Errorless Codes for Over-loaded Synchronous CDMA Systems and Evaluation of Channel Capacity Bounds

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Abstract–**In this paper we introduce a new class of codes for over-loaded synchronous wireless and optical CDMA systems which increases the number of users for fixed number of chips without introducing any errors. Equivalently, the chip rate can be reduced for a given number of users, which implies bandwidth reduction for downlink wireless systems. An upper bound for the maximum number of users for a given number of chips is derived. Also, lower and upper bounds for the sum channel capacity of an overloaded CDMA are derived that can predict the existence of such overloaded codes. Although a high percentage of the overloading factor[2] degrades the system performance in noisy channels, simulation results show that this degradation is not significant.**

I. INTRODUCTION

In a synchronous wireless CDMA system[3], we can obtain errorless transmission by using orthogonal codes (Hadamard codes) under the assumption of noiseless channel; we assume the number of users is less than or equal to the spreading factor (under or fully-loaded cases). In the over-loaded case (when the number of users is more than the spreading factor), such orthogonal codes do not exist; the choice of random codes creates interference that cannot be removed completely, in general, and creates errors in the Multi-User Detection (MUD) receiver [6-8]. The papers that discuss double orthogonal codes for increasing capacity [20-21] are actually nonbinary complex codes (equivalent to m phases for MC-OFDM) and are not really fair for comparison.

Likewise, for under-loaded optical CDMA systems, Optical Orthogonal Codes (OOC) [13] and [19] can be used. Unlike the connotation of the name of OOC, the optical codes are not really orthogonal, but by interference cancellation, we can remove the interference completely. However, for the fully and over-loaded cases, OOC's (with minimal cross-correlation value of $\lambda = 1$) do not exist and similar to the wireless CDMA, the choice of random codes creates interference that, in general, cannot be removed completely.

However, when the channel bandwidth is limited, the over-loaded CDMA may be needed. Most of the research in the over-loaded case is related to code design and Multi-Access Interference (MAI) cancellation to decrease the probability of error. Examples of these types of research are pseudo random spreading (PN) [1-2], design of codes (signatures) that are as orthogonal as possible with respect to the criterion of Total Squared Correlation [3-5], OCDMA/OCDMA (O/O) [9-11], Multiple-OCDMA (MO) [12], PN/OCDMA (PN/O) [9] signature sets, Serial and Parallel Interference Cancellation (SIC and PIC) [14-18].

None of these signatures and decoding schemes guarantees errorless communication in an ideal (noiseless and without near-far effect) synchronous channel. In this paper, we plan to introduce Codes for Over-loaded Wireless (COW) and Codes for Over-loaded Optical (COO) CDMA systems which guarantee errorless communication in an ideal channel and propose an MUD scheme for a special class of these codes. As an example, for a signature length of 64, we have discovered such codes with an overloading factor of about 62% that can be decoded practically in real time, which is also maximum likelihood. However, we have proved the existence of codes with an overloading factor of almost 156% that need to be discovered. These codes are suitable for synchronous Code Division Multiplexing (CDM) in broadcasting, downlink wireless and optical CDMA (assuming chip and frame synch). Alternatively, these codes can be used for the present downlink CDMA systems with much lower chip rate and hence significant bandwidth saving for the operating companies. Using 64 chips, we have also derived an upper bound where the overloading factor cannot be more than 425%.

A method for decoding the over-loaded CDMA signal at the receiver is proposed. The complexity of the decoding depends on the number of chips and the overloading factor. We shall show that for a system that uses 64 chips with a moderate overloading factor of 62%, the decoding complexity is low and can be practical. The implications of these findings are tremendous; it implies that using this system, we can accommodate 104 users with low complexity decoding at the receiver end without any interference. We shall show that the decoding algorithm is Maximum Likelihood and is also robust against additive noise.

Sections II and III discuss the necessary and sufficient conditions for errorless transmission in an over-loaded CDMA system along with methods for constructing large COW and COO codes with high percentage of overloading factor. An upper bound for the number of users for a given signature length is also presented in this section. Channel capacity evaluation for noiseless CDMA is discussed in

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[2] The percentage of the number of users divided by the number of chips minus 1.

[3] In general, by wireless CDMA, we mean the signature codes and the input data consist of binary elements of {±1}; while for optical CDMA systems, the binary elements are {0,1}.

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Section IV. One decoding method is discussed in Section V. Simulation results are summarized in section VI. Finally, conclusion and future work are covered in section VII.

Because of the 5 page limitation, we have omitted all the proofs of the theorems despite the fact that some of the proofs are not trivial.

II. Codes for Over-loaded Wireless (COW) CDMA Systems

For developing COW and COO codes (matrices), we first discuss an intuitive geometric interpretation and then develop the codes mathematically. At a given time the multi-user binary data can be represented by an n-dimensional vector; these vectors can be interpreted as the vertices of a hyper-cube. Each user data is multiplied by a signature of m chips long and finally their summation is transmitted. Thus, the transmitted m-tuple vector is the multiplication of an $m \times n$ matrix (the columns are the signatures of different users) by the input n-dimensional vector. Hence, the hyper-cube vertices are mapped onto points in an m-dimensional space ($m < n$). As long as the points in the m-dimensional space are distinct, the mapping is 1-1 and therefore, we can uniquely decode each received m-tuple vector at the receiver; on the other hand, if these m-tuple vectors are not distinct, the mapping is not 1-1 and the system is not invertible. Consequently, we look for codes that map the vertices of the n-dimensional hyper-cube to distinct points in the m-dimensional space. Most of the over-loaded codes discussed in the literature have not this property and thus any MUD cannot be perfect. We coin the invertible codes, as mentioned in the introduction, as COW and COO codes for wireless and optical applications, respectively. We first develop systematic ways to generate COW codes and then extend it to COO codes.

Lemma 1 We denote the vertices $\{\pm 1\}^n$ of an n-dimensional hyper-cube with the set $\boldsymbol{\mathcal{V}}$. The necessary and sufficient condition for the multiplication of a COW matrix $\mathbf{C}$ with elements of $\boldsymbol{\mathcal{V}}$ to be a 1-1 transformation is $Ker\ \mathbf{C} \cap \{\pm 1, 0\}^n = \{0\}^n$, where $Ker\ \mathbf{C}$ is the null space of $\mathbf{C}$.

Corollary 1

a- A new COW matrix can be generated by multiplying each row or column of the matrix $\mathbf{C}$ by -1.

b- New COW matrices can be generated by the permuting of the columns and rows of the matrix $\mathbf{C}$.

c- By adding an arbitrary row to a COW matrix, we obtain another COW matrix.

By Corollary 1, we can assume that all entries of the first row of a COW matrix are 1.

Theorem 1 Assume that $\mathbf{C}$ is an $m \times n$ COW matrix and $\mathbf{P}$ is an invertible $k \times k$ $\{\pm 1\}$-matrix, then $\mathbf{P} \otimes \mathbf{C}$ is a $km \times kn$ COW matrix, where $\otimes$ denotes the Kronecker product.

In the following theorem, we study the existence of COW matrices with much higher percentage of the overloading factor.

Theorem 2 Assume $\mathbf{C}$ is an $m \times n$ COW matrix and $\mathbf{H}_2 = \begin{bmatrix} +1 & +1 \\ +1 & -1 \end{bmatrix}$. We can add $\lceil (m-1)\log_3 2 \rceil$ columns to $\mathbf{H}_2 \otimes \mathbf{C}$ to obtain another COW matrix.

Note 1 $n/m \to \infty$ as $m \to \infty$.

This observation is a direct result of Theorem 2 since n/m is of order $O(\log m)$. It implies that as the chip rate increases, the number of users grows much faster.

Example 1 Applying Theorem 2 on a 2×2 Hadamard matrix, we first get a 4×5 COW matrix ($\mathbf{C}_{4\times 5}$). In the next try, we can find an 8×13 COW matrix ($\mathbf{C}_{8\times 13}$). According to Theorem 1, $\mathbf{C}_{8\times 13}$ leads to a 64×104 COW matrix by the Kronecker product $\mathbf{H}_8 \otimes \mathbf{C}_{8\times 13}$ (where $\mathbf{H}_8$ is an 8×8 Hadamard matrix); this implies that we can have errorless decoding for 104 users with only 64 chips; i.e., more than %62 overloading factor (we will introduce a suitable decoder for this code in Section VI). However, repetition of Theorem 2 for $\mathbf{C}_{8\times 13}$ shows the existence of a 64×164 COW matrix which implies an overloading factor of about %156.

Note 2 If a user has no data to transmit or its handset is off, we need to transmit signature codes as fill codes. Alternatively, we can design special COW codes that can handle inputs with entries $\{\pm 1, 0\}$.

III. COO for Optical CDMA

We would like to extend the results to optical CDMA, i.e., COO matrices.

Theorem 3 If there is an $m \times n$ COW matrix for the wireless CDMA, then there is an $m \times n$ COO matrix for the optical CDMA.

Corollary 2 Similarly we can show that, if we have a COO matrix with an all 1 row, then we will obtain a COW matrix by substituting the zeros of the COO matrix with -1.

Example 2 As a special case, by Example 1 and Theorem 3, we also have a 64×164 COO matrix.

The theorems for COO matrices are similar to the previous theorems related to COW matrices. In addition, there are a few extra algorithms for the construction of COO matrices as described below.

Theorem 4 If $\mathbf{D}$ is an $m \times n$ COO matrix, then $\mathbf{P} \otimes \mathbf{D}$ is also a $km \times kn$ COO matrix, where $\mathbf{P}$ is an invertible $k \times k$ $\{0,1\}$-matrix.

Corollary 3 If we set $\mathbf{P} = \mathbf{I}$ in the above theorem, then the generated COO matrices are sparse and have low weights that are suitable for optical transmission due to low power [13].

Theorem 5 Suppose $\mathbf{A} = \mathbf{J} - \mathbf{I}$ is an $m \times m$ matrix, and $V_i = [\overbrace{1 \ \cdots \ 1}^{2^i} \ \ \overbrace{0 \ \cdots \ 0}^{m-2^i}]^T$, for $i = 0, \ldots, d$, where $d = \lfloor \log_2 m \rfloor - 2$. If $\mathbf{B} = [V_0 \ \ V_1 \ \ \cdots \ \ V_d]$, then $\mathbf{C} = [\mathbf{A}|\mathbf{B}]$ is an $m \times (m + d + 1)$ COO matrix.

Example 3 Using Theorem 5, we get a 64×69 COO matrix with the structure discussed in the theorem.

The following theorem provides an upper-bound for the overloading factor for a COW matrix.

Theorem 6 If $\mathbf{C} = [c_{ij}]$ is a COW matrix with n columns (users) and m rows (chips), then

$$n \le -m \left(\sum_{i=0}^{n} \frac{\binom{n}{i}}{2^n} \log_2 \frac{\binom{n}{i}}{2^n} \right)$$

where $\binom{n}{i} = \frac{n!}{i!(n-i)!}$.

Example 4 The above upper bound is shown in Fig. 1. This figure shows that we cannot have errorless communication using 64 chips and 336 users. By using a tighter bound which we have not discussed here, the number of users for 64 chips becomes less than 268.

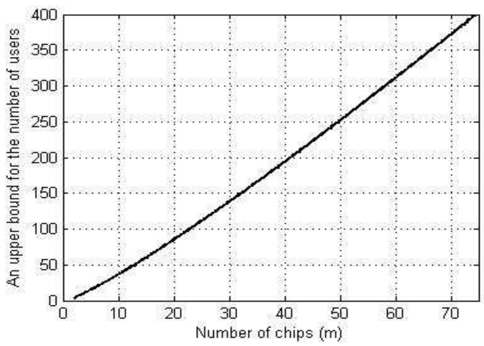


Figure 1. The upper bound of Theorem 6 for the number of users versus the number of chips.

IV. Channel Capacity for Noiseless CDMA Systems

In this section, we shall develop lower and upper bounds for the sum channel capacity [22] of an overloaded CDMA with MUD when there is no additive noise. The only interference is the overloaded users. In this case, the channel is deterministic but not lossless. The interesting result is that the lower bound estimates a region for the number of users n for a given chip rate m such that COW or COO matrices exist. To develop the lower bound, we start by the following assumptions for the wireless case but results are also valid for the optical CDMA:

For a given m and n, let $\boldsymbol{Q}_n = \{\pm 1\}^n$ and $\boldsymbol{\mathcal{F}}_{m,n}$ be the set of functions $f: \boldsymbol{Q}_n \to \mathbb{Z}^m$ defined by $f(X) = \mathbf{M}X$, where $\mathbf{M}$ is an $m \times n$ matrix with entries ± 1 and X is the input multiuser vector as defined before with entries ± 1.

Definition: The sum channel capacity function $\mathcal{C}$ is defined as

$$\mathcal{C} = \max_{f \in \mathcal{F}_{m,n}} \log_2 |f(\boldsymbol{Q}_n)|$$

where $|\ |$ denotes the number of elements of the set. It can be shown that for any matrix $\mathbf{M}$, there exists a probability distribution on $\boldsymbol{Q}_n$ such that $\mathrm{H}\big(f(X)\big) = \log_2 |f(\boldsymbol{Q}_n)|$. Since the channel is deterministic $I(X,Y) = H(Y)$ ($Y = f(X)$ is the received vector corresponding to the transmission of X), the above definition is equivalent to maximizing the mutual information $I(X,Y)$ over all the input probabilities and over all $m \times n$ $\mathbf{M}$ matrices.

Lemma 2

a- $\mathcal{C}(m,n) \le n$

b- $\mathcal{C}(m,n) \le m \log_2(\mathrm{n}+1)$

Lemma 3 If n is divisible by m, then

$$\mathcal{C}(m,n) \ge m \log_2(m+n) - m \log_2 m.$$

To get tighter bounds than the ones given in Lemmas 2 and 3, we need the following theorems:

Theorem 7 (Channel Capacity Lower Bound)

$$\mathcal{C}(m,n) \ge n - \log_2 A(m,n)$$

where $A(m,n) = \sum_{j=0}^{\lfloor n/2 \rfloor} \binom{n}{2j} \left[\binom{2j}{j} / 2^{2j}\right]^m$.

Theorem 8 (Channel Capacity Upper Bound)

$$\mathcal{C}(m,n) \le m\big(1/2 \log_2 n + \log_2 \lambda\big) + 1$$

where λ is the unique positive solution of the equation

$$\big(\lambda\sqrt{n}\big)^m = m e^{-\lambda^2} 2^{n+1}.$$

The plots of the channel capacity upper and lower bounds with respect to n, for a typical value of $m = 64$ is given in Fig. 2(a). Fig. 2(b) is a dual plot with respect to m for a fixed value of $n = 220$. Plots of the channel capacity lower bounds with respect to m and n are given in Fig. 3. The plot of the lower bound from Lemma 3 is not shown since the bound is lower than the one from Theorem 7 (see Fig. 2(a)) for $n < 1000$, however, for large n (> 4000), it is a better lower bound.

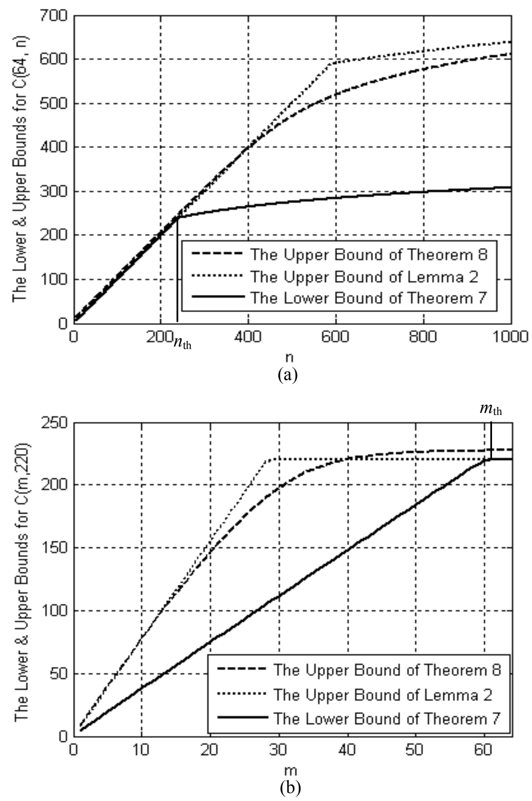
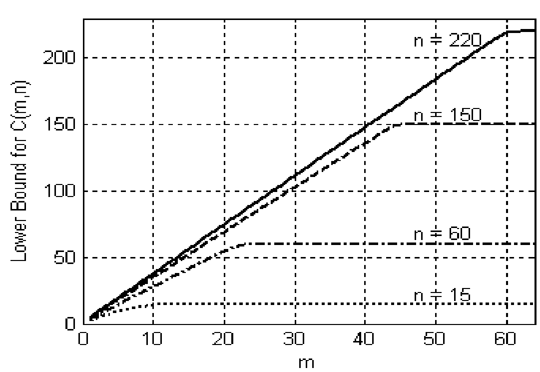
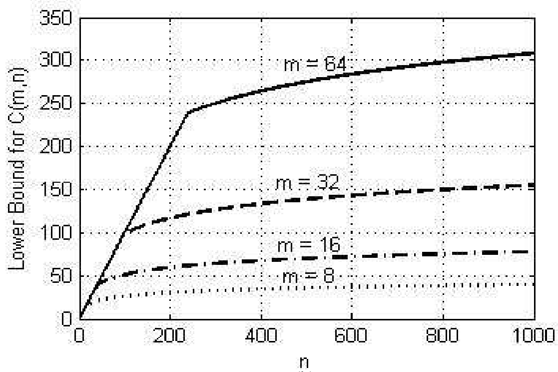


Figure 2. Lower and upper bounds for the sum channel capacity with respect to: (a) the number of users n for $m = 64$, (b) the chip rate m for $n = 220$.



(a)



(b)

Figure 3. Plots of channel capacity lower bounds for various n and m: (a) lower bounds vs. number of users n for a given chip rate m, (b) lower bounds vs. m for a given n.

Interpretation: The lower bounds show interesting and surprising results. The lower bounds essentially show two modes of behavior. In the first mode, the lower bounds for the sum channel capacity (Fig. 2(a) and Fig. 3(a)) are almost linear with respect to n for a given m, which implies the existence of codes that are almost lossless. Since we know that there exist COW (COO) codes that can achieve the sum channel capacity (number of users is equal to the sum channel capacity) without any error, the lower bound is very tight in this region. For small values of m such as 4, we know that the maximum value of $n_{\max}$ such that a COW matrix exists is 5. The sum channel capacity lower bound for $m = 4$ is 4.21 bits, which is within a fraction of integer from 5. Also, for 8×13 COW matrix, the lower bound is 12.164 bits, which is again within a fraction of an integer from 13. We thus conjecture that the maximum number of users for a COW/COO matrix for $m = 64$ is around 239 from Fig. 2(a).

After n increases beyond a threshold value n_{th} (Fig. 2(a)), the channel becomes suddenly lossy and enters the second mode of behavior. This loss is due to the fact that 2^n input points that are mapped to a subset of $(n+1)^m$ points cannot find any empty space and a fraction of them get overlapped (no longer COW or COO condition). Apparently, in the region of interest ($n_{th} < n < 500$), the lower bound is almost linear which implies that with each unit increment of n, the number of input points are doubled and a fixed number of points are overlapped at the output, i.e., $H(X|Y) < \log_2 A(m,n) \approx \log_2 (1.81)^{n-n_{th}} \cong 0.856\ (n - n_{th})$. Also, in this mode the lower bound is very close to $m/2 \log(n+1)$.

Figs. 2(b) and 3(b) show another interesting behavior. Initially, the bound increases almost linearly with m for a given n. This region is related to the case where m the chip rate is much less than the number of users n. The slope of the linear curve is determined by the number of users, e.g., at $n = 220$, the slope is almost 3.7, i.e., for each increment of m, the channel capacity is incremented by about 3.7 bits-much faster than its counterpart in Fig. 3(a). In our case, n behaves like an amplitude or power, while m behaves like frequency. As m increases beyond a threshold (m_{th} in Fig. 2(b)), the sum channel capacity remains almost constant since the capacity cannot be greater than n (Lemma 2). In fact, n is the supremum of the lower bound in this mode. This mode is the lossless case that predicts the existence of COW/COO codes.

V. Decoding

Suppose $\mathbf{D}$ is a COW or COO matrix and Y is the transmitted vector. We wish to find a vector X ($\{\pm 1\}$ for wireless systems and $\{0,1\}$ for optical systems) such that $\mathbf{D}X = Y$. Obviously, the complexity of the decoding is dependent on the overloading factor. The decoding algorithm is as follows.

If $\mathbf{D}_{km \times kn}$ is the Kronecker product of an invertible matrix $\mathbf{P}_{k \times k}$ with a COW matrix $\mathbf{C}_{m \times n}$, then the above problem reduces to solving k matrix equations with dimension of $\mathbf{C}_{m \times n}$ (which is smaller than the dimension of $\mathbf{D}_{km \times kn}$). If $Y = \boldsymbol{D}X = (\mathbf{P} \otimes \mathbf{C})X$, then $(\mathbf{P}^{-1} \otimes \mathbf{I})Y = (\mathbf{I} \otimes \mathbf{C})X$. If $X = [X_1{}^T \ \cdots \ X_k{}^T]^T$ and $(\mathbf{P}^{-1} \otimes \mathbf{I})Y = [Y_1{}^T \ \cdots \ Y_k{}^T]^T$, then $Y_i = \mathbf{C}X_i$ for $i = 1, \ldots, k$, where each X_i's are $n \times 1$ vectors.

For example, the inverse problem for the 64×104 matrix which is found by Kronecker multiplication of an 8×8 Hadamard matrix ($\mathbf{H}_8$) and $\mathbf{C}_{8 \times 13}$ can be handled by solving the inverse problem for $\mathbf{C}_{8 \times 13}$. The inverse problem for $\mathbf{C}_{8 \times 13}$ can easily be done through a look up table with the first column consisting of 13-tuple $\{\pm 1\}$-vectors and the second column is the corresponding 8-tuple vectors. In the presence of noise, we assume each of the 8-tuple sub-vectors of the 64×1 vector generated by $\left(\mathbf{H}_8{}^{-1} \otimes \mathbf{I}\right)Y'$, where Y' is the noisy version of Y at the receiver end, be the vector in the table that is the closest (in the sense of Euclidean metric). This method is equivalent to the Maximum Likelihood (ML) decoding.

VI. Simulation Results

The table look up method has been simulated for the wireless systems. The probability of errors with respect to E_b/N_0 is shown in Fig. 4.

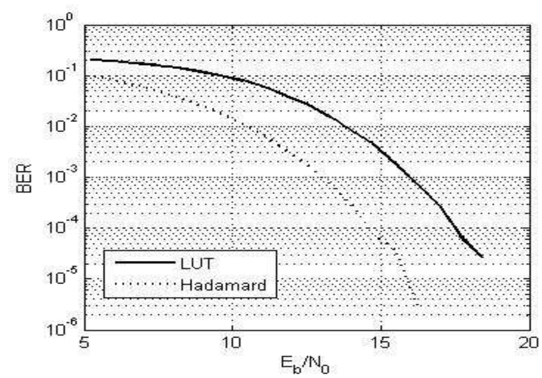


Figure 4. Bit-error-rate versus E_b/N_0. The dashed curve is a Hadamard code with 64 chips and 64 users. The solid curve is the table look up method for a COW code with 64 chips and 104 users.

This figure shows that the look-up table approach for (64,104) over-loaded CDMA is within 3dB of the orthogonal Hadamard (64,64) fully-loaded CDMA.

VII. Conclusion

In this paper, we have shown that there exists a large class of $m \times n$ codes ($m < n$) that are suitable for over-loaded synchronous CDMA both for wireless and optical systems. For a given spreading factor m, an upper bound for the number of users n has been found. For example for $m = 64$, the upper bound predicts a maximum of $n = 336$. We have found a tighter bound not discussed in this paper (5 page limitation) where n is lowered to 268. Mathematically, we have proved the existence of codes of size $(64,164)$. However, since the decoding of such over-loaded codes are not practical, we have developed codes of size $(64,104)$ that are generated by Kronecker product of a Hadamard matrix by a small matrix of size $(8,13)$. The decoding can be done by a look-up table of size 2^{13} rows. This is a ML decoder for such codes with reduced dimension.

A tight lower bound and an upper bound for the sum channel capacity of a deterministic COW or COO channel matrix has been evaluated. The lower bound suggests the existence of codes that can reach the capacity without any errors.

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