One-Hop Throughput of Wireless Networks with Random Connections

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***Abstract*—We consider one-hop communication in wireless networks with random connections. In the random connection model, the channel powers between different nodes are drawn from a common distribution in an i.i.d. manner. In a wireless network with n nodes, a simple one-hop scheme achieving the throughput scaling of order $n^{1/3-\delta}$, for any arbitrarily small $\delta > 0$, is proposed. This scheme is based on maximizing the number of successful concurrent transmissions. Such achievable throughput, along with the order $n^{1/3}$ upper bound derived by Cui et al., characterizes the throughput of one-hop schemes for the class of connection models with distribution having finite mean and variance.**

***Index Terms*—Wireless networks, random connection model, achievable throughput.**

I. Introduction

WIRELESS networks are subject to fundamental limitations in establishing source-destination data sessions. Investigating such limitations, along with discovering potential communication capabilities of wireless networks is of vital importance in designing efficient and practical algorithms for their operation. While Shannon's approach [1] to mathematical analysis of communication systems is the most powerful approach, it is not easily extendable to wireless networks with large number of nodes.

The pioneering work of Gupta and Kumar [2] in 2000 ignited the efforts in characterizing the fundamental communication limits and capabilities of wireless networks. Gupta and Kumar's work, along with subsequent papers ([3], [4] and [5]), establish the order of $\sqrt{n}$ achievable aggregate throughput for wireless networks with multi-hop technology, where n is the number of nodes. However, in another line of research the upper bound of order n is derived for the capacity of wireless networks, by exploiting information-theoretic max-flow min-cut discussions [6]. The remarkable work of Özgür et al. in 2007 resolves such gap between the upper and lower bounds [7]. In fact, by not considering interference as being always harmful, and by exploiting Multiple-Input Multiple-Output (MIMO) techniques, they propose a hierarchical cooperation scheme achieving linear aggregate throughput scaling.

Many of papers on wireless networks capacity use channel models based on distance between nodes, while others use models based on random distributions. As also mentioned in [8], in many scenarios a wireless channel model based on randomness is a more appropriate choice than distance-based models. As an example of such scenario, we can point to the case where randomly moving obstacles block signal propagation, and the distance-based model cannot address such issues. Also, when the network area size is small, the dominating factor in characterizing the channel properties between nodes is the random fluctuations due to fading, rather than the distance-based path-loss effect. In such situations the network is strongly interference-limited, which is best modeled by a random-based channel model. Moreover, many wireless systems employ a unit called Automatic Gain Control (AGC) which compensates for the distance effect. Accordingly, in many scenarios, it is more suitable to use a randomness-based channel model which is called the "Random Connection Model". In such model, the channel power γ between each pair of nodes is represented by a random variable drawn from a common parent probability distribution function (p.d.f.) $f(\gamma)$, and different links are independent.

The first work considering the random connection model in communication over wireless networks is by Gowaikar et al. [8]. They propose a multi-hop scheme achieving linear scaling for a specific case of parent distribution. Their scheme is based on establishing routes in random graphs. Their subsequent paper investigates a model which considers both the geometry and randomness effects [9]. Another important work using the random connection model is the paper by Cui et al. [10]. In their work, one-hop and two-hop communication schemes are investigated. It is shown that, in the class of parent distributions with finite mean and variance, the one-hop throughput is upper bounded by order $n^{1/3}$. Also, for two-hop schemes, they provide upper and lower bounds of order $n^{1/2}$.

While Cui et al. prove that in one-hop schemes, and in the class of parent distributions with finite mean and variance, one cannot surpass the throughput scaling of order $n^{1/3}$, they leave the achievability part unanswered. In this letter, we solve this open problem and propose an scheme achieving the throughput scaling of order $n^{1/3-\delta}$, for any arbitrarily small $\delta > 0$. In order to do this, we propose a simple scheme which establishes concurrent one-hop communications. We will prove that this scheme achieves the mentioned throughput in networks with a specific channel distribution having finite mean and variance. Similar to the achievability part of two-hop communication schemes proved in [10], this throughput is achieved by assuming an specific channel power distribution which is not very practical. However, this result is of great theoretical importance since it completely characterizes the throughput of one-hop schemes for the class of connection models with distribution having finite mean and variance.

The letter structure is as follows. In section II, the network model is explained. In section III, we explain the proposed

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scheme and prove that it achieves throughput of order $n^{1/3-\delta}$. Finally, section IV concludes the letter.

II. Network Model

Consider a wireless network consisting of n nodes. Each node is capable of transmitting and receiving signals simultaneously (i.e., full duplex communication). The nodes follow an on/off strategy. In such strategy, at each time slot, a subset of nodes with m elements are "on" and transmit simultaneously, while other nodes do not transmit any signal. We call the subset of active nodes $\mathbb{S}$. Each node in the network is a source of data for exactly one destination, and also, it is destination of data for exactly one source. Thus, we have n sources (i.e., $S_1, \ldots, S_n$), and n destinations (i.e., $D_1, \ldots, D_n$). Each source node S_i wishes to transmit to the destination node D_i for $i = 1, \ldots, n$. The signal received by D_i at a specific time slot is:

$$y_i = \sum_{j \in \mathbb{S}} h_{j,i} x_j + n_i, \tag{1}$$

where x_j is the signal transmitted by jth source node, and $h_{j,i}$ is the channel gain between S_j and D_i. We define $\gamma_{j,i} \triangleq |h_{j,i}|^2$ to be the channel power, which is a random variable drawn from the parent distribution $f(\gamma)$. In addition, all links are independently and identically distributed (i.i.d.). Finally, n_i is the additive white Gaussian noise at each receiver whose variance is N_0. It should be noted that we assume a quasi-static time-varying channel model. In other words, channel coefficients are changed after each round of communication.

The communication between S_i and D_i is successful if and only if the received Signal to Interference and Noise Ratio ($SINR$) at D_i is above a given threshold level:

$$SINR_i \triangleq \frac{\gamma_{i,i}}{N_0 + \sum_{j \in \mathbb{S}, j \neq i} \gamma_{j,i}} \geq \beta. \tag{2}$$

As explained earlier, such channel mode, also known as the "Random Connection Model" is a very appropriate model in many network scenarios [8], [10].

III. Throughput Achievability of Order $n^{1/3}$

We consider one-hop communication between sources and destinations. At each time slot, the nodes belonging to the active subset $\mathbb{S}$ transmit their signals simultaneously, and the rest of the nodes do not transmit. Then, the corresponding destinations try to decode their message. We define the one-hop throughput of the network as the expected number of successful receptions at each time slot (similar to [10]). Cui et al. have proved that the throughput of such one-hop strategy, when $f(\gamma)$ has finite mean and variance, is upper bounded by order $n^{1/3}$. In this section, we propose an achievable scheme which achieves the throughput of order $n^{1/3-\delta}$ for any arbitrarily small $\delta > 0$. The main result of the letter is stated in the following theorem:

Theorem 1. *There exists a one-hop communication scheme achieving the throughput of order $n^{1/3-\delta}$ in the class of links with finite mean and variance. δ is an arbitrarily small strictly positive real number independent of n.*

Proof:

- Proof Summary

First we propose a link distribution with finite mean and variance. Then, in order to prove the existence of such an scheme we provide a simple scheme which achieves the given throughput. In order to explain the scheme, we let $m \triangleq n^{1/3-\delta}$ transmitters to be active, while the remaining nodes are inactive. Then, we prove that the average number of successful receptions is lower bounded by $m/4$. This will prove that the throughput of the proposed scheme is of order $n^{1/3-\delta}$.

- The Channel Power Distribution

The proposed link distribution is

$$f(\gamma) = \frac{2+\epsilon}{(1+\gamma)^{3+\epsilon}}, \gamma \geq 0, \tag{3}$$

in which ϵ is any arbitrarily small strictly positive real number. Since $\mathbb{E}\{\gamma\} = \frac{1}{1+\epsilon}$ and $\mathbb{E}\{\gamma^2\} = \frac{2}{\epsilon(1+\epsilon)}$, the proposed distribution has finite mean and variance. It should be noted that although ϵ is an small positive real number, the variance is finite due to the fact that ϵ is strictly positive.

- The Set of Active Nodes

Consider source nodes $S_1, \ldots, S_n$ and destination nodes $D_1, \ldots, D_n$. Remember that the channel power between S_i and D_i is $\gamma_{i,i}$. Let us sort $\gamma_{i,i}$'s to get their order statistics as follows:

$$\gamma_{(1),(1)} \leq \gamma_{(2),(2)} \leq, \ldots, \leq \gamma_{(n),(n)}. \tag{4}$$

This will sort the source-destination pairs based on the direct link between them. In other words, $S(i_1) - D(i_1)$ has better direct link compared to $S(i_2) - D(i_2)$ if $i_1 > i_2$. Thus, $S_{(n-i+1)} - D_{(n-i+1)}$ is the source-destination pair which has the ith most powerful channel, $\gamma_{(n-i+1),(n-i+1)}$.

In the proposed scheme, at each time slot, the first m strongest source-destination pairs (i.e., $S_{(k)} - D_{(k)}, k = n - m+1, \ldots, n$) are active, and other nodes are inactive. In other words, at each time slot, sources $S_{(k)}, k = n - m + 1, \ldots, n$ transmit their signals simultaneously, and the corresponding receivers $D_{(k)}, k = n - m + 1, \ldots, n$ attempt to decode their messages. Since we have assumed quasi-static time-varying channels, on average, all nodes will be selected the same number of times. Therefore, fairness issues arising in similar scenarios will not be of major concern in our case.

- Number of Successful Receptions

Define $SINR_{(i)}$ to be the $SINR$ at $D(i)$ and $r \triangleq n-m+1$. If we define M as the number of successful receptions, for the network throughput we have[1]:

$$\begin{aligned}\mathbb{E}\{M\} &= \sum_{k=r}^{n}\mathbb{P}\{SINR_{(k)} \geq \beta\} \\ &\geq m\mathbb{P}\{SINR_{(r)} \geq \beta\} \\ &= m\mathbb{P}\{\gamma_{(r),(r)} \geq \beta(N_0 + \sum_{j=r+1}^{n}\gamma_{(j),(r)})\} \\ &\geq m\mathbb{P}\{\gamma_{(r),(r)} > 2\beta\bar{\gamma}m\} \times \\ &\quad \mathbb{P}\{\beta(N_0 + \sum_{j=r+1}^{n}\gamma_{(j),(r)}) < 2\beta\bar{\gamma}m\},\end{aligned} \tag{5}$$

where $\bar{\gamma} \triangleq \mathbb{E}\{f(\gamma)\}$. The first inequality is due to the fact that $S_{(r)} - D_{(r)}$ has the weakest direct channel power among the active pairs. The last inequality is due to the independence of $\gamma_{(r),(r)}$ and $\beta(N_0 + \sum_{j=r+1}^{n}\gamma_{(j),(r)})$. According to Markov's inequality we have:

$$\mathbb{P}\{\beta(N_0 + \sum_{j=r+1}^{n}\gamma_{(j),(r)}) > 2\beta\bar{\gamma}m\} \leq \frac{\beta(N_0 + (m-1)\bar{\gamma})}{2\beta\bar{\gamma}m} \tag{6}$$

$$\simeq \frac{1}{2},$$

for large m. From (5) and (6) we have:

$$\mathbb{E}\{M\} \geq \frac{m}{2}\mathbb{P}\{\gamma_{(r),(r)} > 2\beta\bar{\gamma}m\}. \tag{7}$$

To continue the proof we should look for the statistical properties of $\gamma_{(r),(r)}$ to analyze $\mathbb{P}\{\gamma_{(r),(r)} > 2\beta\bar{\gamma}m\}$, which appears in (7). Accordingly, we need the following Lemma due to Falk [11]:

Lemma 1. *Suppose $X_1, \ldots, X_n$ are n i.i.d. random variables with the parent distribution $f(x)$. Define $X_{(1)}, \ldots, X_{(n)}$ to be the order statistics of these random variables. Suppose $F(x)$ is the cumulative distribution function (c.d.f.) of the parent distribution, which is absolutely continuous, and for some $\alpha > 0$ we have (known as one of von Mises conditions [12]):*

$$\lim_{x\to\infty} x\frac{f(x)}{1-F(x)} = \alpha. \tag{8}$$

Then, if $i \to \infty$ and $i/n \to 0$ as $n \to \infty$, there exist sequences a_n and $b_n > 0$ such that

$$\frac{X_{(n-i+1)} - a_n}{b_n} \Rightarrow N(0,1), \tag{9}$$

where $\Rightarrow$ stands for convergence in distribution, and $N(0,1)$ is the normal distribution with zero mean and unit variance. Furthermore, one choice for a_n and b_n is:

$$a_n = F^{-1}(1 - \frac{i}{n}), \tag{10}$$

$$b_n = \frac{\sqrt{i}}{nf(a_n)}.$$

Now, we are ready to apply Lemma 1 to the throughput analysis of our scheme. We have the same "intermediate order statistics" problem as the one stated in Lemma 1. Consequently, to apply Lemma 1 to our problem, we set:

$$X_{(r)} = \gamma_{(r),(r)}, \tag{11}$$

and

$$i = m = n^{\frac{1}{3}-\delta}, \tag{12}$$

for any arbitrarily small $\delta > 0$ and independent of n. Also, we observe that the proposed p.d.f. in (3) is absolutely continuous and satisfies the von Mises condition:

$$\lim_{x\to\infty} x\frac{f(x)}{1-F(x)} = 2 + \epsilon > 0. \tag{13}$$

Accordingly, due to Lemma 1 we have:

$$\frac{\gamma_{(r),(r)} - a_n}{b_n} \Rightarrow N(0,1), \tag{14}$$

where

$$\begin{aligned}a_n &= F^{-1}(1 - \frac{i}{n}) \\ &\simeq n^{\frac{1}{3}\frac{2+3\delta}{2+\epsilon}} \\ &= n^{\frac{1}{3}},\end{aligned} \tag{15}$$

where we have put $\delta = \frac{\epsilon}{3}$. Therefore, we have:

$$\begin{aligned}\mathbb{P}\{\gamma_{(r),(r)} > 2\beta\bar{\gamma}m\} &= \mathbb{P}\{\gamma_{(r),(r)} > (2\beta\bar{\gamma}n^{-\delta})n^{1/3}\} \\ &> \mathbb{P}\{\gamma_{(r),(r)} > n^{1/3}\} \\ &= \frac{1}{2},\end{aligned} \tag{16}$$

where the inequality is valid for large-enough n, due to the fact that $\bar{\gamma}$ and β are independent of n. The last equality is a consequence of the result of Lemma 1, which is stated in equation (14). By putting (16) in (7) we will have:

$$\mathbb{E}\{M\} \geq \frac{m}{4}. \tag{17}$$

Remembering that M is the number of successful receptions and $m = n^{1/3-\delta}$, (17) concludes the proof. ■

Finally, with respect to the amount of Channel State Information (CSI) needed in the proposed scheme, it should be mentioned that each receiver is responsible for feeding back to the corresponding transmitter whether it should be active or inactive. Thus, in order to sort channel powers, each receiver should know n channel gains corresponding to the n transmission pairs.

IV. Conclusions and Discussions

In this letter, we have proved that the lower bound of one-hop communication schemes in wireless networks with random connection model, in the class of finite mean and variance channel powers, is $n^{1/3-\delta}$, where $\delta > 0$ is any arbitrarily small strictly positive real number. Our result, combined with the upper bound of $n^{1/3}$ derived by Cui et al., characterizes the throughput of such networks.

The result achieved in this letter is based on a link distribution which is not very practical (similar to the achievability result of Cui et al. for two-hop communications [10]). However, it has an important theoretical implication that the upper bound derived by Cui et al. is tight. The link distribution of interest has the property that it has a heavy tail, while having a

[1] $\mathbb{E}\{.\}$ and $\mathbb{P}\{.\}$ are the expectation operator and probability measure respectively.

finite variance. Therefore, considering other distributions with such overall property can be of interest for further investigation in this framework.

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