# A NOVEL ALGORITHM TO ESTIMATE THE LINE SPECTRAL FREQUENCIES FROM LPC COEFFICIENTS

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## ABSTRACT

A new method to estimate the line spectral frequencies (LSF's) with any desired accuracy is described. LSF's are estimated by locating the zeros of two polynomials derived from LPC inverse filter polynomial on the unit circle in z-plane. The computational load increases as the required accuracy increases. An important feature of algorithm is the ability of compromising the required accuracy and computational load. The method can also be applied to convert the higher order LPC coefficients to associated LSF's. The algorithm does not require a very high precision arithmetic and can be implemented using fixed-point digital signal processors.

## 1. INTRODUCTION

Linear predictive coding (LPC) parameters are widely used in speech coding applications to represent the spectral envelope of a short segment of speech. The direct quantization of LPC parameters is inappropriate due to their relatively large dynamic range and the analysis complexity of potential instability of the LPC all-pole synthesis filter. Line spectral frequency (LSF) representation of LPC parameters proposed by Itakura [1] contains some useful properties such as well-behaved dynamic range, localized spectral sensitivity and simple stability analysis of LPC synthesis filter. These properties make the LSF domain suitable to be used for quantization of LPC parameters [2] even more efficiently than many other alternatives such as reflection coefficients and log area ratios [3]. Representation of LPC parameters in LSF domain requires the evaluation of complex zeros of a polynomial. On the other hand, the root finding task is computationally intensive for an LPC analysis with order higher than 8, [4]. In this paper, we propose a new algorithm which allows a relatively coarse LSF resolution as a trade-off between precision and computational complexity. It even allows coarse estimate of certain unimportant LSF's which can be used to decrease the computational load further. The proposed algorithm is also applicable to convert the higher order LPC coefficients to associated LSF's.

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## 2. LSF REPRESENTATION OF LPC PARAMETERS

In the LPC analysis of speech a short frame of speech is modeled as the result of passing a glottal excitation through an all pole filter $H(z) = \frac{1}{A(z)}$, where $A(z)$ is the inverse filter polynomial given by

$$A(z) = 1 + \sum_{i=1}^{p} a_i z^{-i}. \tag{1}$$

$p$ is the order of LPC analysis and $\{a_i, i = 1, 2, ..., p\}$ are the LPC parameters. The inverse filter polynomial $A(z)$ is used to construct the following two polynomials

$$P(z) = A(z) + z^{-(p+1)}A(z^{-1}) \tag{2}$$

$$Q(z) = A(z) - z^{-(p+1)}A(z^{-1}) \tag{3}$$

and $A(z)$ can be written as

$$A(z) = \frac{P(z) + Q(z)}{2}. \tag{4}$$

The LSF's are defined as the complex zeros of the polynomials $P(z)$ and $Q(z)$. It has been shown in [4] that all zeros of $P(z)$ and $Q(z)$ are interlaced with each other and lie on the unit circle in z-plane. The synthesis all-pole filter is minimum phase as long as the quantized zeros of $P(z)$ and $Q(z)$ are interlaced and reside on the unit circle [4]. The angles of zeros computed on the upper half of the unit circle, from 0 to $\pi$, result in LSF's in the ascending order. Each pair of complex roots of equation $A(z) = 0$ determines a formant frequency or a spectral shaping feature and corresponding bandwidth. According to eq. (4), each zero of $A(z)$ maps into one zero in each of the polynomials $P(z)$ and $Q(z)$. As the two resulting zeros of $P(z)$ and $Q(z)$ get closer in angle, the corresponding zero in $A(z)$ more likely represents a narrow-bandwidth formant, otherwise it indicates a formant with a wider bandwidth or a spectral shaping feature.

## 3. PROBLEM FORMULATION AND THEORETICAL BACKGROUND

The objective is to find the angles of zeros of $P(z)$ and $Q(z)$ on the upper semicircle in z-plane. Multiplying both sides of (2) and (3) by $z^{p+1}$ and using (1), we can write

$$P'(z) = z^{p+1} + 1 + \sum_{i=1}^{p}(a_i + a_{p-i+1})z^{p-i+1} \tag{5}$$

$$Q'(z) = z^{p+1} - 1 + \sum_{i=1}^{p}(a_i - a_{p-i+1})z^{p-i+1} \tag{6}$$

where $P'(z) = z^{p+1}P(z)$ and $Q'(z) = z^{p+1}Q(z)$. $P'(z)$ and $Q'(z)$, or equivalently, $P(z)$ and $Q(z)$ have a zero at $z = -1$ and $z = +1$, respectively, if $p$ is even. The zeros of $P(z)$ and $Q(z)$ on the unit circle are equivalent to the zeros of $P'(z)$ and $Q'(z)$ on the unit circle, respectively. The estimation of LSF's can be achieved by finding the roots of a complex polynomial function $F(z) = 0$ on the upper half of the unit circle where

$$F(z) = z^m + b_1 z^{m-1} + ....... + b_{m-1}z + b_m \tag{7}$$

and

$$\begin{cases} m = & p+1 \\ b_m = & \pm 1 \\ b_i = & a_i \pm a_{p-i+1} \qquad \text{for } i = 1, 2...., m-1. \end{cases} \tag{8}$$

$F(z) = P'(z)$ or $F(z) = Q'(z)$ if plus or minus sign is used in (8), respectively. The approach taken by the proposed algorithm to compute the roots of $F(z) = 0$ is based on finding the number of roots of $F(z) = 0$ that lie inside a closed curve $\Delta$ in z-plane. The method exploites the following concept from complex analysis [5]. The number of roots of a function $F(z) = 0$ enclosed by a closed curve $\Delta$ in the z-plane is equal to the number of times $(n)$ that $F(\Delta)$, i.e., the curve obtained by evaluating the polynomial $F(z)$ along the closed curve $\Delta$, rotates about the origin.

## 4. ALGORITHM DESCRIPTION

Since we are looking for roots lying on the unit circle, the closed curve of interest is taken to be a radial sector $(\Delta)$ consisting of two radial boundaries, i.e., rays at angles $\theta_1$ and $\theta_2$ with $0 < \theta_1 < \theta_2 < \pi$, and a closing arc with radius $R$ greater than one as shown in Fig. (1). The algorithm starts with an initial radial sector in the interval of $\theta = (0, \pi)$. The initial sector is then bisected into two sectors and the number of roots of function $F(z) = 0$ fallen inside each sector is computed. The bisection procedure followed by computation of the number of enclosed roots is repeated until a single root is isolated within a sector, i.e., a radial sector in the interval of $(\theta_1, \theta_2)$ as shown in Fig. (1), such that

$$\theta_2 - \theta_1 \leq \delta, \tag{9}$$

where $\delta$ is the prescribed accuracy. The value of $\delta$ can be determined by the assigned quantization tolerance in encoding the LSF's. The $l^{th}$ isolated root angle is then estimated as $\hat{\theta}_l$ by

$$\hat{\theta}_l = \frac{\theta_1 + \theta_2}{2}. \tag{10}$$

The corresponding estimated line spectral frequency $\hat{f}_l$ can be written as [6]

$$\hat{f}_l = \frac{f_s}{2\pi}\hat{\theta}_l, \tag{11}$$

where $f_s$ is the sampling frequency. Since the radial borders of final enclosing sector may be made as close as a prescribed level of resolution, i.e., $\delta$, the angles of isolated roots or equivalently the LSF's can be determined with any desired accuracy. For instance if a maximum deviation of $\pm 0.5$ from the real values of LSF's is tolerated, the bisecting procedure should be continued until the condition in eq. (9) is satisfied for $\delta = \frac{1 \times 2\pi}{f_s}$. The number of successive bisections can be reduced by allowing coarser LSF estimation (i.e., larger $\delta$) to achieve a considerable savig in computations.
The function $F(z)$ should be evaluated at sufficiently close points on a given sector $\Delta$ traversed in the counter clockwise direction to ensure that the estimated $\widehat{F}(\Delta)$ consisting of a sequence of straight lines approximates $F(\Delta)$ faithfully. The approximation should be close enough so that no complete rotation of $\widehat{F}(\Delta)$ around the origin is left undetected. In this case, the estimated winding number $(n')$ would be equal to the number of roots $(n)$ enclosed by $\Delta$. In order to achieve sufficiently dense set of points on $\Delta$, the complex plane is divided into eight regions, i.e., regions $C_0$ to $C_7$ in Fig. (1). The algorithm starts with two adequately close points on $\Delta$ and reduces the distance between them by a refining procedure until the final points $z_i$ and $z_{i+1}$ are located on $\Delta$ such that $F(z_i)$ and $F(z_{i+1})$ lie in adjacent regions. Fig. (1) shows the mapping process from the complex plane containing the roots of $F(z) = 0$ and $\Delta$ to the complex plane containing the values of $\widehat{F}(z)$ at points $z_i$ and $z_{i+1}$ on $\Delta$. A complete rotation of $\widehat{F}(\Delta)$ around the origin can be recorded when a transition from region seven to region zero (i.e., rotation in

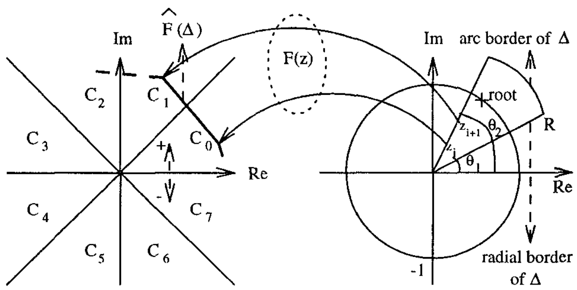


Figure 1: Graphical representation of mapping.

the positive direction) or a transition from region zero to region seven (i.e., rotation in the negative direction) takes place (i.e., the positive and negative arrows in Fig. 1). The estimated number of roots enclosed by $\Delta$ is given by the number of the positive rotations minus the number of the negative rotations. To simplify the evaluation of $F(z)$ over the arc border of $\Delta$, the radius of the closing arc is taken to be much larger than unity (i.e., $R >> 1$) so that the following approximation holds

$$F(z) \approx z^m \qquad \text{for } |z| >> 1. \tag{12}$$

Hence, as the arc border is traversed from $\theta_1$ to $\theta_2$, the contribution to the overall winding number of $\widehat{F}(\Delta)$ around the origin can be simply determined by the change in arguments of $F(z)$ computed at the start and end points of the closing arc, i.e., $m(\theta_2 - \theta_1)$. On the other hand, more points on the radial borders should be mapped onto $\widehat{F}(\Delta)$, as the radius $R$ grows. In order to avoid severe increase in computational complexity and, at the same time, to satisfy the approximation in (12), a compromized value of $R$ should be used. We have found that an arc radius of 2, i.e., $R = 2$, works well. Since only the points lying on radial borders of $\Delta$ are required to be processed, a function $\eta(\theta)$ is defined to numerically compute the winding number of $\widehat{F}(\Delta)$ around the origin as a radial border at angle $\theta$ is traversed from the origin to the arc border. As a result, we can write

$$N = \eta(\theta_1) - \eta(\theta_2), \tag{13}$$

where $N$ is the total winding number of $\widehat{F}(\Delta)$ around the origin due to two radial borders at angles $\theta_1$ and $\theta_2$, excluding the contribution of the arc border. The radial sector $\Delta$ should be processed in the counter clockwise direction and since, by the definition of $\eta(\theta)$, the radial border at angle $\theta_2$ is traversed from the origin outwards, a minus sign is required in (13) in front of $\eta(\theta_2)$. The contribution of the arc border can be simply included by revising the definition of $\eta(\theta)$ and defining a new function $\Phi(\theta)$ as

$$\Phi(\theta) = \eta(\theta) - \frac{m\theta}{2\pi}. \tag{14}$$

The total winding number $n'$ is then estimated by

$$n' = \Phi(\theta_1) - \Phi(\theta_2). \tag{15}$$

$n'$ is the number of roots on the unit circle bounded by the rays at angles $\theta_1$ and $\theta_2$ with $\theta_1 < \theta_2$. The function $F(z)$ is evaluated at any point $z_0$ on the radial border using the recursive algorithm given in Fig. (2).

1. Given $z_0 = | z_0 | e^{j\theta}$, and F(z) as in (7) and (8);
2. Initialization:
   $z_t = z_0$;
3. Recursion:
   For $l = 1, 2, ......, m-1$
   $z_t = z_0(z_t + b_l)$
   Next $l$
   $F(z_0) = z_t + b_m$.

Figure 2: Recursive algorithm to evaluate $F(z)$ at a point, $z_0$.

The proposed root finding algorithm is employed twice per frame to find the LSF's. First, set $F(z) = P'(z)$ and find the LSF's due to the polynomial $P(z)$. Then, set $F(z) = Q'(z)$ and find the LSF's due to the polynomial $Q(z)$.

## 5. EXPERIMENTAL RESULTS

The LPC envelope obtained from the tenth-order LPC analysis of a short frame (20 ms) of speech and the associated LSF's estimated by the proposed algorithm have been depicted in Fig. (3). The values of LSF's derived from the zeros of polynomials $P'(z)$ and $Q'(z)$ are given in Table 1. As it is seen in Table 1, the two groups of LSF's are interlaced. To compute the roots of equation $A(z) = 0$ inside the unit circle, the proposed algorithm was applied to estimate the angles of rays containing the zeros of polynomial $A(z)$. Then, the magnitude of a zero of $A(z)$ was found by locating a point in the interval of $(0, 1)$ on the corresponding ray such that the magnitude of $A(z)$ was minimized. The angle corresponding to each pair of roots is computed in terms of frequency using eq. (11). Each row of Table (2) specifies a pair of roots of $A(z) = 0$ in terms of magnitude and frequency. The 3-dB bandwidth, $B_{3dB}$, associated to each pair of roots was computed using [6]

$$B_{3dB} = -\frac{f_s}{\pi} \ln r \tag{16}$$

where $r$ is the estimated root magnitude. Table (2) represents the $B_{3dB}$ corresponding to each pair of zeros of polynomial $A(z)$. The frequencies corresponding to the angles of first, second, fourth and fifth pair of zeros of $A(z)$ indicate four formant frequencies. It can be seen from Fig. (3) that the bandwidth of a zero of $A(z)$ depends on the closeness of the corresponding LSF's. The fundamental operation required by the proposed algorithm is the computation of $F(z)$ at points located on the radial borders. Since the magnitude of $F(z)$ is only needed to specify the correct region, i.e., $C_0$ to $C_7$ in Fig. (1), the evaluation of $F(z)$ does not require high numerical precision and it can be efficiently implemented on the fixed-point DSP chips. The bisecting operation can be stopped at any point that the prescribed accuracy in computing the LSF's is achieved. Hence, a compromise can be easily made between the required accuracy and the computational complexity in a real-time speech processing system using LSF's.

| Number | LSF's computed from $P'(z)$ in Hz | LSF's computed from $Q'(z)$ in Hz |
|---|---|---|
| 1 | 337.335226 | 426.629629 |
| 2 | 1174.823045 | 1448.346167 |
| 3 | 1847.821164 | 2501.080492 |
| 4 | 2680.609434 | 2832.879546 |
| 5 | 3124.261487 | 3325.409041 |

Table 1: LSF's computed from LPC coefficients.

| Number | Frequency (Angle),Hz | Bandwidth in Hz | Root Magnitude |
|---|---|---|---|
| 1 | 348.614525 | 77.563660 | 0.97 |
| 2 | 1412.628224 | 296.750967 | 0.89 |
| 3 | 2509.539909 | 2090.609819 | 0.44 |
| 4 | 2683.428936 | 130.617333 | 0.95 |
| 5 | 3175.018596 | 268.298417 | 0.90 |

Table 2: Five pair of roots of equation $A(z) = 0$ in terms of equivalent frequency representation of the corresponding angle, root magnitude and $3dB$ bandwidth.

## 6. CONCLUSIONS

In this paper, we described a new algorithm to estimate the LSF's with any desired accuracy. The method determines the number of roots within a given closed sector by estimating the winding number of $F(\Delta)$ around the origin, where $F(z)$ is a polynomial derived from LPC inverse filter polynomial and $\Delta$ is the boundary of a closed radial sector. The sector $\Delta$ is successively bisected into smaller sectors until a single root is isolated within a final sector. The radial angle limit of the final sector is determined by a prescribed accuracy required for the computation of LSF's. The proposed algorithm was also used to estimate the formant frequencies by computing the zeros of $A(z)$. The computational complexity can be easily compromised by allowing a coarse tolerance in resolving the LSF's.

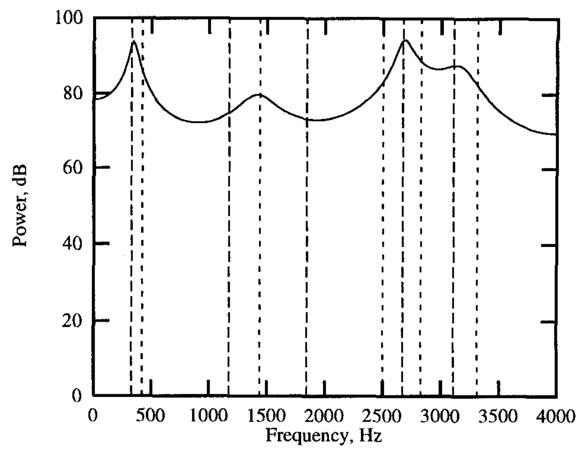


Figure 3: LPC envelope of a frame of speech and associated LSF's.

## 7. REFERENCES


[1] F. Itakura, "Line Spectrum Representation of Linear Predictive Coefficients of Speech Signals," *J. Acoust. Soc. Amer.*, vol. 5, p. S35, April 1975.

[2] K. Paliwal, and B. Atal, "Efficient Vector Quantization of LPC Parameters at 24 Bits/Frame," *in IEEE Trans. Speech and Audio Processing*, vol. 1, pp. 3-14, Jan. 1993.

[3] R. Viswanathan, and J. Makhoul, "Quantization Properties of the Transmission Parameters in Linear Predictive Systems," *in IEEE Trans. Acoust., Speech, Signal Processing*, vol. 23, pp. 309-321, June 1975.

[4] F. K. Soong, and B. Juang, "Line Spectrum Pair (LSP) and Speech Data Compression," *in proc. IEEE Int. Conf. Acoust., Speech, Signal Processing*, vol. 1, pp. 1.10.1-1.10.4, 1984.

[5] P. Henrici, Applied and Computational Complex Analysis, *Wiley*, volume 1, 1974.

[6] L. R. Rabiner and R. W. Schafer, Digital Processing of Speech Signals, *Prentice Hall*, 1978.