Phase-Noise Measurement Using Two Inter-Injection-Locked Microwave Oscillators

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***Abstract*—Phase noise in two mutually coupled oscillators is analyzed by the describing function method, and the after-lock phase noise of the oscillators is calculated in terms of their free-running phase noise. A new phase-noise measurement technique based on inter-injection locking of two similar oscillators is proposed. Experimental results are presented, which confirm the theory. It is shown that in the case of zero phase of coupling coefficient, the system is in the optimum state where the only required parameter for the measurement is the locking bandwidth. In this optimum state, as far as the locking bandwidth is measured correctly, imperfections such as the frequency drift, parameters discrepancy, and nonlinear susceptance of the oscillators have no serious effect on the measurement accuracy. The proposed method is compared to the conventional ones.**

***Index Terms*—Describing function, inter-injection locking, microwave oscillator, phase noise, phase-noise measurement.**

I. Introduction

OSCILLATORS are nonlinear time-varying systems due to the large-signal oscillation within their circuits. This makes the phase-noise analysis in the oscillators a challenging problem, which has been the area of investigation for several decades. There are three major approaches to the problem. The simplest approach to the phase-noise analysis uses a linear time invariant (LTI) model for oscillators. The well-known Leeson model for the phase noise is based on this method [1]. While often of great practical importance, this approach has two major defects. One is that it cannot correctly describe the up conversion of the low-frequency flicker noise into the phase noise around the carrier and the other is that it predicts infinite output power [2], [3]. Using a linear time variant (LTV) model for the oscillator, Hajimiri and Lee [3] has proposed a phase-noise analysis method, which explains this up-conversion phenomenon, but it again fails to correctly predict the phase noise at frequency offsets very close to carrier. To overcome this problem, a nonlinear analysis is required. Among these methods are the harmonic-balance and Monte Carlo methods, which are suitable for numerical calculation of phase noise [4], [5]. Demir [2] presents a general method that can correctly predict the spectrum of the phase noise, but it is more suitable for numerical calculations. Both the LTV and nonlinear approaches are mainly used for the numerical calculation of phase noise and they are not suitable for extracting closed-form relations for the phase noise. In this paper, the describing function concept, first introduced in [6], is used for the analysis of phase noise in two mutually coupled oscillators. Considering noise as a small perturbation around the steady-state point, a linear approximation of the describing functions is used to extract the dynamic equations. This is an LTI method, but since here after-lock phase noise is calculated in terms of the free-running phase noise, some drawbacks of the LTI models do not affect the analysis. For example, it is not required to regard the up-conversion phenomenon because the phase noise is not calculated in terms of the noise sources in the oscillator circuit.

Phase noise in synchronized oscillators has been investigated by several authors. Kurokawa [7] has shown that in the near-carrier offsets, the phase-noise power spectral density of an externally injected oscillator is equal to that of the injected signal. Phase-noise analysis in inter-injection-locked (IIL) oscillators has been performed recently by Shumakher and Eisenstien [8] preceded by Chang *et al.* [9] and Makino *et al.* [10]. It is shown that in an array of N synchronized oscillators, the near-carrier phase noise of the oscillators becomes $1/N$ that of their free-running values. None of the previous analyses are useful for phase-noise measurement by inter-injection locking. In this paper, phase noise in two mutually coupled oscillators is analyzed by a different approach and the after-locked phase noise is calculated in terms of the free-running phase noise of the oscillators, which would be suitable for phase-noise measurement.

Due to the role of phase noise in the design of RF front-end systems, phase-noise measurement is of great importance. The phase detector and delay-line FM discriminator are two widely used methods for the measurement of phase noise. The former is expensive and complicated and the latter is of inferior performance due to the increased loss of long delay line at higher frequencies [11]. In addition, the conventional phase-detector method is not suitable for the phase-noise measurement of relatively unstable oscillators. In these types of oscillators, the oscillation frequency has a relatively wide span of variation requiring a wide loop bandwidth in the conventional phase-detector method, which limits the performance of the phase-noise measurement system. Zhang *et al.* [12] introduced a new method, which is similar to the phase-detector method, but it uses injection locking to synchronize the oscillators. Synchronizing the oscillators by injection locking is much simpler than phase-locked looping them. In addition, it is not required that one of the oscillators have the electronic tuning capability. In the injection-locking method, by approaching the carrier, the phase-noise floor of the system increases with a 20-dB/dec slope. In this paper, inter-injection locking is used to synchronize the oscillators for phase-noise measurement. In "injection locking," one oscillator affects the other, while in "inter-injection locking," the oscillators mutually interact with

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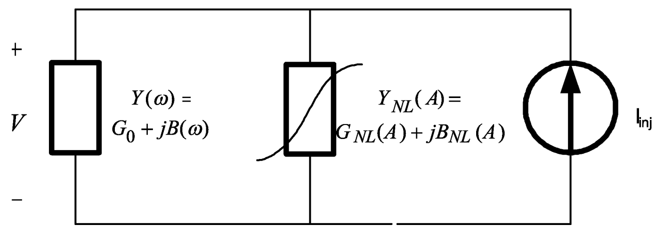

Fig. 1. Describing function model of an externally injected oscillator. G_0 represents the oscillator load.

each other. Thus, the dynamic equations are more complicated and the locking bandwidth is twice that of the injection-locking technique leading to easier synchronization.

In this paper, phase noise in two mutually coupled oscillators is analyzed. Locked oscillator steady-state analysis has been performed in the authors' previous paper [13], [14] and it is not repeated here. In Section II, dynamic equations of an externally injected oscillator are cited, which are the basis for the subsequent analysis. Phase noise in two mutually coupled oscillators is then analyzed and after-lock phase noise is calculated in terms of the free-running phase noise of the oscillators. In Section III, a new phase-noise measurement technique based on inter-injection locking of two similar oscillators is introduced. The results of Section II are used to analyze the proposed system. Experimental results are presented, which validate the analysis. The effect of imperfections such as the frequency drift, parameters discrepancy, and nonlinear susceptance of the oscillators are inspected and the proposed method for the phase-noise measurement is compared to the conventional ones.

II. Phase-Noise Analysis in Two Mutually Coupled Oscillators

A. Dynamics of an Externally Injected Oscillator

When an external signal is injected into a free-running oscillator, the amplitude and phase of the oscillation are perturbed. Dynamic equations describe the behavior of the oscillator under injection. These equations are based on the describing function model of the oscillator [6]. In this model, the oscillator is separated into active (nonlinear) and passive (linear) parts. The active part, which powers up the oscillation, is represented by an amplitude-dependent describing function $Y_{\rm NL}(A)$ and the passive part, which includes the resonator and oscillator load, is modeled by a frequency-dependent admittance $Y(\omega)$. The external signal is modeled by the current source $I_{\rm inj}$ (Fig. 1). It is noteworthy to state that, in practice, the active part is a weak function of frequency, which with good accuracy, can be regarded as frequency independent within the resonator bandwidth.

In the presence of an injected signal, the oscillation is perturbed as

$$V = (A_0 + \delta A)e^{j(\omega_0 t + \theta)}. \tag{1}$$

In (1), θ and δA, respectively, represent the phase and amplitude perturbation around the free-running oscillation point (ω_0, A_0).

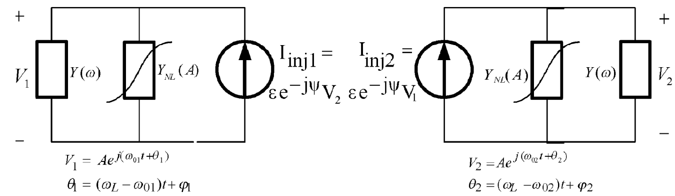

Fig. 2. Describing function model of two inter-injected oscillators.

In the case of weak injection ($|I_{\rm inj}| \ll G_0 A_0$), the dynamic equations have been derived in [15] and are cited here for subsequent reference as follows:

$$\begin{cases} \dfrac{\partial B}{\partial \omega}\dfrac{d\delta A}{Adt} + \dfrac{\partial G_{\rm NL}}{\partial A}\delta A = {\rm Re}\left\{\dfrac{I_{\rm inj}}{V}\right\} \\ \dfrac{\partial B}{\partial \omega}\dfrac{d\theta}{dt} + \dfrac{\partial B_{\rm NL}}{\partial A}\delta A = {\rm Im}\left\{\dfrac{I_{\rm inj}}{V}\right\}. \end{cases} \tag{2}$$

All the derivations in (2) are calculated in the free-running oscillation point. These equations are the basis for the subsequent analysis.

B. Steady State of Two IIL Oscillators

Suppose that two similar oscillators are mutually coupled to each other through a reciprocal path with the coupling coefficient of $\varepsilon e^{-j\psi}$ (Fig. 2). If the free-running frequency difference of the oscillators ($\Delta\omega_0 = \omega_{02} - \omega_{01}$) is less than a specific value, called the locking bandwidth ($\Delta\omega_L$), the oscillators would be synchronized and oscillate with the same frequency (ω_L). Due to the free-running frequency difference, a steady-state phase difference ($\xi_0 = \varphi_2 - \varphi_1$) is produced between the locked oscillators. Steady-state analysis has been performed in [13] and the results are as follows:

$$\Delta\omega_L = \frac{\varepsilon\omega_0}{G_0 Q}\left|\cos(\psi)\right|, \qquad \text{where } Q = \frac{\omega_0}{2G_0}\frac{\partial B}{\partial \omega} \tag{3}$$

$$\sin(\xi_0) = \frac{\Delta\omega_0}{\Delta\omega_L}. \tag{4}$$

In the above relations, ω_0 is the average of the free-running frequency of the oscillators and Q is the external quality factor of the oscillators. In (3) and (4), the effect of the nonlinear susceptance of the oscillator model has been neglected, which will be taken into account later in this paper. According to (4), the after-lock phase difference is a function of the free-running frequency difference of the oscillators. This fact is the key to the phase-noise analysis in the Section II-C.

C. Phase-Noise Analysis in Two Mutually Coupled Oscillators

Existing noise sources in the oscillator circuit perturb the phase of the output signal, thereby produce the phase noise. Noise sources in two free-running oscillators are independent, therefore their phase noises are also independent. In two IIL oscillators, the noise sources in one oscillator affect the phase noise of the other within the locking bandwidth; consequently the phase noise of the oscillators have joint sources and, thus, are correlated. Here, the after-lock phase noise of the oscillators are calculated in terms of their free-running phase noise. This is required for the analysis of the phase-noise measurement setup introduced in Section III. The approach to compute the phase noise in mutually locked oscillators in this paper is different from what has been done in [9]. While in [9] the free-running phase noise and the after-lock phase noise are calculated separately in their corresponding circuit model, in this paper, we compute the after-lock phase noise directly in terms of the free-running phase noise as it will be followed.

In addition to the phase noise (PM noise), amplitude noise (AM noise) is also produced by the existing noise sources in the oscillator circuit. PM noise and AM noise interact with each other through AM-to-PM and PM-to-AM conversions. According to (2), nonlinear susceptance of the oscillator model accounts for these conversions. It is well known that in the near-carrier offsets, AM noise is much smaller than PM noise, hence the amount of AM noise converted to PM noise has a small contribution in the total PM noise [7]. This justifies the neglect of AM noise in PM noise calculations here. By (2), one can easily observe that if AM noise is neglected, there would be no nonlinear susceptance effect in noise calculations. Thus, in order to evaluate the effect of nonlinear susceptance, AM noise should be included in the analysis, as will be done in Section III-C.

Suppose two oscillators are of similar construction. It is assumed that they have the same oscillation amplitude and the same external quality factor, but they may have different phase noises. Once synchronized, they will both oscillate at frequency ω_L and will have a definite after-lock phase difference (ξ_0) due to the difference in their free-running frequencies. The free-running frequency of each oscillator has small fluctuations ($\delta\omega_{\mathrm{F.R.}i}$) around its nominal value (ω_{0i}), which is called the free-running frequency noise. Frequency noise is related to the phase noise in the frequency domain by the relation $\delta\omega_{\mathrm{F.R.}i} = j\omega\delta\varphi_{\mathrm{F.R.}i}$. As was stated above, free-running frequency difference of the oscillators results in a steady-state after-lock phase difference between them. Consequently free-running frequency fluctuations of oscillators are translated into after-lock phase noise ($\delta\varphi_i$). The relationship between $\delta\varphi_i$ and $\delta\omega_{\mathrm{F.R.}i}$ is now extracted. With respect to Fig. 2, one will have

$$\begin{aligned} V_1 &= Ae^{j(\omega_{01}t+\theta_1)} \\ \theta_1 &= (\omega_L-\omega_{01})t+\varphi_1+\delta\varphi_1 \\ V_2 &= Ae^{j(\omega_{02}t+\theta 2)} \\ \theta_2 &= (\omega_L-\omega_{02})t+\varphi_2+\delta\varphi_2. \end{aligned} \tag{5}$$

Note that the oscillators have the same amplitude and the AM noise is ignored here. $\delta\varphi_i$ is the after-lock phase noise of the oscillators. By substituting (5) into (2), one obtains

$$\begin{cases} \dfrac{d[\varphi_1+\delta\varphi_1]}{dt} \\ \quad = \dfrac{\varepsilon}{\frac{\partial B}{\partial\omega}}\sin(\xi_0+\delta\varphi_2-\delta\varphi_1-\psi)+\omega_{01}-\omega_L+\delta\omega_{\mathrm{F.R.1}} \\ \dfrac{d[\varphi_2+\delta\varphi_2]}{dt} \\ \quad = \dfrac{\varepsilon}{\frac{\partial B}{\partial\omega}}\sin(-\xi_0-\delta\varphi_2+\delta\varphi_1-\psi)+\omega_{02}-\omega_L+\delta\omega_{\mathrm{F.R.2}}. \end{cases} \tag{6}$$

In (6), the free-running frequency fluctuations have been introduced into the equations after the substitution of (5) into (2). Since free-running fluctuations are small, (6) is reduced to (7) by Taylor's expansion

$$\begin{cases} \dfrac{d\delta\varphi_1}{dt} = \dfrac{\varepsilon}{\frac{\partial B}{\partial\omega}}\cos(\xi_0-\psi)(\delta\varphi_2-\delta\varphi_1)+\delta\omega_{\mathrm{F.R.1}} \\ \dfrac{d\delta\varphi_2}{dt} = -\dfrac{\varepsilon}{\frac{\partial B}{\partial\omega}}\cos(\xi_0+\psi)(\delta\varphi_2-\delta\varphi_1)+\delta\omega_{\mathrm{F.R.2}}. \end{cases} \tag{7}$$

Equation (7) shows the relation of the after-lock phase noise to free-running frequency noise of the oscillators in the time domain. The term $\varepsilon/(\partial B/\partial\omega)$ can be substituted with $0.5\Delta\omega_{L0}$, which is half the locking bandwidth when $\psi=0$. By this substitution, after Fourier transformation of both sides of (7) and by solving for $\boldsymbol{\delta\varphi_1}$ and $\boldsymbol{\delta\varphi_2}$, one obtains (8), shown at the bottom of this page, where the boldface characters denote the Fourier transform of the time-domain signals and ω is the frequency offset from the carrier. In (8), after-lock phase noise of the oscillators ($\boldsymbol{\delta\varphi_i}$) has been calculated in terms of their free-running phase noise. Investigating (8), it is revealed that, for the frequency offsets much smaller than the locking bandwidth, the after-lock phase noise of the oscillators are the same and equal to the average of the free-running phase noise of the oscillators, and in the frequency offsets much greater than the locking bandwidth, they go back to their free-running values. These results are in accordance with what has been stated in [11]. Equation (8) is used in Section III for the phase-noise measurement by inter-injection locking.

III. Phase-Noise Measurement Using Two IIL Oscillators

A. *Phase-Noise Measurement Setup and Theory*

All the phase-noise measurement techniques use a mixer and a low-pass filter as a phase detector to demodulate the phase

$$\begin{cases} \boldsymbol{\delta\varphi_1} = \dfrac{[j\omega+0.5\Delta\omega_{L0}\cos(\xi_0+\psi)]\boldsymbol{\delta\varphi_{\mathrm{F.R.1}}}+[0.5\Delta\omega_{L0}\cos(\xi_0-\psi)]\boldsymbol{\delta\varphi_{\mathrm{F.R.2}}}}{j\omega+\Delta\omega_{L0}\cos(\xi_0)\cos(\psi)} \\ \boldsymbol{\delta\varphi_2} = \dfrac{[0.5\Delta\omega_{L0}\cos(\xi_0+\psi)]\boldsymbol{\delta\varphi_{\mathrm{F.R.1}}}+[j\omega+0.5\Delta\omega_{L0}\cos(\xi_0-\psi)]\boldsymbol{\delta\varphi_{\mathrm{F.R.2}}}}{j\omega+\Delta\omega_{L0}\cos(\xi_0)\cos(\psi)} \end{cases} \tag{8}$$

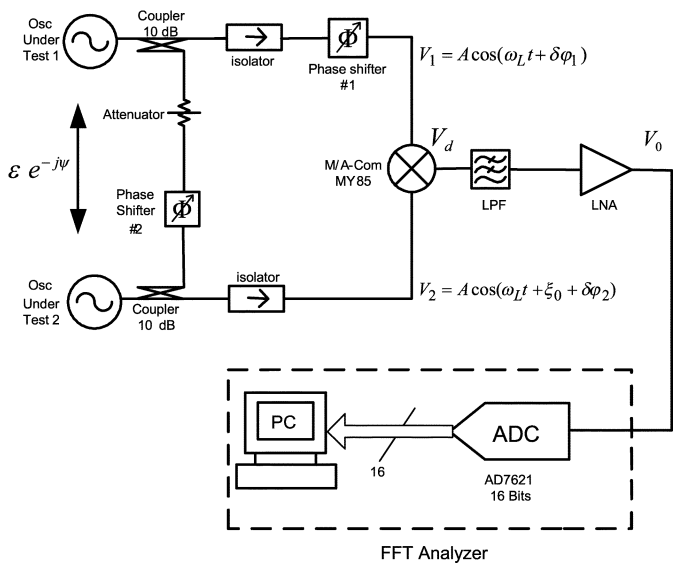

Fig. 3. Proposed phase-noise measurement setup. Two ultra-low-noise AD797 op-amps were used in the construction of the LNA. The LNA gain was set to 70 dB.

noise of the oscillator(s). The signals at the mixer ports should have the same frequency to avoid producing beat note at the mixer IF port. For those methods, which use two oscillators for the measurement, a provision should be made to synchronize the oscillators. The conventional phase-detector method uses a phase-locked loop (PLL) for the synchronization of the oscillators. Zhang *et al.* [12] used injection locking for synchronization. In this paper, inter-injection locking is used to synchronize the oscillators. It is notable that in injection locking, only one oscillator is under injection, while in inter-injection locking, the two oscillators interact with each other. Thus, the dynamic equations are different and the locking bandwidth is twice that of the injection-locking technique leading to an easier synchronization.

The proposed measurement setup is depicted on Fig. 3. The two oscillators are synchronized by inter-injection locking. The injection path consists of directional couplers, an attenuator (which controls the coupling coefficient magnitude) and a phase shifter (which controls the phase of coupling coefficient). The signals of synchronized oscillators are fed into the mixer input ports. Isolators are used to prevent unwanted injection through the mixer ports. If an isolator is used in the injection path, the setup will be converted to injection locking rather than inter-injection locking. Phase shifter 1 is adjusted to make the signals at mixer ports in quadrature when the oscillators are in-phase. It will be shown in Section III-C that this adjustment makes the system insensitive to frequency drift of the oscillators. The signal detected by the mixer is proportional to the after-lock phase-noise difference between the oscillators

$$V_d = K_{\varphi_0}\cos(\xi_0)(\delta\varphi_2 - \delta\varphi_1). \tag{9}$$

Here, $K_{\varphi 0}$ denotes gain of the mixer when $\xi_0 = 0$. Substituting (8) into (9), one obtains

$$\mathbf{V_d} = K_{\varphi_0}\cos(\xi_0)\frac{-j\omega[\boldsymbol{\delta}\varphi_{\mathbf{F.R.1}} - \boldsymbol{\delta}\varphi_{\mathbf{F.R.2}}]}{j\omega + \Delta\omega_{L0}\cos(\xi_0)\cos(\psi)}$$

or

$$\langle\mathbf{V_d^2}\rangle = K_{\varphi_0}^2\cos^2(\xi_0)\frac{\omega^2[\langle\boldsymbol{\delta}\varphi_{\mathbf{F.R.1}}^2\rangle + \langle\boldsymbol{\delta}\,\varphi_{\mathbf{F.R.2}}^2\rangle]}{\omega^2 + \Delta\omega_{L0}^2\cos^2(\xi_0)\cos^2(\psi)}$$

and

$$\mathbf{V_0} = G_{\mathrm{LNA}}\mathbf{V_d}. \tag{10}$$

V_o is the detected signal amplified by the low-noise amplifier (LNA) and finally the fast Fourier transform (FFT) analyzer shows the spectral density of V_o. From (10), the average of the power spectral density of the oscillators can be extracted as follows:

$$S_\varphi = \frac{S_{\varphi\mathrm{F.R.1}} + S_{\varphi\mathrm{F.R.2}}}{2} = \frac{[\omega^2 + \Delta\omega_{L0}^2\cos^2(\xi_0)\cos^2(\psi)]}{2K_{\varphi_0}^2 G_{\mathrm{LNA}}^2\cos^2(\xi_0)\omega^2}S_{V0}$$

where

$$S_{V0} = \left\langle\mathbf{V_o^2}\right\rangle \quad S_{\varphi\mathrm{F.R.}i} = \left\langle\boldsymbol{\delta}\varphi_{\mathbf{F.Ri}}^2\right\rangle, \qquad i = 1, 2. \tag{11}$$

Therefore, the measured signal spectrum (V_o) should be calibrated by (11) to retrieve the average of the free-running phase noise of the oscillators. In this case, the calibration factor is the term that is multiplied by S_{V_o} in (11). In this paper, we call the phase-noise measurement by the inter-injection-locking technique the "IIL method."

It can be noted that if the phase-noise power spectral densities of the oscillators are identical, their average equals their individual values. If the phase noise of one oscillator is much greater than that of the other, the average of their phase noise is 3 dB less than that of the noisier oscillator.

B. Experimental Verification

Two Gunn oscillators at frequencies about 10.5 GHz were used in the setup of Fig. 3 to validate the proposed method. The power level of the oscillators was approximately 11 dBm. The gain of the LNA was set to 70 dB and the detected signal was sampled at a rate of 3-Msamples/s with Analog Devices' 16-bit A/D converter. For each plot, 100 measurements were taken to average out the spurious fluctuations. To verify the measurement validity, five sets of measurement were performed for five different locking bandwidths. The locking bandwidth was changed by altering the magnitude of the coupling coefficient. In Fig. 4, the measured spectra of detected signals V_o, as well as the spectra of the calibrated phase noise, are depicted for all five measurements. It is clear that while the spectra of the detected signals are different for five different locking bandwidths, the spectra of the calibrated phase noise converge to the same values.

To further investigate the validity of our method of measurement, the free-running phase noise of each oscillator was sepa-

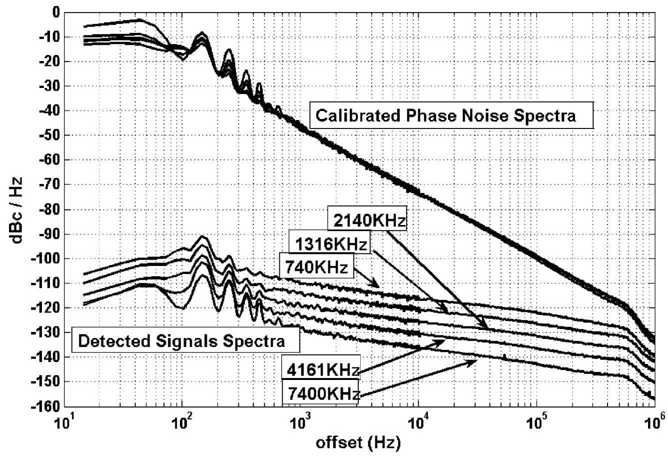

Fig. 4. Results of phase-noise measurement with the IIL technique. The numbers show the value of the locking bandwidth for each measurement. The magnitude of coupling coefficient was altered to change the locking bandwidth. The oscillators center frequencies were approximately 10.5 GHz. The sampling duration was 66.66 ms. The 150-Hz, 250-Hz, etc. spurs are due to the leakage of the ac power line harmonics into the measurement system.

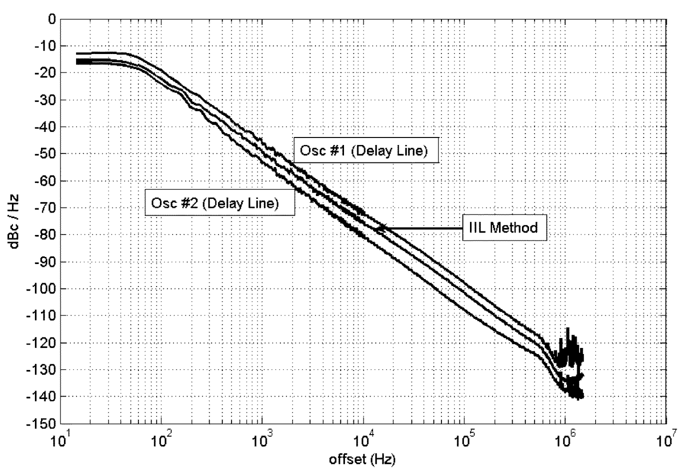

Fig. 5. Phase noise measured by the setup of Fig. 3 compared to the free-running phase noise of the oscillators measured by the delay-line method. The locking bandwidth is 1300 kHz. It is clear that the measured phase noise by the IIL method is average of the free-running phase noise of individual oscillators. The oscillators center frequencies were approximately 10.5 GHz. The sampling duration was 66.66 ms.

rately measured using a 110-ns delay-line method. The free-running phase noise of the oscillators and the phase noise measured by the setup of Fig. 3 are compared in Fig. 5. As is seen in Fig. 5, the measured phase noise by the IIL method is the average of the free-running phase noise of the oscillators as predicted by the theory.

In the measurements of Fig. 4, the locking bandwidth was changed by altering the magnitude of the coupling coefficient (ε). It is also possible to change the locking bandwidth by altering the coupling coefficient phase (ψ). The measurement was repeated with a fixed value of ε and several values of ψ. The results are shown in Fig. 6. Similarly varying the value of ψ changes the level of the detected signal, but the calibrated phase noise is virtually the same for all the measurements.

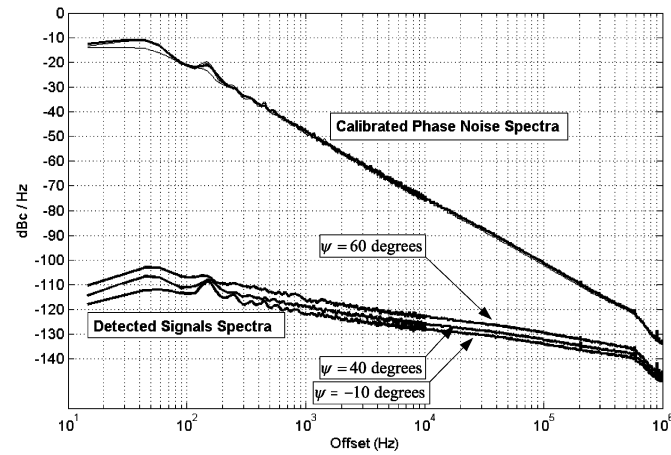

Fig. 6. Results of phase-noise measurement with inter-injection locking technique for several values of ψ. The locking bandwidth for $\psi = 0$ was 1300 kHz. The oscillators center frequencies were approximately 10.5 GHz. The sampling duration was 66.66 ms.

TABLE I
MEASUREMENT ERROR DUE TO THE FREQUENCY DRIFT OF THE OSCILLATORS. THE LOCKING BANDWIDTH IS 1 MHz

Offset (Hz) / Drift (Hz)	100k	300k	500k	800k	1M	
400k	0	0.07	0.16	0.31	0.39	Error due to Frequency Drift (dB)
600k	0.02	0.2	0.46	0.86	1	
800k	0.08	0.59	1.32	2.29	2.76	

C. Measurement Errors

1) Measurement Error Due to the Frequency Drift of the Oscillators: It was shown that a steady-state phase difference (ξ_0) is produced between the synchronized oscillators due to the difference in their free-running frequencies. Practical oscillators have slow free-running frequency drifts due to the oscillator environment. Therefore, during the measurement process, the free-running frequency difference of the oscillators varies, which leads into the variation of the after-lock phase difference. One may anticipate that the measurement becomes erroneous due to the frequency drift of the oscillators. Careful investigation of (11) reveals that ξ_0 is present in the denominator as it is in the numerator; as a result, the calibration factor has a minor sensitivity to ξ_0. Table I shows the measurement error at different frequency offsets due to the free-running frequency drift of the oscillators. The locking bandwidth is assumed to be 1 MHz. It is seen that even in the case of 600-kHz frequency drift, the phase-noise measurement error at frequency offsets up to 1 MHz is less than 1 dB. It can be concluded that, for most practical cases, the frequency drift of the oscillators has a negligible effect on the phase-noise measurement.

2) Measurement Error Induced by Discrepancy in Parameters of the Oscillators: Thus far, it has been assumed that the oscillators have the same amplitude and external quality factors. In practice, even the oscillators with the same circuit design do not have identical amplitudes and external quality factors. Here, the effect of discrepancy in parameters of the oscillators is investigated. Note that since AM noise is much smaller than the PM noise, the nonlinear susceptance of the oscillators have a small

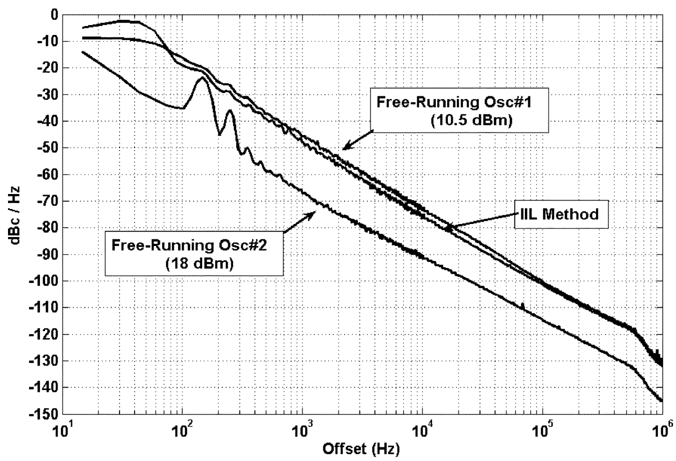

Fig. 7. Phase-noise measurement with the inter-injection technique when the oscillators have 7.5-dB output power level difference. The free-running phase noise of the oscillators was measured by the delay-line method with 110-ns delay. It is clear that the measured phase noise by the IIL method is the average of the free-running phase noise of individual oscillators. The oscillators' center frequencies were approximately 10.5 GHz. The sampling duration was 66.66 ms. The 150- and 250-Hz spurs are due to the leakage of the ac power line harmonics into the measurement system.

role in the interactions and, thus, there is no need to regard the difference in the nonlinear susceptance of the oscillators. For the sake of simplicity, ψ is set to zero.

The steady-state analysis of two oscillators with different amplitudes (A_1 and A_2), which are inter-injection locked to each other, has been worked out in [16]. It is shown that in this case, the locking bandwidth is

$$\Delta\omega_{L0}^{*} = \frac{\varepsilon\omega_0}{2Q}\left(\frac{A_1}{A_2}+\frac{A_2}{A_1}\right), \qquad \text{for } \psi = 0. \tag{12}$$

$\Delta\omega_{L0}^{*}$ denotes the locking bandwidth when $\psi = 0$ and when the oscillators have different amplitudes. Phase-noise analysis when the oscillators have different amplitudes is straightforward following the procedure in Section II-C. The result is that the detected signal by the mixer in Fig. 3 becomes the following:

$$\mathbf{V_d} = K_{\varphi 0}\cos(\xi_0)\frac{-j\omega[\boldsymbol{\delta\varphi}_{\mathbf{F.R.1}}-\boldsymbol{\delta\varphi}_{\mathbf{F.R.2}}]}{j\omega+\Delta\omega_{L0}^{*}\cos(\xi_0)}, \qquad \text{for } \psi = 0. \tag{13}$$

The form of (13) is exactly the same as that of (10) when ψ is zero. The only difference is that $\Delta\omega_{L0}$ is replaced with $\Delta\omega_{L0}^{*}$. In the other words, the difference in the amplitude of the oscillators just alters the locking bandwidth, and if the locking bandwidth is correctly measured and used in (11), no error is induced on the measurement. To practically verify this argument, two oscillators with the output power level difference of 7.5 dB were used in the setup of Fig. 3. The measured locking bandwidth was 1400 kHz. The measurement result is depicted on Fig. 7. It can be seen that even in the case of 7.5-dB output power difference between the oscillators, the measured phase noise by the IIL method is, with a good approximation, the average of the phase noise of the oscillators. It can be concluded that the difference in the output power of the oscillators has a minor effect on the measurement.

If the oscillators have different external quality factors, with the same line of reasoning, one can conclude that the discrepancy in external quality factors of the oscillators just affects the locking bandwidth and (11) remains unchanged. It can be concluded that as far as the locking bandwidth is measured correctly, the discrepancy in parameters of the oscillators does not affect the measurement validity.

3) Measurement Error Induced by Nonlinear Susceptance of Oscillator Model: In the analysis of Section II-C, AM noise was neglected. Since the nonlinear susceptance of oscillator model produces AM-to-PM and PM-to-AM conversion, neglecting AM noise causes B_{NL} to be removed from the equations. In order to determine the effect of B_{NL}, AM noise should be entered into the analysis.

The steady-state analysis of two mutually coupled oscillators, which takes into account the effect of nonlinear susceptance, has been performed in [13]. It is shown that if the phase of the coupling coefficient is zero, B_{NL} does not affect the locking bandwidth and (3) with $\psi = 0$ remains valid. In order to extract the equations for after-lock phase noise, one should substitute (5) into (2) for each of the oscillators. The results are as follows:

$$\begin{cases} \dfrac{\partial B}{\partial\omega}\dfrac{d\delta A_1}{Adt} + \dfrac{\partial G_{\mathrm{NL}}}{\partial A}\delta A_1 \\ \quad = -\varepsilon\sin(\xi_0)(\delta\varphi_2-\delta\varphi_1) \\ \dfrac{\partial B}{\partial\omega}\dfrac{d\delta\varphi_1}{dt} + \dfrac{\partial B_{\mathrm{NL}}}{\partial A}\delta A_1 \\ \quad = \varepsilon\cos(\xi_0)(\delta\varphi_2-\delta\varphi_1) + \dfrac{\partial B}{\partial\omega}\delta\omega_{\mathrm{F.R.1}} \end{cases}$$
$$\begin{cases} \dfrac{\partial B}{\partial\omega}\dfrac{d\delta A_2}{Adt} + \dfrac{\partial G_{\mathrm{NL}}}{\partial A}\delta A_2 \\ \quad = -\varepsilon\sin(\xi_0)(\delta\varphi_2-\delta\varphi_1) \\ \dfrac{\partial B}{\partial\omega}\dfrac{d\delta\varphi_2}{dt} + \dfrac{\partial B_{\mathrm{NL}}}{\partial A}\delta A_2 \\ \quad = -\varepsilon\cos(\xi_0)(\delta\varphi_2-\delta\varphi_1) + \dfrac{\partial B}{\partial\omega}\delta\omega_{\mathrm{F.R.2}}. \end{cases} \tag{14}$$

The free-running frequency fluctuations have been introduced into the equations after the substitution of (5) into (2). It is noteworthy to mention that since AM noise has not been considered in (5), δA_1 and δA_2 in (14) are that portion of AM noise that has been produced by PM-to-AM conversion. By careful investigation of (14), it is revealed that δA_1 and δA_2 have similar expressions and, thus, are equal $\delta A_1 = \delta A_2$. Using this fact and subtracting the second line of each couple of (14) from one another, one obtains

$$\frac{\partial B}{\partial\omega}\frac{d}{dt}[\delta\varphi_1-\delta\varphi_2] = 2\varepsilon\cos(\xi_0)[\delta\varphi_2-\delta\varphi_1] + \frac{\partial B}{\partial\omega}(\delta\omega_{\mathrm{F.R.1}}-\delta\omega_{\mathrm{F.R.2}}). \tag{15}$$

After Fourier transformation and some manipulations, one obtains

$$\boldsymbol{\delta\varphi_1} - \boldsymbol{\delta\varphi_2} = \frac{j\omega[\boldsymbol{\delta\varphi}_{\mathbf{F.R.1}}-\boldsymbol{\delta\varphi}_{\mathbf{F.R.2}}]}{\frac{2\varepsilon}{\frac{\partial B}{\partial\omega}}\cos(\xi_0)+j\omega}. \tag{16}$$

From (3), (9), and (16), the detected signal by the mixer in Fig. 3 is calculated as follows:

$$\mathbf{V_d} = K_{\varphi_0}\cos(\xi_0)\frac{-j\omega[\boldsymbol{\delta}\varphi_{\mathbf{F.R.1}} - \boldsymbol{\delta}\varphi_{\mathbf{F.R.2}}]}{j\omega + \Delta\omega_{L0}\cos(\xi_0)}. \tag{17}$$

The form of (17) is exactly the same as that of (10). In other words, the nonlinear susceptance of the oscillators has no effect on the detected signal and on the calibration process provided that the phase of the coupling coefficient is zero.

One can conclude that the state of $\psi = 0$ is the optimum state, which makes the system insensitive to imperfections such as nonzero nonlinear susceptance and discrepancy in parameters of the oscillators. Note that the locking bandwidth can be conveniently varied by altering the magnitude of the coupling coefficient.

D. Comparison of the Proposed Method With the Conventional Methods

Phase-noise measurement by inter-injection locking of two oscillators is simple and easy to implement. Synchronization of the oscillators by inter-injection locking is relatively easy and the calibration process is straightforward: after measuring the locking bandwidth, one should use (11) for calibrating the phase noise. In contrast, the conventional two-oscillator method is expensive because it requires one of the oscillators to be electronically tunable and, in addition, it uses the PLL to synchronize the oscillators. Synchronization of the oscillators by the PLL is difficult and the loop bandwidth of the PLL limits the application of the method only to the relatively stable oscillators. Thus, one advantage of the proposed method over the conventional phase-detector method is that it can be used for the phase-noise measurement of relatively unstable free-running oscillators where the conventional phase-detector method is difficult to implement. Compared to the conventional delay-line method, phase-noise measurement by the IIL method can be conveniently used for millimeter-wave oscillators where the conventional delay-line method fails to work due to the considerable loss of the delay line.

The phase-noise floor of the proposed method is determined by replacing S_{Vo} in (11) with $S_n = \langle V_n^2 \rangle$, where V_n is the voltage noise floor of the system mainly produced by the noise sources in the mixer and LNA

$$S_{\varphi_{\text{noise floor}}} = \frac{[\omega^2 + \Delta\omega_{L0}^2\cos^2(\xi_0)\cos^2(\psi)]}{2K_{\varphi_0}^2 G_{\text{LNA}}^2\cos^2(\xi_0)\omega^2} S_n. \tag{18}$$

To measure V_n, an RF signal is split and fed into the mixer ports. The phase noise of the signals at the mixer ports are the same and are canceled out at the mixer output. The remaining signal is then the voltage noise floor of the system. The voltage noise floor of the setup of Fig. 3 was measured and its phase-noise floor was calculated for different locking bandwidths, which are depicted in Fig. 8.

According to (8), by decreasing the locking bandwidth, the after-lock phase noises of the oscillators become less correlated and, thus, the strength of the detected signal V_d is increased compared to the system noise floor. This means that the system phase-noise floor is decreased, as is obvious from (18) and Fig. 8. In Fig. 8, approaching the carrier, the phase-noise floor of the proposed method increases with the slope of 20 dB/dec. Since the flicker noise corner frequency of the system was near 1 kHz, the voltage noise floor of the system was rather flat with frequency, therefore, the term ω in the numerator of (18) accounts for this slope. If the same mixer and LNA were used in the conventional phase-detector method, the phase-noise floor would be what is labeled as "PLL" in Fig. 8. It is evident that the phase-noise floor of the proposed method is higher than that of the phase-detector method. This renders the performance of the method introduced in this paper inferior to the conventional phase-detector method in case of small loop bandwidth. However, in Fig. 8, the effect of PLL loop bandwidth has not been taken into account. The phase-noise floor of the conventional phase-detector method drastically increases within the loop bandwidth [11]. Thus, if the oscillator under test has a broad span of frequency deviation, a wide loop bandwidth is required for synchronization, which severely degrades the phase-noise floor. In addition, as far as the oscillators not going out of lock, the phase-noise floor of the setup of Fig. 3 can be improved by decreasing the locking bandwidth. If the same mixer and LNA were used in the conventional delay-line method with a delay of 110 ns (30-m length of ultra-low-loss cable), the phase-noise floor would be what is labeled as the "110 ns delay line" in Fig. 8. It is seen that the proposed method with a locking bandwidth of 1 MHz has a lower phase-noise floor than the delay-line method. A locking bandwidth of 1 MHz is sufficient for most practical cases. Therefore, in general, the performance of the inter-injection technique is superior to the delay-line method.

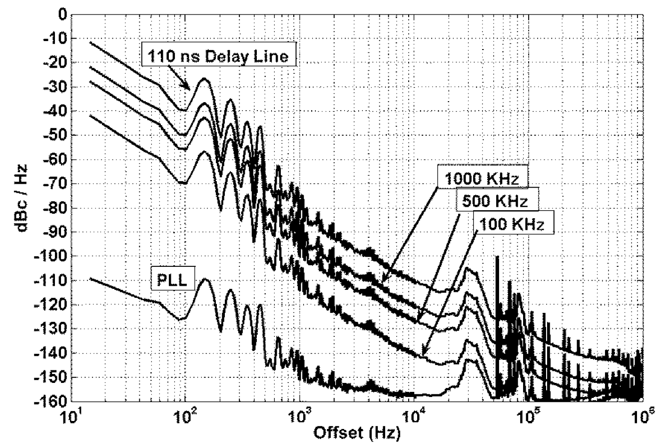

Fig. 8. Phase-noise floor of the setup of Fig. 3 for different locking bandwidths. Phase-noise floor of the phase detector and the delay-line method are also depicted. The spurs are due to the ac line harmonics.

IV. CONCLUSION

The proposed IIL setup, which is based on inter-injection locking of two oscillators, measures the free-running phase-noise average of the oscillators. For most oscillators, the presented theory is accurate for offsets down to approximately 100 Hz where the phase noise of the oscillator can be interpreted as a small perturbation around carrier. Experimental results showed the validity of the proposed technique. It was shown that, in the case of a zero phase-coupling coefficient, the only pre-required parameter for the measurement is the locking bandwidth, and if the locking bandwidth is measured correctly, the measurement results are not vulnerable to error due to imperfections such as the drift, discrepancy in oscillator parameters, and nonlinear susceptance of the oscillators. In contrast to the conventional phase-detector method, the proposed measurement setup and calibration is easy and applicable to even relatively unstable oscillators. The phase-noise floor of the IIL technique is controllable and is lower than that of the delay-line method, but it is higher than that of the phase-detector method with a small loop bandwidth. In addition, the proposed setup is simple and easy to implement with readily available components. The IIL method can be conveniently used for millimeter-wave oscillators where phase-noise measurement by the delay-line method is not practical.

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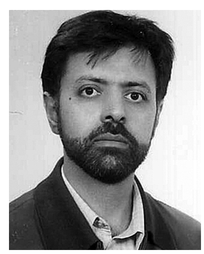

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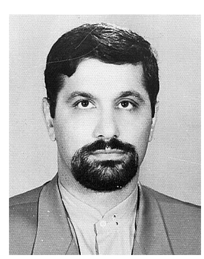

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