A Circuit Model for Analysis of Metal–Insulator–Metal Plasmonic Complementary Split-Ring Resonators

Meisam Bahadori, Ali Eshaghian, and Khashayar Mehrany

Abstract—A circuit model based on the transmission line theory is proposed to analyze the recently introduced metal–insulator–metal (MIM) complementary split-ring resonators (CSRRs). It is shown that integer and noninteger modes of CSRRs can be characterized by transmission line models with short- and open-circuited terminals. The proposed circuit model is then extended to incorporate side-coupling effects between the CSRRs and straight MIM waveguides. Thereby, simple closed-form expressions are provided for the coupling quality factor. It is shown that waveguide resonator structures based on CSRRs at specific resonance frequency and bandwidth can be smaller than waveguide resonator structures based on conventional ring resonators. Thanks to the smaller size of CSRRs, the coupling in the former arrangement is stronger than it is in the latter. Therefore, CSRRs are not required to be in the extreme proximity of the waveguide as in the waveguide ring resonators.

Index Terms—Coupled mode theory (CMT), coupled transmission lines, impedance, metal–insulator–metal, plasmons, split-ring resonators, surface waves.

I. INTRODUCTION

PLASMONIC resonators are capable of confining light to small volumes below the diffraction limit [1], [2]. With such tight confinement, new horizons are opened up for squeezing light into deep sub-wavelength regions of space [3]–[5] with miscellaneous functionalities for on-chip nanophotonics. Various structures for plasmonic ring resonators have been studied in recent years. In 2006, Xiao et al. [6] studied the two-dimensional realization of metal–insulator–metal (MIM) add-drop filters and concluded that metallic loss is a problematic issue in such structures due to its dramatic effect on the quality factor. In the same year, Bozhevolnyi et al. [7] used channel plasmon polaritons (V-grooves) to experimentally investigate a plasmonic waveguide-ring resonator. Next year, they used the same channel plasmon scheme to fabricate and characterize wavelength selective structures such as add-drop filters with a ring resonator [8]. In the same year, Hosseini and et al. [9] analyzed an MIM rectangular ring resonator side-coupled to a waveguide and proposed a plasmonic add-drop filter. In 2009, a dielectric-loaded plasmonic ring resonator was introduced by Holmgaard et al. [10].

In that year, Han et al. [11] proposed aperture coupled MIM ring resonators and argued that aperture-coupling mechanism could be more efficient over the evanescent coupling mechanism. Still, the idea of evanescent coupling was more widely recognized. For instance, the capability of MIM circular ring resonators in realization of wavelength demultiplexing was demonstrated by utilizing evanescent coupling mechanism between a ring and straight waveguides [12]. In 2010, the same idea was further pursued by substituting the circular ring with a rectangular ring [13].

Recently, the structure of MIM complementary split-ring resonator (CSRR) has been proposed [14]. A CSRR is formed by inserting a metallic wall inside a regular MIM ring resonator. This metallic wall disturbs the symmetry of the MIM ring resonator and excludes CSRRs from the category of traveling-wave resonators. Interestingly, CSRRs accommodate non-integer modes in addition to the standard integer modes supported by regular ring resonators. In a more recent modification reported in 2013, Ma and Lee [15] numerically investigated the structure of a circular CSRR and found out that the non-integer resonance modes of these structures are highly sensitive to the width of the metallic nano-wall, as well as its position with respect to a coupling waveguide. They also discussed that the circular geometry of CSRR outperforms the rectangular geometry as the effects of sharp bending are not present in circular CSRRs. Introduction of a narrow metallic-wall in the CSRRs provides a unique way of tuning the coupling efficiency of CSRRs by either changing the width of the metallic wall or changing its position. Regrettably, accurate analysis of these structures is hitherto carried out by the finite-difference time-domain method. Such brute-force numerical methods impose undesirable numerical burden, and provide only a limited insight, if any, of the electromagnetic phenomena inside the structure. Therefore, an approximate yet accurate enough solution for extraction and analysis of the resonant modes is warmly welcome.

The most straightforward approach to have approximate solutions for optical coupling interactions is to call upon the standard coupled-mode theory (CMT) [16]–[18]. Since the dispersion of the metal is usually weak over the desired frequency region, standard CMT is accurate for MIM plasmonic resonators [6]. Unfortunately, however, CMT is still in demand of some numerical calculations. The resonance wavelengths, intrinsic and coupling quality factors for integer and non-integer modes are all needed. Their calculations necessitate numerical simulations. Another analytical method that has received some attention in recent years is the introduction of the circuit theory in
metal-optics [19] and the use of transmission line (TL) concepts in modeling and approximate analysis of MIM devices. The parameters of such models can be determined in terms of geometrical dimensions and waveguide propagation constant, which are readily known [1]. Structures such as splitters [20], [21], junctions [22], [23], stubs [24], [25], and demultiplexers [26] are among those that have been successfully analyzed by the TL models. Recently, the use of coupled TLs has led to the theoretical investigation of side-coupling between parallel MIM waveguides [27], [28]. Furthermore, an extension of this theory has been proposed for the incorporation of the higher order modes at MIM discontinuities [29]. It should be mentioned that TL models have been already employed to extract the resonance frequencies of rectangular CSRRs [14]. However, such analysis has two essential flaws. First, transmission characteristics and resonance quality factors of the CSRR in side-coupled arrangement are yet to be extracted numerically. Second, the proposed model provides limited accuracy since it neglects the effects of the evanescent-coupling (tunneling) between the two sides of the metallic nano-gap [26]. These effects become more prominent for integer modes and in case of small gaps (10–20 nm).

In this paper, for the first time to the best of our knowledge, an approximate and accurate enough analytical model based on TL theory is presented and utilized to obtain the spectral characteristics of the recently proposed MIM CSRRs in side-coupling arrangements. The proposed model takes into account the effects of evanescent coupling between the two sides of the metallic nano-gap and thus is more accurate than the model presented in [14]. The proposed model provides closed-form expressions for resonance wavelengths, the intrinsic and waveguide coupling quality factors.

The rest of this paper is organized as follows: Section II discusses the TL model of an isolated CSRR and gives its resonance frequencies and their corresponding intrinsic quality factors. Section III extends the concept of the TL model to include the side-coupling between a CSRR and a straight waveguide. Thanks to the analytic nature of the model, closed-form expressions are provided for the coupling quality factors in Section IV. In Section V, the proposed TL model is employed to design plasmonic filters with specific resonance frequency and bandwidth. Finally, conclusions are made in Section VI.

II. CIRCUIT MODEL FOR UNCOUPLED MIM CSRR

In this section we present a circuit model for analysis of circular plasmonic CSRRs. The model provides closed-form expressions for the resonance frequency and intrinsic quality factor of the integer and non-integer modes supported by CSRRs.

The structure of a circular MIM CSRR is shown in Fig. 1(a). It can be seen that CSRR is formed when a metallic wall splits the conventional ring resonator structure. The width of the metallic wall, the inner and the outer radii of the ring are denoted by \( W_g \), \( R \), and \( R + r \), respectively. This structure is modeled by two TL segments cascaded to each other. One is needed to represent wave propagation within the split MIM ring, and the other to account for the metallic wall. Naturally, the lengths of the former and of the latter are \( L_r = 2\pi (R + r/2) - W_g \), and \( W_g \), respectively. Since bending loss in plasmonic ring resonators is virtually absent [11], the propagation constant, \( \beta_m \), and the characteristic impedance, \( Z_m \), of the TL segment representing the split ring can be accurately approximated by the propagation constant and the characteristic impedance of a straight MIM waveguide whose width is \( r \). Therefore, the following equations hold [1], [20]:

\[
\tanh \left( \frac{\beta_m^2 - k_0^2 \varepsilon_d}{2} \frac{r}{\lambda_0} \right) = -\frac{\sqrt{\beta_m^2 - k_0^2 \varepsilon_m / \varepsilon_d}}{\sqrt{\beta_m^2 - k_0^2 \varepsilon_m / \varepsilon_d}} \quad (1a)
\]

\[
Z_m \approx r \frac{\beta_m}{\omega \varepsilon_0 \varepsilon_d} \quad (1b)
\]

where \( k_0 \) is the free space wavenumber, \( \varepsilon_d \) is the dielectric permittivity inside the waveguide, and \( \varepsilon_m \) is the complex permittivity of the metal.

Since narrow metallic walls do not cause significant diffraction, the propagation constant \( \beta_m \) and the characteristic impedance \( Z_m \) of the TL segment representing the metallic wall can be written as [26]

\[
\beta_m \approx \sqrt{\beta_m^2 + k_0^2 (\varepsilon_m - \varepsilon_d)}, \quad Z_m \approx r \frac{\beta_m}{\omega \varepsilon_0 \varepsilon_m} \quad (2)
\]

The proposed TL model [see Fig. 1(b)] is then formed by cascading the above-mentioned TL segments in accordance with the geometrical figure of the CSRR. It can be easily shown that the resonance condition in the model is met when the following equation is held:

\[
\cos(\beta_m W_g) \cos(\beta_m L_r) - 1 = (Z_m/Z_m + Z_m/2Z_m) \times \sin(\beta_m W_g) \sin(\beta_m L_r) \quad (3)
\]

It is worth noting that the above-mentioned resonance condition is no different from the standard constructive interference condition for the round trip phase when \( W_g \) approaches zero, i.e. the CSRR becomes a conventional MIM ring resonator

\[
\text{Re} \{\beta_g\} \times L_r = 2m\pi, \quad m \in \mathbb{Z} \quad (4)
\]

For the general case when \( W_g \) is nonzero, the resonance condition given in (3) can be rewritten as a multiplication of two
separate factors
\[
\{ Z_d \tan(\beta_d L_r / 2) + Z_m \tan(\beta_m W_g / 2) \} \\
\times \{ Z_d \cot(\beta_d L_r / 2) + Z_m \cot(\beta_m W_g / 2) \} = 0. \tag{5}
\]

Since the resonance condition is met whenever either of these factors becomes zero, two distinct sets of modes are conceivable for the CSRR. These two sets have even and odd symmetry about the AA’−BB’ axis passing through the metallic wall [see Fig. 1(a)]. The first set of modes obtained by finding the zeros of the first factor in (5) corresponds to the TL model in Fig. 2(a). It can be easily seen that the amplitude of the current phasor in this model is maximum at short-circuit AA’ and BB’ terminals. Since the current distribution in the model represents the magnetic field profile in the structure, the TL model in Fig. 2(a) represents the set of resonant modes whose magnetic field profiles have even symmetry about AA’−BB’ axis. They are referred to as the integer modes because their resonance condition for \( W_g \ll L_r \) is no different from having a round trip phase whose value is an integer multiple of \( 2\pi \) [see (4)]. It is no wonder that integer modes closely resemble modes of a conventional MIM ring.

The second set of modes obtained by finding the zeros of the second factor in (5) corresponds to the TL model in Fig. 2(b). This time, the current phasor is zero at open-circuit AA’ and BB’ terminals and therefore its corresponding magnetic field profile has odd symmetry about AA’−BB’ axis. Since the round trip phase of these modes is not an integer multiple of \( 2\pi \) even for \( W_g \ll L_r \), they are referred to as the non-integer modes [15]. It can be shown that there is always a non-integer mode between two consecutive integer modes. Therefore, resonant modes of CSRR can be ordered as TM\(_1\), TM\(_1.5\), TM\(_2\), TM\(_2.5\), and so forth. It should be noted that the designation of the TE and TM modes in the resonator depends on what is chosen to be the longitudinal direction, which is rather arbitrary. Referring to the \( n \)th order fundamental plasmonic modes of the resonator as TM\(_n\), follows the suit of the previous works [14], [15] but is a rather odd choice because the magnetic field of the resonator is perpendicular to the ring plane. In the cylindrical coordinate system whose origin is at the center of the resonator, the longitudinal direction is perpendicular to the ring and plasmonic modes should be referred to as TE\(_{0n0}\). The reason on account of which the plasmonic modes were originally referred to as TM\(_n\), is probably due to the fact that these modes are formed by the TM modes in the bent MIM waveguide that builds the resonator.

The intrinsic quality factor, \( Q_i \), of the isolated CSRR in Fig. 1(a), can be approximated by resorting to the lumped circuit perspective of the TL models [27]. Each section of the equivalent TL model in Fig. 2 is composed of complex per-unit-length series self-inductance, \( L_s \), and shunt self-capacitance, \( C_s \), calculated as
\[
C_s = \epsilon_0 \varepsilon_{d,m} / r_i \quad L_s = r_i \beta_{d,m}^2 / (\omega^2 \epsilon_0 \varepsilon_{d,m}) \tag{6}
\]
where the subscripts \( d \) and \( m \) refer to the dielectric and metallic regions, respectively. According to the standard definition of the intrinsic quality factor of a resonator, \( Q_i \) is written as
\[
Q_i = \omega_0 (W_E + W_H) / P_{loss} \tag{7}
\]
where \( \omega_0 \) is the resonance frequency, \( W_E, W_H \), and \( P_{loss} \) are the average electrical energy, magnetic energy, and lost power at \( \omega_0 \) in the equivalent models. Using the resonance voltage and current distribution together with the per-unit-length lumped circuit elements in the TL model, \( W_E, W_H \), and \( P_{loss} \) can be written as
\[
W_E = \frac{1}{4} \left( \int_{\text{MIM}} dz + \int_{\text{Wall}} dz \right) \text{Re} (C_s) |V(z)|^2 \tag{8a}
\]
\[
W_H = \frac{1}{4} \left( \int_{\text{MIM}} dz + \int_{\text{Wall}} dz \right) \text{Re} (L_s) |I(z)|^2 \tag{8b}
\]
\[
P_{loss} = \frac{-\omega_0}{2} \left( \int_{\text{MIM}} dz + \int_{\text{Wall}} dz \right) \left\{ \text{Im} (C_s) |V(z)|^2 + \text{Im} (L_s) |I(z)|^2 \right\}. \tag{8c}
\]
Substitution of (8) in (7) yields the following formula for the intrinsic quality factor of the conventional MIM ring:
\[
Q_i^{\text{RADIUS}} = \frac{\text{Re} (C_s |Z_d|^2 + L_s)}{2 \text{Im} (C_s |Z_d|^2 + L_s)} \tag{9}
\]

Thanks to the strong resemblance between the integer modes of the CSRR and modes of the conventional MIM rings, the same formula can be employed as a good approximation for the intrinsic quality factor of the integer modes in CSRR with narrow enough metallic wall.

III. CIRCUIT MODEL FOR SIDE-COUPLED MIM CSRR

A typical CSRR side-coupled to a straight MIM waveguide is shown in Fig. 3(a). The width of the straight waveguide, and the closest distance between the CSRR and the upper interface of the waveguide are denoted by \( w \) and \( d \), respectively. The position of the metallic wall in the CSRR is determined by the angle \( \theta_0 \) as indicated in the figure. Clearly, the strength of coupling between the MIM waveguide and the CSRR depends on \( d \) and the curvature radius of the CSRR. One way to estimate the coupling strength of the structure is to compare the vertical distance between the upper interface of the MIM waveguide and

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Fig. 2. Equivalent transmission line model of the CSRR for (a) integer modes with short-circuited terminals, and (b) non-integer modes with open-circuited terminals.
become zero when $d > D \approx L$.  

$$\delta_L \approx \sqrt{\frac{c}{\omega}}$$  

Inset shows the circuit model for a section of length $dz$ of the coupled transmission line representing the side coupling of the straight waveguide and the CSRR.

different points on the outer circumference of the CSRR against the penetration depth of the metal ($\delta_m$)  

$$\delta_m \approx 1 / \text{Re} \left( \sqrt{\frac{k_m^2 - k_m^2 \varepsilon_m}{R^2}} \right).$$  

Since the coupling becomes too weak when the minimum distance between the CSRR and the MIM waveguide, $d$, is larger than twice the penetration depth ($d > 2\delta_m$), we focus on the case when the coupling is strong ($d < 2\delta_m$). In such a case, it is reasonable to assume that the coupling is almost lost at the point where the decaying tail of the electromagnetic wave confined in the CSRR meets the evanescent tail of the electromagnetic wave confined in the MIM waveguide. There are two such points lying at the intersection of a circle concentric with the CSRR but of radius $R + r + \delta_m$, and a horizontal line at distance $\delta_m$ above the upper interface of the waveguide. The circle and the line mark the decaying tails of the wave in the CSRR and the MIM waveguide, respectively. In accordance with Fig. 3(a), these two points together with the center of the CSRR as vertex subtend the coupling angle $\theta_c$.  

$$\theta_c \approx 2 \cos^{-1} \left( 1 - \frac{D - d}{R + r} \right)$$  

where $D$ is the vertical distance between the upper interface of the MIM waveguide and the most distant point on the outer circumference of the CSRR, which is still coupled to the MIM waveguide  

$$D = \delta_m \left( 2 - \frac{2\delta_m - d}{R + r + \delta_m} \right).$$  

It can be assumed that the length of the circular arc subtending the coupling angle $\theta_c$ is a good estimate for the coupling length  

$$L_c = (R + r/2) \theta_c.$$  

It is worth noting that $\theta_c$ and therefore $L_c$ become zero when $d = 2\delta_m$. Increasing $d$ beyond twice the penetration depth ($d > 2\delta_m$) kills the coupling in our proposed model.  

Now that a good estimate is at hand for the coupling length, the TL model of the CSRR in the previous section can be easily modified to account for the side-coupling effects between the CSRR and the MIM. This is shown in Fig. 3(b) for $\theta_m - \theta_0 < 3\pi/2 - \theta_0/2$. It can be seen that the modified model is still made of cascaded TL segments representing the metallic wall and the split MIM ring. Nevertheless, the TL segment corresponding to the split ring region is partly coupled to the conventional TL model already reported for straight MIM waveguides. The complex per-unit-length series self-inductance $L_s$ and shunt self-capacitance $C_s$ in the TL model of the straight MIM waveguide can be obtained from (9) by replacing $r$ with $w$. The complex per-unit-length series mutual-inductance $L_m$, and shunt mutual-capacitance $C_m$ existing in the coupling region between the CSRR and the MIM, on the other hand, are not uniformly along the coupling region. Fortunately, the non-uniformity of $L_m$ and $C_m$ is practically negligible for $D/d < 2.5$ when the strength of coupling remains more or less constant along the coupling region. In such a case, the non-uniform coupling region shown in Fig. 4(a) can be approximated by the two side-coupled straight MIM waveguides shown in Fig. 5(b). The distance $d_{eff}$ between the straight waveguides of the latter structure is found by calculating the average distance along the coupling length between the CSRR and the MIM waveguide  

$$d_{eff} \approx \frac{\int d(z)p(z)dz}{\int p(z)dz}$$  

where $p(z)$ is a weight function that reflects the non-uniformity of the coupling strength between the CSRR and the waveguide.
One intuitively appealing choice for the weight function $p(z)$ is

$$p(z) = \sin \left( A \frac{z}{R + r} \right), \quad A = \frac{\pi}{2 \sin(\theta_r/2)} \quad (15)$$

whose maximum point at $z = (R + r) \sin(\theta_r/2)$ and minimum points at $z = 0$, and $2(R + r)\sin(\theta_r/2)$ correspond to the physical spots where the coupling strength reaches its maximum and minimum, respectively [see Fig. 4(a)]. The effective distance is then calculated as

$$d_{\text{eff}} \approx d + (R + r) \left( 1 - A \int_0^{\theta_r/2} \cos^2 \alpha \cos(A \sin \alpha) d\alpha \right). \quad (16)$$

It is worth noting that the coupling angle, $\theta_r$, much like the coupling strength, is a function of the frequency, and thus, the effective distance is frequency dependent.

The complex per-unit-length mutual circuit elements of the model, $L_m$ and $C_m$, can then be written in terms of the even and odd supermodes supported by two side-coupled straight MIM waveguides [27] in Fig. 4(b). For practical cases where the width of the straight waveguide is equal to the thickness of the CSRR, the complex mutual capacitance and mutual inductance follow very simple expressions [29]

$$L_m = \left[ \left( \frac{\beta_e}{\beta_o} \right)^2 - 1 \right] L_S \quad (17a)$$

$$C_m = 0.5 \left[ \frac{\beta_o^2}{2\beta_o^2 - \beta_e^2} - 1 \right] C_S \quad (17b)$$

where $\beta_e$ and $\beta_o$ are the propagation constants of the even and odd supermodes between two side-coupled waveguides. The parameters of the TL segments representing the uncoupled region of the CSRR are not different from the parameters of the TL segments representing the isolated CSRR. They are shown in Fig. 3(b). Once the parameters of the model are all obtained, closed form expressions for the transmittance, $S_{21}(\omega)$, and reflection, $S_{11}(\omega)$, can be easily obtained [28].

It is worth noting that the proposed model in Fig. 3(b) is valid as long as the metallic wall does not enter the estimated coupling region. If that happens, the TL model of the side-coupled part must be divided into three segments, two of which representing the side coupling between the CSRR and the waveguide. These two segments are then connected to each other via the TLs that represent the metallic wall and the uncoupled part of the waveguide between the two segments. Regrettably, the coupling between the straight MIM waveguide and the metallic wall in the CSRR cannot be accurately modeled in this manner. Therefore, it is applicable only when the coupling is supported by integer modes.

**IV. COUPLING QUALITY FACTORS IN TERMS OF CIRCUIT ELEMENTS**

Conventionally, the temporal CMT has been used to extract the spectral response of a side-coupled resonator at each of its resonance modes. The estimated transmittance spectrum of the CMT is

$$T(\omega) = \frac{j(\omega - \omega_0) + 0.5\omega_0(1/Q_i - 1/Q_o)}{j(\omega - \omega_0) + 0.5\omega_0(1/Q_i + 1/Q_o)}$$

for a ring resonator (traveling wave resonator) [16], and is

$$T(\omega) = \frac{j(\omega - \omega_0) + 0.5\omega_0(1/Q_i)}{j(\omega - \omega_0) + 0.5\omega_0(1/Q_i + 1/Q_o)}$$

for a CSRR (standing wave resonator) [18]. In these expressions, $\omega_0$, $Q_i$, and $Q_o$ are the resonance frequency, intrinsic quality factor, and waveguide coupling quality factor.

Since $\omega_0$ and $Q_i$ of the CSRR structure are already given in (5) and (7), the coupling quality factor, $Q_w$, can be found by comparing the transmittance spectrum as obtained by the proposed model, i.e. $S_{21}(\omega_0)$, against the transmittance spectrum as obtained by CMT, i.e. $T(\omega_0)$. It can be shown that

$$Q_w = Q_i \frac{|S_{21}(\omega_0)|}{1 - |S_{21}(\omega_0)|}. \quad (20)$$

By following the same approach, the coupling quality factor when the metallic wall is absent ($W_g = 0$) can be written as

$$Q_w = Q_i \frac{1 - |S_{21}(\omega_0)|}{1 + |S_{21}(\omega_0)|} \quad (21)$$

for an over-coupled resonance ($Q_i > Q_o$), and

$$Q_w = Q_i \frac{1 + |S_{21}(\omega_0)|}{1 - |S_{21}(\omega_0)|} \quad (22)$$

for an under-coupled resonance ($Q_i < Q_o$). Since the over- and under-coupled modes are easily distinguished by their distinct phase behavior [18], the phase of $S_{21}(\omega)$ can be readily utilized to determine whether the structure is under-coupled or over-coupled. Obviously, distinction between over- and under-coupled regimes is meaningless when the metallic wall is present; because the resonator becomes standing type. Therefore, there is no critical coupling for the CSRR. The product of the half power bandwidth (HPBW), and the amplitude of the wave transmission at the resonance frequency $\omega_0$ is

$$\text{HPBW} \times |T(\omega_0)| = \omega_0/Q_i \quad (23)$$

and there is always a trade-off between bandwidth and transmission efficiency.

**V. NUMERICAL EXAMPLES**

In this section, the proposed model is employed to design first a critically side-coupled ring resonator structure with a specific resonance frequency, and then a side-coupled MIM CSRR with the same resonance frequency and bandwidth. For each case, the widths of the ring and the waveguide are set to the typical values $r = 50$ nm and $w = 50$ nm. The width of the metallic wall for the CSRR is set to $W_g = 20$ nm [15]. The parameters left to be found are the radius of the ring; $R$, the minimum distance between the ring and the waveguide; $d$, and the position of the metallic gap; $\theta_0$. The constraints of the design are that $d > 10$ nm, $100$ nm $< R < 500$ nm, and $0^\circ < \theta_0 < 90^\circ$.

Throughout this section, the dielectric permittivity is $\varepsilon_r = 1$ (the relative permittivity of vacuum), and the approximate Drude...
model is used for the complex permittivity of the metallic region [11]. The obtained results of the proposed model are always verified by the two-dimensional finite-element method using the COMSOL software. The simulations are carried out with 3 nm triangular mesh.

A. Ring Resonator Side-coupled to a Waveguide

In this subsection, a side-coupled ring resonator is to be designed to support a critically coupled resonance mode at 300 THz ($\lambda_0 = 1 \mu m$). The following steps are taken: First, the TL model for isolated ring resonator is invoked, and the permissible values for the radius of the ring to support a resonance mode at 300 THz are found. The obtained results for the radii and their corresponding mode numbers are tabulated in Table I. It is worth noting that the constraints of the design limit the options to TM$_2$, TM$_3$, and TM$_4$. The lower order mode necessitates ring resonators with small radius, while the higher order modes necessitate ring resonators with spacious size. Interestingly, according to (9), all the permissible mode numbers have the same isolated quality factor, $Q_i$.

To ensure that the critical coupling condition is satisfied, the TL model for side-coupled ring resonator is invoked, and the minimum distance between the ring and waveguide, $d$, is found in such a manner that $Q_i \approx Q_w$. It can be shown that the constraint on $d$ leaves us with only two options, TM$_3$, and TM$_4$. The latter enjoys a larger $d$, and thus is the best choice.

To demonstrate that the designed ring resonator works neatly, the transmission power spectra of the designed structure obtained by the proposed model, CMT, and COMSOL are all compared against each other in Fig. 5. There is a good agreement between all three. The intrinsic quality factor of this resonator is estimated as $Q_i \approx 260$ [see Eq. (9)] and the coupling quality factor is determined as $Q_c \approx 267$ [see Eq. (22)]. Therefore, the overall quality factor would be $Q_{tot} \approx 132$ and the half-power bandwidth is about 2.27 THz ($\Delta \lambda \approx 7.5$ nm). The magnetic field profile of the TM$_4$ resonance mode is also plotted in Fig. 5, which shows that nearly no power passes the ring at 300 THz.

B. Complementary Side-coupled Split-Ring Resonator

CSRR is a standing wave resonator and does not support critical coupling condition. The aim of this subsection is to design a side-coupled CSRR to support a resonance mode at 300 THz with the same bandwidth as in the previous subsection (2.27 THz).

In a similar fashion, the TL model for isolated CSRR is invoked, and the permissible values for the radius of the CSRR to support a resonance mode at 300 THz are found. The obtained results for the radii and their corresponding intrinsic quality factors and mode numbers are tabulated in Table II. The constraints of the design limit the options to TM$_2$, TM$_3$, and TM$_4$ among integer modes and to TM$_{2.5}$, TM$_{3.5}$, and TM$_{4.5}$ among non-integer modes. The non-integer modes are discarded because they are of lower intrinsic quality factor. Once again, all the permissible integer mode numbers have nearly the same isolated quality factor, $Q_i$. Given that the desired bandwidth is fixed at 2.27 THz, $Q_{tot}$ should be 132 and thus the coupling quality factor is needed to be $Q_c \approx 268$. Now, the TL model for side-coupled ring resonator is invoked to find $d$, and $\theta$. The obtained results are also given in Table II. The lower order mode is the best choice because it accommodates the smallest radius ($R = 205$ nm) while maintaining the desired bandwidth. The designed CSRR also enjoys a larger minimum distance, $d$, at the expense of lower transmission efficiency in comparison to the previously designed ring resonator.

VI. CONCLUSION

A comprehensive discussion on developing a TL model for MIM CSRRs and their ring counterparts was made in this paper. The success of the proposed model is indebted to the following facts: the absence of curvature loss, negligible diffraction at the
metallic wall, and small penetration depth of the electromagnetic energy within the metallic region. Thanks to the proposed TL model and the analytical expressions that it provides, the design of side-coupled rings and CSRRs is now fast and simple in view of the fact that no heavy numerical calculation is needed.

It was also pointed out that unlike ring resonators, CSRRs do not have critical coupling. Nevertheless, they can provide the same resonance frequency and the same bandwidth with smaller size, and larger minimum distance between the resonator and the straight waveguide. It is worth noting that the minimum distance between the conventional ring resonator and the straight waveguide is usually quite small to ensure that there is a strong enough coupling.

Finally, it should be noted that since the validity of the proposed model is indebted to the high confinement of the electromagnetic energy, silver can be replaced by gold or any other plasmonic material that guarantees high confinement within the dielectric region in the structure.

References


Authors’ biographies not available at the time of publication.