user is a user-specific binary Walsh–Hadamard code with period 32 multiplied by a common long QPSK random code. Since the channel memory is shorter than the symbol period, ISI is produced only by the adjacent symbols. Therefore, the equalizer feedback filters each only need one tap, i.e., $N_b = 1$.

In Fig. 3, the bit error probability is plotted as a function of the average signal to noise per bit (E_b/N_0) for 2D-RAKE and 2D-DFE for both the deterministic and random cases. We assumed eight active (non-silent) users within the same cell/frequency, but the actual number of users can be more, depending on the service. The channel has $L = 3$ paths and the number of 2D-DFE feed-forward taps, N_f, is equal to four. As can be seen in Fig. 3, as E_b/N_0 increases, BEP of both types of DFE fall faster than RAKE, and the deterministic DFE has the lowest BEP for all values of SNR. At higher values of E_b/N_0, MUI dominates the thermal noise and the performance curve of 2D-RAKE approaches a saturation level, making the performance improvement gained by DFE more significant. To verify our performance analysis, simulation results for bit error rate versus E_b/N_0 is also plotted for 2D-DFE. Simulation has been performed on over 2000 blocks each containing 400 space-time coded symbols, and the channel has been assumed constant during each frame. It can be seen that the simulation results closely match the analytical results.

In Fig. 4, receiver bit error probability has been plotted versus the number of active users K for 2D-RAKE and 2-D deterministic and random DFE, at $E_b/N_0 = 10$ dB. For comparison, BEP of RAKE and deterministic DFE without transmit diversity, but with the same total transmit power, have also been included. We have assumed three paths for channels and four feed-forward taps for all DFE structures. It is observed that as the number of users increases, the performance curves of 2-D and 1-D RAKE converge. This is a predictable result, since all the interfering signals in downlink go through the same channel as the desired signal, hence providing diversity has no effect on the signal-to-interference ratio.

An interesting observation is that 1D-DFE outperforms 2D-DFE in high traffic conditions. This result can be explained by the fact that in a single-input/single-output (SISO) channel, the receiver can, in principle, perfectly equalize the channel into a flat channel, regardless of the underlying modulation and, hence, completely cancel the MUI and ISI caused by multipath. This is what 1D-DFE does in the interference-limited regime. On the other hand, in a multi-input/single-output (MISO) channel, the receiver doesn't have enough degrees of freedom to perfectly equalize the channels from all the transmit antennas without extracting the temporal properties of the space-time coded signal. Implementing the DFE in a receiver with at least the same number of antennas as the transmitter can overcome this problem.

To estimate the required number of filter taps, bit error probability of the deterministic 2D-DFE is plotted in Fig. 5 versus the number of feed-forward taps, N_f, for different numbers of resolvable channel paths, L. In this figure, E_b/N_0 is equal to 10 dB, and eight active users are assumed within the cell. As one expects, increasing the filter lengths reduces the error probability by providing more degrees of freedom for mitigating

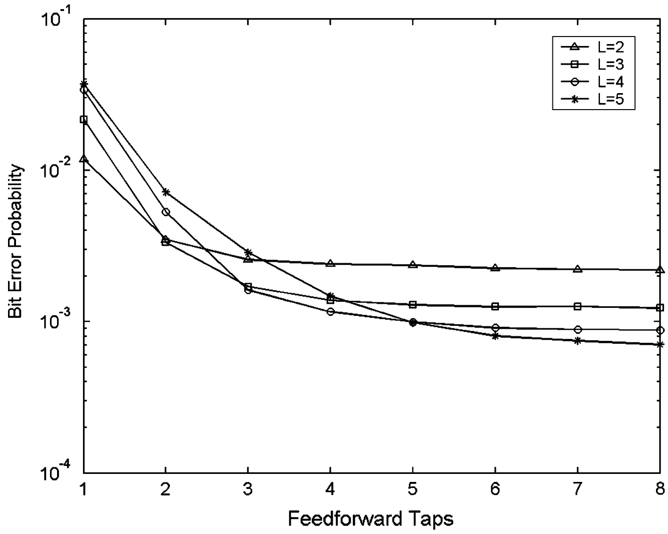

Fig. 5. BEP of deterministic DFE versus N_f for different values of L at $E_b/N_0 = 10$ dB ($K = 5$ users).

the mean-square error. However, for $N_f > 1.5L$, the error probability doesn't change significantly, as signal-to-interference plus noise ratio (SINR) is very low in the additional samples of the received signal. Therefore, $N_f \approx 1.5\,L$ is a reasonable value for the number of feed-forward taps of the DFE.

V. Conclusion

We studied the multiuser wideband CDMA downlink with space-time transmit diversity in a frequency selective channel. In order to suppress interference in addition to providing the antenna and multipath diversity in a single antenna receiver, we used a generalized version of the decision-feedback equalizer proposed in [12]. The advantage of this structure over other schemes with similar complexities is that it balances between the residual noise, ISI, and MUI, so it works well when any of these three effects dominates. Furthermore, this receiver doesn't require a separate block for space-time decoding, and can be implemented with single or multiple antennas. Two cases were assumed for calculating the filter coefficients. As we have shown, even in the random version of the 2D-DFE, which doesn't use knowledge of the spreading code of the user of interest and has much less computational complexity compared to the deterministic version, satisfactory performance improvement is achieved.

Bit error probability of the 2D-DFE structure was calculated using the Gaussian approximation. As demonstrated by the analytical results, the proposed scheme significantly outperforms the standard RAKE, especially at high signal-to-noise ratios where interference dominates. However, at very high interference levels, it was shown show that using transmit diversity cannot improve the performance, unless the degrees of freedom at the receiver is increased, e.g., by multiple antennas, or interference is suppressed by more sophisticated methods such as multiuser detection. As an overall conclusion, the proposed scheme seems to work best in high data rate indoor environments with moderate numbers of active users.

Appendix

Expanding (6) and (7) using (1) and (4) yields

$$\begin{aligned} y_{1,j} = \sum_{k=0}^{K-1}\sum_{l=0}^{L-1}\sum_{i=-1}^{1}\sqrt{\frac{E_k}{2}}\Big[h_l^{(1)}b_k(2(n+i)) \\ + h_l^{(2)}b_k(2(n+i)+1)\Big] \\ \cdot R_{k,i}^{(1)}(t_j - iT_s - \tau_l) + n_{1,j} \end{aligned} \quad (18)$$

$$\begin{aligned} y_{2,j} = \sum_{k=0}^{K-1}\sum_{l=0}^{L-1}\sum_{i=-1}^{1}\sqrt{\frac{E_k}{2}}\Big[h_l^{(2)*}b_k(2(n+i)) \\ - h_l^{(1)*}b_k(2(n+i)+1)\Big] \\ \cdot R_{k,i}^{(2)*}(t_j - iT_s - \tau_l) + n_{2,j} \end{aligned} \quad (19)$$

where

$$R_{k,i}^{(1)}(t) = \int_0^{T_s} a_{0,n}^*(\tau)a_{k,n+i}(\tau+t)\,d\tau \quad (20)$$

$$R_{k,i}^{(2)}(t) = \int_0^{T_s} a_{0,n+N}^*(\tau)a_{k,n+N+i}(\tau+t)\,d\tau \quad (21)$$

are the cross-correlation functions of the desired spreading waveforms, and $n_{1,j}$ and $n_{2,j}$ are the responses of the matched filters to the thermal noise. To compute the coefficients, $\underline{P}$ can be written using (11) as

$$\underline{P} = \begin{bmatrix} E\left\{\underline{Y}_1 b_0^*(2n)\right\}_{N_f\times 1} \\ E\left\{\underline{Y}_2 b_0^*(2n)\right\}_{N_f\times 1} \\ \underline{0}_{N_b\times 1} \\ \underline{0}_{N_b\times 1} \end{bmatrix}. \quad (22)$$

Now, we can use (18) and (19) to compute $E\{\underline{Y}_1 b_0^*(2n)\}$ and $E\{\underline{Y}_2 b_0^*(2n)\}$. For example, the mth element of $E\{\underline{Y}_1 b_0^*(2n)\}$ is equal to

$$E\{y_{1,m}b_0^*(2n)\} = \sqrt{\frac{E_0}{2}}\sum_{l=m+1}^{L-1} h_l^{(1)}E\left\{R_{0,0}^{(1)}(t_m - \tau_l)\right\}. \quad (23)$$

In the deterministic case, the spreading code of the desired user is known, thus $R_{0,0}^{(j)}(t_m - \tau_l)$ is deterministic. On the other hand, in the random case

$$E\left\{R_{0,0}^{(j)}(t_m - \tau_l)\right\} = R_p(t_m - \tau_l) \quad (24)$$

where

$$R_p(t) \stackrel{\text{def}}{=} \int_{-\infty}^{+\infty} p(\tau)p^*(t-\tau)\,d\tau. \quad (25)$$

Matrix $\boldsymbol{A}$ can be written as (26), shown at the top of the next page, where $\boldsymbol{I}$ and $\mathbf{0}$ indicate the identity and zero matrices, respectively. To expand the elements of $\boldsymbol{A}$, it is useful to define the following auxiliary notations. For any $k \neq 0$, we define

$$\begin{aligned} \rho(i,m,n,l,q) \\ \stackrel{\text{def}}{=} E\left\{R_{k,i}^{(j)}(t_m - iT_s - \tau_l)R_{k,i}^{(j)*}(t_n - iT_s - \tau_q)\right\} \end{aligned}$$

$$A = \begin{bmatrix} E\left\{\underline{Y}_1\underline{Y}_1^H\right\}_{N_f\times N_f} & E\left\{\underline{Y}_1\underline{Y}_2^H\right\}_{N_f\times N_f} & E\left\{\underline{Y}_1 b_0^*(2n-2)\right\}_{N_f\times N_b} & E\left\{\underline{Y}_1 b_0^*(2n-1)\right\}_{N_f\times N_b} \\ E\left\{\underline{Y}_2\underline{Y}_1^H\right\}_{N_f\times N_f} & E\left\{\underline{Y}_2\underline{Y}_2^H\right\}_{N_f\times N_f} & E\left\{\underline{Y}_2 b_0^*(2n-2)\right\}_{N_f\times N_b} & E\left\{\underline{Y}_2 b_0^*(2n-1)\right\}_{N_f\times N_b} \\ E\left\{b_0(2n-2)\underline{Y}_1^H\right\}_{N_b\times N_f} & E\left\{b_0(2n-2)\underline{Y}_2^H\right\}_{N_b\times N_f} & I_{N_b} & 0_{N_b} \\ E\left\{b_0(2n-1)\underline{Y}_1^H\right\}_{N_b\times N_f} & E\left\{b_0(2n-1)\underline{Y}_2^H\right\}_{N_b\times N_f} & 0_{N_b} & I_{N_b} \end{bmatrix} \tag{26}$$

$$= \frac{1}{G_p^2}\sum_{w=1-G_p}^{G_p-1}(G_p-|w|)R_p(t_m-iT_s+wT_c-\tau_l) \cdot R_p^*(t_n-iT_s+wT_c-\tau_q)(1-\delta(w)\delta(i)) \tag{27}$$

where $\delta(n)$ is the Kronecker delta function, and the term $(1-\delta(w)\delta(i))$ guarantees the orthogonality of the un-shifted spreading codes [16], i.e., that $R_{k,0}^{(j)}(0)=0$ for $k\neq 0$. Note that (29) is independent of the antenna index j and the user index k. Also

$$\varsigma^{(u,v)}(i,m,n,l,q) \stackrel{\text{def}}{=} E\left\{R_{0,i}^{(u)}(t_m-iT_s-\tau_l)R_{0,i}^{(v)*}(t_m-iT_s-\tau_l)\right\} \tag{28}$$

is equal to $R_{0,i}^{(u)}(t_m-iT_s-\tau_l)R_{0,i}^{(v)*}(t_m-iT_s-\tau_l)$ for the deterministic case and equal to

$$\begin{cases}\rho(i,m,n,l,q)+R_p(t_m-\tau_l)R_p^*(t_n-\tau_q)\delta(i), & \text{for } u=v\\ 0, & \text{for } u\neq v\end{cases} \tag{29}$$

for the random case, where the second term in the first line guarantees that $R_{0,0}^{(j)}(0)=1$. Also, we define $E_I=\sum_{k=1}^{K-1}E_k$ as the sum of the transmit powers of the interfering users.

Using the above notations, as an example, an element on the mth row and nth column of $E\{\underline{Y}_1\underline{Y}_1^H\}$ is given by

$$\begin{aligned} E\left\{y_{1,m}y_{1,n}^H\right\} &= \frac{E_0}{2}\sum_{i=-1}^{1}\sum_{l=0}^{L-1}\sum_{q=0}^{L-1}\left[h_l^{(1)}h_q^{(1)*}+h_l^{(2)}h_q^{(2)*}\right] \\ &\quad\times\varsigma^{(1,1)}(i,m,n,l,q) \\ &\quad+\frac{E_I}{2}\sum_{i=-1}^{1}\sum_{l=0}^{L-1}\sum_{q=0}^{L-1}\left[h_l^{(1)}h_q^{(1)*}+h_l^{(2)}h_q^{(2)*}\right] \\ &\quad\times\rho(i,m,n,l,q)+N_0E\left\{R_{0,0}^{(1)}(t_m-t_n)\right\} \end{aligned} \tag{30}$$

where the first term contains the signal and ISI, the second term represents the MUI, and the last term takes account of the thermal noise. Other elements of $\boldsymbol{A}$ can also be expanded with a similar approach. Note that only $E\{\underline{Y}_1\underline{Y}_1^H\}$ and $E\{\underline{Y}_2\underline{Y}_2^H\}$ contain MUI and noise terms. The filter coefficients for detecting the odd symbols can be derived similarly.

A. Generalizing to the Multi-Rate Case

We have so far assumed that all users have equal symbol rates. In a multi-rate scenario, different users operate at equal chip-rates but not necessarily equal processing gains. However, since we have modeled the interfering symbols and spreading codes as random sequences, we can virtually assume that all users are operating with the same symbol rate as that of the desired user, i.e., $1/T_s$, but with modified symbol energies. So, if user k has a processing gain n_k times that of the desired user, we should replace E_k by $E'_k=E_k/n_k$ in the above equations in order to make them applicable to the multirate case.

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