QoSNC: A Novel Approach to QoS-Based Network Coding for Fixed Networks

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Abstract: **In this paper, we present a decentralized algorithm to find minimum cost quality of service (QoS) flow subgraphs in network coded multicast schemes. The main objective is to find minimum cost subgraphs that also satisfy user-specified QoS constraints, specifically with respect to rate and delay demands. We consider networks with multiple multicast sessions. Although earlier network coding algorithms in this area have demonstrated performance improvements in terms of QoS parameters, the proposed QoS network coding approach provides a framework that guarantees QoS constraints are actually met over the network.**

Index Terms: **Flow optimization, network coding, quality of service (QoS).**

I. INTRODUCTION

Quality of service (QoS) refers to the ability of a network to provide users with the desired quality metrics for services such as video and audio streaming. The desired quality is determined via certain parameters such as delay, throughput, jitter, and reliability. In order to provide users with their required service quality, network operators have to guarantee that these parameters remain within acceptable bounds almost all the times. Due to increasing demand for services with higher quality levels, QoS has been a hot research area in recent years.

Over-provisioning is the first step toward providing QoS. Network operators have to increase their resources so that they could support growing demand for services requiring higher quality. However, economical factors force network operators to seek alternative means of providing QoS.

In order to provide QoS, routing algorithms must be modified in order to support QoS and be able to differentiate between different packet types in the process of finding the best route that satisfies QoS metrics. The goals of QoS routing are in general twofold: selecting routes that satisfy QoS requirement(s), and achieving global efficiency in resource utilization [1]. For a good survey on quality of service and its issues, see [2].

Network coding, first proposed by Ahlswede *et al.* [3], has been used as an alternative for routing algorithm for packet transmission. In traditional routing algorithms, packets are replicated by intermediate nodes (routers). Network coding allows intermediate nodes to code received data and send a function of received packets on the output links. This function could be linear or nonlinear. It is shown that linear network coding leads to the optimum solution in multicast networks [4]. But in general, linear network coding is not optimal [5].

In linear network coding, each node sends a linear combination of received packets to other nodes. Determining network codes is an important issue in linear network coding. Network codes could be designed using the algorithm proposed by Jaggi *et al.* [6] in a polynomial time. Another approach in designing network codes is to use random linear coding (RLC) [7]. Using RLC has an advantage of not relying on any intelligence for designing the network codes. In RLC, we only need to assume a large enough field size so that the receiver is able to decode the received packets correctly with high probability [8].

Network coding has several advantages over simple routing. For example, using network coding we are able to achieve higher throughput [3] mostly in wireline networks [9], lower delays [10], higher reliability [11], and security [12] (for a good survey on network coding see [10] and [13]). Because of these advantages, many algorithms that used routing are being modified to incorporate network coding instead.

However, it should be noted that although by using network coding, one will be able to *improve* quality of service parameters such as throughput, delay, and reliability, providing QoS *guarantees* is a more difficult objective to achieve. Therefore, the main contribution of this paper is to address this issue by introducing algorithms that not only improve QoS metrics, but also ensure that the final solution *satisfies* the given constraints on such metrics.

The algorithms proposed in the literature so far, have not directly addressed the problem of satisfying a given set of QoS constraints. For example, the minimum cost multicast (MCM) algorithm proposed by Lun *et al.* [14] does not directly consider QoS, although it relies on optimization schemes to determine proper flow subgraphs that minimize given cost functions. There are also other algorithms that in a similar fashion maximize network utility instead of minimizing cost [15]. The algorithms proposed in [16] and [17] try to find flow subgraphs that guarantee a minimum rate and maximize a utility function. In that sense, they constitute the closet approaches to the one proposed in this paper.

It should be noted that yet another approach to QoS network coding has also been recently presented. The algorithms that fall into this category, do not look for optimal solutions that minimize a cost function or maximize the network utility. On the

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other hand, they focus on replacing routing with network coding in well-established algorithms used for sending audio/video streams [18]–[22]. Although these methods can not be considered as QoS network coding algorithms, they clearly show that by use of network coding, quality of service parameters can be improved.

In this paper, we propose a new QoS network coding approach to provide quality of service in a network. The suggested algorithm determines flow subgraphs that satisfy specified quality constraints while minimizing total network cost. The most distinctive property of our approach is that we *explicitly* consider link delay and end-to-end delay as metrics for determining flow subgraphs. Moreover, as the main requirement of providing QoS in a network is differentiating between different types of flows, the proposed scheme provides the framework for such differentiation between different flows according to their quality class. In our early works we considered one multicast session with multiple quality classes [23], [24]. Here we address the general case of multiple multicast sessions with multiple quality classes. A set of node-based primal-dual subgradient algorithms are designed that iteratively find the appropriate coding subgraphs that satisfy QoS constraints.

The rest of this paper is organized as follows: In Section II, we explain the problem of QoS network coding for networks with only one multicast session. Section III extends the problem to more general case of multiple multicast sessions. Simulation results are presented in Section IV and finally, Section V concludes the paper and suggests several paths for future works.

II. QUALITY OF SERVICE NETWORK CODING: ONE MULTICAST SESSION

In this section, we present the network model and explain the details of our QoS network coding algorithm. In the beginning, we assume that there is only one multicast session present in the network. Extending the algorithm to multiple multicast sessions will be consequently presented in the next section.

A. Network Model and Notations

We model the network by a directed and connected graph, $G(V, E)$ where V denotes the set of nodes and E represents the set of links. The only multicast source of the network is s. The set of multicast receivers is indicated by T. The source has to provide multicast receivers with their required flows and also guarantee the desired quality.

There are M quality classes in the network. Each class has a minimum required rate, $R^{(c)}$ and a maximum tolerable delay constraint, $D^{(c)}$. The source s will either refuse to send requested flows to the receivers or if it has accepted the request, it has to guarantee that the end-to-end delay of each flow toward each destination is less that $D^{(c)}$ and the rate of flow is at least $R^{(c)}$. The flow toward the destination t that belongs to class c and is passing through link ℓ is indicated by $x_\ell^{(t)(c)}$.

In order to account for the end-to-end delay experienced by network flows, we have assumed each link to have a constant delay of d_ℓ for a given period of time. We have assumed that before running our algorithm, each node has measured the delay of its outgoing links. Therefore, d_ℓ could be viewed as the short term average delay of link ℓ. This delay includes constant propagation delay plus any additional delay caused by MAC layer. Some other types of delay, like the delay caused by congestion, could be handled by including them in the cost function.

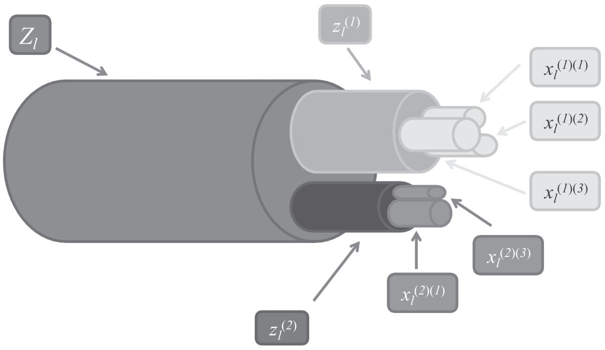


Fig. 1. Network flow notations and their relationship.

Since we use network coding instead of routing, at each node, flows of each class are coded together. However, inter-class coding is not permitted. This is due to the fact that if all classes are coded together into one flow, all classes experience the same quality level and it would not be possible to provide the required quality level for different classes. In each link, we indicate the coded flow of class c by $z_\ell^{(c)}$. These coded flows are sent across each link independently. The total flow passing through link ℓ is indicated by Z_ℓ. Subsequently, we denote the set of all $x_\ell^{(t)(c)}$ by the vector $\underline{x}$ and the set of all $z_\ell^{(c)}$ by the vector $\underline{z}$. Fig. 1 illustrates our notations and their relationship.

B. Problem Formulation

We formulate the quality of service network coding as a network flow optimization problem. We would like to find flow subgraphs such that the final solution satisfies the rate and delay constraints for all classes. If there are more than one solution with these properties, the minimum cost solution will be chosen. The network cost of each link is assumed to be a convex function of total flow (Z_ℓ) over each link. The total cost is the sum of all link cost functions.

It must be noted that the focus of our work is not on designing specific network codes and the focus would be on specifying flow subgraphs that satisfy a given set of constraints. When these subgraphs are determined, one can run any appropriate network coding method such as random linear network coding [7] to obtain a specific network code. It is shown in [14] that these two tasks could be performed separately without affecting the optimality of the final solution.

Therefore, we formulate the QoS network coding flow optimization problem as follows:

$$\min_{\underline{x}} \sum_{\ell \in E} f_\ell(Z_\ell) \tag{1}$$

where

$$Z_\ell = \sum_{c=1}^{M} z_\ell^{(c)},$$

$$z_\ell^{(c)} = \max_{t\in T}\{x_\ell^{(t)(c)}\}.$$

Subject to:

$$0 \le Z_\ell \le a_\ell, \quad \forall \ell \in E, \tag{2a}$$

$$R^{(c)} \le r^{(c)} \quad c = 1,\cdots,M, \tag{2b}$$

$$\sum_{c=1}^{M} r^{(c)} \le \Re, \tag{2c}$$

$$D(t,c) = \max_{p\in P_t} D_p^{(t)(c)} \le\ D^{(c)} \ \ t\in T,\ c=1,\cdots,M \tag{2d}$$

where

$$D_p^{(t)(c)} = \sum_{\ell\in p} D_\ell(x_\ell^{(t)(c)}),$$

$$D_\ell(x_\ell^{(t)(c)}) = \begin{cases} d_\ell & \text{if } x_\ell^{(t)(c)} > 0 \\ 0 & \text{otherwise,}\end{cases}$$

$$\sum_{\ell\in Out(n)} x_\ell^{(t)(c)} - \sum_{\ell\in In(n)} x_l^{(t)(c)} = \sigma_n^{(t)(c)}, \tag{2e}$$

$$\sigma_n^{(t)(c)} = \begin{cases} r^{(c)}, & \text{if } n = s \\ -r^{(c)}, & \text{if } n\in T \\ 0, & \text{otherwise,}\end{cases}$$

$$x_\ell^{(t)(c)} \ge 0,\ \forall \ell\in E,\ t\in T,\ c=1,\cdots,M \tag{2f}$$

In above equations, $f_\ell(Z_\ell)$, a_ℓ and d_ℓ are the cost, capacity and delay of link ℓ, respectively. $R^{(c)}$ is the minimum required rate and $D^{(c)}$ is the maximum tolerable delay of class c. $\Re$ is the max-flow-min-cut rate of the network and $r^{(c)}$ refers to the actual rate of class c flow. P_t is the set of all paths toward sink t and $D_p^{(t)(c)}$ is the delay that a flow from class c experiences toward sink t on path p. Finally, $In(n)$ and $Out(n)$ refer to input and output links of node n respectively.

Constraint (2a) is the capacity constraint. It guarantees that the total flow on link ℓ is less than its capacity. Constraint (2b) indicates that the rate of class c must be greater than the minimum required value determined by the user. Constraint (2c) makes sure that the total rate of all classes is less than the max-flow rate of network. Equation (2d) indicates the delay constraint, ensuring that the end-to-end delay of class c is less than the maximum tolerable delay of the class. End-to-end delay of a flow toward destination t is the maximum delay among all multiple paths from the source to t (P_t). Equation (2e) formulates the flow conservation constraint. Finally, constraint (2f) ensured the non-negativity of $\underline{x}$.

By solving problem (1), we obtain flow subgraphs that satisfy the specified constraints and are also minimum cost among all feasible answers. In the next section, we provide a distributed and simple solution for this problem.

C. Modified Problem

Before we start the detailed explanation of optimization method, we slightly modify the problem definition, in order to simplify the implementation of the optimization algorithm. In the modified problem, we replace

$$z_\ell^{(c)} = \max_{t\in T}\{x_\ell^{(t)(c)}\} \tag{3}$$

by

$$z_\ell^{(c)} \ge \{x_\ell^{(t)(c)}\} \ \ \forall t\in T. \tag{4}$$

Using this technique, we may solve the problem with a primal-dual approach [25]. Now the modified problem will be:

$$\min \sum_{\ell\epsilon E} f_\ell(Z_\ell) \tag{5}$$

subject to:

$$Z_\ell \in \mathbb{Z}, \tag{6a}$$

$$z_\ell^{(c)} \ge x_\ell^{(t)(c)} \ \ \forall t\in T,\ \forall \ell\in E,\ c=1,\cdots,M, \tag{6b}$$

$$x_\ell^{(t)(c)} \in\ \mathbf{X} \tag{6c}$$

where $\mathbb{Z}$ and $\mathbf{X}$ are the set of feasible $\underline{z}$ and $\underline{x}$, respectively. By feasible, it is meant that members of $\mathbb{Z}$ must satisfy the capacity constraint (2a) and members of $\mathbf{X}$ must satisfy constraints (2b), (2d), (2e), and (2f).

D. Distributed Solution

In order to simplify the results, we have assumed that the actual rate of each class is equal to the minimum accepted rate. More specifically, we have assumed that for all classes, $r^{(c)} = R^{(c)}$ and seek the solution that satisfies this condition. Consequently, the constraint (2b) will be removed. Moreover, since $r^{(c)}$ is now fixed, the constraint (2c) becomes a feasibility condition which should be checked before the algorithm is run. In other words the source node should check this condition to see if the problem is feasible. If not, the user's request is withdrawn.

In order to solve the problem (5), the primal-dual decomposition method [25] is used, in which the problem (5) is first broken into two subproblems using primal decomposition. Then, each subproblem is solved using dual Lagrangian method [26]. This approach will result in a simple, distributed solution. Moreover, since we have assumed the cost function to be convex, problem (5) is a convex optimization problem and the duality gap of decomposition method will be zero (see Section 5.2.3 of [27]).

Subsequently, in order to break the problem (5) into two subproblems, we first assume $\underline{x}$ to be constant and solve the problem over $\underline{z}$. Then, the resulting cost function is minimized over $\underline{x}$ [26]. More specifically, the problem (5) is decomposed into the following subproblems:

Subproblem A:

$$\min_{\underline{z}} \sum_{\ell\in E} f_\ell(Z_\ell) \tag{7}$$

subject to:

$$Z_\ell \le a_\ell \ \ \forall \ell\in E, \tag{8a}$$

$$x_\ell^{(t)(c)} \le z_\ell^{(c)} \ \ \forall t\in T,\ \forall \ell\in E,\ c=1,\cdots,M, \tag{8b}$$

and **subproblem B**:

$$\min_{\underline{x}} \sum_{\ell\in E} f^\star(\underline{x}) \tag{9}$$

subject to constraints (2b), (2c), (2d), and (2e). In the above equation, $f^\star$ indicates the solution of subproblem (7).

In order to solve subproblem (7), we use its Lagrangian equivalent. Define:

$$L(\underline{z}, \underline{\lambda}, \underline{\beta}) = \sum_{\ell \in E} f_\ell(Z_\ell) + \underline{\lambda}^T(\underline{Z} - \underline{a}) \\ + \sum_{\ell \in E} \sum_{t \in T} \sum_{c=1}^{M} \beta_{\ell tc}(x_\ell^{(t)(c)} - z_\ell^{(c)}). \tag{10}$$

Then, subproblem (7) is equivalent to:

$$\min_{\underline{z}} L(\underline{z}, \underline{\lambda}, \underline{\beta}) \tag{11}$$

and

$$\max_{\underline{\lambda}, \underline{\beta}} L^\star(\underline{\lambda}, \underline{\beta}) \tag{12}$$

where $L^\star$ is the solution of problem (11).

We could decouple the above solution into L subproblems, one for each link:

$$\min_{\underline{z}} f_\ell(Z_\ell) + \lambda_\ell Z_\ell - \sum_{t \in T} \sum_{c=1}^{M} \beta_{\ell tc} z_\ell^{(c)} \tag{13}$$

and

$$\max_{\underline{\lambda}, \underline{\beta}} f(Z_\ell^\star) + \lambda_\ell(Z_\ell^\star - a_\ell) \\ + \sum_{t \in T} \sum_{c=1}^{M} \beta_{\ell tc}(x_\ell^{(t)(c)} - z_\ell^{\star(c)}). \tag{14}$$

Each node n has to solve the above problems for all of its outgoing links, exchange Lagrange multipliers with its neighbors and repeat this process till convergence. Problems (13) and (14) could be solved using subgradients method as follows [26]:

$$z_\ell^{(c)}(\tau + 1) = [z_\ell^{(c)}(\tau) - \alpha(\tau)(\nabla f_\ell^{(c)} - \sum_{t \in T} \beta_{\ell tc})]_{\underline{z} \in \mathbb{Z}}, \tag{15}$$

$$\lambda_\ell(\tau + 1) = [\lambda_\ell(\tau) + \alpha(\tau)(Z_\ell^\star - a_\ell)]_+, \tag{16}$$

$$\beta_{\ell tc}(\tau + 1) = [\beta_{\ell tc}(\tau) + \alpha(\tau)(x_\ell^{(t)(c)} - z_\ell^{\star(c)})]_+ \tag{17}$$

where $[\ \cdot\ ]_+$ indicates that $\underline{\lambda}$ and $\underline{\beta}$ must be non-negative and $[\ \cdot\ ]_{\underline{z} \in \mathbb{Z}}$ ensures that the updated $\underline{\text{z}}$ lies in the feasible region. $\nabla f_\ell^{(c)}$ represents the (ℓ, c) member of $\nabla f(Z)$. $\alpha(\tau)$ is the algorithm step size, chosen such that convergence of the algorithm is guaranteed.

There are many results on convergence of the gradient/subgradient method with different choices of step-sizes [28]. Although in our algorithm, we have assumed $\alpha(\tau) = \frac{1}{\tau}$ to guarantee the convergence of the subgradient method, any diminishing step-size may be used as well (see Section 6.3.1 of [26]).

If a constant step-size is used, as is more convenient for distributed algorithms, the gradient algorithm converges to the optimal value provided that the step-size is sufficiently small (assuming that the gradient is Lipschitz) [26].

In the same manner, subproblem (9) could be solved using subgradients method. In each iteration, update $\underline{\text{x}}$ and its Lagrange multiplier according to (18) and (19) as follows:

$$x_\ell^{(t)(c)}(\tau + 1) = [x_\ell^{(t)(c)}(\tau) - \alpha(\tau)(\nabla f_\ell^{\star(c)} \\ + \beta_{\ell tc} + \sum_{\substack{p \in P_t \\ \ell \in p}} \upsilon_p^{(t)(c)} d_\ell)]_{x \in \mathbf{X}} \tag{18}$$

and

$$\upsilon_p^{(t)(c)}(\tau + 1) = [\upsilon_p^{(t)(c)}(\tau) + \alpha(\tau)(D_p^{(t,c)} - D^{(c)})]_+ \tag{19}$$

where "$x \in \mathbf{X}$" means that the updated $\underline{\text{x}}$ should be projected onto feasible $\underline{\text{x}}$ region. $\underline{\upsilon}$ is the Lagrange multipliers vector for delay constraint.

To summarize, each node has to perform the following procedure to solve the problem (5):

1. Initialization
2. Solve subproblem (15) for each outgoing link.
3. Update Lagrange multipliers according to (16) and (17).
4. Update $\underline{\text{x}}$ according to (18).
5. Update delay related Lagrange multipliers according to (19).

E. Separable Cost Function

In the previous section, a distributed solution of the problem (5) was presented. So far, convexity is the only assumption that was made about the cost function. But, if the cost function could be decomposed itself, each subproblem could also be decoupled into other subproblems. Here are some examples of these kind of cost functions:

$$f_\ell(Z_\ell) = e_\ell Z_\ell = \sum_{c=1}^{M} e_\ell z_\ell^{(c)} = \sum_{c=1}^{M} f(z_\ell^{(c)}), \tag{20}$$

$$f_\ell(Z_\ell) = \sum_{c=1}^{M} \frac{z_\ell^{(c)}}{a_\ell - z_\ell^{(c)}} = \sum_{c=1}^{M} f(z_\ell^{(c)}) f. \tag{21}$$

The cost function (20) represents energy consumption and the cost function (21) may be considered as a representative of queuing costs. If energy or monetary costs are more important for network operators, we use cost function (20). But if handling queuing delays and congestion is more important, cost function (21) may be used. Please note that since we are designing an algorithm to provide QoS, other lower layer algorithms should be modified as well. As a result, we have assumed that each node has separate queues for different classes. This is clearly shown in cost function (21). This assumption will enable us to have a separable cost function, if queuing delays are to be handled as well.

If the cost function is of the form (20) or (21), then each of L subproblems (13) could be decoupled into M subproblems, where M is the number of classes, as follows:

$$\min_{\underline{z}} f(z_\ell^{(c)}) + \lambda_\ell z_\ell^{(c)} - \sum_{t \in T} \beta_{\ell ct} z_\ell^{(c)}. \tag{22}$$

Each node should solve problem (22) for its outgoing links and derive $z_\ell^{(c)}$, update $\underline{\text{x}}$ and Lagrange multipliers according to (16)–(19), respectively. Then, it has to exchange these multipliers with its neighbors until convergence is achieved.

The above procedure leads to a distributed, simple solution in which each node n must solve at most $M \times outdegree(n)$ where M is the number of quality classes.

F. A Note on QoS Network Coding and Routing

In general, network coding could achieve higher rates and lower delays than pure routing. Naturally, a similar situation also arises in comparison between QoS network coding and QoS routing. As will be shown in the simulation results, there are cases in which QoS routing is unable to deliver flows with desired quality but QoS network coding can.

However, with respect to QoS and minimum cost network coding, one should be cautious about comparing the two algorithms. Since QoS network coding requires differentiation between flows and involves delay and rate constraints, there could be some cases that minimum cost network coding has a solution but QoS network coding does not. These cases are the result of a more stringent delay or rate constraint. For example, consider the network shown in Fig. 2(a). It is clear that minimum cost multicast with rate 2 is feasible in this network. Yet, QoS network coding with the same rate and a delay constraint of 4 units does not lead to any feasible subgraph. Such result is due to the fact that providing flows whose delay is less than 6 units is not possible in the given network.

This problem certainly is not a deficiency of this algorithm or any other algorithm of this kind. The need for quality, which is the inherent requirement of QoS provisioning algorithms, causes such shortcomings. Therefore, one must be careful in comparing these two kinds of algorithms due to this basic difference.

III. EXTENSION TO MULTIPLE MULTICAST SESSIONS

Although in the previous section we assumed that there is only one multicast session present in the network, this may not be the case in general. In a real network, there are many multicast sessions whose receiver sets are not necessarily the same. In this section, we extend our algorithm to networks with multiple multicast sessions.

A. Notations

The network model and other assumptions are the same as before. However, in this case we assume that there are multiple sessions running in the network. The ith multicast session is denoted by its source, s_i. The set of sources is indicated by $S = \{s_1, \cdots, s_Q\}$ where Q is the number of sessions. Each multicast session has its own set of receivers, shown by $T(s_i)$. The set of receivers are not necessarily the same for all multicast sessions but some nodes may be in more than one set, i.e., there could be common receivers among multicast sinks. All sources do not provide all quality classes in general and consequently, each source may provide only a portion of M quality classes for its receivers.

The flow from source s to the destination t that belongs to class c and is passing through link ℓ is indicated by $x_\ell^{(t)(c)(s)}$.

As in the previous case, no inter-class coding is permitted. Moreover, no inter-session coding is allowed. In other words, only same-class flows from same-sessions are coded together. We indicate the coded flow of class c that has originated from the source s by $z_\ell^{(c)(s)}$. These coded flows are sent across each link independently. The total flow of class c on link ℓ is shown by $Z_\ell^{(c)}$. Finally, the total flow passing through link ℓ is indicated by Z_ℓ.

B. Problem Formulation

As in the previous case, we formulate the quality of service network coding problem in a network flow optimization framework where the goal is to find flow subgraphs that satisfy the rate and delay constraints for all classes. Among all solutions with such properties, the minimum cost one will be chosen. The network cost of each link is assumed to be a convex function of total flow (Z_ℓ) over each link. The total cost is the sum of all link cost functions. As a result, the problem would be:

$$\min_{\underline{x}} \sum_{\ell \in E} f_\ell(Z_\ell) \tag{23}$$

where

$$Z_\ell = \sum_{c=1}^{M} \sum_{s \in S} z_\ell^{(c)(s)},$$

$$z_\ell^{(c)(s)} = \max_{t \in T(s)} \{x_\ell^{(t)(c)(s)}\}.$$

Subject to:

$$0 \le Z_\ell \le a_\ell \quad \forall \ell \in E, \tag{24a}$$

$$R^{(c)} \le r^{(c)(s)} \quad c = 1, \cdots, M, \ \forall s \in S, \tag{24b}$$

$$\sum_{c=1}^{M} r^{(c)(s)} \le \Re^{(s)} \quad \forall s \in S, \tag{24c}$$

$$D(t,c,s) = \max_{p \in P_t^s} D_p^{(t)(c)(s)} \le D^{(c)}, \quad \forall t \in T, \ s \in S, \ c = 1, \cdots, M \tag{24d}$$

where

$$D_p^{(t)(c)(s)} = \sum_{\ell \in p} D_\ell(x_\ell^{(t)(c)(s)}), \tag{24e}$$

$$D_\ell(x_\ell^{(t)(c)(s)}) = \begin{cases} d_\ell & \text{if } x_\ell^{(t)(c)(s)} > 0 \\ 0 & \text{otherwise}, \end{cases} \tag{24f}$$

$$\sum_{\ell \in Out(n)} x_\ell^{(t)(c)(s)} - \sum_{\ell \in In(n)} x_l^{(t)(c)(s)} = \sigma_n^{(t)(c)(s)}, \tag{24g}$$

$$\sigma_n^{(t)(c)(s)} = \begin{cases} r^{(c)(s)}, & \text{if } n = s \\ -r^{(c)(s)}, & \text{if } n \in T(s) \\ 0, & \text{otherwise}, \end{cases} \tag{24h}$$

$$x_\ell^{(t)(c)(s)} \ge 0 \ \forall \ell \in E, \ t \in T, \ s \in S, \ c = 1, \cdots, M. \tag{24i}$$

In the above equations, $\Re$ is the max-flow-min-cut rate of the network and $r^{(c)(s)}$ is the actual rate of flows originating from the source s and belonging to class c. P_t^s denotes the set of all multiple paths from the source s toward the destination t.

Constraint (24a) is the capacity constraint. It guarantees that the total flow on link ℓ is less than its capacity. Constraint (24b) indicates that the rate of class c must be greater than the minimum required value determined by the user. Constraint (24c) ensures that the total rate of all classes is less than the max-flow rate of network. Equation (24d) indicates the delay constraint, ensuring that the delay of class c is less than the maximum tolerable delay of the class. As before, the end-to-end delay of a flow belonging to class c and from the source s toward the destination t is the maximum delay of all multiple paths from s to t. Finally, equation (24g) is the flow conservation constraint.

C. Distributed Solution of the Multiple Multicast Sessions Problem

Following the same approach as in the previous section, we could solve (23) in a distributed manner. In summary, the procedure of finding the minimum cost flow subgraph that satisfies delay and rate constraints is as follows:

1. Initialization
2. Solve subproblem (25) for each outgoing link, each class and each session as follows:

$$z_\ell^{(c)(s)}(\tau+1) = \left[z_\ell^{(c)(s)}(\tau) - \alpha(\tau)(\nabla f_\ell^{(c)(s)} - \sum_{t \in T(s)} \beta_{\ell stc}) \right]_{\underline{z} \in \mathbb{Z}} . \tag{25}$$

3. Update Lagrange multipliers according to (26) and (27):

$$\lambda_\ell(\tau+1) = [\lambda_\ell(\tau) + \alpha(\tau)(Z_\ell^\star - a_\ell)]_+, \tag{26}$$

$$\beta_{\ell stc}(\tau+1) = [\beta_{\ell stc}(\tau) + \alpha(\tau)(x_\ell^{(t)(c)(s)} - z_\ell^{\star(c)(s)})]_+. \tag{27}$$

4. Update $\underline{\mathrm{x}}$ according to (28):

$$x_\ell^{(t)(c)(s)}(\tau+1) = \left[x_\ell^{(t)(c)(s)}(\tau) - \alpha(\tau)(\nabla f_\ell^{\star(c)(s)} + \beta_{\ell tcs} + \sum_{\substack{p \in P_t^s \\ \ell \in p}} \upsilon_p^{(t)(c)(s)} d_\ell) \right]_{x \in \mathbf{X}} \tag{28}$$

$$\upsilon_p^{(t)(c)(s)}(\tau+1) = [\upsilon_p^{(t)(c)(s)}(\tau) + \alpha(\tau)(D_p^{(t,c,s)} - D^{(c)})]_+. \tag{29}$$

5. Update delay related Lagrange multipliers according to (30)

$$\upsilon^{(t)(c)(s)}(\tau+1) = [\upsilon^{(t)(c)(s)}(\tau) + \alpha(\tau)(D(t,c,s) - D^{(c)})]_+. \tag{30}$$

6. Repeat till convergence

The aforementioned approach will lead to a distributed solution. In addition, if the cost function is decomposable itself, as in (20) and (21), each node n has to solve at most $Q \times M \times outdegree(n)$, where Q is the number of multicast sessions.

IV. COMPARISON WITH MINIMUM COST MULTICAST APPROACHES

In this section, we investigate the performance of our proposed algorithm based on several simulations. We have considered two different cases, namely one and multiple multicast sessions.

We compare our algorithm with MCM techniques [14]. MCM have shown to be a powerful tool in finding subgraphs with minimum cost in coded multicast networks. Lun *et al.* have shown the advantages of network coding-based MCM over traditional routing based techniques to find minimum cost multicast tree, which is equivalent to solving the Steiner tree problem on directed graphs [31].

When there is only one multicast session, and link delays and differentiation of flows into different quality classes are not taken into account, our algorithm reduces to a minimum cost multicast approach. Therefore, MCM techniques can be considered as special cases of our algorithm. As will be shown in this section, while our algorithm is able to provide QoS for multicast users, its final cost is not much higher than network coding based MCM techniques. In other words, QoSNC enables us to guarantee QoS with only a slightly higher cost in comparison with minimum cost achieved by MCM algorithms.

A. One Multicast Session

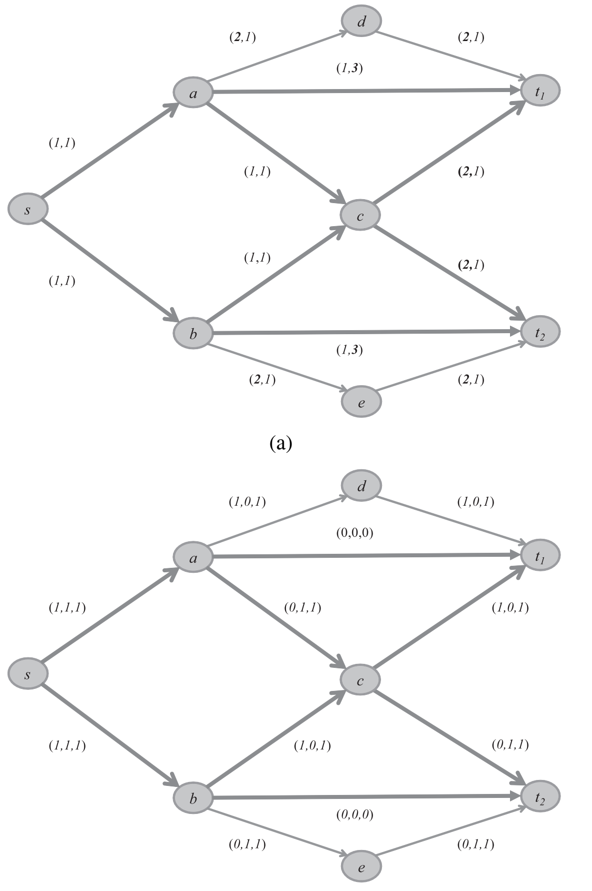
For illustrative purposes, we first simulate the suggested algorithm for some basic cases. Consider the basic network illustrated in Fig. 2(a). Delay and cost of each link is shown in the figure. Assume that there is one multicast source, s, two multicast sinks, t_1 and t_2 and one quality class with desired rate of 2.

Now assume that the delay constraint is 5 units. In this case, there are two possible solutions for the QoS problem with network coding. As a result, total cost becomes decisive and the subgraph with lower cost will be selected as the solution, which is shown in Fig. 2(b). It is obvious that whenever the delay and rate constraints are not tight enough, minimum cost multicast [14] will lead to the same result as the proposed algorithm. Moreover, if delay of network links and division of flows into different quality classes are ignored and only one multicast session is assumed, the solution of problem (5) will become the same as the minimum cost multicast solution.

Please note that end-to-end delay of any routing algorithm will be more than 6 units for one of two multicast receivers. This clearly shows the limitation of routing algorithms in providing QoS. On the other hand, as we have seen, QoS network coding is able to fulfill this task and provide guarantees on both rate and delay.

Now consider the same network but with a more stringent delay constraint of 4 units. In this case, there would be only one feasible solution that satisfies both the delay and rate constraints. This solution is shown in Fig. 3. Obviously, the illustrated subgraph is different from the minimum cost solution. This is due to the fact that the delay of minimum cost solution is greater than the required constraint. Therefore, MCM is not able to provide QoS in this case while QoSNC can.

The previous scenarios were mostly illustrative. To compare our algorithm with minimum cost multicast approaches we have



(a)

(b)

Fig. 2. Minimum cost multicast: (a) Links are marked with their (delay, cost) and (b) each link is marked with $(x_\ell^{(t_1)(1)}, x_\ell^{(t_2)(1)}, z_\ell^{(1)})$.

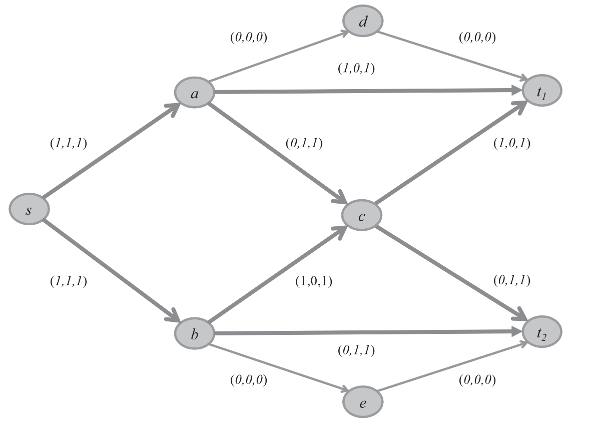


Fig. 3. QoS with network coding.

also evaluated our algorithm over several random topologies. In this case, a random network was generated with different number of nodes and multicast sinks. In each network, the number of nodes, links and multicast sinks are fixed. However, the two ends of each arc, and as a result, the node-arc incidence matrix of the network is random. In addition, the source and sinks are selected randomly in each run. In all cases, we have assumed that each link has a delay of one unit and a cost which is randomly chosen according to a Gaussian distribution with mean 2 and variance 1 (negative costs are truncated to zero). It is assumed that there are two quality classes, the first with rate 1 and the second with rate 2. Multicast sinks have requested one flow from each class. The results for these scenarios are shown in Table 1.

Table 1. Delay of QoS network coding vs. minimum cost multicast.

No. of nodes	Algorithm	Total cost	
		2 sinks	4 sinks
10	MCM	18.25	28.11
	QoSNC	20.22	31.89
20	MCM	25.13	39.55
	QoSNC	27.46	41.59
30	MCM	34.88	61.11
	QoSNC	37.63	63.85

Table 2. Delay of QoS network coding vs. minimum cost multicast.

No. of nodes	Algorithm	Class	2 sinks		4 sinks	
			Delay	Const.	Delay	Const.
10	MCM	1	(2, ***6***)	5	(2, ***6***, 3, 2)	5
		2	(***3***, 2)	2	(***3***, ***5***, 1, 2)	2
	QoSNC	1	(3, 4)	5	(5, 4, 5, 3)	5
		2	(2, 2)	2	(2, 2, 1, 2)	2
20	MCM	1	(6, 3)	8	(***9***, ***11***, 3, 6)	8
		2	(***7***, 2)	4	(2, 2, ***6***, 4)	4
	QoSNC	1	(7, 6)	8	(7, 7, 8, 8)	8
		2	(5, 3)	4	(3, 4, 2, 3)	4
30	MCM	1	(7, ***13***)	11	(6, ***12***, 11, 8)	11
		2	(2, 5)	6	(3, ***7***, 3, 5)	6
	QoSNC	1	(9, 10)	11	(10, 9, 7, 11)	11
		2	(4, 3)	6	(5, 6, 4, 5)	6

As shown in Table 1, the cost function achieved by the proposed algorithm is at most 13.45% higher than the cost obtained by the MCM algorithm, while the latter algorithm does not necessarily satisfy the QoS constraints. Table 2, illustrates the end-to-end delay experienced by each receiver for both algorithms.

It is clear that for QoSNC, the delay for each receiver is less than the desired maximum. But for MCM, some multicast sinks will receive the flow with a delay more than the required maximum value (marked in bold italic form). Finally, in cases where there is a minimum cost subgraph that also achieves the delay and constraints, both algorithms naturally will lead to the same solution.

B. Multiple Multicast Session

There are cases that more than one multicast session are present in the network. In order to examine the performance of our algorithm in such cases, we have considered several random networks in which there are two multicast sources and two quality classes. Source s_1 provides flows from both classes while s_2 only provides flows from the first class. The specifications of classes are the same as before.

In order to compare the cost of our algorithm with that of minimum cost approaches, a random topology is first considered. Then, links delays and classification of flows in different quality classes are ignored and the proposed algorithm is executed over the same topology. In this case, our algorithm yields the minimum cost subgraphs. Simulation results are given in Tables 3 and 4.

From Table 3, it is clear that as in the earlier cases, our algorithm results in subgraphs whose costs are not much higher than

Table 3. Delay of QoS network coding vs. minimum cost multicast.

No. of nodes	Algorithm	Total cost	
		2 sinks	4 sinks
10	MCM	26.28	38.43
	QoSNC	27.61	39.84
20	MCM	44.32	58.64
	QoSNC	46.25	61.54
30	MCM	76.33	90.16
	QoSNC	79.85	94.73

Table 4. Delay of QoS network coding vs. minimum cost multicast.

No. of nodes	Algorithm	Source	Class	2 sinks		4 sinks	
				Delay	Const.	Delay	Const.
10	MCM	1	1	(2, 1)	5	(***8***, 1, 4, 3)	5
			2	(1, ***3***)	2	(***6***, ***4***, 2, ***4***)	2
		2	1	(2, 2)	5	(1, 3)*	5
	QoSNC	1	1	(1, 3)	5	(5, 2, 4, 5)	5
			2	(1, 1)	2	(2, 1, 2, 2)	2
		2	1	(2, 1)	5	(1, 5)*	5
20	MCM	1	1	(***9***, 3)	5	(4, 2, 1, 3)	2
			2	(***5***, ***3***)	2	(1, ***3***, 2, ***3***)	2
		2	1	(3, 5)	5	(3, 1, 1, 1)	5
	QoSNC	1	1	(5, 4)	5	(3, 2, 4, 5)	5
			2	(2, 2)	2	(1, 2, 2, 1)	2
		2	1	(2, 5)	5	(1, 4, 3, 3)	5
30	MCM	1	1	(***7***, 4)	5	(3, ***6***, 5, ***8***)	5
			2	(***9***, 1)	2	(2, ***5***, ***9***, 1)	2
		2	1	(3, 4)	5	(3, ***7***, ***6***, 4)	5
	QoSNC	1	1	(2, 4)	5	(5, 5, 3, 5)	5
			2	(2, 2)	2	(1, 1, 2, 2)	2
		2	1	(5, 3)	5	(4, 5, 3, 5)	5

*As mentioned before, it is not necessary that different sessions have the same number of receivers.

that of minimum cost ones. In the worst case shown in the table, the cost of QoSNC is only 5.1% higher than that of MCM.

Table 4 gives end-to-end delays for the network flows. It is obvious that for the minimum cost approach, delay of some flows is higher than the maximum tolerable threshold (marked in bold italic form). Naturally, these flows will not achieve the required quality level.

V. CONCLUSION AND FUTURE WORKS

We have proposed a new decentralized algorithm that computes minimum cost QoS flow subgraphs in coding-based multicast networks. The subgraphs are determined so that certain user-defined quality measures, such as delay and throughput, are satisfied. The resulting subgraphs are chosen to achieve minimum cost among all subgraphs that satisfy the given quality constraints. We have addressed the problem in networks with multiple multicast sessions and various quality classes, satisfying different rate and delay requirements. The proposed algorithm can handle any convex cost function in addition to user-defined delay and throughput requirements. This makes our algorithm an ideal choice for cases where the cost function is linear (energy consumption) or a function of queueing costs (congestion control).

Extending our work to wireless networks and considering network dynamism such as node arrivals, departures and potential link failures is a subject of our future research.

In addition, the algorithm will be more practical if we include delays caused by congestion into the end-to-end delay shown by constraint (2d). This is also another area for future studies.

Another issue of high practical importance is including pricing and negotiation processes. In QoS provisioning, users make contracts with operators so that the quality of their services is guaranteed. There are cases where network is unable to provide services that satisfy all the required metrics. Then, the user has to either look for another operator or negotiate with the operator to get services with lower qualities and lower costs. Especially, the negotiation and pricing process should be designed carefully when QoS is provided using network coding. tomorrow

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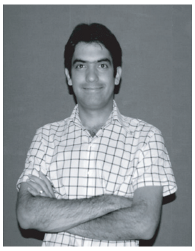

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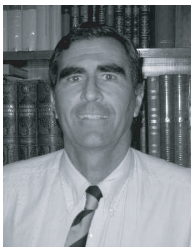

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