Full-Diversity Space-Time-Frequency Coding with Very Low Complexity for the ML Decoder

Mostafa Shahabinejad, *Student Member, IEEE,* and Siamak Talebi, *Member, IEEE*

***Abstract*—Recently proposed full-diversity space-time-frequency block codes (STFBCs) generally suffer from very high computational complexity at the receiver side. In this paper, we introduce a new class of full-diversity STFBCs for quasi-static (QS) channels which features a comparatively low complexity at the receiver. We also demonstrate that our proposed algorithms could offer maximum coding advantage if the transmitter knows partial channel side information. Simulation results also verify that our coding schemes outperform other recently published STFBCs that were considered.**

***Index Terms*—Wireless communication, MIMO-OFDM, quasi-static channels, space-time-frequency coding, fading channels.**

I. Introduction

By combining multiple-input multiple-output (MIMO) and orthogonal frequency-division multiplexing (OFDM), called MIMO-OFDM, not only we can mitigate the inter symbol interference effect, we can also take advantages of diversity. One of the best types of MIMO-OFDM schemes that has been proposed to date is space-time-frequency coding. In general, space-time-frequency block codes (STFBCs) that have been reported so far suffer from high computational complexity at the receiver for their maximum-likelihood (ML) decoders [1]-[3].

In this paper, we aim to unveil a novel class of STFBCs with low complexity for the ML decoder. Our schemes provide the maximum coding advantage when delay and power profiles (DPPs) of the channel are known to the transmitter. In the unknown DPPs case, we first propose artificial DPPs (ADPPs) and then design STFBCs in accordance with these profiles. Simulation results confirm that the proposed schemes outperform other recently proposed schemes.

II. System Model

Consider a STFBC codeword as follows:

$$\mathbf{C}^t = \begin{pmatrix} c_1^t(0) & c_2^t(0) & \dots & c_{M_t}^t(0) \\ c_1^t(1) & c_2^t(1) & \dots & c_{M_t}^t(1) \\ \vdots & \vdots & \ddots & \vdots \\ c_1^t(N-1) & c_2^t(N-1) & \dots & c_{M_t}^t(N-1) \end{pmatrix}, \tag{1}$$

where t=1, 2, ..., τ and $c_i^t(n)$'s are data transmitted by the i^{th} transmit antenna at the n^{th} subcarrier over the t^{th} OFDM block. After applying an N-point inverse fast Fourier transform to each column of $\mathbf{C}^t$ and appending cyclic prefix, transmitter sends the OFDM symbols over M_T antennas for τ successive time slots. For a receiver with M_R antennas in a frequency-selective channel between each pair of transmit and receive antennas, we assume that there are L independent delay paths with the same DPPs. Channel impulse response at the t^{th} OFDM block from transmit antenna i to receive antenna j is given by [1]:

$$h_{i,j}^t(\zeta) = \sum_{l=0}^{L-1} \alpha_{i,j}^t(l)\delta(\zeta - \zeta_l). \tag{2}$$

In (2), ζ_l's are delays, each $\alpha_{i,j}^t(l)$ is a zero-mean complex Gaussian random variable with variance σ_l^2, indicating amplitude corresponding to the l^{th} path of the i^{th} transmit and the j^{th} receive antennas and $\sum_{l=0}^{L-1}\sigma_l^2 = 1$ for normalization purposes. It is supposed that there is no spatial fading correlation between antennas. It is also assumed that the receiver has the channel state information $\alpha_{i,j}^t(l)$ flawlessly. The received signal at the antenna j^{th}, by removing the cyclic prefix and performing fast Fourier transform during the t^{th} OFDM block at the n^{th} frequency tone, is given by:

$$r_j^t(n) = \sum_{i=1}^{M_T} c_i^t(n)H_{i,j}^t(n) + \mathcal{N}_j^t(n), n = 0, 1, ..., N-1, \tag{3}$$

where

$$H_{i,j}^t(n) = \sum_{l=0}^{L-1} \alpha_{i,j}^t(l)\omega^{n\zeta_l}, n = 0, 1, ..., N-1 \tag{4}$$

is the channel frequency response at the n^{th} frequency subcarrier between transmit antenna i and receive antenna j within the t^{th} OFDM symbol duration, $\omega = e^{-j2\pi\frac{BW}{N}}$, where BW is the total bandwidth of the system and N is the number of subcarriers per OFDM block.

III. Proposed STFBCs for MIMO-OFDM Systems

Before introducing our new STFBCs, we need to present a definition:

Definition: *(a row codeword of a space-time block code)*

Consider a 2×2 space-time block code (STBC) codeword as below:

$$\mathbf{X}_{STBC} = \begin{pmatrix} \chi_1 & \chi_2 \\ \chi_3 & \chi_4 \end{pmatrix}, \tag{5}$$

where rows and columns of $\mathbf{X}_{STBC}$ represent numbers of time slots and antennas, respectively; $\chi_1 = \sum_{i=1}^{2}(a_is_i + b_is_i^*)$; $\chi_2 = \sum_{i=3}^{4}(a_is_i + b_is_i^*)$; $\chi_3 = \sum_{i=3}^{4}(c_is_i + d_is_i^*)$; $\chi_4 = \sum_{i=1}^{2}(c_is_i + d_is_i^*)$; a_i's, b_i's, c_i's, and d_i's $\forall i \in \{1,2,3,4\}$ are code's parameters and s_i's $\forall i \in \{1,2,3,4\}$ are symbols chosen from a constellation such as BPSK or QAM. It is worth mentioning that lots of well-known STBCs, such as the Alamouti code [4], the Golden code [5] and the proposed code of [6] could be modeled as (5), with different parameters.

We define χ= $[\chi_1, \chi_2, \chi_3, \chi_4] \in \mathbb{C}^{1\times 4}$ as a row codeword of a STBC.

In what follows, we describe how our proposed STFBCs are constructed in four steps:

STEP I. Create $\chi_k \in \mathbb{C}^{\gamma_{SD}\times 4}, \forall k \in \{1, 2, ..., \lfloor\frac{N}{\gamma_{SD}\Gamma}\rfloor\}$ by sticking γ_{SD} distinct row codewords for each χ_k, where Γ

Manuscript received January 4, 2012. The associate editor coordinating the review of this letter and approving it for publication was W. Hamouda.

This research was supported by the Kerman Telecommunication Research Center, Kerman, Iran.

The authors are with the Department of Electrical Engineering, Shahid Bahonar University of Kerman, Kerman, Iran, and the Advanced Communications Research Institute, Sharif University, Tehran, Iran (e-mail: m.shahabinejad@ieee.org, siamak.talebi@uk.ac.ir).

Digital Object Identifier 10.1109/LCOMM.2012.031212.112648

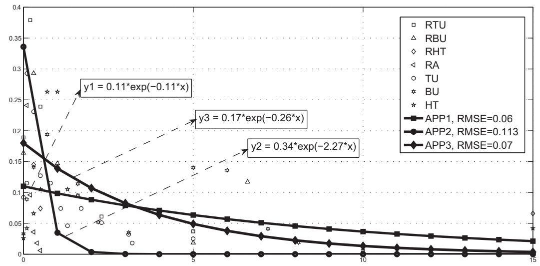

Fig. 1. COST207 channel models and suggested function for the ADPPs.

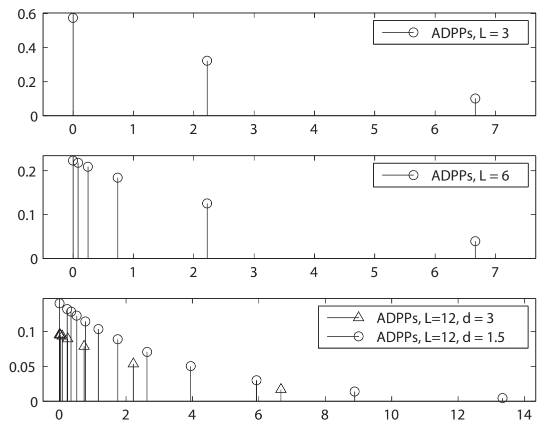

Fig. 2. Suggested ADPPs for different numbers of taps, L={3, 6, 12}.

is a number chosen from set $\{1, 2, ..., L\}$, $\lfloor\cdot\rfloor$ indicates floor operation and γ_{SD} denotes separation distance between the same row codewords.

STEP II. Produce $\mathbf{C}_k \in \mathbb{C}^{\Gamma\gamma_{SD}\times 4}$ by the following equation:

$$\mathbf{C}_k = [\mathbf{1}_{\Gamma\times 1} \otimes \mathbf{I}_{\gamma_{SD}}]\chi_k, k = 1, 2, ..., \lfloor\frac{N}{\gamma_{SD}\Gamma}\rfloor, \qquad (6)$$

where $\otimes$ stands for the tensor product, and $\mathbf{1}_{\Gamma\times 1}$ is a column vector whose elements are all equal to one.

STEP III. If the remainder of $\frac{N}{\gamma_{SD}\Gamma}$ is nonzero, create $\chi' \in \mathbb{C}^{\lfloor\frac{\psi}{\Gamma}\rfloor\times 4}$ by sticking $\lfloor\frac{\psi}{\Gamma}\rfloor$ distinct row codewords, where $\psi = N - \gamma_{SD}\Gamma\lfloor\frac{N}{\gamma_{SD}\Gamma}\rfloor$. Then, generate $\mathbf{C}' \in \mathbb{C}^{\Gamma\lfloor\frac{\psi}{\Gamma}\rfloor\times 4}$ as below:

$$\mathbf{C}' = [\mathbf{1}_{\Gamma\times 1} \otimes \mathbf{I}_{\lfloor\frac{\psi}{\Gamma}\rfloor}]\chi'. \qquad (7)$$

STEP IV. Construct $\mathbf{C} \in \mathbb{C}^{N\times 4}$ as follows:

$$\mathbf{C} = \left[\mathbf{C}_1^T, \mathbf{C}_2^T, ..., \mathbf{C}_{\lfloor\frac{N}{\gamma_{SD}\Gamma}\rfloor}^{\;T}, \mathbf{C}'^T, \mathbf{Z}^T\right]^T, \qquad (8)$$

where $(\cdot)^T$ stands for the transpose operation and $\mathbf{Z} \in \mathbb{C}^{(\psi-\Gamma\lfloor\frac{\psi}{\Gamma}\rfloor)\times 4}$ is a matrix whose components are all zero. Thus, our proposed codes are constructed: the first and the second columns of $\mathbf{C}$ shown in (8) indicate $\mathbf{C}^1$ and similarly, the third and the forth columns of $\mathbf{C}$ indicate $\mathbf{C}^2$. In the appendix, it is shown that the proposed STFBCs achieve full-diversity property upon quasi-static (QS) channels.

***Remark* 1**: For QS channels, a simple comparison between system model of a STBC system (see (1) of [6]) and (3) shows that the receiver complexity of the proposed STFBCs is the same as that of the STBC, which is used to construct them.

Now, we select $\boldsymbol{\gamma_{SD}}$ so as to maximize the coding advantage of the proposed codes. Regarding definition of the coding advantage presented in [1] and equation (16) of the appendix, in order to maximize the coding advantage of the proposed STFBCs, one just needs to maximize *det*($\widehat{\mathbf{R}}_{\mathbf{F}}$), where $\widehat{\mathbf{R}}_{\mathbf{F}}$ is defined in (13). Thus, the coding advantage of the proposed codes depends on $\boldsymbol{\gamma_{SD}}$ as well as on the system bandwidth (*BW*), DPPs and the number of subcarriers. Therefore, if these parameters of the system are known to the transmitter, we can find $\boldsymbol{\gamma_{SD}}$ so that it maximizes the coding advantage. On the other hand, if DPPs are unknown to the transmitter, we design the proposed STFBCs based on the ADPPs. In other words, for the given ADPPs, parameter $\boldsymbol{\gamma_{SD}}$ is specified so that it maximizes the coding advantage of the proposed STFBCs.

In order to obtain ADPPs, COST207 channel models from [7], namely typical urban (TU), bad urban (BU), hilly terrain (HT), reduced TU (RTU), reduced BU (RBU), reduced HT (RHT) and rural area (RA) are used. Assuming that values of delays times represent the independent variables and values of powers represent dependent variables, we find the curve through the data of the different COST207 channel models in a way that minimizes root mean squared error (RMSE) between it and the data of COST207 channel models. Since, in general, the power profile decays exponentially in a typical wireless channel (see Fig. 1), we use the exponential function as a suitable basis for modeling delays powers. Regarding Fig. 1 and the explanations above, ADPPs are obtained through the following steps:

1. Artificial delay profile: As Fig. 1 shows, delays times are concentrated around the origin for practical channel models. Thus, we consider L delays times corresponding to the artificial delay profile as follows:

$$\boldsymbol{\zeta_{ADP}}_{l} = \begin{cases} \mathbf{0}, & l = \mathbf{0}, \\ \frac{\boldsymbol{\zeta_{max}}}{d^{\,L\text{-}l}}, & l = \mathbf{1}, \mathbf{2}, ..., L - \mathbf{1}, \end{cases} \qquad (9)$$

where $\boldsymbol{\zeta_{max}}$ is the length of the cyclic prefix and d=3 for $2 \leq L \leq 8$ and d=1.5 for $L \geq 9$. We chose these values of d by considering different ADPs and comparing them with delays times of practical channel models. For example, regarding artificial delay profiles, we observed that generally the artificial delays times in comparison with COST207 channel models with 6 taps or Stanford University Interim (SUI) channel models [8] with 3 taps are scattered logically. On the other hand, it is seen that totally the artificial delays times in comparison with COST207 channel models are mostly concentrated around the origin in a bad way in the case of channels with 12 taps, which is undesirable (see Fig. 2). Thus, it seems to be logical to want to scatter the values in the delay span for channels with large numbers of delays. Therefore, we scattered different delays times for channels with more than 8 taps.

2. Artificial power profile: For delays times $\boldsymbol{\zeta_{APP}}_{l}$'s specified in (9), we prefer to set powers corresponding to the artificial power profile as follows:

$$\boldsymbol{\sigma_{APP}}_{l}^{\;\mathbf{2}} = \frac{e^{-\mathbf{0.26}\zeta_{ADP_l}}}{\sum_{\boldsymbol{k=0}}^{\boldsymbol{L-1}} e^{-\mathbf{0.26}\zeta_{ADP_k}}}, l = \mathbf{0}, \mathbf{1}, ..., L-\mathbf{1}. \qquad (10)$$

Here, we explain how the trial and error method is used to choose (10). As we know, in any fitting process, one needs a basis function for obtaining the function which could fit the data. In order to model our artificial power profile, we ascertained from Fig. 1 that a good and simple basis could be the exponential function, i.e. $y_{\boldsymbol{APP}} = ae^{-n\times x_{ADP}}$, where a is the constant which is omitted in the normalization process, $x_{\boldsymbol{ADP}}$ is the independent variable (artificial delay profile) and $y_{\boldsymbol{APP}}$ is the dependent variable (artificial power profile). By

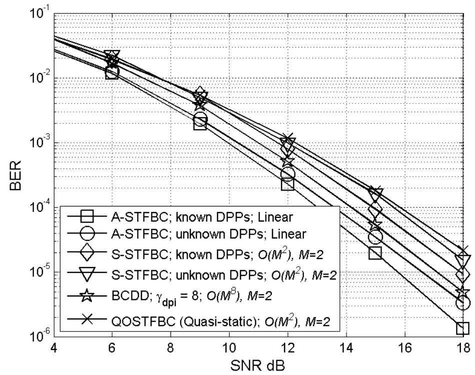

Fig. 3. BER performance, Γ=L=2, delay spread 5 μs, 1 bit/s/Hz.

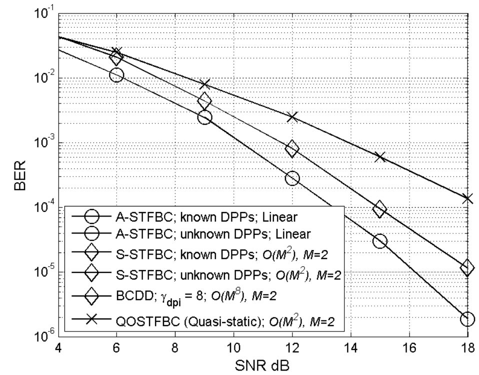

Fig. 4. BER performance, Γ=L=2, delay spread 20 μs, 1 bit/s/Hz.

varying n between [0, 3] in steps of 0.01, we tried to find the curve through the data of the different COST207 channel models in a way that minimizes RMSE between the curve and the data of COST207 channel models. Fig. 1 shows how the resulting curves look like, and for the three different scenarios we note that:

APP1: Three 12-tap channels, namely TU, BU and HT are used to fit the data;

APP2: Four 6-tap channels, namely RTU, RBU, RHT and RA are used to fit the data, and

APP3: Three 6-tap channels, namely RTU, RBU and RHT, and three 12-tap channels, namely TU, BU and HT are used to fit the data.

APP1 and APP3 result in approximately the same RMSEs (0.06 and 0.07 respectively), but APP3 appears to indicate a better estimation of power profile of different COST207 channel models. More clearly, by comparing APP3 against different channel models, APP3 evidently gives a more accurate estimation of different COST207 channel models than that given by APP1.

***Remark* 2**: To the best of our knowledge, there are two permutation methods for the unknown DPPs case proposed in [1] and [3]. With reference to the method in [1], it is seen that the transmitter performs the permutation operation based on a technique which depends on the number of systems' subcarriers, in a random way. As regards the proposition in [3], a fixed permutation solution is offered. In this case, the system is no longer flexible. In other words, this method offers the same permutation under different numbers of channel taps, various bandwidths and different numbers of subcarrier. These methods, in fact, could lead to degradation of the system performance (see Fig. 6 and Fig. 7 of section IV). In contrast, ADPPs seem to present a more logical solution. This is because of the fact that after obtaining the ADPPs based on the number of taps, the code is permuted with due consideration of other system parameters such as system bandwidth as well as number of subcarriers which are available at the transmitter. Simulation results in the next section demonstrate that the ADPPs technique leads to a superior performance in comparison with the two aforementioned permutation methods.

In short, by designing ADPPs, we have tried to approach the problem of the unknown DPPs case more rationally. More specifically, we have built more flexibility into the system by employing ADPPs rather than the random or the fixed permutation methods, as exploited in [1] and [3], respectively.

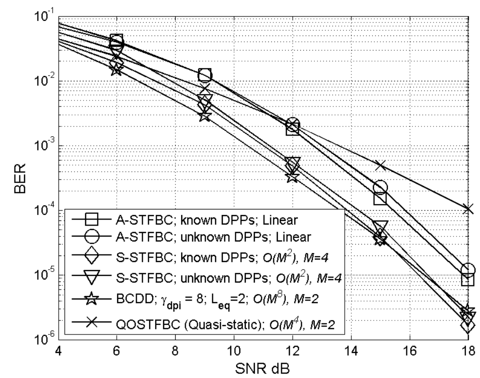

Fig. 5. BER performance, Γ=L=4, 1 bit/s/Hz.

IV. Simulation Results

In our simulations, we consider a QS frequency-selective channel model and a wireless communication system with two transmit and one receive antennas in which QAM constellateions are used to translate bits to symbols. For simulations associated with Fig. 3 to Fig. 5, we also assume that BW = 1 MHz, N = 128 and the length of cyclic prefix is 20 μs. We assess the performance of the new schemes by plotting average bit-error-rate (BER) versus average signal-to-noise-ratio (SNR). Two scenarios for Rayleigh channel models are chosen: 2-ray equal power channels and a 4-ray channel. In the 2-ray channel models, we use two profiles with delay profiles $\{0, 5\}$ μs and $\{0, 20\}$ μs. Power profile $\{0.42, 0.26, 0.18, 0.14\}$ and delay profile $\{0, 6.5, 7.7, 15\}$ μs are used in the 4-ray channel model [3]. In the known DPPs case, for channels with delay spreads 5 μs and 20 μs, values of γ_{SD}'s that maximize the coding advantage are found to be 13 and 3 respectively and in the unknown DPPs case, γ_{SD} is calculated as 10 that maximizes the coding advantage of the ADPPs obtained from equations (9) and (10). Also for the 4-ray channel, γ_{SD}'s are obtained as 30 and 32 for the known and unknown DPPs respectively.

We employed the prominent Alamouti code [4] and also the STBC proposed by Sezginer *et al.* in [6], to construct our STFBCs. We called the consequence STFBCs A-STFBC and S-STFBC, respectively.

It should be noted from Fig. 3 and Fig. 4 that the proposed STFBCs outperform both the quasi-orthogonal space-time-frequency codes (QOSTFBCs) and the block circular delay diversity (BCDD) codes which were introduced in [2] and [3], respectively.

Another important advantage is that in this case one can benefit from very low complexity at the receiver. Stated more

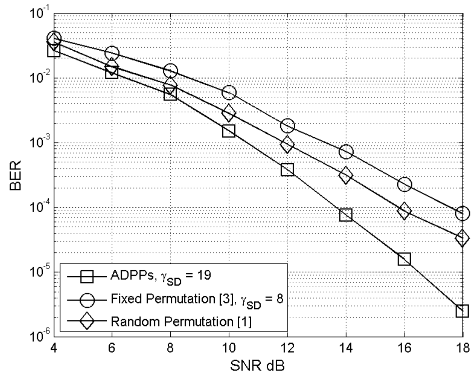

Fig. 6. Different permutation methods for the unknown DPPs case, *BW*=4 MHz, *N*=1024, Γ=L=2, ζ_{max}=20 and delay spread 20 μs.

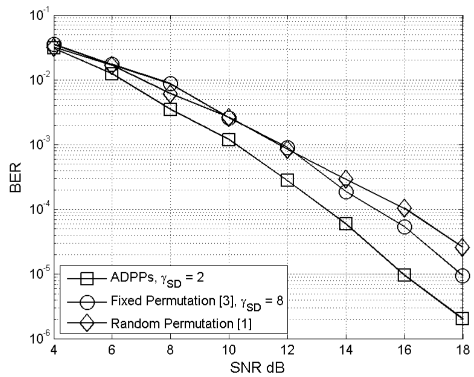

Fig. 7. Different permutation methods for the unknown DPPs case, *BW*=20 MHz, *N*=512, Γ=L=2, ζ_{max}=20 and delay spread 5 μs.

precisely, we have complexity in the orders of $\mathcal{O}(M)$ and $\mathcal{O}(M^2)$ for the A-STFBCs and the S-STFBCs, respectively. Fig. 5 also illustrates that the S-STFBCs offer approximately the same performance as the BCDD and that they both enjoy lower complexity for the ML decoder.

In the following of this section, we will make comparison between the proposed permutation method for the unknown DPPs and those of the random and fixed permutation methods, which were introduced in [1] and [3], respectively. To do that, let us employ the A-STFBC and a wireless communication system with QPSK modulator, two transmit and one receive antennas. As Fig. 6 and Fig. 7 show, for different parameters associated with the channel and system, the proposed ADPPs-based permutation method results in more desirable performances compared with the other two permutation methods.

V. Conclusion

In this paper, we presented new full-diversity STFBCs that offer maximum coding advantage. The proposed algorithm allows the use of a non-complex optimum receiver. Simulation results confirm that the presented models are capable of outperforming other recently reported STFBCs. One other feature of our coding models is their potentials for being implemented in the real-time MIMO systems.

Appendix

In this appendix, we show that our proposed STFBCs could achieve maximum diversity advantage over QS channels. As shown in [1], the maximum achievable diversity of a STFBC is equal to the rank of the following matrix:

$$\mathbf{\Xi} \triangleq \mathbf{\Delta} \circ (\mathbf{R}_{\boldsymbol{\tau}} \otimes \mathbf{R}_{\boldsymbol{F}}). \quad (11)$$

In (11), $\mathbf{\Delta} = (\mathbf{X}^{STFBC} - \widehat{\mathbf{X}}^{STFBC})(\mathbf{X}^{STFBC} - \widehat{\mathbf{X}}^{STFBC})^{\dagger}$, where $\mathbf{X}^{STFBC}$ and $\widehat{\mathbf{X}}^{STFBC}$ are two distinct codewords of a STFBC, $\mathbf{R}_{\boldsymbol{\tau}}$ and $\mathbf{R}_{\boldsymbol{F}}$ are temporal and frequency correlation matrices, respectively, $\circ$ denotes the Hadamard product and $(\cdot)^{\dagger}$ indicates Hermitian. Thus, according to (11) and by simple mathematical operations, one can readily prove that for the proposed STFBCs, the diversity is equal to the rank of the following matrix:

$$\widehat{\mathbf{\Xi}} \triangleq \widehat{\mathbf{\Delta}} \circ (\mathbf{R}_{\boldsymbol{\tau}} \otimes \widehat{\mathbf{R}}_{\boldsymbol{F}}), \quad (12)$$

where, by defining $W^{(a)} \triangleq \sum_{l=0}^{L-1} \sigma_l^2 \omega^{-a\gamma_{SD}\zeta_l}$, $\widehat{\mathbf{R}}_{\boldsymbol{F}}$ could be presented as below:

$$\begin{pmatrix} \mathbf{1} & W^{(\mathbf{1})} & \ldots & W^{(\mathbf{\Gamma-1})} \\ W^{(\mathbf{-1})} & \mathbf{1} & \ldots & W^{(\mathbf{\Gamma-2})} \\ \vdots & \vdots & \ddots & \vdots \\ W^{(-(\mathbf{\Gamma-1}))} & W^{(-(\mathbf{\Gamma-2}))} & \ldots & \mathbf{1} \end{pmatrix} \quad (13)$$

and clearly, for QS channels and two time slots ($\boldsymbol{\tau}$=2), we can consider $\mathbf{R}_{\boldsymbol{\tau}}$ as

$$\mathbf{R}_{\boldsymbol{\tau}} = \begin{pmatrix} \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} \end{pmatrix}. \quad (14)$$

Also in (12), $\widehat{\mathbf{\Delta}}$ is a matrix of size $2\mathbf{\Gamma}\times 2\mathbf{\Gamma}$, whose components of the i^{th} row and the j^{th} column, say $\mathbf{\Delta}_{i,j}$, are obtained from the following equation:

$$\mathbf{\Delta}_{i,j} = \begin{cases} |\delta_1|^2 + |\delta_2|^2, & 1 \le i \le \Gamma, 1 \le j \le \Gamma \\ \delta_1\delta_3^* + \delta_2\delta_4^*, & 1 \le i \le \Gamma, \Gamma+1 \le j \le 2\Gamma \\ \delta_1^*\delta_3 + \delta_2^*\delta_4, & \Gamma+1 \le i \le 2\Gamma, 1 \le j \le \Gamma \\ |\delta_3|^2 + |\delta_4|^2, & \Gamma+1 \le i \le 2\Gamma, \Gamma+1 \le j \le 2\Gamma \end{cases} \quad (15)$$

In (15), $\delta_k = \chi_k - \widehat{\chi}_k, \forall k \in \{1,2,3,4\}$, where χ_k and $\widehat{\chi}_k$ are linear combinations of symbols corresponding to codewords X^{STFBC} and $\widehat{X}^{STFBC}$, respectively. It could be numerically shown that for typical constellations such as QAMs and PSKs, the minimum determinant of $\widehat{\Xi}$ is attained when either δ_2 and δ_3 or δ_1 and δ_4 are zero. Without loss of generality, let us suppose that δ_2 and δ_3 are zero. Therefore, we have:

$$det(\widehat{\Xi}) = (|\delta_1||\delta_4| det(\widehat{\mathbf{R}}_{\boldsymbol{F}}))^2. \quad (16)$$

Similar to the discussion presented in [1, p. 1853], we can infer that $det(\widehat{\mathbf{R}}_{\boldsymbol{F}})$ has a nonzero value. Also δ_1 and δ_4 are nonzeros according to the supposition. Therefore, the proposed STFBCs achieve a diversity advantage equal to $2\mathbf{\Gamma}M_{\boldsymbol{R}}$.

Acknowledgment

The authors would like to thank the anonymous reviewers for their valuable suggestions.

References

[1] W. Su, Z. Safar, and K. J. R. Liu, "Towards maximum achievable diversity in space, time and frequency: performance analysis and code design," *IEEE Trans. Wireless Commun.*, vol. 4, no. 4, pp. 1847–1857, July 2005.
[2] F. Fazel and H. Jafarkhani, "Quasi-orthogonal space-frequency and space-time-frequency block codes for MIMO OFDM channels," *IEEE Trans. Wireless Commun.*, vol. 7, no. 1, pp. 184–192, Jan. 2008.
[3] M. R. Bhavani. S. and K. V. S. Hari, "Systematic construction of linear transform based full-Diversity, rate-one spacetime frequency codes," *IEEE Trans. Signal Process.*, vol. 57, no. 6, pp. 2285–2298, June 2009.
[4] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Sel. Areas Commun.* vol. 16, no. 8, pp. 1451–1458, Oct. 1998.
[5] J.-C. Belfiore, G. Rekaya, and E. Viterbo, "The Golden code: a 2×2 full-rate space-time code with non-vanishing determinants," *IEEE Trans. Inf. Theory*, vol. 51, no. 4, pp. 1432–1436, Apr. 2005.
[6] S. Sezginer, H. Sari, and E. Biglieri, "On high-rate full-diversity 2×2 space-time codes with low-complexity optimum detection," *IEEE Trans. Commun.*, vol. 57, no.5, pp. 1532–1541, May 2009.
[7] G. Stuber, *Principles of Mobile Communication*. Kluwer Academic, 2001.
[8] Channel models for fixed wireless applications, IEEE 802.16a-03/01, June 2003.