Invariant Wideband Spectrum Sensing Under Unknown Variances

Abbas Taherpour, *Student Member, IEEE,* Saeed Gazor, *Senior Member, IEEE*, and Masoumeh Nasiri-Kenari, *Member, IEEE*

***Abstract*—In this paper, we divide a wide frequency range into multiple subbands and in each subband detect whether in a primary user (PU) is active or not. We assume that PU signal at each subband and the additive noise are white zero-mean independent Gaussian random processes with unknown variances. We also assume that at least a minimum given number of subbands is vacant of PU signal and propose an invariant Generalized Likelihood Ratio (GLR) detector. The concept of the grouping of subbands allows faster spectrum sensing of a subset of subbands which may be occupied by a specific PU. Also, we evaluate trade-offs involved in the proposed algorithms by simulation.**

***Index Terms*—Spectrum sensing, cognitive radio, dynamic spectrum sharing, energy detector.**

I. Introduction

RECENT measurements reveal that many portions of the licensed spectrum are not used during significant time periods [1]. Since the number of users and their data rates steadily increase, the traditional fixed spectrum policy is inefficient and is no longer a feasible approach. One proposal for alleviating the spectrum scarcity is allowing the Secondary Users (SU) to exploit the unused spectrum holes over some frequency ranges by using Cognitive Radio (CR) technology [2]. One of the major challenges of implementing this technology is that the CRs must accurately monitor and be aware of the presence of the PUs over a particular spectrum. To address this challenge, several efficient methods have been proposed [3]–[6]. The Energy Detector (ED)(a.k.a. radiometer) is a common method to detect an unknown signal in additive noise [7]. This method is optimal for white Gaussian noise if the noise variance is known. Unfortunately, the performance of the ED is susceptible to errors in the noise variance [8]. It has been shown that to achieve a desired probability of detection under uncertain noise variance, the Signal-to-Noise Ratio (SNR) has to be above a certain threshold [9]. While there has been an intensive work on the spectrum sensing problem in the case of known noise variance, not enough attention has been made to the spectrum sensing under unknown noise variance except [9]–[12].

In this paper, we investigate the spectrum sensing problem in the case of unknown noise variance. Even though due to man-made noise or the other causes, we may interfere with the colored noise [9], [13], in this paper we consider an additive white Gaussian noise. In Section II, we assume that the variance of the additive white zero-mean Gaussian noise is unknown and at least a minimum given number of subbands is vacant of PUs. We also model the signals of the PUs as zero-mean Gaussian processes with unknown variances. In Section III, we derive the GLR detector for each of the subbands. The concept of grouping and the computational complexity of the proposed detector are investigated in Section III-A and Section III-B, respectively. The simulation results and available tarde-offs have been provided in Section IV.

Manuscript received June 11, 2008; revised October 12, 2008 and December 30, 2008; accepted January 5, 2009. The associate editor coordinating the review of this letter and approving it for publication was A. Yener.

A. Taherpour and M. Nasiri-Kenari are with the Department of Electrical Engineering, Sharif University of Technology, Tehran, Iran (e-mail: taherpour@ieee.org).

S. Gazor is with the Department of Electrical and Computer Engineering, Queen's University, Kingston, ON, Canada.

This work was supported by the Iran Telecommunication Research Center (ITRC) and Natural Sciences and Engineering Research Council of Canada (NSERC).

Digital Object Identifier 10.1109/TWC.2009.080768

II. Signal Model and Problem Statement

Suppose that the frequency bandwidth is divided into K subbands. The SU receives L samples for each frequency subband (for example using K-point Fast Fourier Transformation (FFT)). We also assume that the signal samples are independent random variables with complex Gaussian distributions. This assumption, for instance, is valid for Orthogonal Frequency Division Multiplexing (OFDM) signal in which each carrier is modulated by an independent data stream. The detection at the $k^{\text{th}} \in \mathcal{S} \triangleq \{1, 2, \cdots, K\}$ subband is written as:

$$P_k : \begin{cases} \mathbf{X}_k \sim \mathcal{CN}(0, \sigma^2 I_L), & \mathcal{H}_o^k : \quad \text{(vacant)} \\ \mathbf{X}_k \sim \mathcal{CN}(0, \sigma_k^2 I_L), & \mathcal{H}_1^k : \quad \text{(occupied)}. \end{cases} \tag{1}$$

where $\mathcal{CN}(\mathbf{0}, \sigma_k^2 I_L)$ denotes a zero-mean L-dimensional multivariate complex Gaussian distribution, I_L is the $L \times L$ identity matrix, σ^2 and σ_k^2 are the noise and the received signal variance under the condition that a PU is absent and present at k^{th} subband, respectively. We assume that the variance of noise σ^2 is approximately constant across the scanning subbands (white noise). The objective is to determine the status of the subbands $\mathcal{S}$, and partition $\mathcal{S}$ as $\mathcal{S} = \mathcal{S}_v \cup \mathcal{S}_o$, $\mathcal{S}_v \cap \mathcal{S}_o = \phi$, where $\mathcal{S}_v$ and $\mathcal{S}_o$ denote the subsets of indexes which are vacant or occupied by a PU, respectively.

In the context of dynamic spectrum sharing, the false alarm probability P_{fa} indicates the probability that a spectrum hole (a vacant subband) is falsely detected as an occupied band, i.e., P_{fa} represents the percentage of the spectrum holes which are not used. Therefore, the SUs must reduce the false alarm probability P_{fa} as much as possible. On the other hand, the missed detection probability, i.e., $P_m = 1 - P_d$, determines the probability that an occupied subband is mistakenly detected as a spectrum hole. Such a missed detection induces harmful interference for PU. Thus, the missed detection probability must be small enough to avoid perceptible performance loss for the PU. Under white Gaussian noise assumption, it can be easily observed that these probability values are the same for all subbands. From (1), we have K interrelated composite hypothesis testing problems with $K+1$ unknown parameters, i.e., σ^2 and $\{\sigma_k^2\}_{k=1}^K$. In general, we can not find a proper solution for each of the above K problems if we attempt to solve them individually. Thus, since the noise variance σ^2 is a common unknown parameter, we treat these problems altogether. We assume that at least a given minimum number of subbands (M) out of K subband are vacant, i.e., $|\mathcal{S}_v| \geq M$ where $|.|$ is the cardinality of $(.)$. Let $\mathcal{D} \subseteq \mathcal{S}_v$ ($|\mathcal{D}| = M$) denote the indexes of M vacant subbands[1]. This assumption is quite reasonable, since the current dynamic spectrum sharing research is motivated by the fact that many parts of the spectrum are under-utilized in the most of the times which makes the concept of spectrum sharing to be attractive. Our assumption is justified by the same fact. In practice, one may obtain the minimum number of vacant subbands M using the activity history of PUs over the whole spectrum. It is obvious that for a vacant subband we have $\sigma_k^2 = \sigma^2$ and for an occupied subband $\sigma^2 < \sigma_k^2$ as σ_k^2 is the variance of the PU signal plus the noise variance σ^2.

III. PROPOSED GLR DETECTOR

For the k^{th} subband, the Probability Density Functions (PDF) of the observations under the hypotheses $\mathcal{H}_0^k$ and $\mathcal{H}_1^k$ are respectively given by:

$$f(\mathbf{X};\mathcal{H}_0^k) = \frac{1}{(\pi\sigma^2)^{L(|\mathcal{D}\cup\{k\}|)}} \exp\left\{-\frac{1}{\sigma^2}\sum_{m\in\mathcal{D}\cup\{k\}}\|\mathbf{X}_m\|^2\right\} \times \prod_{m\in\mathcal{S}\backslash(\mathcal{D}\bigcup\{k\})} \frac{1}{\pi^L\sigma_m^{2L}} \exp\left\{-\frac{1}{\sigma_m^2}\|\mathbf{X}_m\|^2\right\}, \tag{2}$$

$$f(\mathbf{X};\mathcal{H}_1^k) = \frac{1}{(\pi\sigma^2)^{L(|\mathcal{D}\backslash\{k\}|)}} \exp\left\{-\frac{1}{\sigma^2}\sum_{m\in\mathcal{D}-\{k\}}\|\mathbf{X}_m\|^2\right\} \times \prod_{m\in\mathcal{S}\backslash(\mathcal{D}\backslash\{k\})} \frac{1}{\pi^L\sigma_m^{2L}} \exp\left\{-\frac{1}{\sigma_m^2}\|\mathbf{X}_m\|^2\right\}. \tag{3}$$

where σ_m^2, σ^2 and $\mathcal{D}$ are unknown parameters, $\mathbf{X} = [\mathbf{X}_1, \cdots, \mathbf{X}_K] \in \mathbb{C}^{L\times K}$ is the input data matrix and $\|.\|$ denotes the Euclidian norm of a vector. We substitute the Maximum Likelihood (ML) estimations of these parameters under each hypothesis in order to derive a GLR detector. By maximizing the above PDFs, the ML estimations of σ^2 and σ_i^2 under $\mathcal{H}_0^k$ and $\mathcal{H}_1^k$ are

$$\mathcal{H}_0^k: \quad \begin{cases} \widehat{\sigma^2} = \frac{1}{|\mathcal{D}\cup\{k\}|L}\sum_{m\in\mathcal{D}\cup\{k\}}\|\mathbf{X}_m\|^2 \\ \widehat{\sigma_i^2} = \frac{\|\mathbf{X}_i\|^2}{L}, \quad \forall i\in\mathcal{S}\backslash(\mathcal{D}\bigcup\{k\}) \end{cases} \tag{4}$$

$$\mathcal{H}_1^k: \quad \begin{cases} \widehat{\sigma^2} = \frac{1}{|\mathcal{D}\backslash\{k\}|L}\sum_{m\in\mathcal{D}-\{k\}}\|\mathbf{X}_m\|^2 \\ \widehat{\sigma_i^2} = \frac{\|\mathbf{X}_i\|^2}{L}, \quad \forall i\in\mathcal{S}\backslash(\mathcal{D}\backslash\{k\}). \end{cases} \tag{5}$$

[1]Please note that $\mathcal{S}_v$ denotes the indexes of all vacant subbands, while $\mathcal{D}$ as a subset of $\mathcal{S}_v$ denotes the indexes of M vacant subbands. Since $\mathcal{S}_v$ is not known in advance, with the assumption that $|\mathcal{S}_v| \geq M$, at the first step of the detection algorithm, the subset $\mathcal{D}$ with the cardinality of M is estimated, as will be discussed later.

The noise variance in (4-5) is estimated as the ensemble mean of the energies over $\mathcal{D}$. To find the ML estimation of the subset $\mathcal{D}$, we substitute σ^2 and σ_i^2 from (4) and (5) respectively in (2) and (3) and maximize the results over $\mathcal{D}$. We easily see that the ML estimate of $\mathcal{D}$ (obtained by these optimizations under $\mathcal{H}_0^k$ and $\mathcal{H}_1^k$) for both cases is the set of the indexes of the M smallest energy values in the vector $[\|\mathbf{X}_1\|^2, \|\mathbf{X}_2\|^2, \cdots, \|\mathbf{X}_K\|^2] = \text{diag}[\mathbf{X}^{\mathrm{H}}\mathbf{X}]$. In [12], a similar result for the ML estimate of $\mathcal{D}$ is proven using maximal invariant approach where the signals are treated as parameters in the context of array signal processing. In addition, to satisfy the condition $\sigma^2 \leq \sigma_k^2$ we require the subset $\mathcal{D}$ be chosen such that

$$\frac{1}{|\mathcal{D}\cup\{k\}|}\sum_{m\in\mathcal{D}\cup\{k\}}\|\mathbf{X}_m\|^2 \leq \frac{1}{K}\sum_{m\in\mathcal{S}}\|\mathbf{X}_m\|^2. \tag{6}$$

Interestingly, the above mentioned ML estimate of $\mathcal{D}$ guarantees (6). This result justifies our intuition, as the subbands with lower energy levels are more likely to contain only noise. Thus, the subset $\widehat{\mathcal{D}}$ is selected by sorting the vector of subbands energies. In this operation, an occupied subband may be mistakenly misclassified in $\widehat{\mathcal{D}}$ only if the observed energy in that subband is smaller than at least $|\mathcal{S}| - M + 1$ other subbands. We refer to this unlikely case as a misclassification in which a missed detection may also happen or be prevented in the detection step. By substituting (4) and (5) respectively in (2) and (3), we obtain the following Likelihood Ratio (LR) function:

$$\text{LR}_k(\mathbf{X}) = \frac{f(\mathbf{X};\mathcal{H}_1^k)|_{(5)}}{f(\mathbf{X};\mathcal{H}_0^k)|_{(4)}} = \frac{\left(\frac{1}{|\widehat{\mathcal{D}}\cup\{k\}|L}\sum_{m\in\widehat{\mathcal{D}}\cup\{k\}}\|\mathbf{X}_m\|^2\right)^{|\widehat{\mathcal{D}}\cup\{k\}|L}}{\left(\frac{\|\mathbf{X}_k\|^2}{L}\right)^L\left(\frac{1}{|\widehat{\mathcal{D}}\backslash\{k\}|L}\sum_{m\in\widehat{\mathcal{D}}\backslash\{k\}}\|\mathbf{X}_m\|^2\right)^{|\widehat{\mathcal{D}}\backslash\{k\}|L}}. \tag{7}$$

which can be written in the following form:

$$\text{LR}_k(\mathbf{X}) = \begin{cases} \frac{(M-1)^{(M-1)L}}{M^{ML}}\frac{\left(\sum_{m\in\widehat{\mathcal{D}}}\|\mathbf{X}_m\|^2\right)^{ML}}{\left(\sum_{m\in\widehat{\mathcal{D}}\backslash\{k\}}\|\mathbf{X}_m\|^2\right)^{(M-1)L}}, & \text{for k} \in \widehat{\mathcal{D}} \\ \frac{M^{ML}}{(M+1)^{(M+1)L}}\frac{\left(\sum_{m\in\widehat{\mathcal{D}}\cup\{k\}}\|\mathbf{X}_m\|^2\right)^{(M+1)L}}{\left(\sum_{m\in\widehat{\mathcal{D}}}\|\mathbf{X}_m\|^2\right)^{ML}}, & \text{for k} \notin \widehat{\mathcal{D}}. \end{cases} \tag{8}$$

Thus, the Log-Likelihood Ratio (LLR) function is given by

$$\text{LLR}_k(\mathbf{X}) = \begin{cases} Lc(M) - L\log\left(\lambda_k(\mathbf{X})\left(1-\lambda_k(\mathbf{X})\right)^{M-1}\right), & \text{for k} \in \widehat{\mathcal{D}} \\ Lc(M+1) + L\log\frac{(1+\lambda_k(\mathbf{X}))^M}{\lambda_k(\mathbf{X})}, & \text{for k} \notin \widehat{\mathcal{D}}. \end{cases} \tag{9}$$

where $c(M) = \log\frac{(M-1)^{M-1}}{M^M}$ and $\lambda_k(\mathbf{X})$ is defined by

$$\lambda_k(\mathbf{X}) \triangleq \frac{\|\mathbf{X}_k\|^2}{\sum_{m\in\widehat{\mathcal{D}}}\|\mathbf{X}_m\|^2}. \tag{10}$$

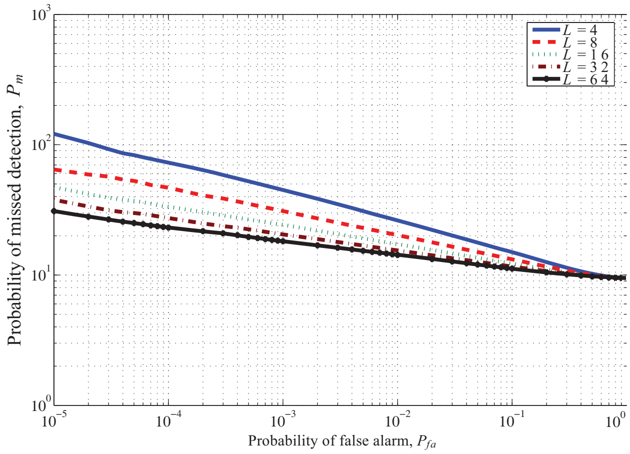

Fig. 1. The threshold versus probability of false alarm P_{fa} for different number of samples L and, $K = 64$, and $M = 16$.

For making decision, we compare $\mathrm{LLR}_k(\mathbf{X})$ to a threshold, i.e., the GLR test is simplified as

$$T^k_{\mathrm{GLR}}(\mathbf{X}) = \tag{11}$$
$$\begin{cases} c(M) - \log\left(\lambda_k(\mathbf{X})\left(1-\lambda_k(\mathbf{X})\right)^{M-1}\right), & \text{for k} \in \widehat{\mathcal{D}} \\ c(M+1) + \log\frac{(1+\lambda_k(\mathbf{X}))^M}{\lambda_k(\mathbf{X})}, & \text{for k} \notin \widehat{\mathcal{D}}. \end{cases} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta$$

In general, the detection threshold, i.e. η, is obtained by solving $F(\eta) = 1 - P_{\mathrm{fa}}$, where P_{fa} denotes the false alarm probability and $F(x)$ is the Cumulative Distribution Function (CDF) of the decision statistic under hypothesis $\mathcal{H}_0$. Also, it is possible to find this threshold by Monte-Carlo simulation method. Since the CDF of the decision statistic in (11) does not have a known closed form, we use the Monte-Carlo simulation method to obtain the threshold. Figure 1 illustrates the plots of the threshold versus the probability of false alarm P_{fa} for different number of samples L. We see that the threshold value η decreases at a given false alarm probability P_{fa} by increasing the number of samples L. From these plots we observe that approximately the logarithm of the threshold value $\log\eta$ is a linear function of $\log P_{\mathrm{fa}}$ as $\log\eta \approx -\alpha(L)\log P_{\mathrm{fa}} + \beta(L)$, where the $\alpha(L)$ and $\beta(L)$ are decreasing functions of L.

Consider the case that an occupied subband k is misclassified, i.e., $k \in \widehat{\mathcal{D}}$. In this case, the decision statistic in (11) can possibly assist us to avoid the missed detection. Please note that for each subbband, regardless of being member of $\hat{\mathcal{D}}$ or not, the decision statistic is constructed by using (11) and the final decision is made about the status, accordingly. That is, $\widehat{\mathcal{D}}$ is not necessarily the subset of subbands which are detected as vacant, and the final decision is based on (11). This case may happen if the number of vacant subbands is relatively small compared with the total number of subbands. Note that the false alarm probability and the corresponding threshold are independent of the unknown parameters, and the detector in (11) is invariant with respect to the noise variance. For instance, under the null hypothesis $\mathcal{H}_0$, $T^k_{\mathrm{GLR}}(\mathbf{X})$ in (11) is invariant with the noise variance σ^2, because $\lambda_k(\mathbf{X}) = \lambda_k(\frac{1}{\sigma}\mathbf{X})$ and hence $T^k_{\mathrm{GLR}}(\mathbf{X}) = T^k_{\mathrm{GLR}}(\frac{1}{\sigma}\mathbf{X})$. In the other words $\frac{1}{\sigma}\mathbf{X}_k \sim \mathcal{CN}(0, I_L)$ under $\mathcal{H}^k_o$ and the decision statistics are identical for the noise variances σ^2 and 1, and thus $T^k_{\mathrm{GLR}}(\mathbf{X})$ is independent of the unknown parameters. This means that a threshold obtained for a given P_{fa} is independent of the SNR value or the noise variance.

A. Grouping of subbands

In the grouping, we assume that the SU for spectrum sensing has knowledge about some adjacent or non-adjacent subbands used simultaneously by a specific PU. That is the SU knows the exact position of this group of subbands by considering a specific part of spectrum which may be occupied by a known application. The spectrum sensing for such a scenario can be considerably enhanced. By grouping, we mean the integration of the information obtained from such a group of subbands. All subbands in a group have a common status regarding the presence or absence of the PU. In the other words, only one binary hypothesis is required for a given group which results in a faster spectrum sensing approach. Let the subset $G \subseteq \mathcal{S}$ denote the indexes of $|G|$ subbands which are used by a specific PU. By following the similar steps as taken for a single subband, it can be easily shown that the GLR detector for this group has the same form of single subband detector in (11) but in this case the variable $\lambda_G(\mathbf{X})$ defined below is used instead of $\lambda_k(\mathbf{X})$

$$\lambda_G(\mathbf{X}) \triangleq \frac{\sum_{k\in G}\|\mathbf{X}_k\|^2}{\sum_{m\in\widehat{\mathcal{D}}}\|\mathbf{X}_m\|^2}. \tag{12}$$

Similarly, the subset $\widehat{\mathcal{D}}$ is the set of the indexes of the M smallest elements of the set $\left\{\|\mathbf{X}_m\|^2\right\}_{m=1}^{K}$ and the threshold is determined by satisfying a given P_{fa}.

B. Computational Complexity

In practice, the proposed GLR detector can be implemented easily and has a low computational complexity (CC) summarized as follows:

- Temporal signal transformation into frequency domain: By using FFT, the CC of this part is $\mathcal{O}(KL\log_2 K)$ (complex multiplication and addition) since K point FFT is employed L times.
- Selecting the subset $\widehat{\mathcal{D}}$: By calculating $\|\mathbf{X}_1\|^2, \|\mathbf{X}_2\|^2, ..., \|\mathbf{X}_K\|^2$ and choosing the indexes of M smallest ones. The CC of this step is in order of $\mathcal{O}(KL)$ (plus CC of finding smallest M numbers out of K numbers which is less than $K\log_2 K$).
- Constructing the parameter $\lambda_k(\mathbf{X})$ in (10) for all subbands: The required CC for this step is only in order of $\mathcal{O}(M)$.
- Calculating the decision statistics in (11) for all subbands: The required CC for this step is in order of $\mathcal{O}(K)$.

The Overall CC order of the algorithm is as low as $\mathcal{O}(KL\log_2 K)$. Obviously the dominant computational cost is imposed by the FFT.

If we use the concept of grouping of subbands, the CC of computing $\lambda_G(\mathbf{X})$ and the decision statistics are in order of the number of groups instead of K, and hence again the dominant computational cost is due to the FFT.

For comparison, for energy detection (ED), similar to the

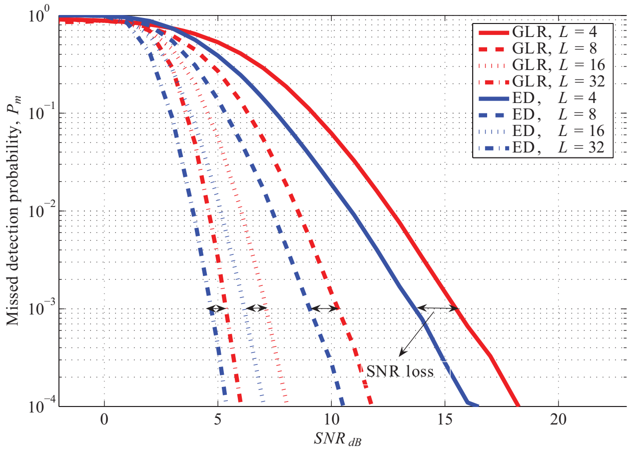

Fig. 2. The probability of missed detection for the GLR detector and the optimal ED versus SNR for different number of samples L and $P_{\text{fa}} = 10^{-2}$, $K = 64$, and $M = 16$.

proposed GLR detector, we need to transform the temporal signal to the frequency domain and then calculate the energies $\|\mathbf{X}_1\|^2, \|\mathbf{X}_2\|^2, \cdots, \|\mathbf{X}_K\|^2$ which their computational complexities are in orders of $\mathcal{O}(KL\log_2 K)$ and $\mathcal{O}(KL)$, respectively. Therefore, the overall CC order of the ED is $\mathcal{O}(KL\log_2 K)$, which is the same as the proposed detector.

IV. Simulation Results

Figure 2 depicts the probability of missed detection P_m of the proposed GLR detector and the optimal ED versus SNR for different number of samples L at a false alarm rate of $P_{\text{fa}} = 10^{-2}$. In order to determine the threshold for a given false alarm probability, we have generated the decision statistic randomly according to the assumed distributions for 10^6 independent trials (in absence of PU signal) and chosen the detection threshold as $100P_{\text{fa}}$ percentile of the generated data, i.e., for $P_{\text{fa}} = 10^{-3}$, $100\times10^{-3} = 0.1\%$ of the generated decision statistic (out of 10^6) are above the determined threshold. For simulation, the locations of M vacant subbands are chosen uniformly out of K available subbands, though in practical wideband spectrum sensing, the neighboring subbands might have correlated spectrum occupancy statistics [2]. As expected, from Figure 2, we can see that the performance of this detector improves by increasing SNR and L. For instance at $P_{\text{m}} = 10^{-3}$, the SNR gaps between the ED and the proposed GLR detector are 2.12dB, 1.06dB, 0.93dB and 0.53dB, respectively for $L = 4, 8, 16$ and 32. This gap is created because the knowledge of the noise variance is not provided to the GLR detector. Using this simulation at $P_{\text{m}} = 10^{-3}$, we conclude that the SNR loss between these detectors is approximately equal to $\text{SNR(GLR)} - \text{SNR(ED)} \approx -0.49\log_2 L + 2.875$ for $L < 64$. For larger number of samples L, the SNR gap is indeed negligible, i.e., for a given probability of missed detection, the SNR loss gap between the optimal ED and GLR detector reduces to almost zero as L increases. This is justified intuitively, as more number of samples results in more information about the noise variance and hence the performance improves. Unfortunately, we can not increase L arbitrarily since L determines the acquisition time (the waiting time-lag before a decision can be made). Thus in practice, we have to make a trade-off between P_{fa} (the spectrum usage efficiency), P_{m} (PU interference protection level) and L (the acquisition time).

[2]please note that under a white Gaussian noise assumption, the method of vacant subband selection does not have any effect on the detection algorithm performance.

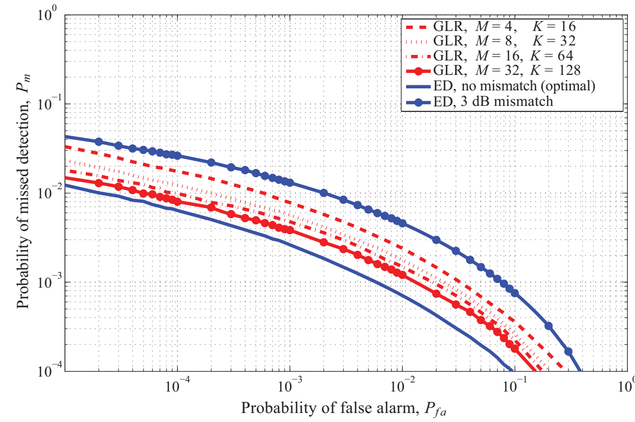

Fig. 3. The effect of K and M on the optimal ED and the proposed GLR detector for SNR=10dB, $L = 8$, and $\frac{K}{M} = 4$.

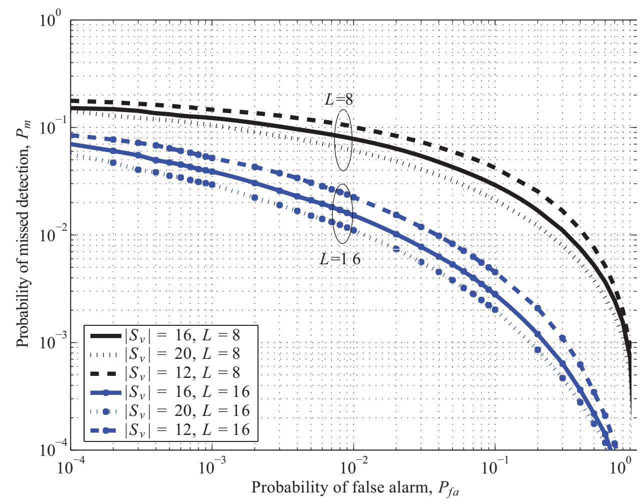

Fig. 4. The effect of over-estimation and under-estimation of vacant subbands M on the performance of the proposed GLR detector for the assumed value $M = 16$, SNR = 5dB and $K = 32$.

Figure 3 illustrates the effect of the number of subbands K and the minimum number of vacant subbands M on the performance of GLR detector, for $L = 8$ and $\frac{M}{K} = \frac{1}{4}$. We observe that the GLR detector performance improves as K and M are increased given a fixed percentage of vacant subbands. We expect that the GLR detector performance approaches to that of the optimal ED as $K \rightarrow \infty$. In this figure, the performance of the ED for a noise variance mismatch of $|10\log_{10}(\frac{\tilde{\sigma}^2}{\sigma^2})| = 3\text{dB}$ is also provided for comparison. For this specific case, the GLR detector and ED perform similarly for 2.4dB mismatch between the true value of the noise variance σ^2 and the value given to the ED $\tilde{\sigma}^2$. Interestingly, when the provided noise variance to the ED is not accurately close to the true value, the GLR detector outperforms the ED.

Figure 4 shows the performance of the proposed GLR

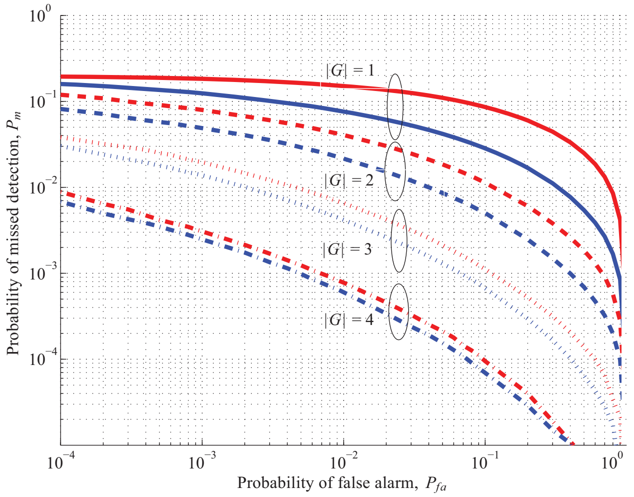

Fig. 5. The effect of the grouping on the performance of optimal ED (blue curves) and GLR detector (red curves) for different number of groups $|G|$ and, SNR=10dB, $L = 4$, $K = 128$, and $M = 32$.

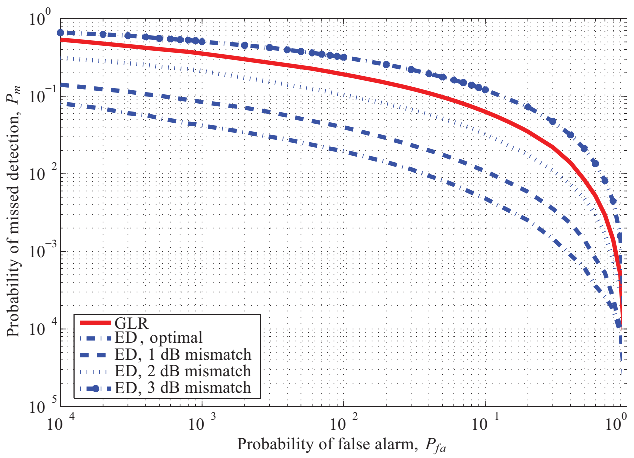

Fig. 6. The effect of mismatch on the performance of the optimal ED and the proposed GLR detector for $L = 4$, SNR = 10dB, $K = 64$, and $M = 16$.

detector when the number of minimum vacant subbands M is over-estimated or under-estimated for number of samples $L = 8$ and $L = 16$ and $K = 32$. As can be observed, when the number of actual vacant subbands is more than the value assumed $(M = 16)$, the better variance estimation is obtained and hence the performance improves. On the other hand, when the actual number of subbands is less than the value assumed $(M = 16)$, the performance is degraded slightly. Also, as can be observed from this figure, by increasing the number of samples, i.e. L, the performance improves. Therefore when the number of vacant subband is not enough, more samples can be used in order to have a better estimation of noise variance and to improve the performance of the GLR detector at the cost of increased sensing time.

Figure 5 depicts the effects of grouping on the performance of the proposed GLR detector. Also, the curves of ED performance are plotted for comparison. We observe that the grouping significantly improves the detection performance, and also by grouping more subbands, the improvement is more. Actually, at high SNRs, we expect, the improvement obtained by grouping to be equivalent to the case where L is increased to $|G|L$; this is also comparable with $10\log_{10}(|G|)$dB increase in SNR. In Figure 6, we compare the proposed GLR detector with the optimal ED under different SNR mismatches 0dB, 1dB, 2dB and 3dB. From this figure, we conclude that the ED outperforms the GLR detector provided that the ED knows the noise variance accurately enough. However, in practice, there is uncertainty about the noise variance. Obviously under unavoidable mismatches, the GLR detector significantly outperforms the ED.

V. Conclusion

In this paper, we have considered the spectrum sensing of several subbands when the background noise is white and has an unknown variance. We have proposed a GLR invariant test for each subband assuming that a minimum given number of subbands is vacant of PU signal. The proposed method has a low computational complexity and is implemented easily. Provided having some knowledge about the bandwidths of potential PUs, we have proposed the grouping approach which improves the detection performance significantly.

References

[1] Federal Communications Commissions, “Spectrum policy task force report (ET Docket No. 02-135)," Nov. 2002.

[2] S. Haykin, “Cognitive radio: brain-empowered wireless communications," *IEEE J. Select. Areas Commun.*, vol. 23, pp. 201-220, Feb. 2005.

[3] A. Taherpour, Y. Norouzi, M. Nasiri-Kenari, A. Jamshidi, and Z. Zeinalpour-Yazdi, “Asymptotically optimum detection of primary user in cognitive radio networks," *IET Commun.*, vol. 1, pp. 1138-1145, Dec. 2007.

[4] G. Ganesan and Y. Li, “Cooperative spectrum sensing in cognitive radio, part II: multiuser networks," *IEEE Trans. Wireless Commun.*, vol. 6, pp. 2214-2222, June 2007.

[5] K. Kim, I. A. Akbar, K. K. Bae, J.-S. Um, C. M. Spooner, and J. H. Reed, “Cyclostationary approaches to signal detection and classification in cognitive radio," in *DySPAN2007*, pp. 212-215, Apr. 2007.

[6] A. Ghasemi and E. Sousa, “Opportunistic spectrum access in fading channels through collaborative sensing," *J. Commun.*, vol. 2, pp. 71-82, Mar. 2007.

[7] H. Urkowitz, “Energy detection of unknown deterministic signals," *Proc. IEEE*, vol. 55, pp. 523-531, Apr. 1967.

[8] A. Sonnenschein and P. M. Fishman, “Radiometric detection of spread-spectrum signals in noise of uncertain power," *IEEE Trans. Aerospace Electron. Systems*, vol. 28, pp. 654-660, July 1992.

[9] R. Tandra and A. Sahai, “SNR qalls for signal detection," *IEEE J. Select. Topics Signal Processing*, vol. 2, no. 1, pp. 4-17, 2008.

[10] A. Taherpour, S. Gazor, and M. Nasiri-Kenari, “Wideband spectrum sensing in unknown white Gaussian noise," *IET Commun.*, vol. 2, pp. 763-771, July 2008.

[11] A. Sahai, N. Hoven, S. Mishra, and R. Tandra, “Fundamental tradeoffs in robust spectrum sensing for opportunistic frequency reuse," submitted to *IEEE J. Select. Areas Commun.*, vol. 1, 2006.

[12] A. A. Tadaion, M. Derakhtian, S. Gazor, and M. R. Aref, “A fast multiple-source detection and localization array signal processing algorithm using the spatial filtering and ML approach," *IEEE Trans. Signal Processing*, vol. 55, pp. 1815-1827, May 2007.

[13] M. Wellens, J. Wu, and P. Mahonen, “Evaluation of spectrum occupancy in indoor and outdoor scenario in the context of cognitive radio," in *Proc. 2nd Conf. Cognitive Radio Oriented Wireless Networks Commun. (CROWNCOM)*, 2007.