IMPULSIVE NOISE REMOVAL FROM IMAGES USING SPARSE REPRESENTATION AND OPTIMIZATION METHODS

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ABSTRACT

In this paper, we propose a new method for impulsive noise removal from images. It uses the sparsity of natural images when they are expanded by mean of a good learned dictionary. The zeros in sparse domain give us an idea to reconstruct the pixels that are corrupted by random-value impulse noises. This idea comes from this reality that noisy image in sparse domain of original image will not have a sparse representation as much as original image sparsity. In this method we assume that the proper dictionary to achieve image in sparse domain is available.

Keywords - Image De-noising, Impulsive noise, Iterative Method, Sparsity.

1. INTRODUCTION

The digital image pixels turn erroneous with maximum or minimum magnitudes called impulses when being acquired by defective sensors or transmitted through faulty radio channels [2]. The salt and pepper noise and the random valued-noise are the two common types of impulsive noises. In the salt and pepper noise, the salt noise is assumed to have the brightest gray level and the pepper noise has the darkest value of the gray level in the image. This assumption can help us to know the corrupted pixels in the images. In these cases the only hard task is to recover the original pixel of the image. But, in the general case of random-valued impulse noise, there is not any pre-assumption about the value of the impulsive noise. Therefore, the image de-noising task in these cases is to detect the corrupted pixels and then correct them by the original pixel of the image. So, image de-noising for random-valued impulse noises is more difficult than fixed salt and pepper image de-noising. In this paper, we focus on the random value impulsive noise. Order Statistics filters in general and the Median Filter as a special case are the most popular nonlinear filters for removing impulse noise [3]. The main disadvantage of these filters is that they manipulate all the pixels of the image, even if they are not noisy; thus, even in low noise density, edge jitter occurs in the image. Different remedies of the median filter have been proposed so far. They are the adaptive median filter [4], the median filter based on homogeneity [5], centre-weighted median filters [6] a generally family called decision-based methods. The so-called "decision-based" methods first identify possible noisy pixels and then replace them by using the median filter or its variants, while leaving all other pixels unchanged. Some of these two-stage methods deal with salt and pepper noise [7] and the others with the case of random value impulse noises [8]. In this paper, first we detect the corrupted pixels in image like decision base methods and then only we will improve the value of these pixels with iterative method which uses the characteristic of image and noise in the sparse domain. we learned a dictionary in order to earn a sparse representation of our image in dictionary domain For instance in our simulation we use K-SVD method [1] to learn dictionary from noisy image so when we compute sparse domain coefficients of signal plus noise, the noise values will appear in all elements of dictionary's atoms and this will encourage us to reduce the noise with cutting down these elements worth.

2. REVIEW OF THE BDND METHOD

In [9] Ng proposed a method, called BDND - Boundary Discriminative Noise Detection - to discover impulsive noise locations. To this end, all the pixels in a specified window that center on the considered pixel are categorized into three groups: low-intensity, medium intensity, and high-intensity. This division is based on two threshold values, T_1 and T_2. In the first step, for each pixel $p_{i,j}$, if $0 \leq p_{i,j} \leq T_1$ the pixel is considered as a low-intensity pixel, and if $T_2 \leq p_{i,j} \leq 255$ as a high-intensity pixel; otherwise the pixel belongs to the medium-intensity group. The first two groups are anointed as corrupted, while the third is thought to be uncorrupted. In the second step, the groups appointed corrupted in the first step are examined again using a tighter window. In the simulation results of [9], the first and second window sizes were empirically determined to be 21×21 and 3×3, respectively. To determine the thresholds for each pixel, the BDND method sorts (in the ascending order) the pixels of the local window into a vector called the sorted vector, S of size$2l + 1$. A vector of intensity differences between adjacent pairs of S defines the difference vector

$$\boldsymbol{D} = [d_1, d_2, \cdots, d_{2l-1}, d_{2l}] \quad (1)$$

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Then, **D** is partitioned into two parts, **D1** and **D2** which are defined by:

$$\boldsymbol{D_1} = [d_1, d_2, \cdots, d_{m_1}, \cdots, d_{l-1}, d_l] \quad (2)$$

$$\boldsymbol{D_2} = [d_{l+1}, d_{l+2}, \cdots, d_{m_2}, \cdots, d_{2l-1}, d_{2l}] \quad (3)$$

Let the position of the maximum elements of $\boldsymbol{D_1}$ and $\boldsymbol{D_2}$ be m_1 and m_2, respectively, then $T_1 = S_{m_1}$ and $T_2 = S_{m_2}$. A binary decision mask is formed for each pixel of the image using the above boundaries. The 0's in this mask correspond to noisy pixels whereas 1's correspond to uncorrupted pixels. After finding the noisy pixels, this method uses a modified Noise adaptive soft-switching median (NASM) filter in order to obtain the reconstructed image.

3. THE PROPOSED METHOD

Our proposed method will be performed in two steps:

1. Finding Corrupted Pixels.
2. De-noising Using Sparse Representation.

3.1. Finding Corrupted Pixels

In the first step by using BDND method [9] we find a mask to separate the corrupted pixels from uncorrupted pixels. In mathematical form we can formulate: $\mathrm{Y} = \mathrm{M} \odot \mathrm{I_c}$ Where $\mathrm{I_c}$ is corrupted image. M is a matrix whose elements are 0 or 1. $m_{i,j}$ equals 1 if the pixel $\mathrm{I}_{c_{i,j}}$ is uncorrupted, otherwise it will be 0 and act like a mask. The operator $\odot$ is defined as matrices element by element product namely $y_{i,j} = m_{i,j} \times \mathrm{I}_{c_{i,j}}$. According to BDND this part generates a 2-D soft-decision mask. Based on the value of the pixel with respect to its neighbors (which is indicative of the noise level), a real number between 0 and 1 is produced. Values close to 1 represent uncorrupted pixels, while values close to 0 represent corrupted ones. For the sake of simplicity we consider these values round toward the nearest integer's value namely zero and one here. Similar to the BDND method, a local window of size $W \times W$ is formed around each pixel of the image. Vectors **D1** and **D2** are formed as suggested by [9]. A soft-decision function is formed using boundary values T_1 and T_2 and the maximum values of **D1** and **D2**, (d_{m_1} and d_{m_2} respectively), in every local window. The function, $f_{i,j}(p)$corresponding to the window surrounding the pixel in location (i,j) is defined as the following function .

$$f_{i,j}(p) = \begin{cases} 0 & p \le T_1 - d_{m_1} \\ \left[\!\left[\frac{p - b_1 + d_{m_1}}{d_{m_1}}\right]\!\right] & T_1 - d_{m_1} < p \le T_1 \\ 1 & T_1 < p \le T_2 \\ \left[\!\left[\frac{b_2 + d_{m_2} - p}{d_{m_2}}\right]\!\right] & T_2 < p \le T_2 + d_{m_2} \\ 0 & p > T_2 + d_{m_2} \end{cases}$$

Let the intensity of the $(\mathrm{i,j})^{\mathrm{th}}$pixel be $p_{i,j}$, then the value of the soft-decision mask for that specific pixel is:

$$Mask_{i,j} = f_{i,j}(p)$$

The operator $[\![.]\!]$ returns the zero or one according to proximity of input to each of them. After locating the noisy pixels, we can use any median filtering method in order to obtain the reconstructed image, but in our method we want to evaluate this step with sparse component analysis. But for better result in output, the initialization parameters of iterative method in the next step are chosen the median filtering of corrupted pixels in this step.

3.2. De-noising Using Sparse Representation

In the second step we use an iterative method to improve the corrupted pixels value toward their real values using sparse representation method. Because we know that the real image has a sparse representation over our dictionary we define following criteria for de-noising:

$$P_0: \ \min_S \|\mathrm{S}\|_0 \quad \text{s.t} \ \ \mathrm{Y} = \mathrm{M} \odot \mathrm{I_c} \ \ \text{and} \ \ \mathrm{Y} \cong \mathrm{AS} \quad (4)$$

Where A is the learned-dictionary which is determined by a dictionary learning method, Note that we now switch to an analysis –type prior in (4). The solution of such an optimization problem can be obtained through an iterative thresholding algorithm called Morphological Component Analysis (MCA) [10]:

$$I^{(n+1)} = \mathrm{Z}_{\mathrm{A},\lambda_\mathrm{n}}(I^{(n)} + \mathrm{Y} - \mathrm{M} \odot I^{(n)}) \quad (5)$$

Where the nonlinear operator $\mathrm{Z}_{\mathrm{A},\lambda_\mathrm{n}}(\mathrm{z})$ consists in

1. Decomposing the signal z in the dictionary A to derive the coefficients S, z =AS
2. Threshold the coefficient $\hat{\mathrm{S}} = \mathrm{Th}(\mathrm{S}, \lambda_\mathrm{n})$, where the thresholding operator *Th* can be either a hard threshold or a soft threshold.
3. Reconstruct $\hat{\mathrm{z}}$ from the threshold coefficients $\hat{\mathrm{S}}$.

We do these three steps simultaneously by mean of following optimization task to evaluate the sparse coefficients, the value of ε depend on density of impulsive noise and its variance. [1]

$$P_0: \ \min_S \|S\|_0 \ \ \text{s.t.} \ \left\|I^{(n)} + \mathrm{Y} - \mathrm{M} \odot I^{(n)} - \mathrm{AS}\right\|_2^2 \le \varepsilon \quad (6)$$

The above optimization task can be converted to optimize the Lagrangian:

$$P_0: \ \min_S \|S\|_0 + \lambda \left\|I^{(n)} + \mathrm{Y} - \mathrm{M} \odot I^{(n)} - \mathrm{AS}\right\|_2^2 \quad (7)$$

Hereafter, motivated by the recently stated work of Mohimani, et al. [11]. We seek to find the sparsest possible answer without such a replacement and instead, attempt to relax the replacing $l^0 - norm$ by a continuous, differentiable cost function, say:

$$F(S) = \sum_i \exp\left(-s_i^2 / 2\sigma^2\right) \quad (8)$$

This function tends to count the number of zeros elements of a vector. So, as stated in [11] the above problem can be converted to:

$$P_0: \quad \min_S (n - F(S)) + \lambda \|\mathrm{Y} - \mathrm{AS}\|_2^2 \quad (9)$$

In above optimization problem constrain becomes a penalty and the parameter λ is dependent on ε. Solution of this problem was recently proposed in [12] and it is shown that for a proper choice of λ, these two problems are equivalent. The brief description of algorithm for finding the sparse coefficients in presence of noise that is described above is depicted in Fig.1.

-Initialization: Let S = $\lambda(I + A^T A)^{-1} A^T$Z
(This is equivalent to the solution when the σ tends to be infinty) i.e.: $\min_S \|S\|_2 + \lambda\|Z - AS\|_2^2$ In this case :
Z = $I^{(n)} + Y - M \odot I^{(n)}$

-Choose a suitable decreasing sequence for σ ,$[\sigma_1, \cdots, \sigma_K]$
-for n=1, $\cdots, K$:

1. Let $\sigma = \sigma_n$.
2. Find $S_\sigma^{opt} = \min_S (n - F(S)) + \lambda\|Z - AS\|_2^2$
   Z = $I^{(n)} + Y - M \odot I^{(n)}$
   Using any kind of optimization tool, say steepest descend with fixed number of iterations.

-Finally answer is $S = S^{opt}$

Fig1. Algorithm for finding the sparse coefficients in presence of noise

As we saw the second step is performed in two iterative subroutines which the first improve the corrupted pixels value in each iteration and another that is performed inside the former computes the sparsest coefficients the sparse domain for image. In short, we gathered the overall algorithm in brief and depicted in Fig2.

-First Step: **Finding the corrupted pixels**

In this step we compute a mask to separate the corrupted pixels from uncorrupted pixels. M = Mask;
Initialize $I^{(0)}$ = MadianFiltering(M, Y)

-Second Step: **De-noising Using Sparse Representation**

Loop n = 1 to L

- Iterative Updating:
  Z = $I^{(n)} + Y - M \odot I^{(n)}$
- Sparse Representation:
  P_0: $\min_S (n - F(S)) + \lambda\|Z - AS\|_2^2$

➢ In this stage the algorithm in Fig.1. is performed to compute the answer for optimization task in P_0; and substitute $I^{(n+1)} = A.S^{opt}$

End Loop
-Final answer is $I^{(L+1)}$.

Fig2. Impulsive Noise Removal from Image Using Sparse Representation and Optimization Method

4. EXPERIMENTAL RESULTS

In this section we perform several experiments to test the proposed algorithm and compare it with other image de-noising techniques. The default number of Loop iteration L is assumed to be 5 iterations. Furthermore, according to our simulations and [9], the size of the window is specified to be 21×21. In addition, we set the maximum number of the iterations in sparse representation phase equals 3 and lambda λ in our simulation was between 2.9 and 4 according to density of impulsive noise. In the first experiment, we compare our method with NASM and BDND [9] and Recursive Detection-Estimation method (RDE) [13] for different intervals of fixed-valued noise and also random noise. To this aim, we use the test image "Lena" of size 512×512 corrupted with 80% noise. As it is observed in Table 1, the proposed method outperforms other methods when the modeled noise interval is wider but in tight range noise interval it does better than two first mentioned method and a little weaker than RDE method in terms of PSNR but our method in aspect of computational complexity is much more better than RDE because in RED we must compute several convolution in each iteration. For obtaining each PSNR value (10), we repeated our simulations several times and calculated the average of the results and using the original image as reference. It can be observed that our method outperforms the BDND method in all of the tested pictures. In Fig. 3, we show that our method yields superior subjective quality and image detail preservation. To achieve this aim, we use 40% corrupted image "Lena" and "Boat".

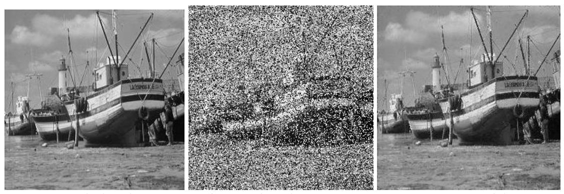
$$\text{PSNR} = 10\log_{10}\left(\frac{255^2}{\frac{1}{m\times n}\sum_{i,j}(I_{i,j}-\widehat{I_{i,j}})^2}\right) \qquad (10)$$

In the second experiment, we compare our proposed method with several well-known de-noising methods for various percentages of impulsive noise. These methods include: CSAM Filter [5], SDROM [15], PMCW [16], BDND [9] , Lue method [14] and CMF[17]. According to Figure 4, our method like the previous work of our lab, RDE has a substantial improvement in image de-noising in salt and pepper noise case with noise percentage between 10% to 30%. The advantage of the proposed method over the RDE is time consumption.

Table 1. Comparison of our method, the BDND and RDE algorithm with 80% noise density on test image Lena.

Noise Range		PSNR(dB)			
Low-Intensity Noise	High – Intensity Noise	NASM	BDND	RDE	**Our Method**
0-9	246 -255	15.432	18.632	21.16	19.31
0-59	196-255	17.283	16.871	20.7	18.813
0-100	146-255	16.067	14.95	17.87	18.12
0-255		15.824	12.83	15.93	16.503

(a)Original (b) Noisy image (c) recovered image



(a)Original (b) Noisy image (c) recovered image

Fig. 3. Reconstruction result for test image Lena and boat by our proposed method, for 40% noise achieved PSNR=35.12 dB and PSNR=30.86.

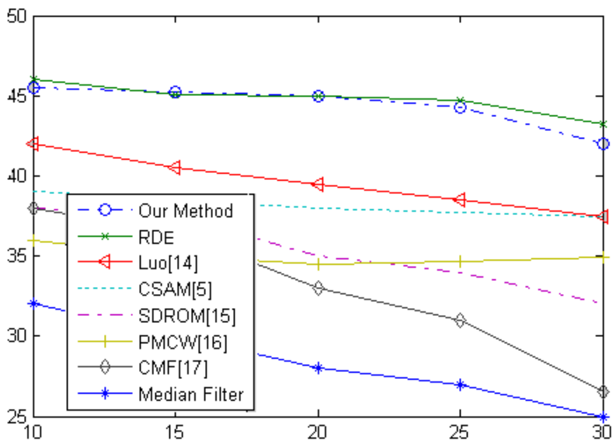


Fig. 4 Comparison of our method with different algorithms at various noise ratios (10-30) for test image Lena vs. PSNR value (25-50)

5. CONCLUSION

In this paper, we proposed a new method to suppress impulsive noise from corrupted images. This was accomplished in two steps: first, we use the boundaries proposed in the BDND method to find the location of impulsive noise and to form a soft-decision mask and an initial image to run our iterative method. This mask indicates which pixels are corrupted and which are not. In the second step, by using the obtained mask and the iterative approach we implement our method according to sparse component analysis foundation and then image is reconstructed. To achieve better results, ones can use better optimization tools in their computation and more important is to change the de-noising algorithm to one with soft decision judgments to reconstruct image. Our simulations show that our approach yields substantial improvement of the recovered image in comparison with the other well-known techniques.

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