

# A New Outer Bound for a Class of Interference Channels with a Cognitive Relay and a Certain Capacity Result

Hossein Charmchi, Ghosheh Abed Hodtani, and Masoumeh Nasiri-Kenari

**Abstract**—The interference channel with a cognitive relay is a variation of the classical two-user interference channel in which a relay aids the transmission among the users. The relay is assumed to have genie-aided cognition: that is it has full, a-priori, knowledge of the messages to be transmitted. We obtain a new outer bound for this channel model and prove capacity for a class of channels in which the transmissions of the two users are non interfering. This capacity result improves on a previous result for the Gaussian case in which the capacity was proved to within a gap of 3 bits/s/Hz.

**Index Terms**—Semi-deterministic interference channel with a cognitive relay, Parallel channel with a cognitive relay.

## I. INTRODUCTION

COGNITION is defined as the ability of a wireless node to overhear simultaneous communications taking place over the network and adapt its transmission strategies to the acquired knowledge. Information theory considers the limiting case in which the cognitive node is able to obtain perfect knowledge of the messages transmitted over the network: although idealistic, this approach provides an outer bound to the limiting performance of a cognitive system. The first cognitive model considered in the literature is the the cognitive interference channel of [1] and studied in several subsequent publications (please see [2] and references therein). The Cognitive Interference Channel (CIFIC) is obtained from the classical Interference Channel (IFC) by providing one user with full and non-causal knowledge of the other user's message.

In recent years, various extension of the CIFIC has been considered in the literature. In this letter, we consider one such extension: the Interference Channel with a Cognitive Relay (IFC-CR). This model is obtained by adding a relay node to the IFC which has full non-causal message knowledge of the message of both two users. The IFC-CR was first introduced in [3], where an achievable rate region was proposed. In [4], the rate region was improved upon and a sum-rate outer bound was also provided. In [5], where the IFC-CR was referred to as “Broadcast Channel with Cognitive Relays”, an achievable rate region containing previously proposed regions was derived. In [6], a more general achievable rate region was provided that included the region of [5] as a special case. The first outer bound for the non-Gaussian IFC-CR was derived in [7].

Manuscript received August 12, 2012. The associate editor coordinating the review of this letter and approving it for publication was T. Oechtering.

H. Charmchi and M. Nasiri-Kenari are with the Department of Electrical Engineering, Sharif University of Technology, Tehran, Iran (e-mail: hcharmchi@alum.sharif.edu, mnasiri@sharif.edu).

G. A. Hodtani is with the Department of Electrical Engineering, Ferdowsi University of Mashhad, Mashhad, Iran (e-mail: hodtani@um.ac.ir).

Digital Object Identifier 10.1109/LCOMM.2013.011113.121808

In [7], the authors also derived an outer bound for a class of semi-deterministic channels similar to the one introduced in [8]. This bound was further tightened for a high-SNR linear deterministic approximation of the Gaussian channel [9]. The authors of [10] considered a symmetric linear deterministic model whose capacity was shown in almost all parameter regimes. In [11], a new outer bound for the general IFC-CR, an outer bound for the strong interference regime, and its achievability in the very strong interference regime were presented. A special case of IFC-CR, termed as Parallel Channels with a Cognitive Relay (PC-CR) was studied in [12] where capacity was determined to within 3 bits/s/Hz.

A more realistic but harder to study channel model is an IFC with an additional relay that causally obtains the message of users [13]–[15].

**Our work:** In this letter, we derive a new outer bound for a class of semi-deterministic IFC-CRs. This outer bound is to be capacity region when the semi-deterministic IFC-CR reduces to a (semi-deterministic) PC-CR. The capacity for the semi-deterministic PC-CR was previously only known to within 3 bits/s/Hz in the subclass of Gaussian PC-CR, in which the channel outputs are obtained as a linear combinations of the channel inputs plus a Gaussian distributed noise term [12]. Our result establishes the capacity of this model exactly.

The rest of the paper is organized as follows. In Section II, the channel model is introduced. In Section III, we review related results available in the literature. In Section IV, we provide a new upper bound leading to the capacity of Gaussian PC-CR. Section V concludes the paper.

## II. CHANNEL MODEL AND NOTATIONS

We denote random variables and their realizations by upper-case and lower-case letters, respectively. The probability distribution of random variable  $X$  is denoted by  $p_X(x)$ , where  $x \in \mathcal{X}$ . In the following, we drop the subscripts of probability distributions if the arguments of the distributions are lower-case versions of the corresponding random variables.

In discrete and memoryless IFC-CR model depicted in Fig. 1, we consider finite input alphabets  $\mathcal{X}_1, \mathcal{X}_2, \mathcal{X}_c$ , finite output alphabets  $\mathcal{Y}_1, \mathcal{Y}_2$ , and transition probability function  $p(y_1, y_2 | x_1, x_2, x_c)$ . The channel is memory-less and time-invariant. A  $((2^{nR_1}, 2^{nR_2}), n)$  code for this channel model includes two message sets  $\mathcal{M}_i = \{1, \dots, 2^{R_i}\}$ , three encoding functions  $f_i^{\text{enc}} : \mathcal{M}_i \rightarrow \mathcal{X}_i^n$ , and  $f_c^{\text{enc}} : \mathcal{M}_1 \times \mathcal{M}_2 \rightarrow \mathcal{X}_c^n$  and two decoding functions  $g_i^{\text{dec}} : \mathcal{Y}_i^n \rightarrow \mathcal{M}_i$ ,  $i = 1, 2$ . In the above,  $R_i$  denotes the transmission rate of sender  $i$ . Assume that in transmitter  $i$ , the message index  $M_i$  is selected uniformly from the message set  $\mathcal{M}_i$  and decoder  $i$  estimates the transmitted message index as  $\hat{M}_i = g_i^{\text{dec}}(Y_i^n)$ . Then, the

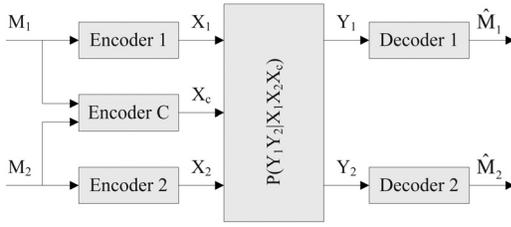


Fig. 1. The interference channel with a cognitive relay.

average probability of error is computed as:

$$P_e^{(n)} = \frac{1}{2^{n(R_1+R_2)}} \sum_{(M_1, M_2) \in \mathcal{M}_1 \times \mathcal{M}_2} \Pr \left\{ \bigcup_i g_i^{dec}(Y_i^n) \neq M_i \mid (M_1, M_2) \text{ sent} \right\}. \quad (1)$$

$(R_1, R_2)$  is said to be achievable if there exists a sequence of  $((2^{nR_1}, 2^{nR_2}), n)$  codes with  $P_e^{(n)} \rightarrow 0$ . The capacity region is the closure of the region of all achievable rate pairs.

### III. PREVIOUS RELATED WORKS

In the following theorems,  $U_1$  and  $U_2$  are auxiliary random variables and  $Q$  is the time-sharing random variable which is independent of all other random variables and uniformly distributed on  $[1, n]$ . A general outer bound for IFC-CR has been derived in [11] as follows.

**Theorem 1. [11, Theorem 1]:** *If  $(R_1, R_2)$  lies in the capacity region of the IFC-CR, then for any input distribution  $p(q, x_1, x_2, x_c, u_1, u_2)$  that factors as*

$$p(q)p(x_1|q)p(x_2|q)p(x_c|x_1, x_2, q)p(u_1, u_2|x_1, x_2, x_c, q) \quad (2)$$

the following must hold:

$$R_1 \leq I(Y_1; X_1 X_c | X_2 Q) \quad (3a)$$

$$R_1 \leq I(Y_1; U_2 X_1 | Q) \quad (3b)$$

$$R_2 \leq I(Y_2; X_2 X_c | X_1 Q) \quad (3c)$$

$$R_2 \leq I(Y_2; U_1 X_2 | Q) \quad (3d)$$

$$R_1 + R_2 \leq I(Y_1; X_1 X_c | U_1 X_2 Q) + I(Y_2; U_1 X_2 | Q) \quad (3e)$$

$$R_1 + R_2 \leq I(Y_2; X_2 X_c | U_2 X_1 Q) + I(Y_1; U_2 X_1 | Q) \quad (3f)$$

$$R_1 + R_2 \leq I(Y_1; U_1 | Q) + I(Y_2; U_2 | Q) \quad (3g)$$

$$R_1 + R_2 \leq I(Y_1; X_1 X_2 X_c | Q) + I(Y_2; X_2 X_c | Y_1 X_1 Q) \quad (3h)$$

$$R_1 + R_2 \leq I(Y_2; X_1 X_2 X_c | Q) + I(Y_1; X_1 X_c | Y_2 X_2 Q). \quad (3i)$$

An inner bound for IFC-CR is derived in [12] as follows.

**Theorem 2. [12, Theorem 5]:** *Any rate pair  $(R_1, R_2)$  is achievable for IFC-CR if for all input distributions of the form*

$$p(q)p(u_1, x_1|q)p(u_2, x_2|q)p(x_c|u_1, u_2, x_1, x_2, q) \quad (4)$$

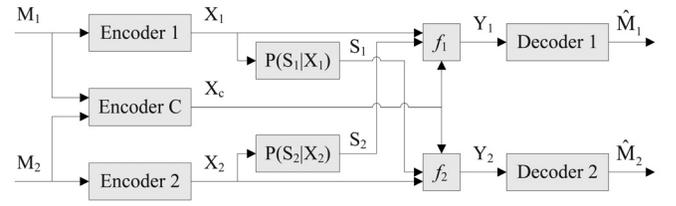


Fig. 2. A family of semi-deterministic IFC-CRs.

satisfy:

$$R_1 \leq I(Y_1; X_1 U_1 | U_2 Q) \quad (5a)$$

$$R_2 \leq I(Y_2; X_2 U_2 | U_1 Q) \quad (5b)$$

$$R_1 + R_2 \leq I(Y_1; X_1 | U_1 U_2 Q) + I(Y_2; X_2 U_1 U_2 | Q) \quad (5c)$$

$$R_1 + R_2 \leq I(Y_2; X_2 | U_1 U_2 Q) + I(Y_1; X_1 U_1 U_2 | Q) \quad (5d)$$

$$R_1 + R_2 \leq I(Y_1; X_1 U_2 | U_1 Q) + I(Y_2; X_2 U_1 | U_2 Q) \quad (5e)$$

$$2R_1 + R_2 \leq I(Y_1; X_1 U_1 U_2 | Q) + I(Y_1; X_1 | U_1 U_2 Q) + I(Y_2; X_2 U_1 | U_2 Q) \quad (5f)$$

$$R_1 + 2R_2 \leq I(Y_2; X_2 U_1 U_2 | Q) + I(Y_2; X_2 | U_1 U_2 Q) + I(Y_1; X_1 U_2 | U_1 Q). \quad (5g)$$

### IV. MAIN RESULTS

In the spirit of [7], [8], and [16], we consider a family of IFC-CRs depicted in Fig. 2, in which the inputs  $X_1$  and  $X_2$  are first passed through independent and discrete memory-less channels  $p(s_1|x_1)$  and  $p(s_2|x_2)$  to generate outputs  $S_1 \in \mathcal{S}_1$  and  $S_2 \in \mathcal{S}_2$ ; then the received signals are generated using two deterministic functions:

$$Y_1 = f_1(X_1, X_c, S_2) \quad (6a)$$

$$Y_2 = f_2(X_2, X_c, S_1) \quad (6b)$$

for which we assume the following invertibility conditions for all distributions of  $X_1$ ,  $X_2$ , and  $X_c$ :

$$H(Y_1 | X_1 X_c) = H(S_2 | X_1 X_c) = H(S_2 | X_c) \quad (7a)$$

$$H(Y_2 | X_2 X_c) = H(S_1 | X_2 X_c) = H(S_1 | X_c). \quad (7b)$$

That is when (6) and (7) hold,  $S_1$  and  $S_2$  are deterministic functions of  $(Y_2, X_2, X_c)$  and  $(Y_1, X_1, X_c)$ , respectively.

In [7], it was assumed that the interferences  $S_1$  and  $S_2$  were generated as  $S_1 = g_1(X_1, Z_2)$  and  $S_2 = g_2(X_2, Z_1)$ , where  $g_1$  and  $g_2$  are deterministic functions and  $Z_1$  and  $Z_2$  are “noise” random variables independent of the inputs. With this assumptions, an upper bound was derived that was not possible to be directly written in the single-letter form.

#### A. A new outer bound

We provide an upper bound that can be directly single-letterized.

**Theorem 3.** *If  $(R_1, R_2)$  lies in the capacity region of the IFC-CR, then for any input distribution  $p(q, x_1, x_2, x_c, u_1, u_2)$  that*

factors as (2), the following must hold:

$$R_1 \leq H(Y_1|X_2Q) - H(S_2|X_2Q) \quad (8a)$$

$$R_2 \leq H(Y_2|X_1Q) - H(S_1|X_1Q) \quad (8b)$$

$$R_1 + R_2 \leq H(Y_1|U_1X_2Q) + H(Y_2|Q) - H(S_1|X_1Q) - H(S_2|X_2Q) + I(S_1; X_1|Q) \quad (8c)$$

$$R_1 + R_2 \leq H(Y_2|U_2X_1Q) + H(Y_1|Q) - H(S_2|X_2Q) - H(S_1|X_1Q) + I(S_2; X_2|Q) \quad (8d)$$

$$R_1 + R_2 \leq H(Y_1|U_1Q) + H(Y_2|U_2Q) - H(S_1|X_1Q) - H(S_2|X_2Q) + I(S_1; X_1|Q) + I(S_2; X_2|Q) \quad (8e)$$

$$2R_1 + R_2 \leq H(Y_1|U_1X_2Q) + H(Y_1|Q) + H(Y_2|U_2Q) - H(S_1|X_1Q) - 2H(S_2|X_2Q) + I(S_1; X_1|Q) + I(S_2; X_2|Q) \quad (8f)$$

$$R_1 + 2R_2 \leq H(Y_2|U_2X_1Q) + H(Y_2|Q) + H(Y_1|U_1Q) - H(S_2|X_2Q) - 2H(S_1|X_1Q) + I(S_2; X_2|Q) + I(S_1; X_1|Q), \quad (8g)$$

where  $(U_1, U_2) \in \mathcal{S}_1 \times \mathcal{S}_2$  and  $(U_1, U_2)$  are conditionally distributed with  $(X_1, X_2, X_c, Q)$  as

$$p(u_1, u_2|x_1, x_2, x_c, q) = p_{S_1|X_1, Q}(u_1|x_1, q)p_{S_2|X_2, Q}(u_2|x_2, q). \quad (9)$$

*Proof:* From the theorem assumptions and considering Fig. 2, the Markov chain  $U_1U_2 - X_1X_2X_c - S_1S_2Y_1Y_2$  holds and  $(U_i, X_i)$ ,  $i = 1, 2$ , has a similar distribution as  $(S_i, X_i)$ .

We use (3a) to prove (8a):

$$\begin{aligned} R_1 &\leq I(Y_1; X_1X_c|X_2Q) \\ &\stackrel{a}{=} H(Y_1|X_2Q) - H(S_2|X_1X_2X_cQ) \\ &\stackrel{b}{=} H(Y_1|X_2Q) - H(S_2|X_2Q), \end{aligned} \quad (10)$$

where the equality in “a” follows from expanding the mutual information and replacing  $Y_1$  with  $S_2$  in the second term because of the assumed determinism of the model. The equality in “b” follows from the independence of  $S_2$  from  $(X_1, X_c)$  conditioned on  $X_2$ .

Equations (8c), (8e) and (8f) are proved as follows:

$$\begin{aligned} n(R_1 + R_2 - 2\epsilon_n) &\stackrel{a}{\leq} I(M_1; Y_1^n U_1^n M_2) + I(M_2; Y_2^n) \\ &\stackrel{b}{\leq} I(M_1; U_1^n) + I(M_1; Y_1^n | U_1^n M_2) + I(Y_2^n; X_1^n X_2^n X_c^n) \\ &\stackrel{c}{=} H(U_1^n) - H(U_1^n | M_1 X_1^n) + H(Y_1^n | U_1^n M_2 X_2^n) \\ &\quad - H(Y_1^n | U_1^n M_1 M_2 X_1^n X_2^n X_c^n) + H(Y_2^n) \\ &\quad - H(Y_2^n | X_1^n X_2^n X_c^n) \\ &\stackrel{d}{\leq} H(S_1^n) - H(S_1^n | X_1^n) + H(Y_1^n | U_1^n X_2^n) \\ &\quad - H(S_2^n | X_2^n) + H(Y_2^n) - H(S_1^n | X_1^n) \\ &\leq n(H(Y_1|U_1X_2Q) + H(Y_2|Q) - H(S_1|X_1Q) \\ &\quad - H(S_2|X_2Q) + I(S_1; X_1|Q)), \end{aligned} \quad (11)$$

where “a” follows from Fano’s inequality [17]. The inequality in “b” follows from the independence of  $M_1$  and  $M_2$  in the first term, and then adding  $(X_1^n X_2^n X_c^n)$  to the second term and

removing  $M_2$  from the result (because of the independence of  $Y_2^n$  from  $M_2$  conditioned on the channel inputs). The equality in “c” follows from the fact that  $X_i^n$  is a function of  $M_i$  and  $X_c^n$  is a function of  $(M_1, M_2)$ . The inequality in “d” follows from the assumed similar characteristics of  $S_i$  and  $U_i$ , the assumed determinism of the model, and the independence of  $S_i^n$  from all other variables conditioned on  $X_i^n$ .

Using similar arguments, we have

$$\begin{aligned} n(R_1 + R_2 - 2\epsilon_n) &\leq I(M_1; Y_1^n U_1^n) + I(M_2; Y_2^n U_2^n) \\ &= I(M_1; U_1^n) + I(M_1; Y_1^n | U_1^n) + I(M_2; U_2^n) \\ &\quad + I(M_2; Y_2^n | U_2^n) \\ &\leq H(U_1^n) - H(U_1^n | M_1 X_1^n) + H(Y_1^n | U_1^n) \\ &\quad - H(Y_1^n | M_1 U_1^n X_1^n X_2^n X_c^n) + H(U_2^n) - H(U_2^n | M_2 X_2^n) \\ &\quad + H(Y_2^n | U_2^n) - H(Y_2^n | M_2 U_2^n X_1^n X_2^n X_c^n) \\ &= H(S_1^n) - H(S_1^n | X_1^n) + H(Y_1^n | U_1^n) - H(S_2^n | X_2^n) \\ &\quad + H(S_2^n) - H(S_2^n | X_2^n) + H(Y_2^n | U_2^n) - H(S_1^n | X_1^n) \\ &\leq n(H(Y_1|U_1Q) + H(Y_2|U_2Q) - H(S_1|X_1Q) \\ &\quad - H(S_2|X_2Q) + I(S_1; X_1|Q) + I(S_2; X_2|Q)) \end{aligned} \quad (12)$$

and

$$\begin{aligned} n(2R_1 + R_2 - 3\epsilon_n) &\leq I(M_1; Y_1^n U_1^n M_2) + I(M_1; Y_1^n) + I(M_2; Y_2^n U_2^n) \\ &\leq I(M_1; U_1^n) + I(M_1; Y_1^n | U_1^n M_2) + I(Y_1^n; X_1^n X_2^n X_c^n) \\ &\quad + I(M_2; U_2^n) + I(M_2; Y_2^n | U_2^n) \\ &\leq H(U_1^n) - H(U_1^n | M_1 X_1^n) + H(Y_1^n | U_1^n M_2 X_2^n) \\ &\quad - H(Y_1^n | U_1^n M_1 M_2 X_1^n X_2^n X_c^n) + H(Y_1^n) \\ &\quad - H(Y_1^n | X_1^n X_2^n X_c^n) + H(U_2^n) - H(U_2^n | M_2 X_2^n) \\ &\quad + H(Y_2^n | U_2^n) - H(Y_2^n | M_2 U_2^n X_1^n X_2^n X_c^n) \\ &\leq H(S_1^n) - H(S_1^n | X_1^n) + H(Y_1^n | U_1^n X_2^n) - H(S_2^n | X_2^n) \\ &\quad + H(Y_1^n) - H(S_2^n | X_2^n) + H(S_2^n) - H(S_2^n | X_2^n) \\ &\quad + H(Y_2^n | U_2^n) - H(S_1^n | X_1^n) \\ &\leq n(H(Y_1|U_1X_2Q) + H(Y_1|Q) + H(Y_2|U_2Q) \\ &\quad - H(S_1|X_1Q) - 2H(S_2|X_2Q) + I(S_1; X_1|Q) \\ &\quad + I(S_2; X_2|Q)). \end{aligned} \quad (13)$$

(8b), (8d) and (8g) can be proved in a similar way. ■

#### B. A modified version for a known inner bound

**Theorem 4. (Modified version of Theorem 2):** Any rate pair  $(R_1, R_2)$  that for all input distributions of the form (4) satisfy

$$R_1 \leq H(Y_1|U_2Q) - H(S_2|U_2X_cQ) \quad (14a)$$

$$R_2 \leq H(Y_2|U_1Q) - H(S_1|U_1X_cQ) \quad (14b)$$

$$R_1 + R_2 \leq H(Y_1|U_1U_2Q) + H(Y_2|Q) - H(S_1|U_1X_cQ) - H(S_2|U_2X_cQ) \quad (14c)$$

$$R_1 + R_2 \leq H(Y_2|U_2U_1Q) + H(Y_1|Q) - H(S_2|U_2X_cQ) - H(S_1|U_1X_cQ) \quad (14d)$$

$$R_1 + R_2 \leq H(Y_1|U_1Q) + H(Y_2|U_2Q) - H(S_1|U_1X_cQ) - H(S_2|U_2X_cQ) \quad (14e)$$

$$2R_1 + R_2 \leq H(Y_1|U_1U_2Q) + H(Y_1|Q) + H(Y_2|U_2Q)$$

$$-H(S_1|U_1X_cQ) - 2H(S_2|U_2X_cQ) \quad (14f)$$

$$R_1 + 2R_2 \leq H(Y_2|U_2U_1Q) + H(Y_2|Q) + H(Y_1|U_1Q)$$

$$-H(S_2|U_2X_cQ) - 2H(S_1|U_1X_cQ) \quad (14g)$$

is achievable for the channel model of Fig. 2.

*Proof:* To obtain (14) from (5), it is sufficient to assume  $X_c$  to be a deterministic function of  $U_1$  and  $U_2$  (note that  $X_1$  and  $X_2$  and therefore  $U_1$  and  $U_2$  are known at the cognitive relay) and use (6) and (7). ■

### C. The capacity region of the semi-deterministic PC-CR model

In order to reach to the capacity of the semi-deterministic PC-CR model, we first compute the difference between the inner bound in Theorem 4 and a larger version of the outer bound in Theorem 3. Note that if we replace any  $X_i$ ,  $i = 1, 2$ , in the conditioned parts of the positive entropy terms of (8) by  $U_i$ , we obtain a larger outer bound and since

$$H(S_i|U_iX_cQ) - H(S_i|X_iQ) = I(S_i; X_i|U_iX_cQ), \quad (15)$$

we can write the difference between this larger outer bound ( $R^o$ ) and the inner bound of Theorem 4 ( $R^i$ ) as follows:

$$R_1^o - R_1^i \leq I(S_2; X_2|U_2X_cQ) \quad (16a)$$

$$R_2^o - R_2^i \leq I(S_1; X_1|U_1X_cQ) \quad (16b)$$

$$R_1^o + R_2^o - R_1^i - R_2^i \leq \min \{I(S_1; X_1|Q), I(S_2; X_2|Q)\} \\ + I(S_2; X_2|U_2X_cQ) + I(S_1; X_1|U_1X_cQ) \quad (16c)$$

$$2R_1^o + R_2^o - 2R_1^i - R_2^i \leq 2I(S_2; X_2|U_2X_cQ) \\ + I(S_1; X_1|U_1X_cQ) + I(S_1; X_1|Q) + I(S_2; X_2|Q) \quad (16d)$$

$$R_1^o + 2R_2^o - R_1^i - 2R_2^i \leq I(S_2; X_2|U_2X_cQ) \\ + 2I(S_1; X_1|U_1X_cQ) + I(S_2; X_2|Q) + I(S_1; X_1|Q). \quad (16e)$$

From the above equations, we see that the difference between the inner and outer bounds depends on the capacity of interference channels defined by  $p(s_1|x_1)$  and  $p(s_2|x_2)$ . Therefore, when the users transmissions do not interfere with each other, the difference will be zero and the outer bound will be achievable.

### D. The Gaussian PC-CR model

In the literature, there are possibly extensions of discrete alphabet channels results to continuous alphabet channels. Here we extend our capacity region of the PC-CR to its Gaussian version as a practically important model. The ‘‘Gaussian’’ PC-CR model is investigated in [12]. Similar to [8], it can be shown that the semi-deterministic channel model of Fig. 2 is more general than the ‘‘Gaussian’’ IFC-CR model which can be expressed in standard form as [6]:

$$Y_1 = |h_{11}|X_1 + |h_{1c}|X_c + h_{12}X_2 + Z_1 \quad (17a)$$

$$Y_2 = |h_{22}|X_2 + |h_{2c}|X_c + h_{21}X_1 + Z_2 \quad (17b)$$

where  $Z_1$  and  $Z_2$  are the Gaussian noise and  $h_i \in \mathbb{C}$ , for  $i \in \{11, 1c, 12, 2c, 21\}$ , are the channel coefficients. For

this channel, it is sufficient to define  $S_1$  and  $S_2$  as

$$S_2 = h_{12}X_2 + Z_1 \quad (18a)$$

$$S_1 = h_{21}X_1 + Z_2 \quad (18b)$$

in order to describe the Gaussian case in the form of the semi-deterministic model of Fig. 2. By assuming  $h_{12}$  and  $h_{21}$  to be zeros, we reach to the Gaussian PC-CR model.

## V. CONCLUSION

In this letter an interference channel with an additional cognitive relay has been investigated. For semi-deterministic channel model, a new outer bound has been derived and it is shown that this the outer bound is capacity when the transmission of the two users are non interfering. This capacity result has been extended to the Gaussian model in which the channel outputs are obtained as a linear combination of the channel inputs plus an additional Gaussian noise term.

## REFERENCES

- [1] N. Devroye, P. Mitran, and V. Tarokh, ‘‘Achievable rates in cognitive radio channels,’’ *IEEE Trans. Inf. Theory*, vol. 52, pp. 1813–1827, May 2006.
- [2] S. Rini, D. Tuninetti, and N. Devroye, ‘‘New inner and outer bounds for the memoryless cognitive interference channel and some new capacity results,’’ *IEEE Trans. Inf. Theory*, vol. 57, pp. 4087–4109, July 2011.
- [3] O. Sahin and E. Erkip, ‘‘On achievable rates for interference relay channel with interference cancellation,’’ in *Proc. 2007 Asilomar Conference on Signals, Systems and Computers*.
- [4] S. Sridharan, S. Vishwanath, S. Jafar, and S. Shamai, ‘‘On the capacity of cognitive relay assisted Gaussian interference channel,’’ in *Proc. 2008 IEEE Int. Symp. Information Theory*, pp. 549–553.
- [5] J. Jiang, I. Maric, A. Goldsmith, and S. Cui, ‘‘Achievable rate regions for broadcast channels with cognitive relays,’’ in *Proc. 2009 IEEE Information Theory Workshop*, pp. 500–504.
- [6] S. Rini, D. Tuninetti, N. Devroye, and A. Goldsmith, ‘‘On the capacity of the interference channel with a cognitive relay.’’ Available: <http://arxiv.org/abs/1107.4600v1>, 2011.
- [7] S. Rini, D. Tuninetti, and N. Devroye, ‘‘Outer bounds for the interference channel with a cognitive relay,’’ in *Proc. 2010 IEEE Information Theory Workshop*, pp. 1–5.
- [8] E. Telatar and D. Tse, ‘‘Bounds on the capacity region of a class of interference channels,’’ in *Proc. 2007 IEEE Int. Symp. Information Theory*, pp. 2871–2874.
- [9] A. Avestimehr, S. Diggavi, and D. Tse, ‘‘A deterministic approach to wireless relay networks,’’ in *Proc. 2007 Allerton Conf. Commun., Control and Comp.*
- [10] A. Dytso, N. Devroye, and D. Tuninetti, ‘‘On the capacity of the symmetric interference channel with a cognitive relay at high SNR,’’ in *Proc. 2012 Int. Conference on Communications*.
- [11] S. Rini, D. Tuninetti, N. Devroye, and A. Goldsmith, ‘‘The capacity of the interference channel with a cognitive relay in strong interference,’’ in *Proc. 2011 IEEE Int. Symp. Information Theory*, pp. 2632–2636.
- [12] S. Rini, D. Tuninetti, and N. Devroye, ‘‘Capacity to within 3 bits for a class of Gaussian interference channels with a cognitive relay,’’ in *Proc. 2011 IEEE Int. Symp. Information Theory*, pp. 2627–2631.
- [13] O. Sahin and E. Erkip, ‘‘Achievable rates for the Gaussian interference relay channel,’’ in *Proc. 2007 IEEE Globecom Coriference*.
- [14] O. Sahin, O. Simeone, and E. Erkip, ‘‘Interference channel with an out-of-band relay,’’ *IEEE Trans. Inf. Theory*, vol. 57, pp. 2746–2764, May 2011.
- [15] Y. Tian and A. Yener, ‘‘Symmetric capacity of the Gaussian interference channel with an out-of-band relay to within 1.15 bits.’’ Available: <http://arxiv.org/abs/1010.6290v2>, 2012.
- [16] A. A. El Gamal and M. H. M. Costa, ‘‘The capacity region of a class of deterministic interference channels,’’ *IEEE Trans. Inf. Theory*, vol. 28, pp. 343–346, Mar. 1982.
- [17] T. M. Cover and J. A. Thomas, *Elements of Information Theory*, 2nd edition. Wiley-Interscience, 2006.