Interference Cancellation in Overloaded Optical CDMA Systems Using Unipolar Walsh Codes

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***Abstract*—In this article, the authors introduce and propose an overloaded Synchronous Optical Code Division Multiple Access (SOCDMA) based on Unipolar Walsh Code (UWC) for fiber-optic communication systems. Given L as the number of chips, it is shown that $2L-2$ active users can be supported by the proposed system. In this system the users of the network are categorized into two groups. Users of each group transmit at the same power level but different from the level of the other group's users. Using the UWC properties we propose a simple receiver that cancels multiple-user interference (MUI) completely. Moreover, we obtain the optimal power level of each group in order to minimize the probability of error for a given average transmit power. We demonstrate that the proposed receiver has an acceptable performance compared to the optimum multiuser receiver.**

I. Introduction

Optical Code-Division Multiple-Access (OCDMA) systems have attracted much attention due to the establishment of Passive Optical Networks (PON) as one of the popular solutions for the residential access. In OCDMA systems, all resources such as time and frequency are available to the all users simultaneously and each user is assigned a unique signature sequence to be distinguished from other users. Two cases are considered in OCDMA system, synchronous and asynchronous. In Synchronous OCDMA (SOCDMA) systems, a time synchronization between users is required. However, SOCDMA systems have higher spectral efficiency compared to asynchronous OCDMA systems and more users can be supported in SOCDMA systems.

OCDMA systems are also classified into coherent and incoherent systems. Unlike in the coherent case, the signature sequences in the incoherent OCDMA systems are chosen from unipolar set $\{0,1\}$. Several signature sequences have been proposed for incoherent OCDMA systems such as Optical Orthogonal Codes (OOC) [1] for the asynchronous case and modified prime sequences [2] for SOCDMA systems. Due to the unipolar signalling in incoherent OCDMA systems, the signature sequences can not be orthogonal even in the synchronous case. Therefore, Multiple-User Interference (MUI) limits the performance of incoherent OCDMA systems.

Many detectors have been proposed to alleviate MUI in the OCDMA systems. Some of them are single user detectors such as correlator detector [1], hard limiter correlator [3], and chip level detector [4]. In the single user detectors, simplicity at the receiver is achieved at the expense of degrading the performance as the number of users increases. To overcome this problem, a number of multiuser detectors have been introduced. Verdu [5] has proposed an optimal detector which can be used in OCDMA systems. However, the optimum detector is prohibitively complex in practical systems. In order to reduce the complexity of the optimum detector, extensive research efforts [6]-[8] are devoted to develop suboptimum receivers. Most notable ones include the decorrelating detector, the Minimum Mean-Squared Error (MMSE) detector, and the successive and parallel interference cancellation [9]-[11].

Recently, a set of overloaded SOCDMA codes was proposed in [12], [13], where the number of users is greater than the number of chips. Using overloaded SOCDMA codes in communication networks will result in more efficient use of bandwidth. These codes are uniquely detectable for overloaded SOCDMA systems named Codes for Overloaded Optical CDMA (COO). A simple Maximum Likelihood (ML) detection algorithm was proposed for some COO codes derived by the knroneker method [12]. However, for general uniquely detectable codes, there is no simple receiver to detect the user's information.

In this paper, we propose an overloaded SOCDMA system based on Unipolar Walsh Codes (UWC). In our proposed scheme, the users are divided into two groups and users in each group transmit their codes at the same power but different from the other group. Using UWC properties we propose a simple receiver that completely removes the MUI. Furthermore, we obtain the system performance due to the thermal noise and also obtain the optimal power level of each group to minimize the Bit Error Rate (BER) for a predefined average transmit power. The rest of this paper is organized as follows: Section II describes our system model. Section III presents the proposed receiver structure. Simulation results are investigated and discussed in Section IV. Finally, Section V concludes the paper.

II. System Model

In this paper, a common On-Off Keying (OOK) OCDMA system is considered. Each user transmits its corresponding information bits employing UWCs as its signature sequence to be discussed in this section. In the following, we describe our structure model from two different aspects; namely, the designed codes and the transmitter structure.

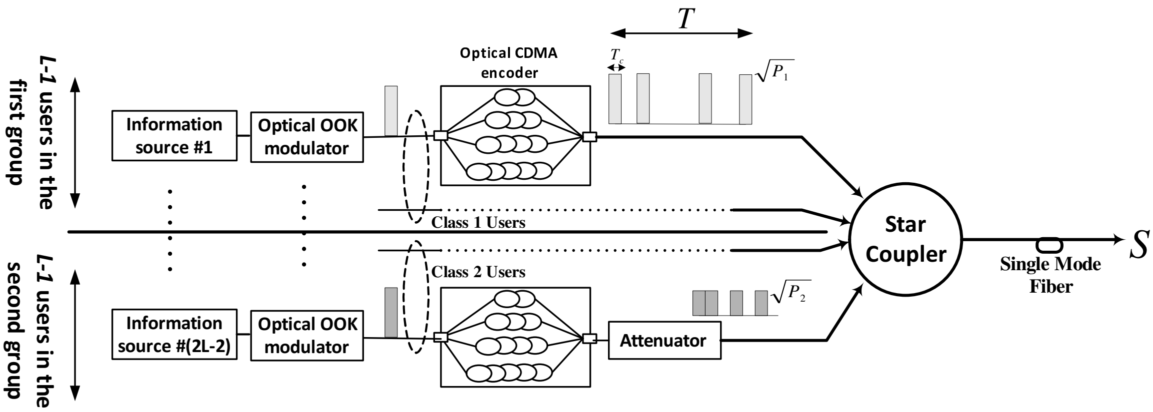

Fig. 1. A schematic diagram of the SOCDMA system based on UWC.

A. Designed Codes

A UWC is a family of $\{0,1\}$ sequences of length L where the number of 1's in each sequence is $\frac{L}{2}$. UWCs are obtained from a Hadamard matrix generated by the following recursive algorithm [14]

$$\mathbf{H_1} = [0] \quad (1)$$

$$\mathbf{H_2} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \qquad \mathbf{H_{2L}} = \begin{bmatrix} \mathbf{H_L} & \mathbf{H_L} \\ \mathbf{H_L} & \mathbf{H_L^c} \end{bmatrix}, \quad (2)$$

where $L = 2^P, (P \geq 1)$ is the length of these codes and P represents the number of recursions; the matrix $\mathbf{H_L^c}$ is the binary complement of matrix $\mathbf{H_L}$. Based on the recursive algorithm, L codes can be generated. However, since the first row of the Hadamard matrix $(\mathbf{H_L})$ contains all zeros, it cannot represent any information; this leaves us with $L-1$ codes. For the rest of this paper, the underloaded UWC matrix is denoted as $\mathbf{U_{L-1}}$ where the number of available codes is less than the code length. For simplicity, we assume that all $L-1$ signature sequences have normalized energy; thus, the amplitude of UWC is divided by $\frac{1}{\sqrt{2^{P-1}}}$. The overloaded UWC matrix is generated as follows

$$\mathbf{C_{2(L-1)}} = \begin{bmatrix} \mathbf{U_{L-1}} \\ \mathbf{U_{L-1}^c} \end{bmatrix}. \quad (3)$$

Each row of $\mathbf{C_{2(L-1)}}$ represents a UWC; thus, $2(L-1)$ UWCs with the code length of L are obtained. The loading factor, i.e., the ratio of the number of users to the code length, is approximately equal to 2. Let C_i be the ith row vector of $\mathbf{C_{2(L-1)}}$. From (3), it can be observed that C_i is the complement of C_j if $|i-j| = L$. Therefore, the correlation function, $\Phi_{i,j}$, for any pair of code sequences C_i and C_j in a discrete manner is presented as follows

$$\Phi_{i,j} = \begin{cases} 1 & i = j \\ 0 & |i-j| = L \\ 0.5 & \text{otherwise} \end{cases} \quad (4)$$

From (4), the correlation between any two sequences which are not complementary, is 0.5. This property can be used to cancel the MUI.

B. Transmitter Structure

In this subsection, the transmitter structure of the overloaded SOCDMA system employing the proposed UWC is proposed. The fiber-optic CDMA system considered in this paper consists of $2L-2$ active users communicating synchronously through a passive star coupler. To transmit its corresponding information bits each user employs the UWC stream as the signature sequence. Moreover, the users are divided into two groups. Half of the users transmit the generated UWC at the power level P_1 while the rest of the users transmit at the power level P_2. Figure 1 shows the principle structure of the proposed system. As depicted in Fig. 1, the output bit of each information source is modulated into OOK signal by an optical modulator. The optical OOK signal is then passed to an optical CDMA encoder that maps each bit into a UWC. The optical CDMA encoders are realized by the simple tapped delay lines. Then the encoded lightwave from all users are coupled into the single-mode fiber channel by a passive star coupler. To simplify the analysis, we ignore the fiber loss and splitting effect of all passive devices such as star coupler and optical CDMA encoders. Therefore, the discrete-time model for the received signal can be described by

$$\begin{aligned} \mathbf{S} &= \sum_{i=1}^{L-1} \sqrt{B}_i b_i \mathbf{C}_i + \sum_{i=L}^{2(L-1)} \sqrt{B}_i b_i \mathbf{C}_i \\ &= \sqrt{P}_1 \sum_{i=1}^{L-1} b_i \mathbf{C}_i + \sqrt{P}_2 \sum_{i=L}^{2(L-1)} b_i \mathbf{C}^c_{1+mod(i,L)}, \end{aligned} \quad (5)$$

where B_i, b_i, and $\mathbf{C}_i$ represent the optical intensity, the transmitted bit, and the code sequence of the ith user, respectively. In the next section, we describe the proposed receiver structure for such systems.

III. The Proposed Receiver

In this section, we describe the proposed receiver structure for the SOCDMA system. In the proposed receiver, data of any two users with complementary codes are decoded simultaneously. As it can be observed from Fig. 2, the received signal is divided into two equal parts by a $1:2$ optical splitter, then

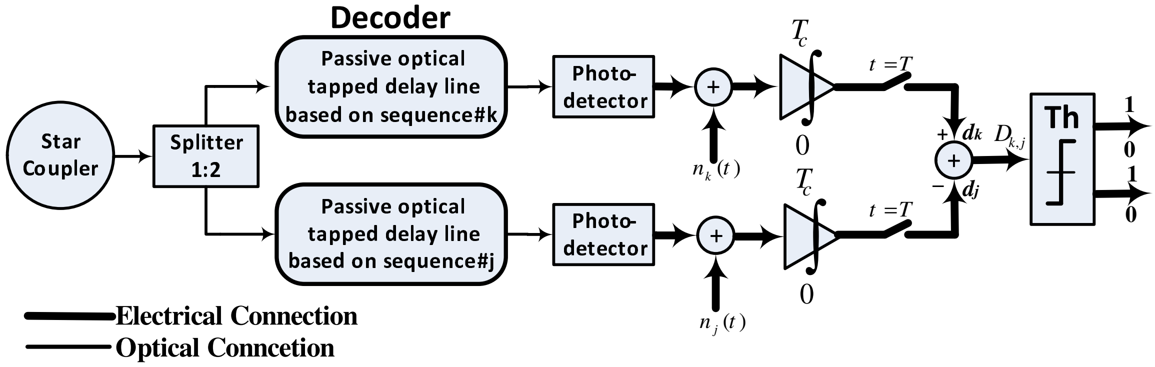

Fig. 2. System model of the proposed receiver structure.

the split signals are fed into two optical matched filters. These optical matched filters are implemented by fiber tapped delay lines [1]. In the upper branch the injected signal is correlated with the kth spreading sequence, while in the lower branch, the fed signal is correlated with the jth spreading sequence which is the complementary of the kth spreading sequence. The outputs of these matched filters are converted into the electrical signals employing two photodetectors. The photodetectors are modeled as an ideal square-law device [15], with quantum efficiency equal to one. During the photodetection process, several noises such as beat noise, thermal noise, and shot noise are generated. It has been shown that the beat noise can be ignored by averaging through the detection in the incoherent OCDMA systems [16]. Furthermore, we assume that the dominant noise source in the receiver is thermal noise and shot noise contribution is negligible. Therefore, the photodetector noise can be modeled as additive Gaussian noise. Following the squaring of the input light in each branch, we proceed by processing the detected light. In most digital optical systems, especially for receivers in the direct detection, the processing part is an integrate-and-dump device with integration time T_c. We denote T_c to be the chip duration of the UWC. Thus, the output of the upper and lower branches after sampling are obtained as follows

$$\begin{aligned} d_k &= \mathbf{S}\mathbf{C}_k^T\mathbf{C}_k\mathbf{S}^T + n_k \\ &= \overbrace{P_1 b_k}^{\text{desired term}} + \overbrace{0.5P_1 \sum_{i=1,i\neq k}^{L-1} b_i + 0.5P_2 \sum_{i=L,i\neq k+L}^{2(L-1)} b_i}^{\text{MUI term}} \\ &+ \overbrace{n_k}^{\text{noise term}}, n_k = \frac{1}{T_c}\int_0^{T_c} n_k(t)\,\mathrm{d}t, \end{aligned} \quad (6)$$

$$\begin{aligned} d_j &= \overbrace{P_2 b_j}^{\text{desired term}} + \overbrace{0.5P_1 \sum_{i=1,i\neq k}^{L-1} b_i + 0.5P_2 \sum_{i=L,i\neq k+L}^{2(L-1)} b_i}^{\text{MUI term}} \\ &+ \overbrace{n_j}^{\text{noise term}}, n_j = \frac{1}{T_c}\int_0^{T_c} n_j(t)\,\mathrm{d}t, \end{aligned} \quad (7)$$

where $n_k, n_j \sim \mathcal{N}(0, \sigma^2)$ are independent Gaussian random variables. In addition, $\sigma^2 = \frac{4k_B T_0 B_R}{R_L}$, where $B_R = 1/(2T_c)$ is the receiver bandwidth, k_B is Boltzmans constant, T_0 is the temperature, and R_L is the load resistance. In these expressions, the first term represents the desired signal, the second and third terms are the MUI noise and the last term denotes the thermal noise. The output of the upper and lower branches in (6) and (7) can be rewritten as follows

$$\begin{cases} d_k = P_1 b_k + I_{k,j} + n_k \\ d_j = P_2 b_j + I_{k,j} + n_j \end{cases} \quad (8)$$

where $I_{k,j}$ represents the MUI noise. As shown in (8), based on the correlation properties of UWC, the MUI noise is the same for both branches. Therefore, by subtracting photodetectors' outputs (d_k and d_j), the MUI noise is cancelled out as seen in Fig. 2. Let $D_{k,j}$ be the decision variable as

$$D_{k,j} = d_k - d_j. \quad (9)$$

The decision variable is then passed to the decision unit which is the comparator device. In the decision unit, the two transmitted bits are decoded simultaneously. In Appendix A, it is shown that the simple receiver is an optimum detector based on the observation signals d_k and d_j. Furthermore, we show in Appendix B that the optimal values of P_1 and P_2 to minimize the probability of error for a fixed average optical transmit power are related by $P_1 \simeq 2P_2$. Finally, the decision rule can be summarized as follows

$$(b_k, b_j) = \begin{cases} (0,1) & \text{if} \quad D_{k,j} \leq -0.5P_2 \\ (0,0) & \text{if} \quad -0.5P_2 < D_{k,j} \leq 0.5P_2 \\ (1,1) & \text{if} \quad 0.5P_2 < D_{k,j} \leq 1.5P_2 \\ (1,0) & \text{if} \quad 1.5P_2 \leq D_{k,j} \end{cases} \quad (10)$$

IV. Simulation and Numerical Results

In this section, the numerical and the simulation results of the proposed overloaded SOCDMA system are presented.

Fig. 3 shows the BER performance of the proposed receiver compared to the optimum multiuser receiver versus the average SNR. The average SNR is given by $SNR_{av} = (P_1^2+P_2^2)/2\sigma^2$. The code length, L, is considered to be 16 and the number of users is 30. It should be noted that the optimum multiuser

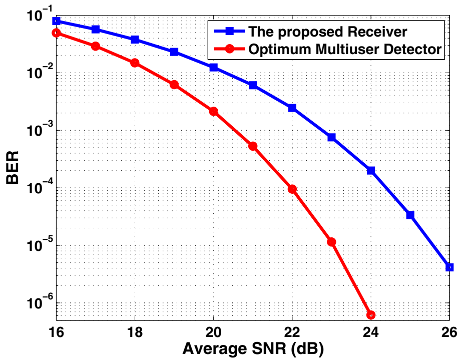

Fig. 3. BER for the proposed receiver and for the multiuser receiver versus average SNR.

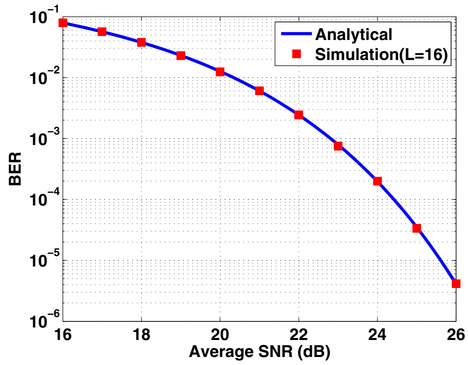

Fig. 4. BER versus average SNR for analytical and simulation results.

receiver performs an exhaustive search to determine the most likely combination of transmitted bits [5]. In the proposed receiver, the noise components at the output of each matched filter are uncorrelated and Gaussian, therefore they are statistically independent. Consequently, the observed noise power at the output of the proposed receiver is 3dB higher than the noise power observed in the optimum multiuser receiver. This can also be observed from Fig. 3 that the required SNR for the proposed receiver is 3dB higher than the optimum multiuser receiver to get the same BER. However, in the optimum multiuser receiver we need to search over 2^{2L-2} possible combinations of the transmitted bits that is much more complex than our proposed receiver.

The analytical result is compared with the simulation for the proposed receiver in Fig. 4. To this end, we consider the code length and the number of users to be 16 and 30, respectively. As shown in Fig. 4, the analytical results are confirmed by simulation.

V. Conclusion

In this paper, we have introduced an overloaded SOCDMA system based on UWCs for fiber optic communication systems. In this system we divide active users into two groups. The members within each group transmit their information at the same power level which is different from the other group members. The receiver design based on the UWC properties not only mitigates the MUI completely but also benefits from a simple structure. Moreover, the optimal power level of each group has been obtained to minimize the BER for a given average transmit power. The performance of the proposed system and the optimum multiuser detector has been compared. It has been shown that the required SNR for the proposed receiver is 3dB higher than the optimum multiuser receiver to achieve the same BER. However, the complexity of the proposed receiver is lower than that of the multiuser detector. Furthermore, the accuracy of the analytical results are verified by the simulation.

Appendix A

In this appendix, we describe the optimum decision rule based on the observation signals, d_k and d_j. For this development, the decision rule is made such that the probability of a correct decision is maximized for equally likely input signals. Thus, the decision rule is defined as follows

$$\arg \max_{\{b_k, b_j\}} P(d_k, d_j | b_k, b_j). \tag{A.1}$$

The probability $P(d_k, d_j | b_k, b_j)$ can be expressed as

$$\begin{aligned} P(d_k, d_j | b_k, b_j) &= \sum_{I_{k,j}} P(d_k, d_j | b_k, b_j, I_{k,j}) P(I_{k,j}) \\ &= \sum_{I_{k,j}} P(d_k | b_k, I_{k,j}) P(d_j | b_j, I_{k,j}) P(I_{k,j}) \\ &= \sum_{I_{k,j}} \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-1}{2\sigma^2}(d_k - P_1 b_k - I_{k,j})^2} \times \\ &\frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-1}{2\sigma^2}(d_j - P_2 b_j - I_{k,j})^2} P(I_{k,j}). \end{aligned} \tag{A.2}$$

In (A.2), the independency of the signals at the correlator outputs is used. Moreover, for a large number of interfering users, the probability density function of MUI is approximated by a Gaussian distribution, $P(I_{k,j}) \sim \mathcal{N}(\mu_I, \sigma_I^2)$, appealing to central limit theorem arguments.

By taking the natural logarithm of (A.2) and using the approximation $\ln \sum e^{x_i} \simeq \max(x_i)$, the decision rule can be simplified as

$$\begin{aligned} &\arg \max_{\{b_k, b_j\}} \ln P(d_k, d_j | b_k, b_j) = \\ &\arg \max_{\{b_k, b_j\}} \Big\{ \arg \max_{\{I_{k,j}\}} \Big\{ -\frac{(I_{k,j} - \mu_I)^2}{2\sigma_I^2} \\ &- \frac{(d_k - P_1 b_k - I_{k,j})^2 + (d_j - P_2 b_j - I_{k,j})^2}{2\sigma^2} \Big\} \Big\} \end{aligned} \tag{A.3}$$

The MUI value which maximizes the inner bracket term of (A.3) is given by

$$I_m = \frac{\mu_I \sigma^2 + \sigma_I^2 \big((d_k + d_j) - (P_1 b_k + P_2 b_j) \big)}{2\sigma_I^2 + \sigma^2} \tag{A.4}$$

As the MUI noise is the dominant noise in the incoherent OCDMA systems, we can assume that $\sigma^2 \ll \sigma_I^2$. Therefore, (A.4) is simplified as

$$I_m \simeq \frac{(d_k + d_j) - (P_1 b_k + P_2 b_j)}{2} \tag{A.5}$$

Finally, substituting (A.5) into (A.3) results in a decision rule as

$$\arg\max_{\{b_k,b_j\}} \log P(d_k, d_j|b_k, b_j) \equiv$$
$$\arg\min_{\{b_k,b_j\}} \left[(d_k - d_j) - (P_1 b_k - P_2 b_j)\right]^2 \quad \text{(A.6)}$$

APPENDIX B

In this appendix we obtain the optimal values of P_1 and P_2 to minimize the BER for a fixed average transmit power. From (8), the decision variable can be represented as

$$D_{k,j} = d_k - d_j = (P_1 b_k - P_2 b_j) + (n_k - n_j). \quad \text{(B.1)}$$

Since the noise components n_k and n_j are uncorrelated and Gaussian, they are statistically independent. Consequently, (B.1) can be rephrased as follows

$$D_{k,j} = d_k - d_j = (P_1 b_k - P_2 b_j) + n, n \sim \mathcal{N}(0, 2\sigma^2). \quad \text{(B.2)}$$

The optimum threshold is chosen such that the BER is minimized. Without loss of generality, assume that $P_1 > P_2$. Therefore, the optimum decision rule is as follows

$$(b_k, b_j) = \begin{cases} (0,1) & D_{k,j} \le -0.5P_2 \\ (0,0) & -0.5P_2 < D_{k,j} \le 0.5(P_1 - P_2) \\ (1,1) & 0.5(P_1 - P_2) < D_{k,j} \le P_1 - 0.5P_2 \\ (1,0) & P_1 - 0.5P_2 \le D_{k,j} \end{cases} \quad \text{(B.3)}$$

By using the obtained optimum threshold, the error probability is calculated as follows

$$P_e = \frac{1}{4}\left[P(e|(0,1)) + P(e|(0,0)) + P(e|(1,1))\right.$$
$$\left. + P(e|(1,0))\right] = Q\left(\frac{P_2}{2\sqrt{2}\sigma}\right) + \frac{1}{2}Q\Big(\frac{P_1 - P_2}{2\sqrt{2}\sigma}\Big), \quad \text{(B.4)}$$

where $P(e|b_k, b_j)$ is the conditional error probability when the transmitted bits are b_k and b_j, respectively.

The goal is to find the power allocated to each user such that the BER is minimized where the total transmitted power is constrained by $P \ge \sum_{i=1}^{2(L-1)} B_i$ where B_i represents the optical intensity of the ith user. Since the users in the first group transmit at the power level P_1 and the users in the second group transmit at the power level P_2, the power constraint can be simplified as $P_1 + P_2 \le \frac{2P}{L-1}$. Hence, the optimum power allocation is given by the following optimization problem

$$\arg \min_{\{P_1,P_2\},\text{s.t. } P_1+P_2 \le \frac{2P}{L-1}} \Big(Q\big(\frac{P_2}{2\sqrt{2}\sigma}\big) + 0.5Q\big(\frac{P_1 - P_2}{2\sqrt{2}\sigma}\big)\Big). \quad \text{(B.5)}$$

The constraint minimization problem can be converted into an unconstrained optimization problem by introducing the Lagrange multiplier λ as follows

$$\Lambda(P_1, P_2, \lambda) = \int_{\frac{P_2}{2\sqrt{2}\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}}\,\mathrm{d}u$$
$$+ 0.5\int_{\frac{P_1-P_2}{2\sqrt{2}\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}}\,\mathrm{d}u + \lambda\Big(P_1 + P_2 - \frac{2P}{L-1}\Big). \quad \text{(B.6)}$$

The optimum allocated power is obtained by solving the following simultaneous equations

$$\begin{cases} \frac{\partial\Lambda}{\partial P_2} = \frac{-1}{2\sqrt{2}\sigma}\frac{1}{\sqrt{2\pi}}\exp\left[\frac{-1}{2}\big(\frac{P_2}{2\sqrt{2}\sigma}\big)^2\right] \\ -\frac{1}{4\sqrt{2}\sigma}\frac{1}{\sqrt{2\pi}}\exp\left[\frac{-1}{2}\big(\frac{P_1-P_2}{2\sqrt{2}\sigma}\big)^2\right] + \lambda P_2 = 0 \\ \frac{\partial\Lambda}{\partial\lambda} = P_1 + P_2 - \frac{2P}{L-1} = 0 \\ \frac{\partial\Lambda}{\partial P_1} = \frac{-1}{4\sqrt{2}\sigma}\frac{1}{\sqrt{2\pi}}\exp\left[\frac{-1}{2}\big(\frac{P_1-P_2}{2\sqrt{2}\sigma}\big)^2\right] + \lambda P_1 = 0 \end{cases}$$

The above equations are solved numerically and the optimum power level is $P_1 \simeq 2P_2$.

ACKNOWLEDGMENT

This work was supported by the Iranian National Science Foundation (INSF) and the Iran Telecom Research Center (ITRC).

REFERENCES

[1] J. A. Salehi, "Code division multiple-access techniques in optical fiber networks Part I: Fundamental principles," *IEEE Trans. Communications*, vol. 37, no. 8, pp. 824-833, Aug. 1989.

[2] G. C. Yang and W. C. Kwong, *Prime Codes With Application to CDMA Optical and Wireless Networks.* MA: Artech House, 2002.

[3] J. A. Salehi and C. A. Brackett, "Code division multiple-access techniques in optical fiber networks. II: Systems performance analysis," *IEEE Trans. Communications*, vol. 37, no. 8, pp. 834-842, Aug. 1989.

[4] H. M. Shalaby, "Chip-level detection in optical code division multiple access," *IEEE J. Lightwave Technology*, vol. 16, pp. 1077-1087, June 1998.

[5] S. Verdu, "Multiple-access channels with point process observation: optimum demodulation," *IEEE Trans. Inform. Theory*, vol. IT-32, pp. 642-651, Sept. 1986.

[6] M. Brandt-Pearce and B. Aazhang, "Multiuser detection for optical code division multiple access," *IEEE Trans. Communications*, vol. 42, pp. 1801-1810, Feb/Mar/Apr. 1994.

[7] C. Goursaud, A. Julien-Vergonjanne, C. Aupetit-Berthelemot, J. Cances, and J. Dumas, "DS-OCDMA receivers based on parallel interference cancellation and hard limiters," *IEEE Trans. Communications*, vol. 54, pp.1663-1671, Sept. 2006.

[8] P. Petal and J. Holtzman, "Analysis of a simple succesive interference cancellation scheme in a DS/CDMA system," *IEEE Journal on selected areas in Communications* , vol. 12, pp.796-807, June 1994.

[9] S. Verdu, *Multiuser Detection.* Cambridge Press, 1998.

[10] L. B. Nelson and H. V. Poor, "Performance of multiuser detection for optical CDMA-Part I: Error probabilities," *IEEE Trans. on Commun.*, vol. 43, pp. 2803-2811, Nov. 1995.

[11] A. Duel-Hallen, J. Holtzman, and Z. Zvonar, "Multiuser Detection for CDMA Systems," *IEEE Personal Communications Magazine*, pp. 46-58, April 1995.

[12] P. Pad, F. Marvasti, K. Alishahi, and S. Akbari, "A Class of Errorless Codes for Overloaded Synchronous Wireless and Optical CDMA Systems," *IEEE Trans. Inform. Theory*, vol. 55, pp. 2705-2715, June 2009.

[13] K. Alishahi, S. Dashmiz, P. Pad, and F. Marvasti, "Design of Signature Sequences for Overloaded CDMA and Bounds on the Sum Capacity With Arbitrary Symbol Alphabets," *IEEE Trans. Inform. Theory*, vol. 58, pp. 1441-1468, March 2012.

[14] E. H. Dinan and B. Jabbari, "Spreading codes for direct sequences CDMA and wideband CDMA cellular networks," *IEEE Comm. Magazine*, pp. 48-54, Sept. 1998.

[15] G. Einarsson, *Principles of Lightwave Communications.* New York: McGraw-Hill, 1996.

[16] X. Wang and K. Kitayama, "Analysis of Beat Noise in Coherent and Incoherent Time-Spreading OCDMA," *IEEE J. Lightwave Technology*, vol. 22, no. 10, pp. 2226-2235, October 2004.