

Analysis and Recovery of Sample-and-Hold and Linearly Interpolated Signals with Irregular Samples

Farokh Marvasti, *Senior Member, IEEE*, and Tsung Jen Lee

Abstract—A modular method is suggested to recover a band-limited signal from the sample-and-hold and linearly interpolated (or, in general, an n th-order-hold) version of the irregular samples. The approach is an extension of the work of one of the authors for the sample-and-hold signals with uniform samples. In the present paper, we apply a coordinate transform technique and show a practical way of finding an approximate inverse coordinate mapping. As a byproduct, we also derive some sufficient conditions on the nonuniform samples for the modular technique to be valid.

I. INTRODUCTION

DIGITAL-to-analog converters are based on sample-and-hold (S&H) or linear interpolation (LI) and low-pass filtering; this process creates some distortion at the Nyquist rate. To alleviate this problem, a modular method of the recovery of a signal from its sampled-and-held version was described in [1]. The technique is shown in Fig. 1. The input signal, $s(t)$, is the S&H or the LI signal which is mixed with the harmonics of the sampling frequency, $\cos(2\pi t/T)$. The output of the mixers are summed up and then low pass filtered. The resulting signal $\hat{x}(t)$ is a good approximation to the original band-limited signal $x(t)$. This approximation becomes exact when the number of harmonics (N) approach infinity. Simulations and comparison of this method for S&H and LI with iterative methods for reducing the distortion are given in [2]. Basically, this modular technique is comparable to the iterative method for LI signals.

It is our intention to generalize this method to the S&H and LI version of nonuniform samples. The problem of nonuniform sampling becomes important when we have jitter in the uniform sampling process of an analog signal or when some of the uniform samples are lost in the communication channel and at the receiver we get a set of nonuniform samples. There are several methods to recover a signal from its nonuniform samples [3]–[6]. One technique is to transform nonuniform samples into a set of uniform samples and then low-pass filter the samples in the new coordinate [3], [4]. The most difficult part of

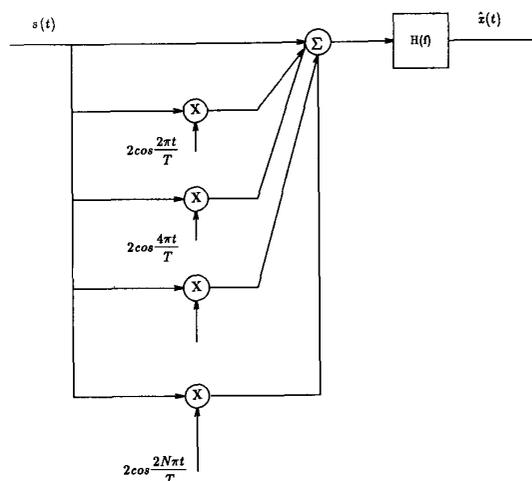


Fig. 1. Modular method to compensate for the S&H distortion for uniform samples.

this method is the inverse coordinate transform, which is not known in general. Other methods, such as nonlinear and iterative techniques are described in [5] and [6]. For a tutorial review of the general theorems related to this area, see [7].

The contributions of this paper are twofold: 1) the derivation of a modular method to recover a band-limited signal from the S&H, LI or, in general, an n th-order-hold version of irregular samples, and 2) the derivation of practical methods to approximate the inverse coordinate mapping required in applying a coordinate transform technique.

II. STATEMENT OF THE PROBLEM AND ITS SOLUTIONS

Assume $x(t)$ to be a band-limited signal. Let us take the sample-and hold signal $s(t)$ (or in general, an n th-order hold) of a set of nonuniform samples $\{x(t_k)\}$ as shown in Fig. 2. Let us take 1-1 coordinate transformation such as the one used by Papoulis [3], i.e.,

$$t = \tau - \theta(\tau) \quad (1)$$

such that when $\tau = nT$, $t = t_n$.

The function $\theta(\tau)$ in (1) could be any function such that $\theta(nT) = \mu_n = nT - t_n$ is the deviation of nonuniform

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F. Marvasti is with the Department of Electrical Engineering, King's College, University of London, London, U.K.

T. J. Lee is with the Department of Electrical and Computer Engineering, Illinois Institute of Technology, Chicago, IL 60616.

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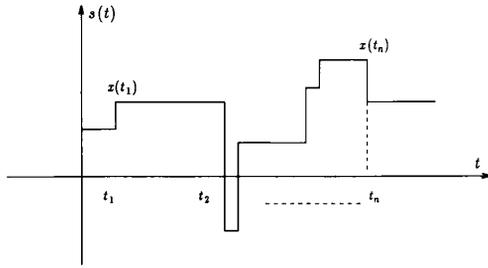


Fig. 2. Sample and hold with variable width for nonuniform samples.

samples from a set of uniform samples. This coordinate transformation maps $x(t)$ into a new function $g(\tau)$ such that

$$x(t) = x(\tau - \theta(\tau)) = g(\tau) \quad \text{and} \quad x(t_n) = g(nT). \quad (2)$$

Assuming that the sample-and-hold signal $s(t)$ is likewise mapped into $s_1(\tau)$, we have [1], [8]

$$\sum_{i=-\infty}^{\infty} S_1(f - i/T) = \sum_{i=-\infty}^{\infty} G(f - i/T) \quad (3)$$

where $S_1(f)$ and $G(f)$ are Fourier transforms of $s_1(\tau)$ and $g(\tau)$, respectively. The above equation can be proved from the fact that $s_1(nT) = g(nT)$ and the summations in the above equation are the Fourier transforms of the ideal impulsive samples of $s_1(\tau)$ and $g(\tau)$ (uniform sampling theory). Assuming $g(\tau)$ is almost a band-limited function with a bandwidth W ,¹ and if the uniform sampling interval $T = (1/2W)$, we have

$$\sum_{i=-\infty}^{\infty} S_1(f - i/T) = G(f), \quad |f| < W. \quad (4)$$

If the original signal is squarely integrable, i.e., $x(t) \in L_2$, then (see Appendix B) $S_1(f)$ is a decaying function and the summation in (4) can be approximated by a finite sum

$$\sum_{i=-N}^N S_1(f - i/T) \approx G(f), \quad |f| < W. \quad (5)$$

In the τ domain, we have

$$h(\tau) * \left\{ s_1(\tau) \left[1 + 2 \sum_{i=1}^N \cos \frac{2\pi i \tau}{T} \right] \right\} \approx g(\tau) \quad (6)$$

where $h(\tau)$ is a low-pass filter with a cutoff frequency equal to W .

The implementation of (6) would resemble that of Fig. 1. Equation (6) can be rewritten in the time domain if τ can be written as

$$\tau = \gamma(t). \quad (7)$$

¹Papoulis assumes small deviation μ_n to assure that $g(\tau)$ is approximately band limited. But if $g(\tau)$ is truly band limited, then $x(t)$ cannot be band limited. However, if the nonuniform sampling deviation, μ_n , is small and $x(t)$ is band limited, then $g(\tau)$ is "almost" band limited.

The above equation is the inverse of (1). Equation (7) can be written as (see Appendix A for the proof and the conditions)

$$\tau = \gamma(t) = t - \phi(t) \quad (8)$$

where $\phi(t)$ is a bounded function. Now, substituting (8) in (6), and noting that $s_1[\gamma(t)] = s(t)$, $g[\gamma(t)] = x(t)$, and $h(\gamma(t)) = \text{sinc} \left\{ \left[t - \phi(t) \right] / T \right\}$, we get

$$\text{sinc} \left[\frac{t - \phi(t)}{T} \right] * \left\{ s(t) \left[1 + 2 \sum_{i=1}^N \cos \left[\frac{2\pi i t}{T} - \frac{2\pi i}{T} \phi(t) \right] \right] \right\} \approx x(t). \quad (9)$$

Fig. 3 is the modular representation of (9); this figure shows that the modules consist of phase modulated (PM) signals as opposed to sinusoidal signals as shown in Fig. 1. The only question left is how to find $\phi(t)$. Appendix A shows an implicit relationship between $\phi(t)$ and $\theta(\tau)$. This calculation is, however, difficult to implement. This is because the uniform samples of $\phi(t)$ correspond to unknown nonuniform points τ_n .

One relatively simple method to derive $\phi(t)$ from $\theta(\tau)$ shall be discussed here. Instead of finding $\phi(t)$ (8) from $\theta(\tau)$ (1), we can directly find $\phi(t)$ (Fig. 3 shows that we do not need to find $\theta(\tau)$ for that implementation). The conditions on $\phi(t)$ are as follows:

1) From (8), we have $\phi(t_n) = t_n - nT = -\mu_n$, (deviation of nonuniform samples).

2) The mapping between $\phi(t)$ and $\theta(\tau)$ should be 1-1. There are some sufficient conditions on nonuniform deviations (μ_n) or maximum value of $\theta(\tau)$, as explained in Appendix A, that can guarantee 1-1 mapping. These conditions are

$$\sum_n \mu_n^2 < \frac{3T^2}{\pi^2}$$

$$\text{and/or} \quad \left(|\mu_n| < \theta_{\max} < \frac{T}{\pi} \right).$$

Now to find a $\phi(t)$, we proceed as follows [9]:

$$\delta(t - t_k) = |\dot{q}(t)| \delta[q(t)] \quad (10)$$

where $\delta(\)$ is an impulse and

$$q(t) = t - kT - \phi(t) \quad (11)$$

and $\phi(t)$ is the function to be determined such that

$$\begin{cases} q(t_k) = t_k - kT - \phi(t_k) = 0 \\ q(t \neq t_k) \neq 0 \\ \dot{q}(t_k) = 1 - \dot{\phi}(t_k) \neq 0. \end{cases} \quad (12)$$

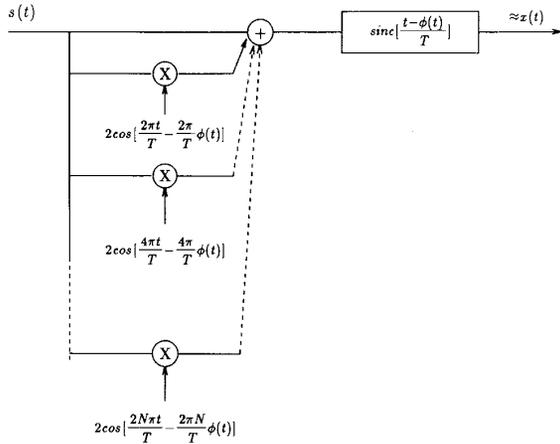


Fig. 3. Modular method for signal recovery from S&H version of non-uniform samples.

From (11) and (10), we have (for more details, see [3], [7])

$$\begin{aligned} \text{comb}(\cdot) &= \sum_k \delta(t - t_k) \\ &= |1 - \dot{\phi}(t)| \sum_k \delta[t - nT - \phi(t)]. \end{aligned} \quad (13)$$

The Fourier series expansion of (13) with respect to $\tau = t - \phi(t)$ is

$$\begin{aligned} \text{comb}(\cdot) &= |1 - \dot{\phi}(t)| \\ &\cdot \left\{ 1 + \sum_{n=1}^{\infty} 2 \cos \left[\frac{2\pi n}{T} t - \frac{2\pi n}{T} \phi(t) \right] \right\}. \end{aligned} \quad (14)$$

Since $\phi(t_n) = t_n - nT = -\mu_n$, if we make the realistic assumption that μ_n is finite, the average number of non-uniform samples per unit time is $(1/T) = 2W$. Hence a band-limited function $\phi(t)$ can be found such that its bandwidth is less than or equal to W . By assuming that the amplitudes of $\phi(t)$ and thus $\dot{\phi}(t)$ are small, which are the cases when the deviation $\phi(t_k) = -\mu_k$ is small, the overlap of phase modulated (PM) spectral components in (14) are negligible, and we can find $\phi(t)$ and $\cos[(2\pi/T)t - (2\pi/T)\phi(t)]$ by low-pass filtering and bandpass filtering the $\text{comb}(\cdot)$ signal represented in (14), respectively. The procedure is shown in the block diagram of Fig. 4. The harmonics of the PM signal in Fig. 3 can be generated from the fundamental component $\cos[(2\pi/T)t - (2\pi/T)\phi(t)]$. (Generation of these harmonics from the $\text{comb}(\cdot)$ function might be inaccurate since as n increases, there is more overlap in the frequency domain.) The last stage of Fig. 3 is a convolution with a filter with an impulse response of $\text{sinc}[(2\pi t/T) - (2\pi\phi(t)/T)]$ as derived in Fig. 4.

The same analysis holds for an n th-order hold such as linear interpolation. Equation (3) is still valid except that S_1 is the Fourier transform of the mapping of the n th-order-hold version of the original signal $s(t)$. Equa-

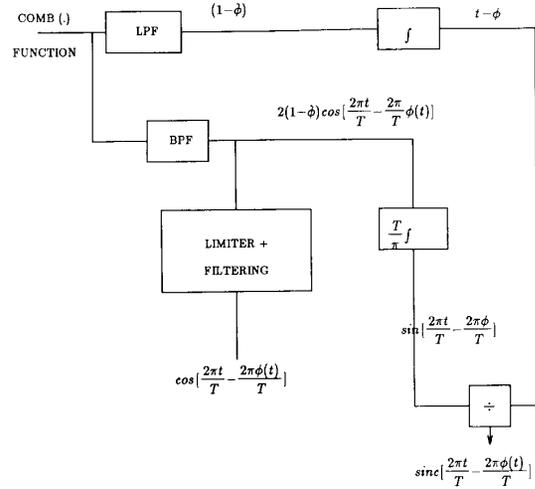


Fig. 4. Block diagram for generating the functions represented in Fig. 3.

tion (5) is still valid since an analysis similar to Appendix B shows that $S_1(f)$ is a decaying function. Thus, (6), (9), and Fig. 3 remain the same. Since an n th-order signal is a better approximation of the original signal, in general, we expect that $S(f)$ and $S_1(f)$ to decay faster and hence smaller number of modules (N) will be needed.

III. SIMULATION RESULTS

In order to compare our results to the uniform sampling case, we use the same signal as in [2]. Some of the results for the uniform sampling case are shown in Fig. 5. This figure shows that the addition of one harmonic does not necessarily improve the recovery but the additions of 50 harmonics significantly improves the systems. Fig. 6 shows the improvement of signal recovery when one harmonic is added for a linearly interpolated signal of a set of nonuniform samples. The nonuniform samples satisfy the Nyquist rate on the average and the random deviation of nonuniform samples from uniform position is less than $T/4$. Table I shows the mean-square error (MSE) for S&H and LI signals derived from a set of nonuniform samples that satisfy condition A-10 as shown in Figs. 7 and 8. This table reveals that most of the improvement is in the inclusion of the first harmonic. Compared to the low-pass filtering of the S&H or the LI signals (which is denoted as zero harmonic), the addition of one harmonic improves the MSE by about 3 dB at the Nyquist rate. At three times the Nyquist rate, we still get over 3 dB improvement for the S&H signal. For the LI signal, the MSE is already low for the zero harmonic (0.73 dB). However, the addition of one harmonic reduces the MSE by a factor of 50%. Addition of more harmonics, depending on the case, might improve or degrade the MSE. This is due to the approximations we made in deriving (9). The results in Table I correspond to the case when $\cos[(2\pi t/T) - (2\pi\phi(t)/T)]$ is derived from Fig. 4, which is the result of the analysis represented in (14). Another method is the

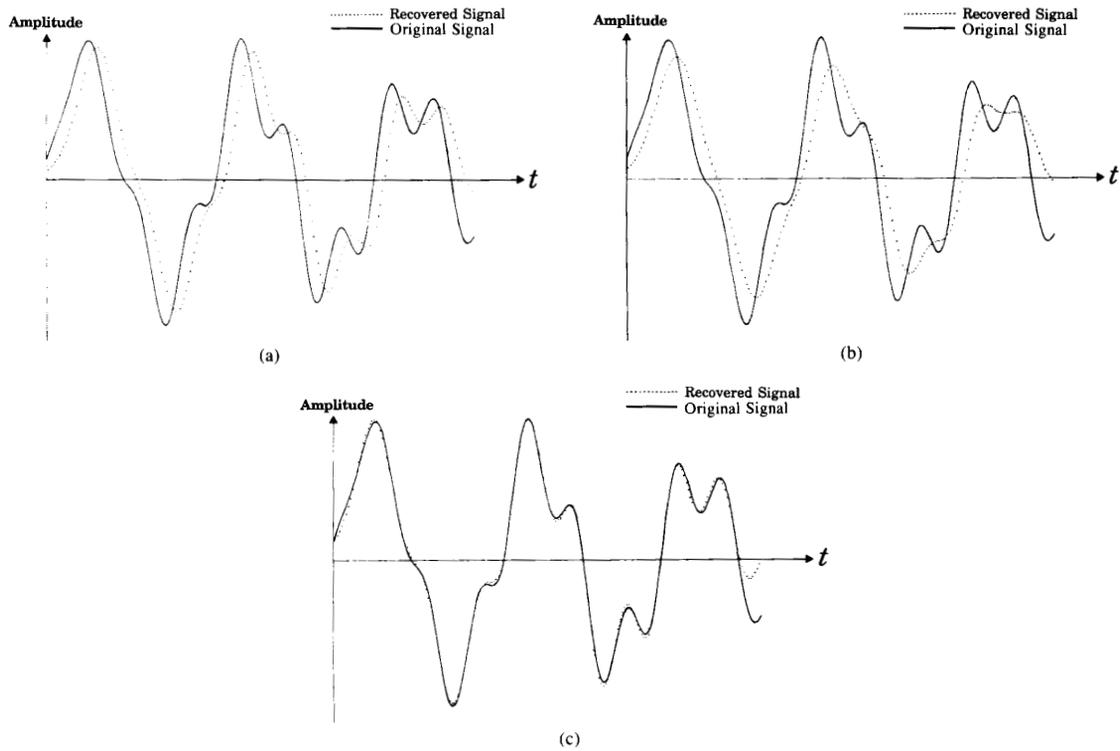


Fig. 5. The harmonic method using uniform samples at the Nyquist rate. The solid line is the original signal; the dotted line is the recovered signal: (a) The result after low-pass filtering the S&H signal. (b) The result after adding one harmonic. (c) The result after adding 50 harmonics.

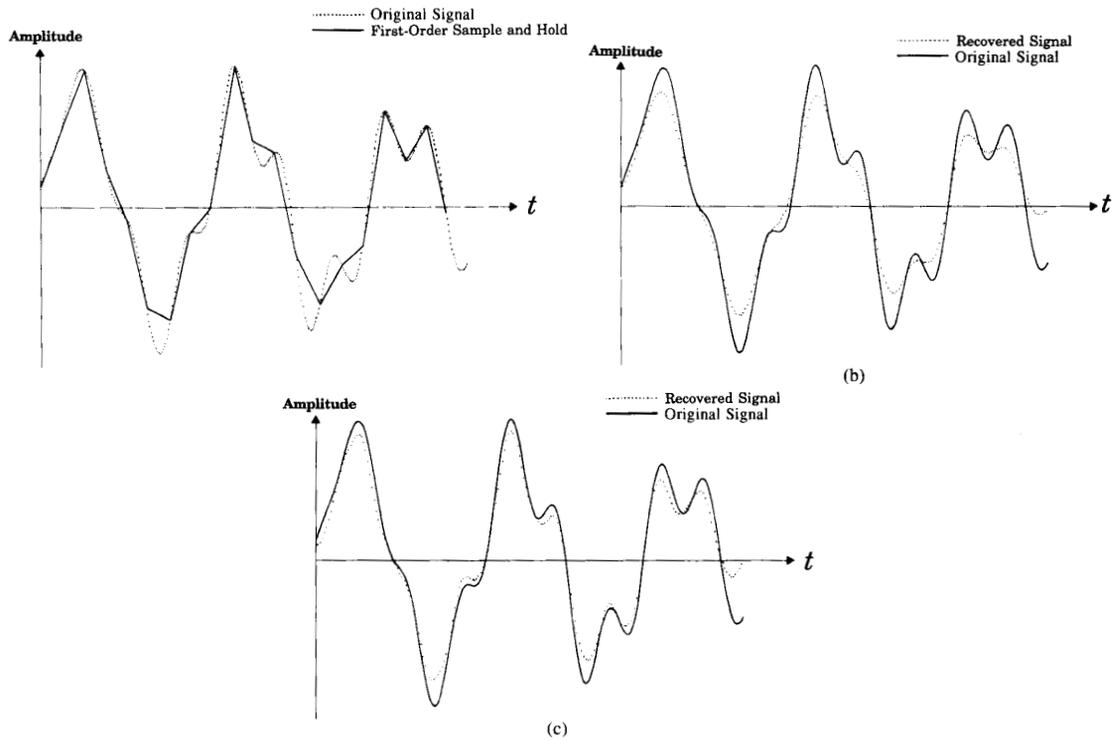


Fig. 6. The harmonic method using nonuniform samples at the Nyquist rate: (a) The original and LI signal. (b) The result after low-pass filtering the LI signal. (c) The result after adding one harmonic.

TABLE I
MEAN-SQUARE ERROR (IN DECIBELS) FOR S&H AND LI AT THE NYQUIST AND THREE TIMES
THE NYQUIST RATE. ($\phi(t)$) IS DERIVED BY THE PROCEDURE SHOWN IN FIG. 4)

No. of Harmonics	S&H Nyquist	LI Nyquist	S&H $3 \times$ Nyquist	LI $3 \times$ Nyquist
0	7.05	5.20	5.94	0.73
1	4.40	2.66	2.65	0.35
5	4.65	2.62	2.60	0.40
15	3.43	2.45	3.15	0.46
25	2.68	2.38	3.32	0.57

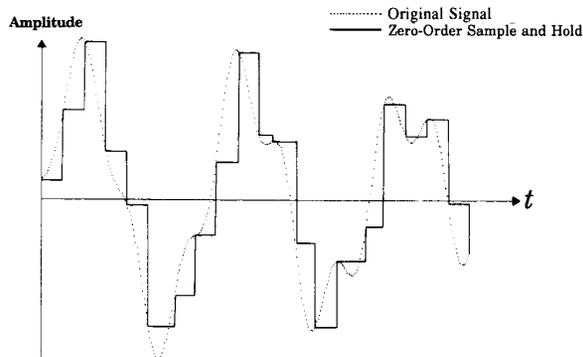


Fig. 7. The S&H signal of a set of nonuniform samples at the Nyquist rate.

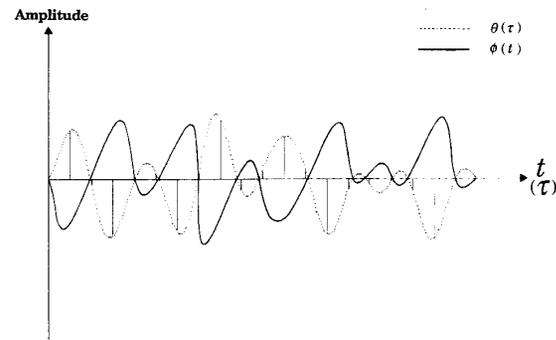


Fig. 9. The plot of $\phi(t)$ and $\theta(\tau)$ versus t and τ , respectively.

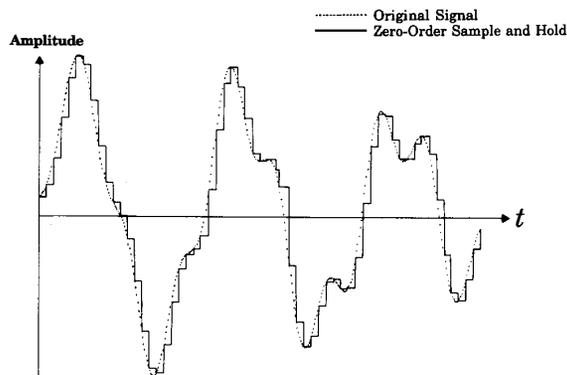


Fig. 8. The S&H signal of a set of nonuniform samples at three times the Nyquist rate.

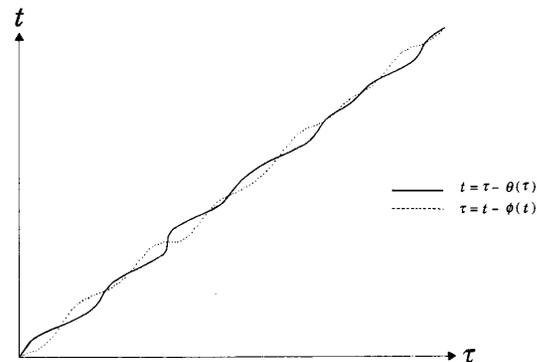


Fig. 10. The plot of $t = \tau - \theta(\tau)$ and its inverse $\tau = \gamma(t) = t - \phi(t)$.

use of (A-4) and (A-7). We first derive $\theta(\tau)$ from (A-7) and then use (A-4) to derive $\phi(t)$; $\cos [(2\pi t/T) - (2\pi/T)\phi(t)]$ is then generated. As (A-5) implies, uniform points of $\theta(nT)$ correspond to nonuniform points of $\phi(t_n)$ and vice versa. This requires a kind of interpolation to get $\phi(t)$ at uniform intervals. The relationship between $\phi(t)$ and $\theta(\tau)$ are shown in Fig. 9. This relationship is derived from the fact that $t = \tau - \theta(\tau)$ and its inverse $\tau = \gamma(t)$ have symmetry along the ramp function as shown in Fig. 10. If we use this method to calculate $\phi(t)$, the results are sometimes better and sometimes worse as shown in Table II. However, this method is computationally intensive and we recommend the procedure shown in Fig. 4.

Comparing the results shown in the above tables to the iterative techniques discussed in [6] (Table I), we see similar results. In both techniques the greatest improvement is in the first iteration or the addition of the first harmonic. The convergence in both cases is slow. However, due to some approximations in deriving the modular method (see footnote 1), the results do not get indefinitely better with the addition of more harmonics. This is not true for the iterative method. The limitation there is only due to round off errors in computer simulations. In general, the iterative technique outperforms the modular method at the expense of longer computer time. The modular method as depicted in Fig. 3 is implemented in parallel and therefore can be processed in real time.

TABLE II
COMPARISON OF MSE FOR TWO DIFFERENT METHODS TO DERIVE $\phi(t)$. THE FIRST AND THE THIRD COLUMNS ARE FROM TABLE I. THE SECOND AND THE LAST COLUMNS ARE CALCULATED WHEN $\phi(t)$ IS DERIVED FROM $\theta(t)$

No. of Harmonics	$\phi(t) \rightarrow$ Fig. 4	$\theta(\tau) \rightarrow \phi(t)$	$\phi(t) \rightarrow$ Fig. 4	$\theta(\tau) \rightarrow \phi(t)$
	S&H Nyquist	S&H Nyquist	LI Nyquist	LI Nyquist
0	7.05	7.05	5.20	5.20
1	4.40	4.35	2.66	2.58
5	4.65	4.99	2.62	2.57
15	3.43	4.80	2.45	2.56
25	2.68	4.90	2.38	2.59

IV. CONCLUSIONS

We extended the modular technique that was developed for the recovery of uniform S&H and LI to the nonuniform case. Besides the sample-and-hold and linear interpolation, this technique, assuming (4) holds, can be generalized to any n th order-hold interpolation.

We have also shown a practical method to determine the inverse coordinate transform suggested by [3], [4].

APPENDIX A

From (1), we have

$$t = \tau - \theta(\tau). \quad (\text{A-1})$$

The inverse of (A-1) is

$$\tau = \gamma(t). \quad (\text{A-2})$$

$\gamma(t)$ in (A-2) exists because the mapping in (A-1) is assumed to be 1-1. We are interested to see under what conditions $\gamma(t)$ can be written as

$$\tau = \gamma(t) = t - \phi(t) \quad (\text{A-3})$$

where $\phi(t)$ is a bounded function. Substituting (A-3) in (A-1), we get

$$\phi(t) = -\theta(\tau) \quad (\text{A-4})$$

where τ in the above is a dependent variable of t as given in (A-3). At nonuniform instants, the above equation becomes

$$\phi(t_k) = -\theta(kT). \quad (\text{A-5})$$

Now, we show that for $\gamma(t)$ to be written in the form shown in (A-3), we need the following condition:

$$\dot{\phi} < 1 \quad \text{and} \quad \frac{d\theta(\tau)}{d\tau} < 1. \quad (\text{A-6})$$

The proof is as follows: For (A-1) to be 1-1, the function t versus τ should be a monotonically increasing function. This implies that the derivative is always positive, i.e.,

$$\frac{dt}{d\tau} = 1 - \frac{d\theta(\tau)}{d\tau} > 0$$

$$\text{or} \quad \frac{d\theta(\tau)}{d\tau} < 1.$$

Likewise, from (A-3), we can derive $\dot{\phi} < 1$. Therefore, we have shown that condition (A-6) guarantees a 1-1 mapping between (A-1) and its inverse (A-3).

Now, we can find the conditions on the deviation of nonuniform samples ($\mu_n = nT - t_n$) based on condition (A-6). Since $\theta(\tau)$ in (1) is any function such that $\theta(nT) = \mu_n$, let us take $\theta(\tau)$ to be band limited to W as defined by

$$\theta(\tau) = \sum_n \mu_n \text{sinc}(2\tau W - n). \quad (\text{A-7})$$

According to the Bernstein inequality [10], the slope of a band-limited signal is related to the bandwidth and the maximum amplitude of the signal, i.e.,

$$\left| \frac{d\theta(\tau)}{d\tau} \right| \leq 2\pi\theta_{\max} W \quad (\text{A-8})$$

where θ_{\max} is the maximum amplitude of $|\theta(\tau)|$ in (A-7). At the Nyquist interval $T = (1/2W)$, from condition (A-8) if

$$\theta_{\max} < \frac{T}{\pi} \rightarrow \left| \frac{d\theta(\tau)}{d\tau} \right| < 1. \quad (\text{A-9})$$

That is, condition (A-9) is a sufficient condition to satisfy (A-6). Since $\mu_n = \theta(nT) \leq \theta_{\max}$, we derive the requirement that

$$|\mu_n| < \frac{T}{\pi}. \quad (\text{A-10})$$

The above equation implies that if condition (A-9) is satisfied (which implies conditions (A-8) and (A-6) are satisfied), the nonuniform samples cannot deviate by more than T/π from the uniform positions.² Unfortunately, condition (A-10) is not a sufficient condition since if (A-10) is satisfied, there is no guarantee that (A-9) is satisfied.

To get a sufficient condition on the nonuniform sampling deviations, μ_n , we try to get a different bound than the Bernstein inequality shown in (A-8). We represent the slope of $\theta(\tau)$ as the inverse Fourier transform and then

²It is interesting to compare this to the sufficient condition $|\mu_n| < (T/4)$ for the nonuniform samples to have Lagrange interpolation, see [7] for details.

invoke the Schwarz's inequality [11]; the result is

$$\begin{aligned} \left| \frac{d\theta(\tau)}{d\tau} \right| &= \left| \int_{-W}^W j2\pi f \Theta(f) e^{j2\pi f \tau} df \right| \\ &\leq \left[\int_{-W}^W |\Theta(f)|^2 df \int_{-W}^W (2\pi f)^2 df \right]^{1/2} \end{aligned} \quad (\text{A-11})$$

where $\Theta(f)$ is the Fourier transform of $\theta(\tau)$. The above equation can be written as

$$\left| \frac{d\theta(\tau)}{d\tau} \right| \leq \frac{2\pi W}{\sqrt{3}} \sqrt{E_\theta 2W} \quad (\text{A-12})$$

where E_θ is the energy of θ given by

$$E_\theta = \int \theta^2(\tau) d\tau = T \sum_n \theta^2(nT) = T \sum_n \mu_n^2. \quad (\text{A-13})$$

Substituting (A-13) in (A-12), we get

$$\left| \frac{d\theta(\tau)}{d\tau} \right| \leq \frac{2\pi W}{\sqrt{3}} \left[\sum_n \mu_n^2 \right]^{1/2}. \quad (\text{A-14})$$

To satisfy (A-6), from (A-14), a sufficient condition would be

$$\sum_n \mu_n^2 < \frac{3T^2}{\pi^2} \quad (\text{A-15})$$

where T is the Nyquist interval ($T = [1/2W]$).

The above sufficient condition shows that (A-10) could be violated as long as (A-15) is satisfied.

APPENDIX B

In this Appendix we would like to show that if $x(t)$ is an L_2 function, the sampled-and-held version of $x(t)$, i.e., $s(t)$ and the mapped function $s_1(\tau)$ are L_2 . If we can prove this, then according to the Parseval energy theorem $\int |S_1(f)|^2 df$ is finite and hence $S_1(f)$ is a decaying function with respect to f . To prove $s(t)$ is an L_2 function, we proceed as follows (see Fig. 2):

$$E_{s(t)} = \int s^2 dt = \sum_k (t_{k+1} - t_k) x^2(t_k) \quad (\text{B-1})$$

where $E_{s(t)}$ is the energy of $s(t)$. Assuming the samples satisfy the condition $t_{k+1} - t_k \leq M$ for all k , we have

$$E_{s(t)} \leq M \sum_k x^2(t_k). \quad (\text{B-2})$$

On the other hand, the set $\{t_k\}$ is a stable sampling set if [7]

$$\sum_k x^2(t_k) \leq B \int x^2 dt \quad (\text{B-3})$$

where B is a fixed constant for all values of t_k . From (B-2) and (B-3), the following equation is derived:

$$E_{s(t)} \leq BM \int x^2 dt. \quad (\text{B-4})$$

Since we have assumed that $x(t)$ belongs to an L_2 space, the inequality (B-4) proves that $s(t)$ is also an L_2 function.

Now, we show that after mapping $s(t)$ to $s_1(\tau)$, $s_1(\tau)$ also belongs to an L_2 space. The proof is as follows.

From (8), we have

$$\begin{aligned} \int_{-\infty}^{\infty} s_1^2(\tau) d\tau &= \int_{-\infty}^{\infty} s^2[t - \phi(t)](1 - \dot{\phi}) dt \\ &= \int_{-\infty}^{\infty} s^2(t)(1 - \dot{\phi}) dt. \end{aligned} \quad (\text{B-5})$$

From (A-6), we know

$$|\dot{\phi}(t)| < 1. \quad (\text{B-6})$$

From (B-5) and (B-6), we get

$$\int_{-\infty}^{\infty} s_1^2(\tau) d\tau < 2 \int_{-\infty}^{\infty} s^2(t) dt. \quad (\text{B-7})$$

Since $s(t)$ belongs to L_2 , we have thus proved that $s_1(\tau)$ also belongs to L_2 . Now, from the Parseval energy theorem, we know that $\int_{-\infty}^{\infty} |S_1(f)|^2 df$ is finite and therefore $S_1(f)$ is a decaying function with respect to f .

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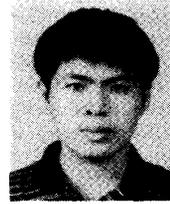


Farokh Marvasti (S'72-M'74-SM'83) received the B.S., M.S., and Ph.D. degrees, all in electrical engineering from Rensselaer Polytechnic Institute in 1973.

Since 1972 he has worked for Graphic Sciences, Singer-Kearfott, and AT&T Bell Laboratories. The last year he has been a consultant on video compression techniques. He was a Professor at Sharif University of Technology, Tehran, Iran, from 1976 to 1985, where he did extensive consulting to telecommunication and power companies.

He was a Visiting Professor at the University of California, Davis, from 1984 to 1985. He was an Associate Professor at the Illinois Institute of Technology from 1987 to 1991. He is now with the Department of Electrical Engineering, University of London, King's College. He is the author of the monograph *A Uniform Approach to Zero-Crossings and Nonuniform Sampling of Single and Multidimensional Signals and Systems* (Nonuniform Publications, Oak Park, IL). He has also contributed two chapters on nonuniform sampling theory to the book, *Shannon Sampling Theory and*

Interpolations II, edited by Dr. R. Marks, II (Springer-Verlag) and a chapter in the *Encyclopedia of Telecommunications*, edited by F. Froehlich. He is also the Editor on data communications for the IEEE TRANSACTIONS ON COMMUNICATIONS.



Tsung Jen Lee was born in Taiwan on February 24, 1963. He received the B.S. degree in marine engineering from Tamkang University, Taiwan, in 1985 and the M.S. degree in electrical engineering from the Illinois Institute of Technology in May 1991. His interests are in signal theory and signal processing in communication systems.