IET Communications

Research Article

Achievable rate regions for many-to-one Gaussian interference channel with a fusion centre

ISSN 1751-8628
Received on 11th July 2013
Accepted on 30th November 2014
doi: 10.1049/iet-com.2014.0590
www.ietdl.org

Mahdi Baianifar, Hamid Behroozi ✉

Department of Electrical Engineering, Sharif University of Technology, Tehran, Iran
✉ *E-mail: behroozi@sharif.edu*

Abstract: This paper considers a many-to-one Gaussian interference channel with a fusion centre (FC) where there is a K -user interference channel in which only one relay (receiver) faces interference while the remaining K-1 receivers are interference free. All the relays communicate information about their observed sequence to the FC through noiseless links at communication rate R_0. First, by analysing traditional relaying schemes, i.e., Decode and Forward, Compress and Forward and Compute and Forward, three rate-regions for this setting are derived. Then, based on nested lattice codes, a new achievable rate-region is provided. Based on the proposed scheme, one can design a transmission scheme that can recover both integer and non-integer linear combination of messages. Numerical examples show that if channel gains are integer, the proposed scheme performs similarly to the compute-and-forward scheme. In the case of non-integer channel gains, the proposed scheme outperforms other relaying schemes at high signal-to-noise ratios. Finally, it is shown that if the channel gains are larger than one and if the rate of each relay-to-FC link equals to the capacity of an AWGN channel, then the proposed scheme can achieve the capacity region in high SNR regime.

1 Introduction

Interference is an impartible issue in wireless networks where a number of independent users try to communicate with each other via a common channel, which is called an interference channel (IC). Each user aims to communicate only with its corresponding receiver, but each communication interferes the others. For this channel, the best achievable rate-region to date is because of Han and Kobayashi [1]. Their work is based on the rate-splitting at the transmitters. In [2], it has been shown that the class of degraded Gaussian IC is equivalent to the class of Z (one-sided) IC. Sason derives an achievable rate-region for two-user Gaussian IC-based on the modified time (or frequency) division multiplexing approach [3]. Etkin and co-workers [4] by deriving new outer bounds show that a very simple Han–Kobayashi type scheme can achieve rates within 1 bit/s/Hz of the channel capacity for all values of the channel parameters. In [5], an achievable rate-region for the Gaussian interference relay channel is obtained. Their coding strategy is a combination of dirty-paper coding, beamforming and interference reduction techniques at the relay. The two-user Gaussian IC with a full-duplex relay is considered in [6] and two new upper bounds on the achievable sum-rate are derived using genie-aided approaches. The achievable approach in [6] includes the block Markov encoding combined with the Han–Kobayashi scheme at the transmitters and the compute-and-forward scheme at the relay. A symmetric K-user Gaussian IC with K transmitters and K receivers is considered in [7] where a very-strong interference regime is derived. It is shown that if the channel gain satisfies a condition, the capacity-region of the IC is similar to the case when no interference is present. The Gaussian interference relay channel with a potent relay is considered in [8], where it is shown that the capacity region is asymptotically equivalent to the case when the relay–destination links are noiseless and orthogonal to other links. In [9], improved outer bounds on the capacity region of general IC are derived. By using these bounds, it is shown that by treating interference as noise, the sum-capacity of the two-user Gaussian IC is achieved in a low-interference regime. In [10], two building-block models of interference-limited wireless networks are studied. In the first case, a single long-range transmitter interferes with multiple parallel short-range transmissions, and, in the second case, multiple-short-range transmitters interfere with a single-long-range receiver.

In this paper, a special case of a data gathering distributed network, modelled by a many-to-one Gaussian IC is considered. In this model, a firm's Chief Executive Officer (CEO) is interested in the message sequence (W_1, W_2, …, W_K). The target message sequence cannot be received directly. The CEO deploys a team of K agents (relays) to receive this message sequence. Agents communicate information about their observed sequence to a fusion centre (FC) through noiseless channels at communication rate R_0. This rate constraint comes from the restrictions on the resources such as bandwidth and power that are available at the agents. Ignoring the FC, the reduced channel is considered in [11] where only one receiver faces interference. The discrete memoryless many-to-one IC is considered in [12]. For this channel, a noisy interference regime, that is, a regime where the random coding by treating interference as noise achieves the sum-capacity is identified. In [13] by obtaining achievable rate-regions and an outer bound over the sum-rate, it is shown that the gap between the upper bound and the achievable sum-rate is related to the number of users; that is $\log_2(K-1)$ where K is the number of users.

Here, based on nested lattice codes, we provide a new achievable rate-region for this channel. Lattice codes are known to outperform random codes for certain channel models, see, for example, [14, 15]. Erez and Zamir [16] show that lattice codes can achieve the capacity of an additive white Gaussian noise (AWGN) channel. Also, lattice codes achieve the rate-regions of DF and CF schemes for the relay channel [17]. Nazer and Gastpar [15] by using nested lattice codes, present a new scheme, referred to as 'compute-and-forward' scheme, which is based on the decoding linear combination of messages. In [18], a two-way relay channel is considered where a compress-and-forward (CF) relaying strategy using nested lattice codes is proposed. It is shown that the achievable rates of the proposed scheme are higher than those provided by the DF strategy in some regions.

In this work, we use nested lattice codes to decode linear combination of messages, for both integer or non-integer coefficients. To reach this goal, we propose a nested lattice code and apply the channel gains in generating our codewords. We also derive rate-regions for standard CF, decode-and-forward and compute-and-forward strategies. We show that if the channel gains are larger than 1 and if the rate of the relay-to-FC link equals to the capacity of an AWGN channel, then our scheme achieves the capacity region in the high SNR regime. Moreover, regardless of all channel parameters, the achievable rate-region is within a constant gap of the outer bound.

This paper is organised as follows. Section 2 provides the system model and some preliminaries on lattice codes. In Section 3, achievable rate-regions of classical relaying schemes as well as our proposed scheme are derived. To characterise performance of different relaying strategies, in Section 4 we first present a trivial outer bound on the achievable rate to the FC. Then, we evaluate the achievable symmetric rate of each scheme where a symmetric rate is a common rate at which all relays are operating. In Section 5, using numerical examples, comparison between the outer bound and performance of the encoding schemes for many-to-one Gaussian IC with a FC is presented. Our proposed strategy is shown to outperform classical strategies such as decode-and-forward, CF and recently introduced compute-and-forward scheme in case of non-integer channel gains. Section 6 concludes the paper.

2 System model and lattice preliminaries

2.1 System model

We consider a special case of Gaussian relay networks, in which many transmitters communicate their messages to a common receiver, called a fusion centre. In this model, a CEO deploys a team of K relays to receive the message sequence. Relays communicate information about their observed sequence to the FC through noiseless channels at communication rate R_0. This rate constraint comes from the restrictions on the resources such as bandwidth and power that are available at the relays. The channel model is depicted in Fig. 1. In this model, interference is experienced by only one relay while the remaining $K-1$ relays are interference free, that is, each relay is received from its intended transmitter only. By removing the FC node, this system is reduced to a Gaussian many-to-one IC considered in [11]. The channel can be described as follows

$$\boldsymbol{Y}_1 = \sum_{j=1}^{K} h_{1j}\boldsymbol{X}_j + \boldsymbol{Z}_1 \tag{1}$$

$$\boldsymbol{Y}_i = h_{ii}\boldsymbol{X}_i + \boldsymbol{Z}_i, \quad \forall i = 2, 3, \ldots, K \tag{2}$$

where $\boldsymbol{X}_i$ denotes the channel input, $\boldsymbol{Z}_i$ represents an AWGN with mean zero and variance N, and h_{ij} $(i, j = 1, 2, \ldots, K)$ denotes the constant channel gain from transmitter j to relay i. Without loss of generality, we assume that $h_{11} = 1$.

Each encoder based on the message W_i generates a codeword $\boldsymbol{X}_i$. Note that $|\mathcal{W}_i| = 2^{nR_i}$. The transmitted sequence is average-power limited to $P > 0$, that is

$$\frac{1}{n}\sum_{j=1}^{n} |\boldsymbol{X}_{ii}[j]|^2 \le P, \quad \text{for } i = 1, 2, \ldots, K \tag{3}$$

The received signal at each relay is denoted by $\boldsymbol{Y}_i$. Based on this

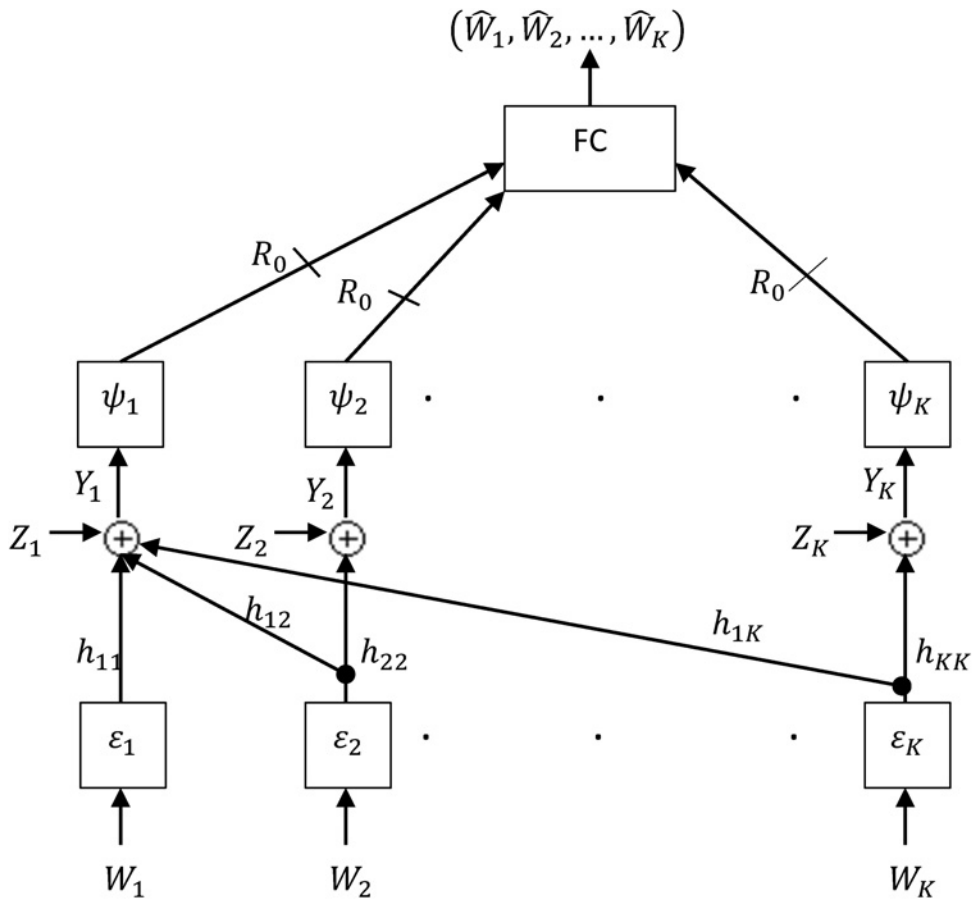

Fig. 1 *System model: a many-to-one Gaussian IC with a FC*

sequence, each relay generates a codeword $\boldsymbol{X}_{ri}$, that is

$$\boldsymbol{X}_{ri} = \psi_i(\boldsymbol{Y}_i) \quad (4)$$

where $\psi_i(.)$ is the encoding function at the relay i. Then, relay i sends $\boldsymbol{X}_{ri}$ to the FC through noiseless limited-rate link. Based on $\boldsymbol{X}_{ri}$s, the FC estimates the message sequence of the transmitters.

The average error probability is defined as (see (5))

$$P_e^{(n)} = \frac{1}{2^{n(R_1+R_2+\cdots+R_K)}} \sum_{w_1,w_2,\ldots,w_k} \Pr\{\hat{W}_1 \neq W_1 \text{ or } \cdots \text{ or } \hat{W}_K \neq W_K | (W_1, \ldots, W_K) \text{ is sent}\} \quad (5)$$

where $(W_1, \ldots, W_K)$ is assumed to be uniformly distributed over $\{1, 2, \ldots, 2^{nR_1}\} \times \cdots \times \{1, 2, \ldots, 2^{nR_K}\}$ and $(\hat{W}_1, \ldots, \hat{W}_K)$ is the estimated message sequence at the FC. The rate tuples $(R_1, \ldots, R_K)$ are achievable if there exists a sequence of length-n code $C^n(R_1, \ldots, R_K)$ such that $P_e^{(n)} \to 0$ as $n \to \infty$ [19].

2.2 Lattice definition

A lattice is a discrete subgroup of $\mathbb{R}^n$, which is closed under addition and subtraction operations, that is, if $\boldsymbol{x}, \boldsymbol{y} \in \Lambda$ then $\boldsymbol{x} \pm \boldsymbol{y} \in \Lambda$. A lattice can be specified by using a generator matrix, $G \in \mathbb{R}^{n\times n}$, as follows

$$\Lambda = \{\lambda = G\boldsymbol{x}: \boldsymbol{x} \in \mathbb{Z}^n\} \quad (6)$$

The fundamental Voronoi region of lattice Λ is defined as

$$\mathcal{V} = \{\boldsymbol{x} \in \mathbb{R}^n: ||\boldsymbol{x}|| \leq ||\boldsymbol{x} - \boldsymbol{c}|| \;\; \forall \boldsymbol{c} \in \Lambda\} \quad (7)$$

where $||\boldsymbol{x}||$ is the Euclidean norm. Note that we have $\mathbb{R}^n = \Lambda + \mathcal{V}$.

The nearest neighbour quantiser of a lattice corresponding to $\mathcal{V}$ is defined as

$$Q_{\mathcal{V}}(x) = \boldsymbol{y}, \quad \boldsymbol{y} \in \Lambda, \quad \text{if } ||\boldsymbol{x} - \boldsymbol{y}|| \leq ||\boldsymbol{x} - \boldsymbol{y}'||, \quad \forall \boldsymbol{y}' \in \Lambda \quad (8)$$

The 'modulo-Λ operation' with respect to lattice Λ is defined as

$$\boldsymbol{x} \bmod \Lambda = \boldsymbol{x} - Q_{\mathcal{V}}(\boldsymbol{x}) \quad (9)$$

The second moment of Λ is defined as the second moment per dimension of a uniform distribution over $\mathcal{V}$, that is

$$\sigma^2(\Lambda) = \frac{1}{\text{Vol}(\mathcal{V})} \cdot \frac{1}{n} \int_{\mathcal{V}} ||\boldsymbol{x}||^2 \, d\boldsymbol{x} \quad (10)$$

and the normalised second moment of a lattice can be expressed as

$$G(\Lambda) = \frac{\sigma^2(\Lambda)}{\text{Vol}(\mathcal{V})^{(2/n)}} \quad (11)$$

Lattice Λ is nested in lattice Λ_C if $\Lambda \subseteq \Lambda_C$. A nested lattice code is defined by using nested lattice partition $\Lambda \subseteq \Lambda_C$ in which Λ_C is used to provide codewords and Λ is used for shaping [16]. The coding rate of a nested lattice code is given by

$$R = \frac{1}{n} \log\left(\frac{\text{Vol}(\Lambda)}{\text{Vol}(\Lambda_C)}\right) \quad (12)$$

Interested reader can refer to [20, 21, 15, 16] to obtain a comprehensive review on lattices.

3 Achievable rate-regions

In this section, we derive achievable rate-regions for transmission over a many-to-one Gaussian IC with a FC.

3.1 Compress-and-forward

In this scheme, we can treat the effect of compression as noise [14]. We can easily show that the variance of the equivalent noise at each link equals to

$$\widetilde{N}_1 = \frac{\sum_{i=1}^K h_{1i}^2 P + N}{2^{2R_0} - 1} \quad (13)$$

$$\widetilde{N}_i = \frac{P + N}{2^{2R_0} - 1}, \quad i = 2, 3, \ldots, K \quad (14)$$

and hence for message recovery without error, we must have [14]

$$R_1 \leq \frac{1}{2} \log\left(1 + \frac{(\sum_{i=1}^K h_{1i} P + N)(2^{2R_0} - 1)}{\sum_{i=1}^K h_{1i} P + 2^{2R_0} N}\right)^+ \quad (15)$$

$$R_i \leq \frac{1}{2} \log\left(1 + \frac{P(2^{2R_0} - 1)}{P + 2^{2R_0} N}\right)^+, \quad i = 2, 3, \ldots, K \quad (16)$$

where

$$x^+ = \max(x, 0)$$

3.2 Decode-and-forward

In this scheme, we decode messages at the relays and then send them to the FC through limited-rate links. At relay 1, we have a K-user MAC and at other relays, we have only an AWGN channel. For each message W_i at the transmitter i, we generate 2^{nR_i} Gaussian codewords $\boldsymbol{x}_i(w_i)$ and send $\boldsymbol{x}_i$'s over the channel. At each relay node, we use jointly typical decoding. Thus, at relay 1, for correct recovery of all messages, we must have [19]

$$\sum_{S \subseteq \{1,\ldots,k\}} R(S) \leq \frac{1}{2} \log\left(1 + \sum_{i \in S} \frac{h_{1i}^2 P}{N}\right)^+ \quad (17)$$

At relay i, $i \in \{2, \ldots, k\}$, we have an AWGN channel and therefore, for correct recovery of message i, any rate R_i satisfying the following constraint is achievable

$$R_i \leq \frac{1}{2} \log\left(1 + \frac{h_{ii}^2 P}{N}\right)^+, \quad \forall i = 2, 3, \ldots, K \quad (18)$$

Now, relay i has message of transmitter i and encodes it to a codeword $\boldsymbol{x}_{ri}(w_i)$ and sends it to the FC. Since the rate of limited-rate link is R_0, for correct recovery of this codeword, we must have

$$R_i \leq R_0, \quad \forall i = 1, \ldots, K \quad (19)$$

Thus, from (17)–(19), we obtain the achievable rate-region for the

IET Commun., 2015, Vol. 9, Iss. 6, pp. 880–887

DF scheme as follows

$$\bigcup \begin{cases} \sum_{S\subseteq\{1,\ldots,k\}} R(S) \le \frac{1}{2}\log\left(1+\sum_{i\in S}\frac{h_{1i}^2 P}{N}\right)^+ \\ R_i \le \frac{1}{2}\log\left(1+\frac{h_{ii}^2 P}{N}\right)^+, \quad \forall i=2,3,\ldots,K \\ R_i \le R_0, \quad i=1,\ldots,K \end{cases} \tag{20}$$

3.3 Compute-and-forward

By applying the compute-and-forward scheme, introduced in [15], a linear combination of codewords with integer coefficients is decoded at relay 1. For correct estimation of a linear combination of messages, we have the following constraint on rate R_i [15]

$$R_i \le \frac{1}{2}\log\left(\left(||\boldsymbol{a}||^2 - \frac{P(\boldsymbol{h}^{\mathrm{T}}\cdot\boldsymbol{a})^2}{N+P||\boldsymbol{h}||^2}\right)^{-1}\right)^+ \tag{21}$$

for $i = 1, 2, \ldots, K$ if $a_i \neq 0$. The vector of channel gains is denoted by $\boldsymbol{h} = [h_{11}, h_{12}, \ldots, h_{1K}]^{\mathrm{T}}$ while the vector $\boldsymbol{a} = [a_1, \ldots, a_K] \in \mathbb{Z}^K$ represents the desired coefficients for the linear combination of messages at relay 1.

At the other relays, we have an AWGN channel. Thus, if we have the following constraint on R_i

$$R_i \le \frac{1}{2}\log\left(1+\frac{h_{ii}^2 P}{N}\right)^+, \quad \forall i=2,3,\ldots,K \tag{22}$$

then, we can recover message W_i at relay i $\forall i = 2, 3, \ldots, K$. Decoded massages of all relays are received at the FC through the limited-rate links with rate R_0. Thus, from (21) and (22), we can see that for correct recovery of all messages at the FC, we must have

$$R_1 \le \min\left(\frac{1}{2}\log\left(\left(||\boldsymbol{a}||^2 - \frac{P(\boldsymbol{h}^{\mathrm{T}}\cdot\boldsymbol{a})^2}{N+P||\boldsymbol{h}||^2}\right)^{-1}\right), R_0\right)^+, \quad \forall a_1 \neq 0 \tag{23}$$

$$R_i \le \left(\frac{1}{2}\log\left(\left(||\boldsymbol{a}||^2 - \frac{P(\boldsymbol{h}^{\mathrm{T}}\cdot\boldsymbol{a})^2}{N+P||\boldsymbol{h}||^2}\right)^{-1}\right)\right)^+, \quad \forall i=2,\ldots,K \text{ if } a_i \neq 0 \tag{24}$$

$$R_i \le \min\left(R_0, \frac{1}{2}\log\left(1+\frac{h_{ii}^2 P}{N}\right)\right)^+, \quad i=2,3,\ldots,K \tag{25}$$

Note that, for correct functionality of this scheme, we must have $a_1 \neq 0$. Since the FC knows messages of all users except message 1, with having the linear combination of messages, it can also recover the message of user 1. From [15], we know that if the real channel gains are close to integer numbers, then this scheme performs well.

3.4 Our proposed scheme

In this section, we introduce a new scheme to recover the linear combination of messages. In contrast to the compute-and-forward scheme, we can recover non-integer linear combination of messages in our proposed scheme. In the following, we describe our scheme.

First, we assume that $1 \le h_{12} \le h_{13} \le \cdots \le h_{1K}$. Then, from [1], we can construct the following lattice chain

$$\Lambda_K \subseteq \cdots \subseteq \Lambda_3 \subseteq \Lambda_2 \subseteq \Lambda_1 \subseteq \Lambda_c \tag{26}$$

such that lattice Λ_c is Poltyrev good while lattices Λ_i for $i = 1, 2, \ldots, K$ are simultaneously Rogers good and Poltyrev good and have the following second moments

$$\sigma^2(\Lambda_i) = h_{1i}^2 P \tag{27}$$

where for simplicity, we assume $h_{11} = 1$. Here, we also assume that $h_{1i} \ge 1$. Existence of such ensemble is considered in [21, 22]. For the other cases, that is, $h_{1i} < 1$ for some i, we can apply a similar approach as considered in [23].

3.4.1 Encoding: To encode messages, we generate the following codebooks

$$\mathcal{C}_i = \{\Lambda_c \bmod \Lambda_i\} \triangleq \{\Lambda_c \cap \mathcal{V}_i\}, \quad i \in \{1, 2, \ldots, K\} \tag{28}$$

Message of node i, $\{1, 2\ldots, 2^{nR_i}\}$ is one-to-one mapped to $\mathcal{C}_i$. Node i, for transmitting a message, chooses $\boldsymbol{V}_i \in \mathcal{C}_i$ associated with the intended message and sends

$$\boldsymbol{X}_i = \frac{1}{h_{1i}}[\boldsymbol{V}_i - \boldsymbol{D}_i] \bmod \Lambda_i \tag{29}$$

where $\boldsymbol{D}_i$ is uniformly distributed over $\mathcal{V}_i$. By the crypto lemma in [24], we know that $\boldsymbol{X}_i$ is uniformly distributed over $\mathcal{V}_i$ and is independent of $\boldsymbol{V}_i$.

3.4.2 Decoding: At the first relay, we have

$$\boldsymbol{Y}_1 = \boldsymbol{X}_1 + \sum_{i=2}^{K} h_{1i}\boldsymbol{X}_i + \boldsymbol{Z}_1 \tag{30}$$

Upon receiving $\boldsymbol{Y}_1$, the first relay computes

$$\hat{\boldsymbol{Y}} = \left[\alpha\boldsymbol{Y}_1 + \sum_{i=1}^{K}\boldsymbol{D}_i\right] \bmod \Lambda_1 \tag{31}$$

which can be simplified as

$$\begin{aligned} \hat{\boldsymbol{Y}} &= \left[\alpha\left(\boldsymbol{X}_1 + \sum_{i=2}^{K} h_{1i}\boldsymbol{X}_i + \boldsymbol{Z}_1\right) + \sum_{i=1}^{K}\boldsymbol{D}_i\right] \bmod \Lambda_1 \\ &= \left[\sum_{i=1}^{K}\boldsymbol{V}_i - ([\boldsymbol{V}_1 - \boldsymbol{D}_1] + \cdots + [\boldsymbol{V}_K - \boldsymbol{D}_K]) \right. \quad (32) \\ &\quad \left. + \alpha\left(\boldsymbol{X}_1 + \sum_{i=2}^{K} h_{1i}\boldsymbol{X}_i + \boldsymbol{Z}_1\right)\right] \bmod \Lambda_1 \\ &= \left[\sum_{i=1}^{K}\boldsymbol{V}_i - ([\boldsymbol{V}_1 - \boldsymbol{D}_1] \bmod \Lambda_1 + \cdots + [\boldsymbol{V}_K - \boldsymbol{D}_K] \bmod \Lambda_K) \right. \\ &\quad \left. + \alpha\left(\boldsymbol{X}_1 + \sum_{i=2}^{K} h_{1i}\boldsymbol{X}_i + \boldsymbol{Z}_1\right)\right] \bmod \Lambda_1 \quad (33) \\ &= \left[\sum_{i=1}^{K}\boldsymbol{V}_i + (\alpha - 1)\left(\boldsymbol{X}_1 + \sum_{i=2}^{K} h_{1i}\boldsymbol{X}_i\right) + \alpha\boldsymbol{Z}_1\right] \bmod \Lambda_1 \\ &= [\boldsymbol{T} + \boldsymbol{Z}_{\mathrm{eff}}] \bmod \Lambda_1 \quad (34) \end{aligned}$$

where (32) results from adding and subtracting $\sum_{i=1}^{K}\boldsymbol{V}_i$, (33) comes from the definition of the modulo operation and the fact that $\Lambda_K \subseteq \cdots \subseteq \Lambda_3 \subseteq \Lambda_2 \subseteq \Lambda_1$, and (34) follows from (29). Thus, similar to [18] we obtain

$$\hat{\boldsymbol{Y}} = [\boldsymbol{T} + \boldsymbol{Z}_{\mathrm{eff}}] \bmod \Lambda_1$$

in which

$$\boldsymbol{T} = \left[\boldsymbol{V}_1 + \sum_{i=2}^{K} \boldsymbol{V}_i\right] \bmod \Lambda_1 \tag{35}$$

$$\boldsymbol{Z}_{\text{eff}} = \left[(\alpha - 1)\left(\sum_{i=1}^{K} h_{1i}\boldsymbol{X}_i\right) + \alpha \boldsymbol{Z}_1\right] \bmod \Lambda_1 \tag{36}$$

Since $\boldsymbol{X}_i$ is independent of $\boldsymbol{V}_i$ (using the crypto lemma) then, we conclude that $\boldsymbol{T}$ is independent of $\boldsymbol{Z}_{\text{eff}}$ and is uniformly distributed over $\mathcal{C}_1$ [21]. For minimising the variance of the effective noise, we choose α as follows

$$\alpha_{\text{MMSE}} = \frac{\sum_{i=1}^{K} h_{1i}^2 P}{\sum_{i=1}^{K} h_{1i}^2 P + N} \tag{37}$$

then, variance of the effective noise satisfies

$$\frac{1}{n}\mathbb{E}\left\{||\boldsymbol{Z}_{\text{eff}}||^2\right\} \le \frac{\sum_{i=1}^{K} h_{1i}^2 P}{\sum_{i=1}^{K} h_{1i}^2 P + N} N \tag{38}$$

Finally, at the decoder, we use the Euclidean lattice decoding [16], which finds the closest point to $\hat{\boldsymbol{Y}}$ in Λ_C

$$\hat{\boldsymbol{T}} = \mathcal{Q}_{\Lambda_C}(\hat{\boldsymbol{Y}}) \tag{39}$$

Then, for the probability of error we have

$$P_{\text{e}} = \Pr(\hat{\boldsymbol{T}} \ne \boldsymbol{T}) = \Pr(\boldsymbol{Z}_{\text{eff}} \notin \mathcal{V}_c) \tag{40}$$

Similar to the argument in [21], we have the following theorem

Theorem 1: Let

$$R_1^* \le \frac{1}{2}\log\left(\frac{1}{\sum h_{1i}^2} + \frac{P}{N}\right)^+ \tag{41}$$

For any $R_1 \le R_1^*$ and a lattice partition chain described in (26), the probability of error (40) is bounded by

$$P_{\text{e}} \le \text{e}^{-n\left(E_p\left(2^{2(R_1^* - R_1)}\right) - O_n(1)\right)} \tag{42}$$

where E_{P} is the Poltyrev exponent [25].

Proof: The proof is similar to the proof of Theorem 3 in [21]. As we know if we set

$$\mu = \frac{\text{Vol}(\Lambda_c)^{(2/n)}}{2\pi e \sigma_{\text{eff}}^2} \tag{43}$$

then, if $\mu \ge 1$ the probability of error will decade exponentially with n, as n goes to infinity [16], in which we have

$$\sigma_{\text{eff}}^2 = \frac{\sum_{i=1}^{K} h_{1i}^2 P}{\sum_{i=1}^{K} h_{1i}^2 P + N} N \tag{44}$$

where (44) is because of (38).

By using (12) and (43), we have

$$\begin{aligned} \mu &= \frac{\text{Vol}(\Lambda_1)^{(2/n)}}{2\pi e 2^{2R_1}\sigma_{\text{eff}}^2} \\ &= \frac{P}{2\pi e G(\Lambda_1) 2^{2R_1}\sigma_{\text{eff}}^2} \qquad (45) \\ &= \frac{P}{2^{2R_1}\sigma_{\text{eff}}^2} \qquad (46) \end{aligned}$$

where (45) is because of (11) and we have (46) since Λ_1 is Rogers good. Then by substituting (44) into (46) and considering $\mu \ge 1$ we have

$$R_1 \le \frac{1}{2}\log\left(\frac{1}{\sum h_{1i}^2} + \frac{P}{N}\right)^+$$

which completes the proof. □

For decoding at relay i ($i = 2, 3, \ldots, K$), first the relay multiplies the received sequence, $\boldsymbol{Y}_i' = h_{1i}\boldsymbol{X}_i + \boldsymbol{Z}_i'$, by (h_{1i}/h_{ii}) to obtain

$$\begin{aligned} \boldsymbol{Y}_i &= h_{1i}\boldsymbol{X}_i + \boldsymbol{Z}_i' \\ &= \left[\boldsymbol{V}_i + \boldsymbol{D}_i\right] \bmod \Lambda_i + \boldsymbol{Z}_i \end{aligned}$$

where $\boldsymbol{Z}'_i = (h_{1i}/h_{ii})\boldsymbol{Z}_i$. Then, the relay computes

$$\begin{aligned} \hat{\boldsymbol{Y}}_i &= \left[\beta \boldsymbol{Y}_i' + \boldsymbol{D}_i\right] \bmod \Lambda_i \\ &= \left[\beta h_{1i}\boldsymbol{X}_i + \beta \boldsymbol{Z}_i' + \boldsymbol{D}_i\right] \bmod \Lambda_i \\ &= \left[\boldsymbol{V}_i + \beta h_{1i}\boldsymbol{X}_i + \beta \boldsymbol{Z}_i' - (\boldsymbol{V}_i - \boldsymbol{D}_i)\right] \bmod \Lambda_i \\ &= \left[\boldsymbol{V}_i + (\beta - 1)h_{1i}\boldsymbol{X}_i + \beta \boldsymbol{Z}_i'\right] \bmod \Lambda_i \\ &= \left[\boldsymbol{V}_i + \boldsymbol{Z}_{\text{eff}_i}\right] \bmod \Lambda_i \end{aligned} \tag{47}$$

where (47) is based on the distributive law for the modulo operation and

$$\boldsymbol{Z}_{\text{eff}_i} = \left[\beta \boldsymbol{Z}_i' - (1 - \beta)h_{1i}\boldsymbol{X}_i\right] \bmod \Lambda_i \tag{48}$$

Since $\boldsymbol{X}_i$ is independent of $\boldsymbol{V}_i$ (by the crypto lemma), then, we conclude that $\boldsymbol{V}_i$ is independent of $\boldsymbol{Z}_{\text{eff}_i}$ and is uniformly distributed over $\mathcal{C}_i$. For minimising the variance of the effective noise, we choose β as follows

$$\beta_{\text{MMSE}_i} = \frac{P}{P + (1/h_{ii}^2)N} \tag{49}$$

Finally, the relay performs the Euclidean lattice decoding [16], which finds the closest point to $\hat{\boldsymbol{Y}}_i$ in Λ_c

$$\hat{\boldsymbol{V}}_i = \mathcal{Q}_{\Lambda_c}(\hat{\boldsymbol{Y}}_i) \tag{50}$$

Then, the probability of error is

$$P_{\text{e}} = \Pr(\hat{\boldsymbol{V}}_i \ne \boldsymbol{V}_i) = \Pr\left(\boldsymbol{Z}_{\text{eff}_i} \notin \mathcal{V}_{\Lambda_c}\right) \tag{51}$$

Similar to the argument in [2, 18], the following rate is achievable

$$R_i \le \frac{1}{2}\log\left(1 + \frac{h_{ii}^2 P}{N}\right)^+, \quad i = 1, 2, \ldots, K \tag{52}$$

The proof is similar to the proof of Theorem 3 in [21] and is removed here.

IET Commun., 2015, Vol. 9, Iss. 6, pp. 880–887

Thus, by the proposed scheme, we estimate the linear combination (not necessarily an integer linear combination) of messages at relay 1, while we estimate message of transmitter i at relay i for $i = 2, 3, \ldots, K$. For correct recovery of messages, from (41) and (52), we obtain the following rate-region

$$R_1 \le \frac{1}{2}\log\left(\frac{1}{\sum h_{1i}^2} + \frac{P}{N}\right)^+ \tag{53}$$

$$R_i \le \frac{1}{2}\log\left(1 + \frac{h_{ii}^2 P}{N}\right)^+, \quad \forall i = 2, 3, \ldots, K \tag{54}$$

On the other hand, there exists a limited-rate link with rate R_0 from relay i to the FC. As a result, for correct recovery of linear combination of messages at relay 1 and other messages at relay $i = 2, 3, \ldots, K$, the communication rate R_i should satisfy

$$R_i \le R_o, \quad \forall i = 1, 2, \ldots, K \tag{55}$$

Thus, the achievable rate-region for many-to-one Gaussian IC with a FC is derived using (53)–(55).

4 Outer bound and symmetric rate

In this section, we first provide an outer bound on the achievable rate-region. Since there is a limited-rate link with rate R_o from each relay to the FC, an outer bound on the achievable rate-region is given by

$$R_i \le R_o, \quad i = 1, 2, \ldots, K \tag{56}$$

This trivial outer bound is used for performance comparison among different transmission schemes. In order to compare different schemes, we consider the achievable symmetric rate of each scheme. A symmetric rate is a common rate at which all users are operating.

In the following, we provide the symmetric rate of each scheme presented in Section 3:

- Compress-and-forward (see (57))

- Decode-and-forward

$$R \le \min\left(R_0, \ \frac{1}{2K}\log\left(1 + \frac{\sum_{i=1}^{K} h_{1i}^2 P}{N}\right)\right)^+ \tag{58}$$

This rate follows from the fact that the dominant term in (17) is

$$\frac{1}{2K}\log\left(1 + \frac{\sum_{i=1}^{K} h_{1i}^2 P}{N}\right)$$

- Compute-and-forward

$$\begin{aligned} R \le \min\Bigg(&\frac{1}{2}\log\left(\left(||\boldsymbol{a}||^2 - \frac{P(\boldsymbol{h}^{\mathrm{T}} \cdot \boldsymbol{a})^2}{N + P||\boldsymbol{h}||^2}\right)^{-1}\right) \\ &\frac{1}{2}\log\left(1 + \frac{h_{ii}^2 P}{N}\right), \ R_0\Bigg)^+, \quad i = 2, 3, \ldots, K \end{aligned} \tag{59}$$

- Our proposed scheme

$$R \le \min\left(R_0, \ \frac{1}{2}\log\left(\frac{1}{\sum h_{1j}^2} + \frac{P}{N}\right), \ \frac{1}{2}\log\left(1 + \frac{h_{ii}^2 P}{N}\right)\right)^+ \tag{60}$$

for $i = 2, 3, \ldots, K$.

Theorem 2: For the many-to-one Gaussian IC with a FC, when channel gains are larger than 1 and in the high SNR regime, our proposed scheme can achieve the symmetric capacity region if the rate of each limited-rate link equals to an AWGN channel capacity.

Proof: Assume that relay i sends its information to the FC at its maximum rate, that is, the rate of each link from relay i to the FC equals to an AWGN channel capacity

$$R_0 = \frac{1}{2}\log(1 + \mathrm{SNR})$$

where SNR = (P/N). Then, from (56) and (60), we obtain

$$R \le \frac{1}{2}\log(1 + \mathrm{SNR}) \tag{61}$$

$$\begin{aligned} R \le \min\Bigg(&\frac{1}{2}\log(1 + \mathrm{SNR}), \ \frac{1}{2}\log\left(\frac{1}{\sum h_{1j}^2} + \mathrm{SNR}\right) \\ &\frac{1}{2}\log\left(1 + h_{ii}^2 \times \mathrm{SNR}\right)\Bigg)^+, \quad i = 2, 3, \ldots, K \end{aligned} \tag{62}$$

in the high SNR regime and when the channel gains are larger than 1, (61) and (62) are simplified to

$$R \le \frac{1}{2}\log(\mathrm{SNR}) \tag{63}$$

$$R \le \frac{1}{2}\log(\mathrm{SNR}) \tag{64}$$

and hence the proof is complete. □

For the CF scheme, from (57) and by considering $R_0 = (1/2)\log(1 + \mathrm{SNR})$, we have

$$R \le \min\left(\frac{1}{2}\log\left(1 + \frac{\mathrm{SNR}\left(1 + \mathrm{SNR}\sum h_{1i}^2\right)}{\mathrm{SNR} + 1 + \mathrm{SNR}\sum h_{1i}^2}\right), \frac{1}{2}\log\left(1 + \frac{\mathrm{SNR}(\mathrm{SNR})}{1 + 2\mathrm{SNR}}\right)\right)^+ \tag{65}$$

in the high SNR regime and when channel gains are larger than 1, (65) reduces to

$$R \le \frac{1}{2}\log\left(\frac{\mathrm{SNR}}{2}\right) \tag{66}$$

For the DF scheme, by considering (58) it can be seen that it is related to the number of users

$$R \le \min\left(\frac{1}{2}\log(1 + \mathrm{SNR}), \ \frac{1}{2K}\log\left(1 + \mathrm{SNR}\sum_{i=1}^{K} h_{1i}^2\right)\right)^+ \tag{67}$$

$$R \le \min\left(\frac{1}{2}\log\left(1 + \frac{\left(\sum_{i=1}^{K} h_{1i}^2 P + N\right)(2^{2R_0} - 1)}{\sum_{i=1}^{K} h_{1i}^2 P + 2^{2R_0} N}, \ \frac{1}{2}\log\left(1 + \frac{P(2^{2R_0} - 1)}{P + 2^{2R_0} N}\right)\right)\right)^+ \tag{57}$$

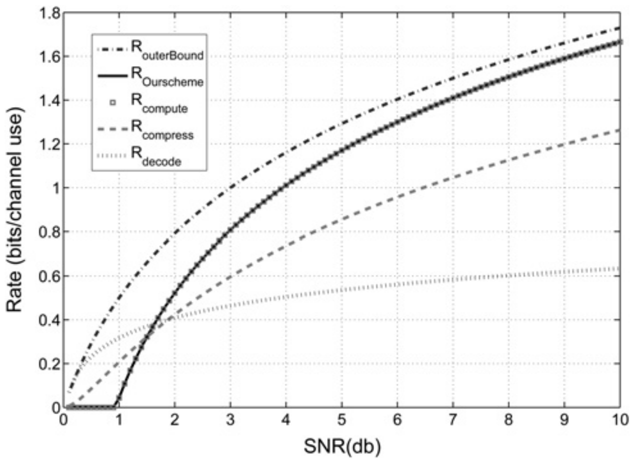

Fig. 2 *Comparison between the outer bound and performance of the encoding schemes for many-to-one Gaussian IC with a FC (integer case)*

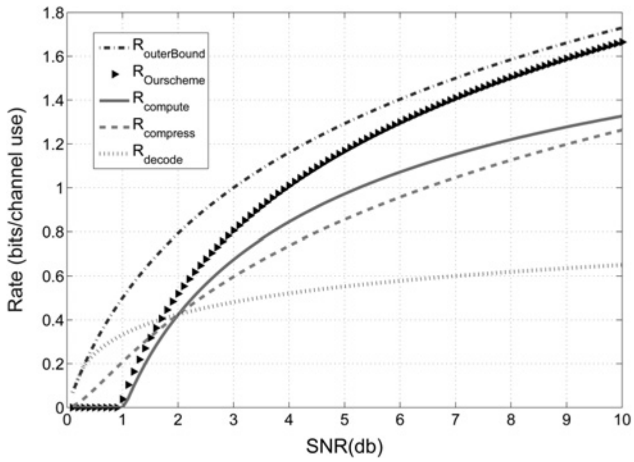

Fig. 3 *Comparison between the outer bound and performance of the encoding schemes for many-to-one Gaussian IC with a FC (non-integer case)*

For the compute-and-forward scheme, when channel gains are larger than 1, using (59) we have

$$R \le \min\left(R_0,\ \frac{1}{2}\log\left(\left(||\boldsymbol{a}||^2 - \frac{P(\boldsymbol{h}^{\mathrm{T}} \cdot \boldsymbol{a})^2}{N + P||\boldsymbol{h}||^2}\right)^{-1}\right)\right)^+ \tag{68}$$

that is related to parameter $\boldsymbol{a}$. It is shown that when channel gains are integer values, this scheme have a good performance. For this case, we set $\boldsymbol{a} = \boldsymbol{h}$ and then we have

$$R \le \min\left(\frac{1}{2}\log(1 + \mathrm{SNR}),\ \frac{1}{2}\log\left(\frac{1}{||\boldsymbol{h}||^2} + \mathrm{SNR}\right)\right)^+ \tag{69}$$

When the channel gains are larger than 1 and in the high SNR regime, (69) is simplified to

$$R \le \frac{1}{2}\log(\mathrm{SNR}) \tag{70}$$

5 Numerical results

In this section, using numerical examples, we compare performance of the encoding schemes, presented in Section 4, for the many-to-one Gaussian IC with a FC. We assume that the rate of each link between the relays and the FC equals to $R_0 = (1/2)\log(1 + (P/N))$. In the first example, we assume that the channel gains are integer-valued, which are set as $\boldsymbol{h} = \boldsymbol{a} = [1,\ 1,\ 1,\ 2,\ 3]$. Achievable rates of different relaying schemes are presented in Fig. 2. In the second example, we assume that the channel coefficients are non-integer, that is, $\boldsymbol{h} = [1,\ 1.2,\ 1.9,\ 2,\ 2.9]$, while $\boldsymbol{a} = [1,\ 1,\ 2,\ 2,\ 3]$ for the compute-and-forward scheme and $\boldsymbol{a} = \boldsymbol{h}$ for our proposed scheme. Achievable rates of different relaying schemes as well as the outer bound are shown in Fig. 3. We observe that, our proposed scheme can achieve the outer bound in the high SNR regime. For integer-valued channel gains, both the proposed scheme and the compute-and-forward scheme outperform classical relaying strategies such as decode-and-forward and CF. In case of non-integer channel gains, our proposed strategy performs superior to the classical strategies (decode-and-forward and CF), as well as to the compute-and-forward scheme. Although in compute-and-forward scheme, the relay is free to choose the linear coefficients, the non-integer penalty is because of the mismatch between real channel coefficients and the integer coefficients of the linear combination. This is the idea implemented in our proposed scheme and the superior performance of our scheme is because of using the channel gains in constructing the transmitted sequences.

6 Conclusion

In this paper, a special case of a data gathering distributed network, modelled by a many-to-one Gaussian IC with a FC, was evaluated where only one receiver faces interference. Based on nested lattice codes, we proposed a transmission scheme and obtained an achievable rate-region. Here, we used the fact that for arbitrary numbers P_1 and P_2 where $P_1 > P_2$, we can find two nested lattices Λ_1 and Λ_2 such that $\Lambda_1 \subseteq \Lambda_2$ and the second moment of Λ_1 and Λ_2 is P_1 and P_2, respectively [1, 2]. We used this property to design a transmission scheme that can recover non-integer linear combination of messages (in contrast to the regular CF scheme where only an integer linear combination of messages is recovered). For comparison, the achievable rate-regions of various classical relaying schemes, that is, decode-and-forward, CF and compute-and-forward schemes, are also derived. By analytical and numerical analysis, we showed that if the rate of each relay-to-FC link equals to the capacity of an AWGN channel, and if channel gains are larger than 1, then we can achieve the capacity region in the high SNR regime.

7 Acknowledgments

The authors thank Shahab Ghasemi-Goojani for numerous helpful discussions that have certainly improved the quality of this paper. They also sincerely thank the Editor and the anonymous reviewers for giving us constructive and helpful comments.

8 References

1 Han, T.S., Kobayashi, K.: 'A new achievable rate region for the interference channel', *IEEE Trans. Inf. Theory*, 1981, **27**, (1), pp. 49–60
2 Costa, M.H.M.: 'On the Gaussian interference channel', *IEEE Trans. Inf. Theory*, 1985, **31**, (5), pp. 607–615
3 Sason, I.: 'On achievable rate regions for the Gaussian interference channel', *IEEE Trans. Inf. Theory*, 2004, **50**, (6), pp. 1345–1356
4 Etkin, R.H., Tse, D.N.C., Wang, H.: 'Gaussian interference channel capacity to within one bit', *IEEE Trans. Inf. Theory*, 2008, **54**, (12), pp. 5534–5562
5 Sahin, O., Erkip, E.: 'On achievable rates for interference relay channel with interference cancelation'. Proc. of Forty First Annual Asilomar Conf. on Signals, Systems and Computers, Pacific Grove, California, November 2007, pp. 805–809
6 Chaaban, A., Sezgin, A.: 'Achievable rates and upper bounds for the interference relay channel'. Record of the Forty Fourth Asilomar Conf. on Signals, Systems and Computers (ASILOMAR), 2010 Conf., 2010, pp. 267–271
7 Sridharan, S., Jafarian, A., Vishwanath, S., Jafar, S.: 'Capacity of symmetric K-user Gaussian very strong interference channels'. Global Telecommunications Conf., IEEE GLOBECOM 2008, November 2008, pp. 1–5
8 Tian, Y., Yener, A.: 'The Gaussian interference relay channel: improved achievable rates and sum rate upperbounds using a potent relay', *IEEE Trans. Inf. Theory Special Issue Interference Netw.*, 2011, **57**, (5), pp. 2865–2879

9 Annapureddy, V., Veeravalli, V.: 'Gaussian interference networks: sum capacity in the low-interference regime and new outer bounds on the capacity region', *IEEE Trans. Inf. Theory*, 2009, **55**, (7), pp. 3032–3050.

10 Jovicic, A., Wang, H., Viswanath, P.: 'On network interference management'. Information Theory Workshop, IEEE ITW'07, September 2007, pp. 307–312

11 Bresler, G., Parekh, A., Tse, D.: 'The approximate capacity of the many-to-one and one-to-many Gaussian interference channels', *IEEE Trans. Inf. Theory*, 2010, **56**, (9), pp. 4566–4592

12 Cadambe, V.R., Jafar, S.A.: 'Interference alignment and a noisy interference regime for many-to-one interference channels', Available at: http://www.arxiv.org/abs/0912.3029, 2009

13 He, X., Yener, A.: 'The Gaussian many-to-one interference channel with confidential messages'. Proc. IEEE ISIT, June 2009, pp. 2086–2090

14 Nazer, B., Gastpar, M.: 'The case for structured random codes in network capacity theorems', *IEEE Trans. Inf. Theory*, 2008, **19**, (4), pp. 455–474

15 Nazer, B., Gastpar, M.: 'Compute-and-forward: Harnessing interference through structured codes', *IEEE Trans. Inf. Theory*, 2011, **57**, (10), pp. 6463–6486

16 Erez, U., Zamir, R.: 'Achieving 1/2 log(1 + SNR) on the AWGN channel with lattice encoding and decoding', *IEEE Trans. Inf. Theory*, 2004, **50**, (22), pp. 2293–2314

17 Song, Y., Devroye, N.: 'Lattice codes for the Gaussian relay channel: decode-and-forward and compress-and-forward', *IEEE Trans. Inf. Theory*, 2013, **59**, (8), pp. 4927–4948

18 Smirani, S., Kamoun, M., Sarkiss, M., Zaidi, A., Duhamel, P.: 'Achievable rate regions for two-way relay channel using nested lattice coding', Available at: http://www.arxiv.org/abs/1311.5360, 2013

19 Cover, T.M., Thomas, J.A.: 'Elements of information theory' (John Wiley & Sons, New York, 2006, 2nd edn.)

20 Erez, U., Litsyn, S., Zamir, R.: 'Lattices which are good for (almost) everything', *IEEE Trans. Inf. Theory*, 2005, **51**, (16), pp. 3401–3416

21 Nam, W., Chung, S.-Y., Lee, Y.H.: 'Nested lattice codes for Gaussian relay networks with interference', *IEEE Trans. Inf. Theory*, 2011, **57**, (12), pp. 7733–7745

22 Krithivasan, D., Pradhan, S.S.: 'A proof of the existence of good nested lattices' Available at: http://www.eecs.umich.edu/techreports/systems/cspl/cspl-384.pdf

23 Ghasemi-Goojani, S., Behroozi, H.: 'Nested lattice codes for Gaussian two-way relay channels', Available at: http://www.arxiv.org/pdf/1301.6291v1.pdf, January 2013

24 Forney, G.D.: 'On the role of mmse estimation in approaching the information theoretic limits of linear Gaussian channels: Shannon meets wiener'. Proc. 41st Ann. Allerton Conf, Monticello, IL, October 2003

25 Poltyrev, G.: 'On coding without restrictions for the AWGN channel', *IEEE Trans. Inf. Theory*, 1994, **40**, (9), pp. 409–417