

# A Novel Iterative Digital Down Converter

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**Abstract**—The digital radio receivers often have fast analog to digital converters delivering vast amount of data. However, in many cases, the signal of interest represents a small proportion of that bandwidth. A Digital Down Converter (DDC) is a filter that extracts the signal of interest from the incoming data stream. In this paper we first introduce an algorithm based on FFT which can be applied for simultaneous frequency shifting and decimating of Intermediate Frequency (IF) band signals, then a simplified iterative algorithm is suggested to improve the quality of reconstructed baseband signal.

**Index Terms**—Digital down converter, iterative algorithm, multi channel receiver.

## I. INTRODUCTION

Digitally implemented modules often have the advantages of a smaller size, less power dissipation, and lower cost than analog modules. The digitally-implemented down conversion block in a receiver is called a digital down converter (DDC) [1]. The DDCs have wide applications in digital wideband receivers in wireless communication systems. A DDC converts an IF band signal into a baseband signal by using a mixer in the digital region and a low-pass filter after the analog-to-digital conversion.

In many applications the signal of interest may not be at the optimum part of the spectrum for processing, for example in communication systems the signal band may be narrow, i.e. in KHz, but the signal band could be centered at IF frequencies, i.e. in MHz. If the signal is sampled according to the Nyquist criteria, twice the highest frequency, the data rate for the IF signal will be very high. Processing the data at this high rate is both difficult and expensive in terms of the amount of hardware required. A DDC will select the frequency band of interest and shift it down in the frequency domain, which allows the data rate required to retain all the information to be much lower, and consequently reduces the complexity of any further processing. The major advantages of DDC include the increased stability over temperature and time and complete elimination of some impairment due to analog circuits (e.g., mismatches between analog I and Q channels).

The conventional DDC consists of a mixer, a cascade integrator-comb (CIC) and two decimators. The desired channel is translated to baseband using the digital mixer comprised of multipliers and a direct digital synthesizer (DDS) [2]. The sampling rate of the signal is then adjusted by a multi-stage, multi-rate filter consisting of a CIC filter and maybe two polyphase FIR filters for more decimation. Though simple and multiplierless, CIC filters suffer from non-flatness in the passband (high passband drop). This is particularly true when the information bandwidth is not enough narrow (relative to the sampling rate) or the order of the CIC filter is high. Another disadvantage of the CIC filters is that the integrators of the CIC work in high sampling rate, which demands for high-speed accumulators [3].

To improve the passband characteristics and overcome the passband drop of the CIC filter, a compensation filter is often required after the CIC filter which is discussed in [3]. A programmable digital down converter structure consisting of seven programmable decimation FIR filter stages have been presented in [4]. Because CIC filter is not used in this structure, there is no need to compensate the passband loss and we have the complete control of the passband and the stopband characteristics. A DDC structure which is based on a bandpass Sigma Delta processor, complex polyphase decimating filter and CORDIC baseband phase rotator is discussed in [5].

In this paper we introduce an algorithm based on FFT for simultaneous frequency shifting and decimating of IF signals, in addition our proposed method has this ability to provide multi-channel down converting. First we propose an analytical discussion which describes our algorithm, then we suggest an iterative algorithm to improve the quality of the reconstructed signal.

## II. THE DDC ALGORITHM BASED ON FFT

In this section we analytically discuss the digital down converting algorithm based on FFT. Let  $x(t)$  be a base-band signal with a bandwidth of  $2w$ , then passband signal,  $y(t)$ , is given by:

$$y(t) = \text{Re}\{x(t)e^{j2\pi f_c t}\} \quad (1)$$

where  $f_c$  is the carrier frequency.  $y[n]=y[nT_{s1}]$  is the discrete-time representation of  $y(t)$  where  $T_{s1}$  is the sampling rate and is chosen in a way that:

$$T_{s1} < \frac{1}{2(f_c + w)} \quad (2)$$

Choosing  $N$  points of the discrete-time signal is equivalent to multiplying the continuous time signal by a rectangular

window with the duration  $T_{s2}=NT_{s1}$ . The value of N should be chosen due to the bandwidth of the baseband signal and the sampling rate to satisfy the Nyquist condition, thus it is set to  $(2wT_{s1})^{-1}$ .

$$y_r(t) = y(t)\Pi\left(\frac{t}{T_{s2}}\right) \quad (3)$$

The Fourier transform of  $y(t)$  can be obtained in terms of the baseband signal,  $x(t)$ , i.e.,

$$y(t) = \frac{1}{2}\left(x(t)e^{j2\pi f_c t} + x^*(t)e^{-j2\pi f_c t}\right) \quad (4)$$

$$Y(f) = \frac{1}{2}\left(X(f - f_c) + X^*(-f - f_c)\right) \quad (5)$$

$$Y_r(f) = \frac{T_{s2}}{2}\left(X(f) \otimes \text{sinc}(T_{s2}f)\Big|_{f=f-f_c} + X^*(-f) \otimes \text{sinc}(T_{s2}f)\Big|_{f=f+f_c}\right) \quad (6)$$

where  $\otimes$  denotes to the convolution operator, thus the N point-FFT of a discrete-time signal can be obtained from its Fourier transform as follows:

$$Y_r[k] = \frac{1}{T_{s1}} \sum_m Y_r\left(\frac{k}{NT_{s1}} - \frac{m}{T_{s1}}\right) \quad (7)$$

Since  $x(t)$  is bandlimited and  $y_r(t)$  is a real signal,  $y_r[k]$  can be calculated as:

$$Y_r[k] = \frac{T_{s2}}{2T_{s1}} X(f) \otimes \text{sinc}(T_{s2}f)\Big|_{\frac{k}{NT_{s1}} - f_c} \quad k = 0, 1, \dots, \left\lfloor \frac{N}{2} \right\rfloor \quad (8)$$

Considering the following statement:

$$S(f) = T_{s2}X(f) \otimes \text{sinc}(T_{s2}f) \quad (9)$$

The maximum of  $S(f)$  occurs at  $f=0$  with the value of:

$$\begin{aligned} \max_f S(f) &= T_{s2} \int X(f) \text{sinc}(-T_{s2}f) df \\ &= T_{s2} \iint x(t) e^{-j2\pi ft} dt \text{sinc}(-T_{s2}f) df \\ &= \int x(t) \int T_{s2} \text{sinc}(-T_{s2}f) e^{-j2\pi ft} df dt \\ &= \int x(t) \Pi\left(\frac{t}{T_{s2}}\right) dt \end{aligned} \quad (10)$$

According to (8), the maximum value of  $Y_r[k]$  occurs when  $|k/NT_{s1} - f_c|$  is minimized; we show this minimum value by  $\Delta f$ . If  $\Delta f=0$ , according to (10), the maximum value of FFT becomes:

$$\max_k Y_r[k] = \frac{1}{2T_{s1}} \int x(t) \Pi\left(\frac{t}{T_{s2}}\right) dt \quad (11)$$

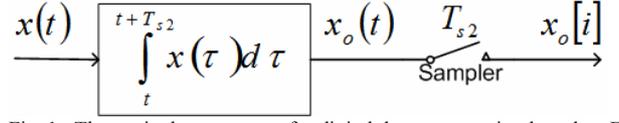


Fig. 1. The equivalent structure for digital down converting based on FFT

In case that  $\Delta f \neq 0$  the maximum value is:

$$\begin{aligned} \max_k Y_r[k] &= \frac{T_{s2}}{2T_{s1}} \int X(f - \Delta f) \text{sinc}(-T_{s2}f) df \\ &= \frac{T_{s2}}{2T_{s1}} \iint x(t) e^{j2\pi \Delta f t} e^{-j2\pi ft} dt \text{sinc}(-T_{s2}f) df \\ &= \frac{1}{2T_{s1}} \int x(t) e^{j2\pi \Delta f t} \int T_{s2} \text{sinc}(-T_{s2}f) e^{-j2\pi ft} df dt \\ &= \frac{1}{2T_{s1}} \int x(t) e^{j2\pi \Delta f t} \Pi\left(\frac{t}{T_{s2}}\right) dt \end{aligned} \quad (12)$$

Hence in our proposed algorithm we apply N point-FFT to the passband signal and then in each blocks we find the maximum value of the FFT. This value for the  $i^{th}$  block is called  $x_o[i]$  given by

$$x_o[i] = \frac{1}{2T_{s1}} \int_{(i-1)T_{s2}}^{iT_{s2}} x(t) e^{j2\pi \Delta f t} dt \quad (13)$$

where  $\Delta f$  is the minimum value of  $\left| \frac{k}{NT_{s1}} - f_c \right|$  for all integer values of  $k$  from 0 to  $\lfloor N/2 \rfloor$ . In case that the carrier frequency is one of the FFT points, the output is the integral of the baseband signal, so our proposed algorithm can be modeled as shown in Fig. 1. Since the sampling rate is relatively high in comparison to  $2w$ , the  $x_o[i]$  is a good approximation for the baseband samples. In the next section an iterative algorithm has been proposed to improve the SNR of the reconstructed base band signal.

### III. ITERATIVE ALGORITHM

The iterative algorithm [6], [7] is given by:

$$x_{k+1} = \lambda G\{x(t)\} + (I - \lambda G)\{x_k(t)\} \quad (14)$$

where  $\lambda$  is the relaxation parameter that determines the convergence rate.  $x_k(t)$  is the  $k^{th}$  iteration and  $x_0(t)$  can be any function of time. However,  $x_0(t) = G\{x(t)\}$  can be a good choice to achieve faster convergence. In general, G can be either a linear or a non-linear operator. Defining the operators  $\hat{G} = \lambda G$  and  $E = I - \hat{G}$ , we can rewrite (14) as

$$x_{k+1} = \hat{G}x + (I - \hat{G})x_k \quad (15)$$

#### IV. SIMULATION RESULTS

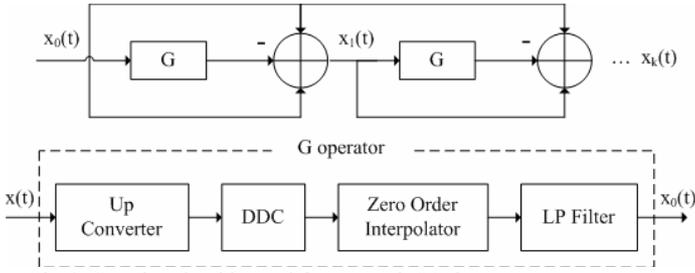


Fig. 2. Iterative algorithm and G operator in DDC .

It is straightforward to show that (15) can be written as:

$$x_k(t) = (E^k + E^{k-1} + \dots + E + I)x_0(t) \quad (16)$$

If  $G$  is a linear operator, we have:

$$E^k + E^{k-1} + \dots + E + I = \frac{I - E^{k+1}}{I - E} \quad (17)$$

If the norm of operator  $E$  satisfies  $\|E\| < 1$ , by increasing the number of iterations  $k$ , (15) approaches the inverse system  $\hat{G}^{-1}$ , therefore,  $x_k(t)$  converges to  $x(t)$ . By setting  $\lambda$  appropriately, we can satisfy this constraint in general. In our case, operator  $G$  is illustrated in Fig. 2.

The iterative algorithm introduces high complexity, because it is needed for the operator  $G$  to be repeated in the each iteration, but in case of DDC we have this possibility to reduce the computational complexity by combining up-converting and DDC modules.

$$\begin{aligned} x_{PB}[i, n] &= \text{Re} \left\{ x[i, n] e^{j2\pi \frac{f_c}{f_s} n} \right\} \\ &= \frac{x[i, n] e^{j2\pi \frac{f_c}{f_s} n} + x^*[i, n] e^{-j2\pi \frac{f_c}{f_s} n}}{2} \end{aligned} \quad (18)$$

In case that the carrier frequency corresponds to one of the FFT points we have  $f_c/f_s = k/N$  where  $k$  is an integer, thus the DDC output for the  $i^{\text{th}}$  block can be calculated as:

$$X_o[i, k] = \sum_{n=0}^{N-1} x_{PB}[i, n] e^{-j\frac{2\pi}{N} kn} \quad (19)$$

Substituting (18) in (19) we have:

$$\begin{aligned} X_o[i, k] &= \frac{1}{2} \sum_{n=0}^{N-1} x[i, n] \\ &\quad + \frac{1}{2} \left( \sum_{n=0}^{N-1} x[i, n] e^{-j\frac{2\pi}{N} (-2k)n} \right)^* \end{aligned} \quad (20)$$

if we define  $a = (-2k \bmod N)$  we have:

$$X_o[i, k] = \frac{1}{2} (X[i, 0] + X[i, a]^*) \quad (21)$$

where  $X[i, k]$  denotes the  $k^{\text{th}}$  FFT-coefficient of the  $i^{\text{th}}$  block in  $x[n]$ , so we just need to calculate two FFT-coefficients in each iteration.

The simulations have been done for random band limited signals with bandwidth of 24 KHz and  $f_c = 20\text{MHz}$ , where the length of each FFT frame is  $N=4096$  and the sampling rate is 100MHz. The absolute values of the baseband signal (solid) and the reconstructed one based on FFT algorithm (dash) have been shown in Fig. 3. In this case, the SNR of the reconstructed signal is about 12dB. To improve the quality of the reconstructed signal, the iterative algorithm has been applied and the output SNR versus number of iterations is shown in Fig. 4. As seen for the single channel, it is possible to reconstruct the baseband signal perfectly by using the iterative DDC.

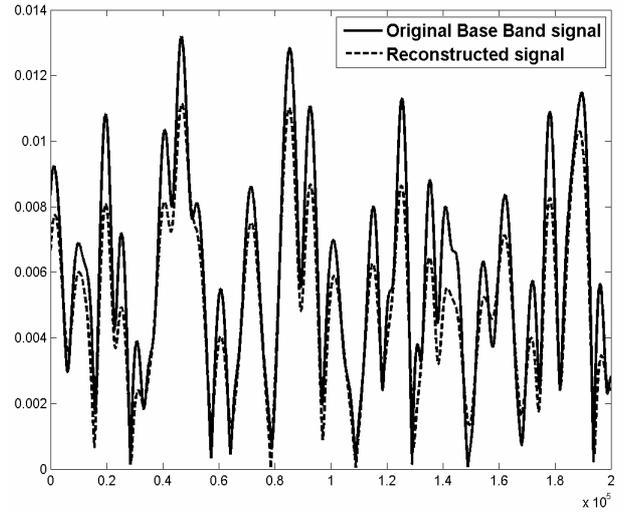


Fig. 3. The original and reconstructed baseband signal using FFT-based DDC method.

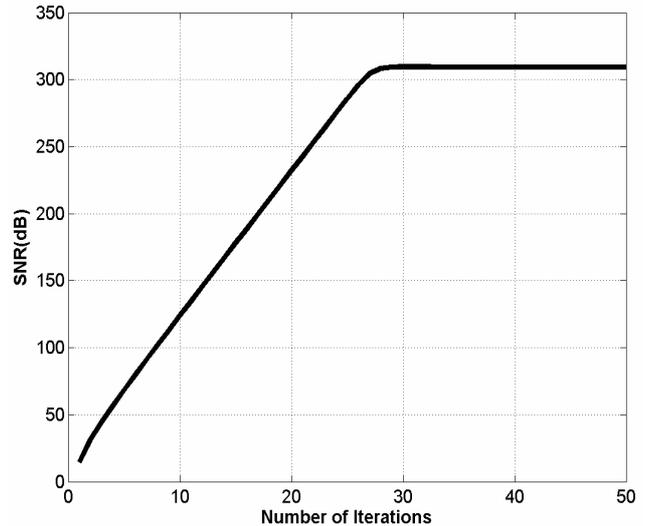


Fig. 4. SNR versus number of iterations for single channel DDC.

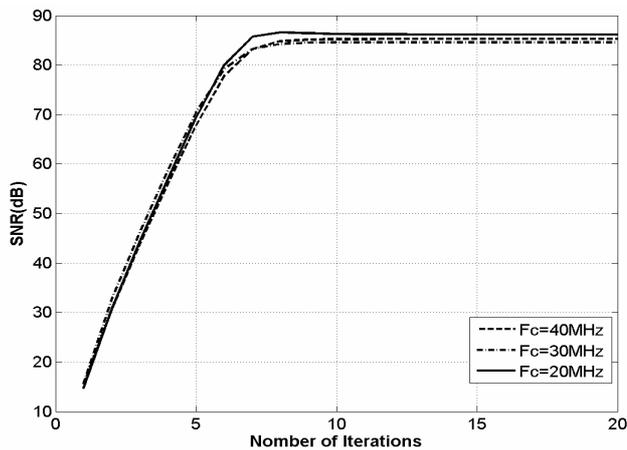


Fig. 4. SNR versus number of iterations for three- channel DDC.

It is our intention to apply the proposed digital down converter for several channels simultaneously. In Fig. 5 we have compared our algorithm for three channels where  $f_c=20, 30, 40$  MHz and all channels have the same bandwidth of 24 KHz. As seen by increasing the number of carriers because of the interference of adjacent channels, the performance of the iterative algorithm reduces. However this type of DDC does not have the disadvantages of CIC filters in the conventional DDCs and has the capability of simultaneous frequency shifting and decimating of IF signals.

## V. CONCLUSION

In this paper we have analytically discussed the digital down converter based on FFT which has the capability of simultaneous frequency shifting and decimating of IF signals. Moreover our system can be applied to down convert multi channels in the same time. To improve the performance of the FFT-based DDC a simplified version of the iterative algorithm has been suggested. In case of single channel by using the iterative DDC, reconstruction of baseband signal can be perfectly performed, but for multi-channel DDC, because of channel interferences, perfect reconstruction is not achievable. We are currently studying the performance improvement of multi-channel DDC using a modified interactive iterative method.

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