Errorless Codes For CDMA Systems with Near-Far Effect

M. H. Shafinia, P. Kabir, P. Pad, S. M. Mansouri, F. Marvasti
Advanced Communications Research Institute (ACRI),
Department of Electrical Engineering, Sharif University of Technology
Tehran, Iran
Email: {shafinia, pouryakabir, pedram_pad, m_mansouri}@ee.sharif.edu and marvasti@sharif.edu

Abstract—In this paper we propose a new model for the near-far effect in a CDMA system. We derive upper and lower bounds for the maximum near-far effect for errorless transmission. Using these bounds, we propose some near-far resistant codes. Also a very low complexity ML decoder for a subclass of the proposed codes is suggested.

I. INTRODUCTION

In a DS-CDMA system, every user is assigned a signature vector for transmitting data through a common channel (in time and frequency). In the channel all the transmitted vectors are added up and the resultant vector is observed at the receiver. In a synchronous CDMA system in the presence of noise, the channel model is

$$Y = \boldsymbol{C}X + N \qquad (1)$$

where $\boldsymbol{C}$ is the signature matrix, X is the user input vector with $\{\pm 1\}$-entries and N is the additive noise vector.

Without perfect power control, the assumption of receiving $+1$ or -1 from each user is no more valid. The channel model for such near-far effect is

$$Y = \boldsymbol{CA}X + N \qquad (2)$$

where $\boldsymbol{A} = \mathrm{diag}(a_1, \cdots, a_n)$ is the matrix of the received amplitude corresponding to each user.

Most of the works on compensating the near-far effect in CDMA are about introducing new methods for decoding such as MMSE in combination with Successive Interference Cancellation (SIC) [1] and blind adaptive interference suppression [2]. In [3], the method of Isolation Bit Insertion (IBI) is introduced for combating the near-far effect. In [4], the authors have found upper and lower bounds for near-far resistance of an MMSE detector. These methods are based on estimating the received power for each user. But in highly overloaded systems (when the number of users is much more than the chip rate), the conventional algorithms for estimating the user received power do not work properly because of the high multi user interference. Thus, the previously proposed methods for combating the near-far effect suddenly collapse; the near-far robustness does not only depend on the decoding scheme but also on the signature sequences we use.

In this paper we propose a class of signature codes and decoding algorithm that is robust against near-far effect without estimating the user received powers. In recent works [5-6], new types of codes are introduced for overloaded CDMA that guarantee errorless communication in noiseless channels. We plan to extend the idea of those papers to introduce new types of codes which are near-far resistant and guarantee errorless communication in a noiseless channel. Simulation results show that these codes are robust in noisy channels or when the near-far effect is more than the predicted value. In addition, we introduce low complexity ML decoders for a subclass of these codes.

The coding/decoding scheme proposed in this paper is suitable for CDMA systems with very high fluctuations of received powers at the receiver. This scheme is discussed in the worst case and in fact is a lower bound for its real performance. This means that although the proposed scheme shows an acceptable performance, its performance will be better in practice.

In the next section, we introduce the channel model; section III contains some upper and lower bounds for the near-far effect for errorless transmission. In Section IV, we propose a construction method for a class of errorless near-far robust codes. Section V includes the ML decoder and a suboptimum decoder for a subclass of these codes. Simulation results are discussed in Section VI. Summary and future works are covered in section VII.

II. THE CHANNEL MODEL

In a CDMA system with no near-far effect, the diagonal matrix $\boldsymbol{A}$ discussed in the previous section is the identity matrix. For modeling the near-far effect in a CDMA channel, we assume that the received power of each user is bounded. Thus, every entry of the diagonal of the matrix $\boldsymbol{A}$ belongs to the interval $[1-\eta, 1+\eta]$ for an $\eta < 1$ (because of the symmetry among the users, η can be assumed to be independent from the user index). We define the Near-Far Factor (NFF) of a CDMA system by $10\log\frac{\min p_i}{\max p_i}$, where $\max p_i$ and $\min p_i$ are the maximum and the minimum received power for each user. Thus, in our model, we have $\mathrm{NFF} = 20\log\frac{1-\eta}{1+\eta}$.

A CDMA system with near-far effect with a noisy channel, can be modeled as

$$Y = \boldsymbol{C}(X + Z) + N \qquad (3)$$

where $\boldsymbol{C}$ is an $m \times n$ signature matrix with normalized columns, X is an $n \times 1$ user data vector. Z is a random vector with entries belonging to the interval $[-\eta\,,\eta]$, and N is the noise vector. It is equivalent to (2), if we set a_i's random variables with values in $[-\eta\,,\eta]$. In this paper, we concentrate on the noiseless case so the model reduces to

$$Y = \boldsymbol{C}(X + Z) \qquad (4)$$

A natural problem arises about the maximum near-far effect in a CDMA system that does not affect unique decodability. Obviously, it depends on the signature matrix $\boldsymbol{C}$. According to [5], errorless communication is possible if the matrix $\boldsymbol{C}$ introduces a one-to-one mapping on $\{\pm 1\}^n$. In the presence of near-far effect, errorless communication is possible if the 2^n m-dimensional shapes $\boldsymbol{C} \cdot (X + [-\eta\,,\eta]^n)$, $X \in \{\pm 1\}^n$ are mutually disjoint (Fig. 1).

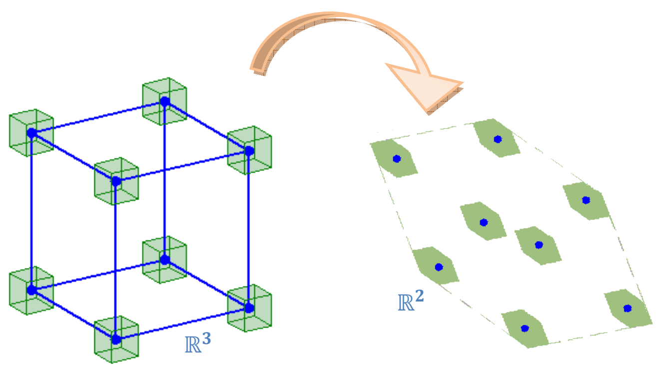


Fig. 1. The 3-dimensional data vectors with near-far effect and the corresponding 2-dimensional received vectors.

Definition Define

$$\eta_{\sup}(\boldsymbol{C}) = \sup\{\eta | \boldsymbol{C} \cdot (X + [-\eta\,,\eta]^n) \text{ are disjoint s for all } X \in \{\pm 1\}^n\}.$$

consequently,

$$\mathrm{NFF}_{\inf} = 20 \log \frac{1 - \eta_{\sup}}{1 + \eta_{\sup}}.$$

In the following section, we find upper and lower bounds for $\eta_{\sup}(\boldsymbol{C})$.

III. Upper and Lower Bounds Theorems

In this section, we will find an upper bound for η under the assumption of errorless transmission.

Theorem 1 For every m and n we have,

$$\eta_{\sup}(\boldsymbol{C}_{m \times n}) \leq \frac{1}{2^{\frac{n}{m}} - 1} \qquad (5)$$

Proof: Assume that the image of the cube $[-1, +1]^n$ by the linear transformation $\boldsymbol{C}X$ is $\mathcal{F}$. Suppose that the m-dimensional volume of $\mathcal{F}$ is v. Since the channel is errorless, the n-dimension cube with the side length 2η around the points $\{\pm 1\}^n$ must be mapped to non-overlapping shapes. Because all these shapes are placed in the image of the cube with side length $2 + 2\eta$, we have

$$\text{Volume}\,(\boldsymbol{C} \cdot [-1 - \eta, +1 + \eta]^n) \geq 2^n \text{ Volume}\,(\boldsymbol{C} \cdot [-\eta, \eta]).$$

Thus,

$$(1 + \eta)^m v \geq 2^n \eta^m v. \blacksquare$$

In the following, we derive a lower bound for η for the signature matrix ($\boldsymbol{C}$).

Theorem 2 For any norm $\|\ \|$ on $\mathbb{R}^m$,

$$\eta_{\sup}(\boldsymbol{C}) \geq \frac{\min\limits_{X \in S} \|\boldsymbol{C}X\|}{\max\limits_{X \in \{\pm 1\}^n} \|\boldsymbol{C}X\|} \qquad (6)$$

where S is the set of all non-zero elements of $\{0, \pm 1\}^n$.

Proof: If there exists $X_1, X_2 \in \{\pm 1\}^n$ such that $\boldsymbol{C} \cdot (X_1 + [-\eta\,,\eta]^n) \cap \boldsymbol{C} \cdot (X_2 + [-\eta\,,\eta]^n) \neq \emptyset$, there exists $Z_1, Z_2 \in [-\eta\,,\eta]^n$ such that $\boldsymbol{C}(X_1 + Z_1) = \boldsymbol{C}(X_2 + Z_2)$. Thus, $\boldsymbol{C}\left(\frac{X_1 - X_2}{2}\right) = \boldsymbol{C}\left(\frac{Z_1 - Z_2}{2}\right)$. Hence, the equation is formed as $\boldsymbol{C}X^* = \boldsymbol{C}Z^*$ where $X^* \in S$ and $Z^* \in [-\eta\,,\eta]^n$. For the proof, we show that if $\eta = \frac{\min_{X \in S} \|\boldsymbol{C}X\|}{\max_{X \in \{\pm 1\}^n} \|\boldsymbol{C}X\|} - \varepsilon$, for any $\varepsilon > 0$, there are no such X^* and Z^*. Obviously,

$$\max\nolimits_{X \in [-\eta, \eta]^n} \|\boldsymbol{C}X\| = \eta \max\nolimits_{X \in [-1,1]^n} \|\boldsymbol{C}X\|$$

and because the set $\boldsymbol{C} \cdot [-1\,,1]^n$ is the convex hull of $\boldsymbol{C} \cdot \{\pm 1\}^n$

$$\max_{X \in [-1,1]^n} \|\boldsymbol{C}X\| = \max_{X \in \{\pm 1\}^n} \|\boldsymbol{C}X\|.$$

Thus,

$$\max_{X \in [-\eta, \eta]^n} \|\boldsymbol{C}X\| = \left(\frac{\min\limits_{X \in S} \|\boldsymbol{C}X\|}{\max\limits_{X \in \{\pm 1\}^n} \|\boldsymbol{C}X\|} - \varepsilon\right) \max_{X \in \{\pm 1\}^n} \|\boldsymbol{C}X\| < \min_{X \in S} \|\boldsymbol{C}X\|$$

which means there are not $X^* \in S$ and $Z^* \in [-\eta\,,\eta]^n$ that $\boldsymbol{C}X^* = \boldsymbol{C}Z^*$.Q.E.D. $\blacksquare$

This theorem proves that errorless transmission is possible for any

$$\eta < \sup_{\|\ \| \text{ is norm}} \left(\frac{\min\limits_{X \in S} \|\boldsymbol{C}X\|}{\max\limits_{X \in \{\pm 1\}^n} \|\boldsymbol{C}X\|}\right)$$

Example 1 For the matrix

$$\boldsymbol{C}_{2 \times 4} = \begin{bmatrix} 1 & 0 & 0.5\sqrt{3} & -0.5 \\ 0 & 1 & 0.5 & 0.5\sqrt{3} \end{bmatrix},$$

the upper bound of Theorem 1 says that $\eta_{\sup}(\boldsymbol{C}_{2 \times 4}) \leq 0.33$. We simulated the lower bound of Theorem 2 for 100 random norms on $\mathbb{R}^2$ and found that $\eta_{\sup}(\boldsymbol{C}_{2 \times 4}) \geq 0.18$. Actually for generating a random norm, we chose a random matrix $\boldsymbol{A}$ and different $p > 1$, define $\|X\|_p^{\boldsymbol{A}} = \|\boldsymbol{A}X\|_p$ where $\|\ \|_p$ denotes the l^p-norm. Computer simulation show that $\eta_{\sup}(\boldsymbol{C}_{2 \times 4}) \cong 0.21$ ($\mathrm{NFF}_{\inf} \cong -3.75$dB), which is very close to both upper and lower bounds.

Example 2 Consider a binary matrix. We have calculated the upper and lower bounds of $\eta_{\sup}(\boldsymbol{C}_{8\times 13})$ for the 8×13 COW matrix suggested in [5]. The upper bound of Theorem 1 is 0.48, the lower bound of Theorem 2 obtained by evaluating over 100 random norms is 0.13 and the simulation results show that $\eta_{\sup}(\boldsymbol{C}_{m\times n}) \cong 0.23$ ($\mathrm{NFF}_{\inf} \cong -4$dB).

IV. Constructing Large Signature Matrices from Small Ones

Using the lower bound in the previous section, we can find small size near-far resistant signature matrices that guarantee errorless communication in noiseless channels. But evaluating this lower bound for large size matrices needs a huge amount of computations and is not practical. For this reason, in this section we propose a method for constructing large near-far robust matrices from small ones.

Theorem 3 Suppose $\boldsymbol{P}$ is an invertible $k\times k$ matrix, then we have

$$\eta_{\sup}(\boldsymbol{P}\otimes\boldsymbol{C}_{m\times n}) = \eta_{\sup}(\boldsymbol{C}) \qquad (7)$$

where $\otimes$ is the Kronecker product.

Proof: Assume $\eta > \eta_{\sup}(\boldsymbol{C})$. According to the proof of Theorem 2, there exist $X^* \in S$ and $Z^* \in [-\eta,\eta]^n$ such that $\boldsymbol{C}X^* = \boldsymbol{C}Z^*$. Let $\bar{X} = [X^{*T} \ \ \dots \ \ X^{*T}]^T \in \{0,\pm 1\}^{kn}$ and $\bar{Z} = [Z^{*T} \ \ \dots \ \ Z^{*T}]^T \in [-\eta,\eta]^{kn}$, we have,

$$\begin{aligned}(\boldsymbol{P}^{-1}\otimes\boldsymbol{I})(\boldsymbol{P}\otimes\boldsymbol{C})\bar{X} &= (\boldsymbol{I}\otimes\boldsymbol{C})\bar{X} = (\boldsymbol{I}\otimes\boldsymbol{C})\bar{Z}\\ &= (\boldsymbol{P}^{-1}\otimes\boldsymbol{I})(\boldsymbol{P}\otimes\boldsymbol{C})\bar{Z}.\end{aligned}$$

Thus, $(\boldsymbol{P}\otimes\boldsymbol{C})\bar{X} = (\boldsymbol{P}\otimes\boldsymbol{C})\bar{Z}$. Consequently, $\eta_{\sup}(\boldsymbol{P}\otimes\boldsymbol{C}) \le \eta_{\sup}(\boldsymbol{C})$. Now, consider $\eta > \eta_{\sup}(\boldsymbol{P}\otimes\boldsymbol{C})$ there exist non-zero $\bar{X} \in \{0,\pm 1\}^{kn}$ and $\bar{Z} \in [-\eta,\eta]^{kn}$ such that $(\boldsymbol{P}\otimes\boldsymbol{C})\bar{X} = (\boldsymbol{P}\otimes\boldsymbol{C})\bar{Z}$. Thus, $(\boldsymbol{I}\otimes\boldsymbol{C})\bar{X} = (\boldsymbol{P}^{-1}\otimes\boldsymbol{I})(\boldsymbol{P}\otimes\boldsymbol{C})\bar{X} = (\boldsymbol{P}^{-1}\otimes\boldsymbol{I})(\boldsymbol{P}\otimes\boldsymbol{C})\bar{Z} = (\boldsymbol{I}\otimes\boldsymbol{C})\bar{Z}$. Let X^* and Z^* be the first n entries of $\bar{X}$ and $\bar{Z}$, respectively. We have $\boldsymbol{C}X^* = \boldsymbol{C}Z^*$, which means $\eta_{\sup}(\boldsymbol{P}\otimes\boldsymbol{C}) \ge \eta_{\sup}(\boldsymbol{C})$. ■

Note Using Theorems 3 and 2, for every invertible matrix $\boldsymbol{P}$, we derive another lower bound

$$\eta_{\sup}(\boldsymbol{C}) \ge \frac{\min\limits_{X\in\{0,\pm 1\}^{kn}-\{0\}^{kn}} \|(\boldsymbol{P}\otimes\boldsymbol{C})X\|}{\max\limits_{X\in\{\pm 1\}^{kn}} \|(\boldsymbol{P}\otimes\boldsymbol{C})X\|}. \qquad (8)$$

In the next section, we propose a very low complexity ML decoder for a subclass of the proposed codes.

V. Maximum Likelihood (ML) Decoder for a Class of the Proposed Near-Far Resistant Codes

In [5] a very low complexity method for decoding COW codes is proposed. Here we try to extend those ideas to decode near-far resistant matrices. In this section we present a suboptimum decoding method that has a very low computational complexity. Also, we study the conditions under which this decoder is ML.

As (3), we model a synchronous CDMA system with near-far effect in an AWGN channel as

$$Y = \boldsymbol{C}(X+Z) + N \qquad (9)$$

Here N is a Gaussian vector with zero mean and auto-covariance matrix $\sigma^2\boldsymbol{I}$ ($\boldsymbol{I}$ is the identity matrix).

Since each entry of Z is a random number belonging to $[-\eta,\eta]$, for the worst case, it has uniform distribution in the interval $[-\eta,\eta]$ (a compact support random variable has maximum entropy when its distribution is uniform [7]). Also, we assume that the time samples of Z are independent. This assumption is much worse than what occurs in practice because in practical situations the power of each user is constant at least for a short period of time and thus the time samples are correlated.

For overloaded systems, the generalized central limit theorem suggests that we can approximate $W = \boldsymbol{C}Z + N$ with a Gaussian vector with zero mean and auto-covariance matrix $\frac{\eta^2}{3}\boldsymbol{C}\boldsymbol{C}^T + \sigma^2\boldsymbol{I}$. This approximation becomes better as the overloading factor grows [8]. Thus, hereafter, we model the channel as

$$Y = \boldsymbol{C}X + W \qquad (10)$$

where W is a zero mean Gaussian random vector with covariance matrix $\frac{\eta^2}{3}\boldsymbol{C}\boldsymbol{C}^T + \sigma^2\boldsymbol{I}$.

Similar to [5-6], we prove a lemma that significantly decreases the complexity of the decoding problem of Theorem 3.

Lemma 1 Suppose $\boldsymbol{P}$ is an invertible matrix and $\boldsymbol{D}_{km\times kn} = \boldsymbol{P}_{k\times k}\otimes\boldsymbol{C}_{m\times n}$. The decoding problem of a system with the signature matrix $\boldsymbol{D}$ can be decoupled to k decoding of a system with the signature matrix $\boldsymbol{C}$.

Proof: Suppose $Y = (\boldsymbol{P}\otimes\boldsymbol{C})X + W$. Multiplying both sides by $\boldsymbol{P}^{-1}\otimes\boldsymbol{I}_m$, we have

$$(\boldsymbol{P}^{-1}\otimes\boldsymbol{I}_m)Y = (\boldsymbol{I}_k\otimes\boldsymbol{C})X + (\boldsymbol{P}^{-1}\otimes\boldsymbol{I}_m)W.$$

Since $\boldsymbol{I}_k\otimes\boldsymbol{C}$ is a block diagonal matrix, the decoding problem of $(\boldsymbol{P}^{-1}\otimes\boldsymbol{I}_m)Y$ is in fact k disjoint decoding problems. Now, if the matrix $\boldsymbol{P}$ is unitary, then $\boldsymbol{P}^{-1}\otimes\boldsymbol{I}_m$ is also a unitary matrix and thus the distribution of the vector $(\boldsymbol{P}^{-1}\otimes\boldsymbol{I}_m)W$ is the same as the distribution of W. Consequently, the ML decoding of these k small size problems is equivalent to the ML decoding of the big size matrices. ■

The following lemma reduces the decoder complexity even more.

Lemma 2 Suppose $\boldsymbol{C}_{m\times n}$ is full rank. The decoding problem for a system with the signature matrix $\boldsymbol{C}$ can be performed by 2^{n-m} Euclidean distance calculations instead of 2^n.

Proof: Since $\boldsymbol{C}$ is full rank, we can assume that $\boldsymbol{C} = [\boldsymbol{A}|\boldsymbol{B}]$ where $\boldsymbol{A}$ is an $m\times m$ invertible matrix. Thus, $Y = \boldsymbol{A}X_1 + \boldsymbol{B}X_2 + W$ where X_1 and X_2 are $m\times 1$ and $(n-m)\times 1$ vectors, respectively. Since $\boldsymbol{A}^{-1}Y = X_1 + \boldsymbol{A}^{-1}\boldsymbol{B}X_2 + \boldsymbol{A}^{-1}W$

and $\mathrm{sign}(V)$ (that is obtained by substituting the positive entries of a vector V by $+1$ and the negatives by -1) is the nearest $\{\pm 1\}$-vector to V, equations

$$\hat{X}_2 = \underset{Z}{\mathrm{argmin}} \|(\boldsymbol{A}^{-1}Y - \boldsymbol{A}^{-1}\boldsymbol{B}Z) - \mathrm{sign}(\boldsymbol{A}^{-1}Y - \boldsymbol{A}^{-1}\boldsymbol{B}Z)\|_2$$

$$\hat{X}_1 = \mathrm{sign}\left(\boldsymbol{A}^{-1}Y - \boldsymbol{A}^{-1}\boldsymbol{B}\hat{X}_2\right)$$

minimize $\left\|\boldsymbol{A}^{-1}Y - \boldsymbol{A}^{-1}\boldsymbol{B}\hat{X}_2 - \hat{X}_1\right\|_2$ which is a suboptimum decoder. This method needs 2^{n-m} Euclidean norm calculations. It is worth mentioning that if $\boldsymbol{A}$ is a unitary matrix, then the above algorithm is optimum [5]. ■

This lemma suggests a decoder with lower complexity; however, in general, this is a suboptimum decoder unless $\boldsymbol{A}$ is unitary and $\boldsymbol{C}\boldsymbol{C}^T$ is a scaled identity matrix (this means that its rows must be orthogonal to each other, i.e., a WBE matrix [9]).

Example 3 From Theorem 3 and Example 1, $\boldsymbol{D}_{64\times128} = \boldsymbol{I}_{32} \otimes \boldsymbol{C}_{2\times4}$ ($\boldsymbol{C}_{2\times4}$ is the matrix in Example 1) is a matrix with $\eta_{\mathrm{sup}}(\boldsymbol{D}_{64\times128}) \cong 0.21$, i.e., $\mathrm{NFF}_{\mathrm{inf}}(\boldsymbol{D}_{64\times128}) \cong -3.7$ dB. Since $\boldsymbol{I}_{32}$ is a unitary matrix, the first two columns of $\boldsymbol{C}$ is also a unitary matrix and the rows of $\boldsymbol{C}$ are orthogonal to each other, lemmas 1 and 2 result in ML decoder of a system with signature matrix $\boldsymbol{D}_{64\times128}$. The proposed method of decoding needs 32×2^2 Euclidean norm calculations instead of 2^{128} such calculations in direct implementation of the ML decoder. The performance of this code in an AWGN channel is simulated in the next section.

Example 4 Similar to Example 3 for $\boldsymbol{D}_{64\times104} = \boldsymbol{H}_8 \otimes \boldsymbol{C}_{8\times13}$, where $\boldsymbol{H}_8$ is an 8×8 Hadamard matrix and $\boldsymbol{C}_{8\times13}$ is the matrix in Example 2, $\eta_{\mathrm{sup}}(\boldsymbol{D}_{64\times104}) \cong 0.23$, i.e., $\mathrm{NFF}_{\mathrm{inf}}(\boldsymbol{D}_{64\times104}) = -4$ dB. The advantage of this matrix is that its entries are ± 1 (it is in fact a COW matrix). Since $\frac{1}{\sqrt{8}}\boldsymbol{H}_8$ is a unitary matrix, according to Lemma 1, the ML decoder of $\boldsymbol{D}_{64\times104}$ can be implemented by 8 ML decoder of $\boldsymbol{C}_{8\times13}$. This means significant reduction in the complexity of the ML decoder; 8×2^{13} Euclidean norm calculations instead of 2^{104} such calculations. However, using Lemma 2, we obtain a suboptimum decoder with 8×2^5 Euclidean norm calculations. This decoder is not ML because the rows of $\boldsymbol{C}_{8\times13}$ are not orthogonal but it is somewhat suboptimum.

VI. Simulation Results

We have simulated two overloaded binary $(64, 128)$ and $(64, 104)$ CDMA systems. The code matrices used for these simulations are $\boldsymbol{D}_{64\times104}$ and $\boldsymbol{D}_{64\times128}$ which are introduced in Examples 3 and 4. The advantage of the system with 104 users is that its signature matrix is binary antipodal which is practically favorable. In our simulations we have assumed the near-far effect for each user is a white random process, i.e., there is no correlation among the time samples. Obviously, this scenario is much worse than what occurs in practical situations. The advantage of this decoding method is that we have assumed that the receiver does not have any knowledge about the received user powers. Notice that in such highly over-loaded systems, conventional methods for estimating the received powers do not work. However, the performance of the system in the presence of noise and near-far effect is very impressive. Simulations are performed for various values of NFF. As it can be predicted, Figs. 2 and 3 show that for NFF more than $\mathrm{NFF}_{\mathrm{inf}}$ the Bit-Error-Rate (BER) tends to zero as E_b/N_0 grows and for NFF less than $\mathrm{NFF}_{\mathrm{inf}}$, BER saturates and errorless transmission is not possible. The simulation results show the robustness of the proposed codes against additive noise and near-far effects.

Both systems employ the proposed decoder with low complexity. The decoder is ML for the system with 128 users and is sub-optimum for the system with 104 users.

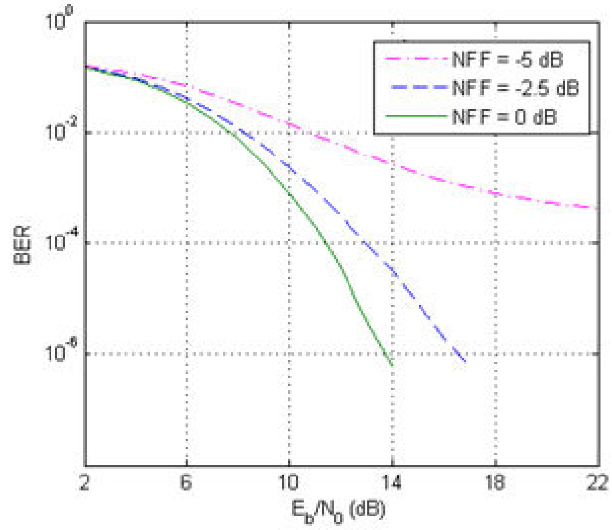
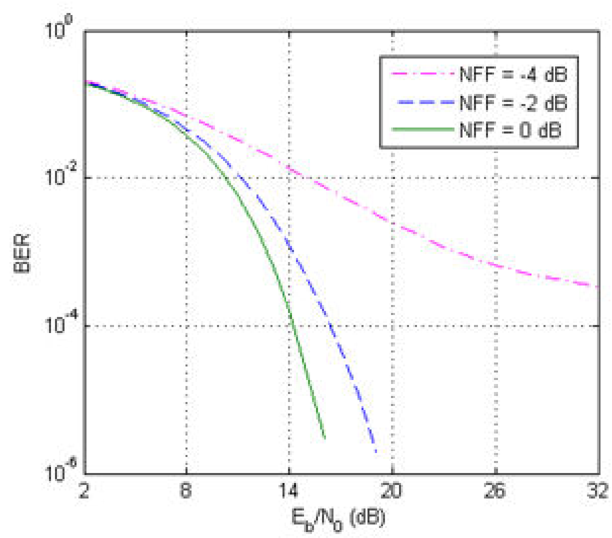


Fig. 1. BER versus $\boldsymbol{E_b/N_0}$ for a binary CDMA system with $\boldsymbol{64}$ chips and $\boldsymbol{104}$ users (binary signatures/binary inputs).

The following simulation is for a non-binary $\boldsymbol{D}_{64\times128}$ matrix. Because this matrix constructed by Kronecker product of identity matrix with the 2×4 WBE matrix, the decoder of Lemma 2 in the previous section is an ML decoder.



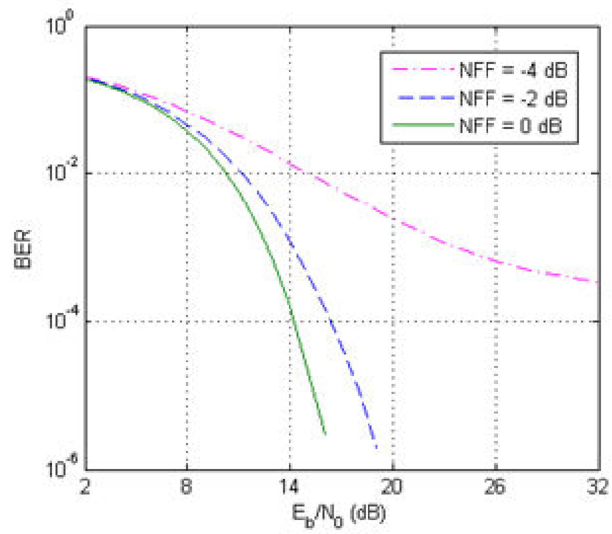


Fig. 3. BER versus $\boldsymbol{E_b/N_0}$ for a CDMA system with $\mathbf{64}$ chips and $\mathbf{128}$ users (non-binary signatures/binary inputs).

VII. CONCLUSION AND FUTURE WORKS

In this paper, we have considered highly overloaded CDMA systems that are robust against near-far effects. The systems are errorless in noiseless channels provided that the near-far effect is lower than a desired value. Also, we have proposed an optimum and suboptimum decoder for a class of the proposed codes. Upper and lower bounds have been derived for the maximum value of near-far effect that does not affect the errorless property of the system. The assumptions in this paper are much worse than what actually occurs in a real situation. We assume that the received user powers are white random processes (there is no correlation among the time samples). However, under such a bad condition, the proposed systems show acceptable performances as demonstrated in Figs. 2 and 3.

For future work, we plan to consider the correlation among the time samples of the near-far effects in practical situations. This assumption will improve the performance of the system. Future work should focus on discovering codes that can tolerate more near-far effects; also the derivation of the sum channel capacity of a CDMA system [10] with near-far effects is very useful.

REFERENCES

[1] Y. Cho and J. H. Lee, "Analysis of an Adaptive SIC for Near-Far Resistant DS-CDMA," *IEEE Transactions on Communications*, vol. 46, No. 11, pp. 1429-1432, 1998.

[2] U. Madhow, "Blind Adaptive Interference Suppression for the Near-Far Resistant Acquisition and Demodulation of Direct-Sequence CDMA Signals," *IEEE Transactions on Signal Processing*, vol. 45, no. 1, pp. 124-136, 1997.

[3] F. C. Zheng, S. K. Barton, "Near-Far Resistant Detection of CDMA Signals via Isolation Bit Insertion," *IEEE Transactions on Communications*, vol. 43, no. 2/3/4, pp. 1313-1317, 1995.

[4] U. Madhow and M. L. Honig, "On the Average Near–Far Resistance for MMSE Detection of Direct Sequence CDMA Signals with Random Spreading," *IEEE Transactions on Information Theory*, vol. 45, no. 6, pp. 2039-2045, 1999.

[5] P. Pad, F. Marvasti, K. Alishahi, and S. Akbari, "A class of errorless codes for over-loaded synchronous wireless and optical CDMA systems," *IEEE Trans. on Inform. Theory*, vol. 55, no. 6, pp. 2705-2715, June 2009.

[6] P. Pad, M. Soltanolkotabi, S. Hadikhanlou, A. Enayati, and F. Marvasti, "Errorless Codes for Over-loaded CDMA with Active User Detection" *ICC'09*, Dresden, Germany, June 2009.

[7] T. Cover and J. Thomas, *Elements of Information Theory*, 2nd ed., New York: Wiley, 2006.

[8] R. Durrett, *Probability: Theory and Examples*, 3rd ed., Thomson, 2005.

[9] M. Rupf and J. L. Massey, "Optimum Sequences multisets for synchronous code-division multiple-access channels," *IEEE Trans. on Inform. Theory*, vol. 40, pp. 1261-1266, Jul. 1994.

[10] K. Alishahi, F. Marvasti, V. Aref, and P. Pad, "Bounds on the Sum Capacity of Synchronous Binary CDMA Channels," *IEEE Trans. On Inform. Theory*, vol. 55, no. 8, pp. 3577-3593, Aug. 2009.