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Blind wideband spectrum sensing in cognitive radio networks based on direction of arrival estimation model and generalised autoregressive conditional heteroscedasticity noise modelling

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Abstract: A new method for wideband spectrum sensing in cognitive radio networks is proposed. Since the problem of estimating the number of occupied channels can be considered as the problem of estimating the number of signal sources in array signal processing, so the model used for direction of arrival (DOA) estimation is utilised here for spectrum-sensing modelling. In the proposed algorithm, wideband spectrum is divided into subchannels, where each subchannel resembles a sensor in array processing for DOA estimation. Furthermore, the detection problem in practical situations is complicated because noise is most likely non-Gaussian and non-stationary unlike the assumption of previously presented algorithms. Therefore, in the proposed algorithm, a generalised autoregressive conditional heteroscedasticity model is used to model the additive noise. The number and locations of the occupied subchannels will be jointly estimated using the maximum likelihood approach. The introduced method is blind as there is no need for initial information about the primary users' signals and noise variance. The results indicate the efficiency of the proposed method.

1 Introduction

Cognitive radio (CR) technology has been revealed to overcome the low utilisation of spectrum by licensed (primary) users. In this regard, unlicensed (secondary) users should detect the spectrum successively to avoid interference. So far, many spectrum-sensing algorithms have been proposed. The energy detector in which the variance of the received signal is computed and then compared with a threshold is considered as the simplest method. Cyclostationary feature of the received signal is used in cyclostationarity-based detector to detect the primary user's signal. Noise uncertainty that relates to the inaccurate estimation of noise variance is a certain shortcoming of spectrum sensing using energy detector. Consequently, several blind algorithms that eliminate the need for initial information about the primary users' signals and noise variance were proposed [1].

Most of the introduced algorithms are narrowband methods and are formulated to solve binary hypothesis tests [2–16]. In narrowband spectrum-sensing methods, binary hypothesis test is used for each subchannel. In these algorithms, each subchannel is appraised individually to assess the existence of the primary users. The received signal in each subchannel at the tth snapshot, $y(t)$, can be expressed as

$$y(t) = \begin{cases} w(t) & H_0 \\ As(t) + w(t) & H_1 \end{cases} \tag{1}$$

where $As(t)$ is the received signal of the primary user and $w(t)$ is the additive white Gaussian noise (AWGN). H_0 and H_1 are two hypotheses that indicate whether primary user's signal exists or not in each subchannel.

In [2–6], blind narrowband spectrum-sensing algorithms are formulated as generalised likelihood ratio test (GLRT). The algorithm derived in [2, 3] is arithmetic to geometric mean (AGM) of eigenvalues of covariance matrix of multiple antennas' received signals. The blind spectrum-sensing algorithms proposed in [7–10] are based on the information theory. Among them, in [7–9], the authors use information-theoretic criterion (ITC) with the assumption on having multiple receive antennas. In [7, 8], the derived algorithm is AGM of eigenvalues of the covariance matrix of multiple antennas' received signals plus a penalty term.

On the other hand, in the wideband spectrum-sensing algorithms, multiple hypothesis test is deployed to sense the spectrum. The presence of primary users' signals is detected by considering all subchannels. Most of the wideband estimation methods sense the spectrum by optimising a predefined cost function [17, 18].

Blind wideband spectrum-sensing algorithms proposed in [19–22] are GLRT-based and the method introduced in [23] is ITC-based. In [19, 20], the authors proposed some GLRT-based algorithms to detect the presence of orthogonal frequency division multiplexing (OFDM) primary users' signals. The algorithms presented in the mentioned works are functions of arithmetic and geometric means of the OFDM subcarriers or cyclic prefix. In the first stage of the two-step algorithms introduced in [23], the estimation of the number of occupied subchannels is obtained using ITC. In the second step, the energies of signals of subchannels are sorted and the subchannels with the highest energies are chosen as the occupied ones.

Wideband spectrum-sensing algorithms are proposed to discriminate noise from primary user's signal by analysing the received signals in all subchannels. As a result, in CR networks the received signal is modelled in two cases. In the occupied subchannels, one or more primary users' signals along with noise exist in each subchannel whereas in the vacant subchannels there is only noise. This approach resembles the model that is employed to estimate the direction of arrival (DOA) of the signal source [23]. In array processing to estimate the number and locations of signal sources, an array of sensors is considered to process the received signals impinging on sensors. If we assume that the signal received at the sensor node in array processing resembles the signal that exists in each subchannel of CR, then we can use DOA estimation model for spectrum-sensing problem.

Furthermore, the additive noise is generally assumed to have Gaussian distribution because of the central limit theorem. Consequently, the algorithms introduced so far are optimised against Gaussian noise. However, noting the practical situation [24], measurements of ambient noise show that we have non-Gaussian and non-stationary characteristics. The time-varying noise variance, which is the characteristic of realistic environments, leads us to use an appropriate model for noise. The performance of previously presented methods, which assume Gaussian model for noise, may degrade when non-Gaussian and non-stationary noise or interference is present. Generalised autoregressive conditional heteroscedasticity (GARCH) model [25] is widely used for modelling economic time series and because of time-varying variance presentation and integration of past observations into present, it has the potential to model non-Gaussian and non-stationary noise. GARCH model is also appropriate to model the heavy tail processes with finite variances. In this paper, we use GARCH model because of modelling non-Gaussianity and non-stationarity of noise in CR networks.

In this study, we initially propose a model for system that is derived from the model used for DOA estimation [23]. In our method, the number and locations of the occupied subchannels will be jointly estimated using the maximum likelihood (ML) approach. To sense the spectrum, first we propose an algorithm to estimate the locations of the occupied subchannels in the spectrum for all possible number of occupied subchannels. Subsequently, we propose an algorithm to estimate the number of occupied subchannels using the previously estimated locations. To estimate the locations of occupied subchannels, we use ML estimation. It is known [26] that the ML estimation method tends to overestimate the number of occupied subchannels. Therefore, we utilise ML estimation and ITC to estimate the number of occupied subchannels. Signal and noise subspaces are utilised to compute the likelihood function.

As stated, GARCH model is utilised to formulate the additive noise with the assumption that the noise of each subchannel is independent from the signals and also from the noises of other subchannels. In order to estimate the correlation matrix of the occupied subchannels of the received signal, we use sample cross-correlation among the subchannels for non-diagonal elements of the correlation matrix and utilise GARCH model for diagonal ones. The proposed method is blind, as we do not need any initial information about the primary users' signals and noise variance.

The rest of this paper is organised as follows: In Section 2, we describe the GARCH model that is essential for the system model. In Section 3, we illustrate a model for the spectrum-sensing problem. In Section 4, we explain the proposed spectrum-sensing algorithm. ML formulation is explained in Section 5. Simulation results are presented in Section 6. Finally, in Section 7, we conclude the paper.

2 GARCH model

GARCH model was first developed by Bollerslev [25]. Heteroscedasticity can be considered as time-varying variance, that is, volatility. Conditional implies a dependence on the observations of the immediate past, and autoregressive incorporates past observations into the present. GARCH models are statistical models that are more popular in the economics. Engle [27] introduced autoregressive conditional heteroscedasticity (ARCH) process in order to allow the conditional variance to change over time as a function of past errors leaving on conditional variance constant. GARCH process is a general form of ARCH process and is a time series modelling technique that uses past variances to forecast future variances.

A series d_k is considered as GARCH(p, q) process if it can be written as

$$d_k = \sigma_k \varepsilon_k, \quad k \in \mathbb{Z} \tag{2}$$

where ε_k is a sequence of independent and identically distributed (i.i.d.) random variables that have Gaussian distribution with zero mean and variance equal to 1. In the above equation, σ_k is a non-negative process such that

$$\sigma_k^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i d_{k-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{k-j}^2, \quad q > 0, \; p \geq 0 \tag{3}$$

where

$$\begin{aligned} &\alpha_0 > 0; \quad \alpha_i \geq 0 \quad 1 \leq i \leq q; \\ &\beta_j \geq 0 \quad 1 \leq j \leq p; \quad \sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j > 1 \end{aligned} \tag{4}$$

For $p = 0$, d_k reduces to the ARCH(q) process [25]. In ARCH (q) processes, the conditional variance is specified as a linear function of past sample variances only, whereas GARCH(p, q) process allows lagged conditional variances to enter as well [25].

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3 Proposed system model

We consider a CR network with a single secondary user and M primary users operating over a wideband spectrum that arise the possibility of the correlation of the signals received in adjacent subchannels. The number of occupied subchannels is Q, where $Q \leq M$. Fig. 1 shows the spectrum.

We can formulate the following model for the received signal vector $\boldsymbol{y}(k) = [y_1(k), y_2(k), \ldots, y_M(k)]^{\mathrm{T}}$ at the kth snapshot as

$$y(k) = As(k) + w(k), \quad k = 1, \ldots, K \tag{5}$$

where $\boldsymbol{A} = \{a_{ij}\}_{M \times Q}$ is the $M \times Q$ occupancy matrix that represents the number and locations of the occupied subchannels. The jth element of the vector $\boldsymbol{s}(k) = [s_1(k), s_2(k), \ldots, s_Q(k)]^{\mathrm{T}}$ denotes the received signal from the jth occupied subchannel. $\boldsymbol{w}(k) = [w_1(k), w_2(k), \ldots, w_M(k)]^{\mathrm{T}}$ is the $M \times 1$ vector consisting of subchannels noises and K is the number of snapshots. Each primary user can occupy more than one subchannel, which guarantees the wideband spectrum-sensing assumption. Hence, the signals of subchannels can have correlations with each other.

If we consider $\boldsymbol{A}$ as the steering matrix and $\boldsymbol{s}(k)$ as the signal sources, then the problem of estimating the number and location of the occupied subchannels can be considered as the problem of estimating the number of signal sources like the techniques used in DOA estimation [23]. In DOA estimation Q users' signals are impinging on an array with M sensors as shown in Fig. 2.

The received signals in DOA system can be written as follows (see (6))

where $y_j(k)$, $j = 1, \ldots, M$ is the received signal in the jth sensor node at the kth snapshot, $s_i(k)$ $i = 1, \ldots, Q$ is the ith signal impinging on the nodes, and $w_j(k)$ is the noise of the jth sensor. φ_i is the direction arrival of $s_i(k)$ and a_{ij} is the response of the ith sensor to the jth impinging signal. In DOA estimation problem, the purpose is to estimate φ_is and the number of sources (Q).

Considering DOA estimation model for $\varphi = 0$, we can use it in spectrum sensing. The received signals of subchannels at the kth snapshot ($y_j(k)$s) can be written as

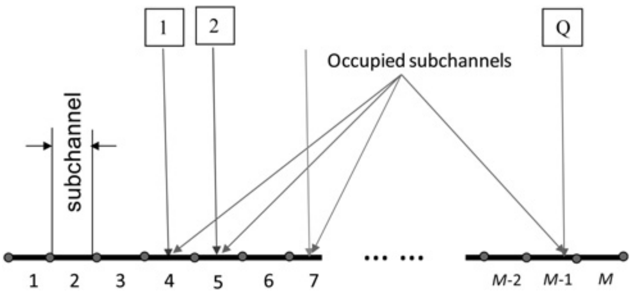

Fig. 1 *Illustration of the wideband spectrum sensing for M subchannels and Q occupied subchannels*

$$\begin{bmatrix} y_1(k) \\ y_2(k) \\ \vdots \\ y_M(k) \end{bmatrix} = \underbrace{\begin{bmatrix} a_{11} & a_{12} & & a_{1Q} \\ a_{21} & a_{22} & & a_{2Q} \\ \vdots & \vdots & \ddots & \vdots \\ a_{M1} & a_{M2} & & a_{MQ} \end{bmatrix}}_{A} \begin{bmatrix} s_1(k) \\ s_2(k) \\ \vdots \\ s_Q(k) \end{bmatrix} + \begin{bmatrix} w_1(k) \\ w_2(k) \\ \vdots \\ w_M(k) \end{bmatrix} \tag{7}$$

where $a_{ij} \in \{0, 1\}$ is a binary value that denotes the absence (0), or presence (1), of the jth primary user in the ith subchannel. $s_j(k)$ represents the jth primary user's signal, and $w_j(k)$ is the noise of the jth subchannel. As in DOA estimation, the aim in spectrum sensing is to estimate a_{ij}.

Since there are $M - Q$ unoccupied subchannels, there are $M - Q$ all-zero rows in the matrix $\boldsymbol{A}$. To simplify the problem, instead of the occupancy matrix $\boldsymbol{A}$, we define the occupancy vector $\boldsymbol{\Psi}$ as

$$\boldsymbol{\Psi} = [\psi_1, \psi_2, \ldots, \psi_M]^{\mathrm{T}} \tag{8}$$

where $\boldsymbol{\Psi}$ is a vector of zeros and ones; that is, $\psi_i \in \{0, 1\}$ where ψ_i expresses the absence or existence of the primary user in the ith subchannel.

$w_j(k)$ in (7) represents the jth subchannel noise with time-varying variance that is modelled as GARCH(p, q) which is defined as follows

$$\varepsilon_{n,j}(k) = \eta(k)\sigma_{n,j}(k), \quad \eta(k) \sim N(0, 1) \tag{9}$$

$$\sigma^2_{n,j}(k) = \alpha^2_{n,0} + \sum_{i=1}^{q} \alpha^2_{n,i}\varepsilon^2_{n,j}(k-i) + \sum_{i=1}^{p} \beta^2_{n,i}\sigma^2_{n,j}(k-i) \tag{10}$$

where $\varepsilon_{n,j}(k)$ in (9) is a discrete-time stochastic process, $\eta(k)$ is a sequence of i.i.d. random variables with zero mean and unit variance and N denotes the standard normal probability density function. $\sigma^2_{n,j}(k)$ is the jth subchannel noise variance at the kth snapshot. In (10), α_is and β_is are the parameters of GARCH model with the conditions $\alpha_0 > 0$, $\alpha_j \geq 0$ for $j = 1, \ldots, q$ and $\beta_j \geq 0$ for $j = 1, \ldots, p$ and $\sum_{i=1}^{q} \alpha_i + \sum_{i=1}^{p} \beta_i \geq 1$ as mentioned before.

The variance of the received primary user's signal (noiseless) is added to the $\alpha^2_{n,0}$ in (10). Hence, the diagonal elements of the covariance matrix of $\boldsymbol{y}$ (i.e. R_{yy}) can also be

$$\underbrace{\begin{bmatrix} y_1(k) \\ y_2(k) \\ \vdots \\ y_M(k) \end{bmatrix}}_{\mathbf{y}(k) \quad M\times 1} = \begin{bmatrix} a_{11}e^{-j\omega_1\varphi_1} & a_{12}e^{-j\omega_1\varphi_2} & & a_{1Q}e^{-j\omega_1\varphi_Q} \\ a_{21}e^{-j\omega_2\varphi_1} & a_{22}e^{-j\omega_2\varphi_2} & & a_{2Q}e^{-j\omega_2\varphi_Q} \\ \vdots & \vdots & \ddots & \vdots \\ a_{M1}e^{-j\omega_M\varphi_1} & a_{M2}e^{-j\omega_M\varphi_2} & & a_{MQ}e^{-j\omega_M\varphi_Q} \end{bmatrix}_{M\times Q} \underbrace{\begin{bmatrix} s_1(k) \\ s_2(k) \\ \vdots \\ s_Q(k) \end{bmatrix}}_{\mathbf{s}(k) \quad Q\times 1} + \underbrace{\begin{bmatrix} w_1(k) \\ w_2(k) \\ \vdots \\ w_M(k) \end{bmatrix}}_{\mathbf{w}(k) \quad M\times 1} \tag{6}$$

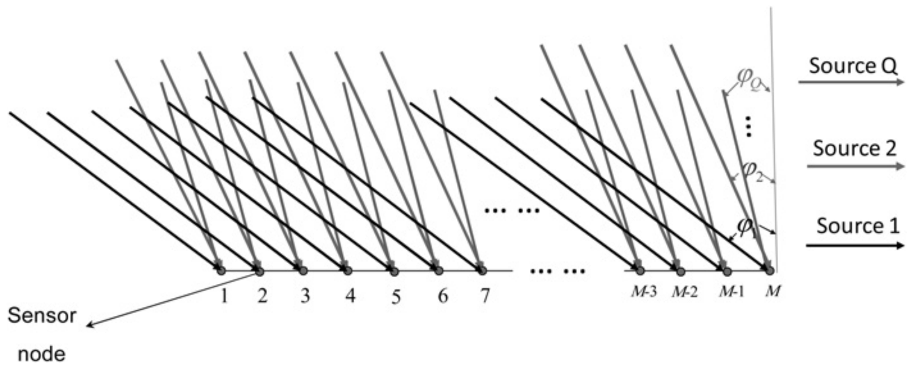

Fig. 2 *Illustration of DOA estimation with M sensor nodes and Q users' signals*

written in the same way as (9) and (10), that is

$$\varepsilon_j(k) = \eta(k)\sigma_j(k), \quad \eta(k) \sim N(0, 1) \tag{11}$$

$$\sigma_j^2(k) = \alpha_{j,0}^2 + \sum_{i=1}^{q} \alpha_{j,i}^2 \varepsilon_j^2(k-i) + \sum_{i=1}^{p} \beta_{j,i}^2 \sigma_j^2(k-i) \tag{12}$$

where $\varepsilon_j(k)$ is the discrete-time stochastic process of the jth subchannel at the kth snapshot and $\sigma_j^2(k)$ is the jth subchannel received variance. $\Phi_{G,j} = [\alpha_{j,0}, \alpha_{j,1}, \ldots, \alpha_{j,q}, \beta_{j,1}, \beta_{j,2}, \ldots, \beta_{j,p}]$ is defined as the vector of unknown parameters used to describe GARCH model parameters for the jth subchannel. Since (12) is a recursive function, we should define initial values for $\varepsilon_j^2(k)\ k = 0, \ldots, 1-q$ and $\sigma_j^2(k)\ k = 0, \ldots, 1-p$. For simplicity, all the initial values are chosen to be equal to $\varepsilon_j^2(1)$ [28]; that is, $\varepsilon_j^2(0) = \cdots = \varepsilon_j^2(1-q) = \sigma_j^2(0) = \cdots = \sigma_j^2(1-p) = \varepsilon_j^2(1)$.

4 Proposed algorithm

In this section, we will consider the problem of estimating the presence of the primary users' signals in subchannels by using an optimisation algorithm to maximise the likelihood function. To do so, we estimate the occupancy vector $\boldsymbol{\Psi}$ using the ML estimator as

$$\hat{\Psi} = \arg\max_{\psi}[P(\boldsymbol{y}|\boldsymbol{\Psi}, \boldsymbol{\Phi})] \tag{13}$$

where P is the likelihood function of $\boldsymbol{\Psi}$ for the received signal $\boldsymbol{y}$ and $\boldsymbol{\Phi}$ represents the vector of the (unknown) model parameters. We replace $\boldsymbol{\Phi}$ by its ML estimate namely $\hat{\Phi}$ and since logarithm is a monotonic function, we equivalently rewrite the optimisation problem of (13) as

$$\hat{\Psi} = \arg\min_{\psi}\left\{-\log[P(\boldsymbol{y}|\boldsymbol{\Psi}, \hat{\Phi})]\right\} \tag{14}$$

The optimal estimate of $\boldsymbol{\Psi}$ can be found by applying an exhaustive ML search. In this case, the number of possible choices for $\boldsymbol{\Psi}$ increases exponentially by increasing M. As stated, the straightforward using of ML approach for spectrum sensing considering GARCH model and joint probability density function as likelihood function, increases the system complexity due to the large number of unknown parameters. Therefore, to simplify the log likelihood function derived from the joint probability density function, we deploy subspace decomposition of the received signal. Subspace decomposition can be used to decompose $y(k)$ in (7) into its signal subspace vector and noise subspace vectors [29]. To be able to write $y_{ss}(k)y_{ss}^{\mathrm{H}}(k)$ and $y_{nn}(k)y_{nn}^{\mathrm{H}}(k)$ in terms of $\boldsymbol{y}(k)y^{\mathrm{H}}(k)$, first, we define $B_1^{(\psi)}$ and $B_2^{(\psi)}$ as follows. If we assume that the $\{i, j, \ldots, k\}$th and $\{i_c, j_c, \ldots, k_c\}$th elements of the occupancy vector $\boldsymbol{\Psi}$ to be all ones and all zeros, respectively, then $B_1^{(\psi)}$ and $B_2^{(\psi)}$ will be the matrices that contain the $\{i, j, \ldots, k\}$th and $\{i_c, j_c, \ldots, k_c\}$th rows of $\boldsymbol{I}$, respectively, where $\boldsymbol{I}$ is an $M \times M$ unitary matrix and $\{i, j, \ldots, k\}U\{i_c, j_c, \ldots, k_c\} = \{1, 2, \ldots, M\}$. We demonstrate this procedure by an example as follows

$$\psi_{\text{typical}} = [\overset{i}{1}, \overset{i_c}{0}, \overset{j}{1}, \overset{j_c}{0}, \ldots, \overset{k}{1}, \overset{k_c}{0}]_{1\times M}$$

$$\rightarrow I = \begin{bmatrix} i = [1 & 0 & 0 & 0 & 0 & \cdots & 0] \\ i_c = [0 & 1 & 0 & 0 & 0 & \cdots & 0] \\ j = [0 & 0 & 1 & 0 & 0 & \cdots & 0] \\ j_c = [0 & 0 & 0 & 1 & 0 & \cdots & 0] \\ \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ k = [0 & 0 & 0 & 0 & 0 & \cdots & 0] \\ k_c = [0 & 0 & 0 & 0 & 0 & \cdots & 1] \end{bmatrix}_{M\times M}$$

$$\rightarrow \begin{cases} B_1^{(\psi)} = \begin{bmatrix} i \\ j \\ \vdots \\ k \end{bmatrix} \\ B_2^{(\psi)} = \begin{bmatrix} i_c \\ j_c \\ \vdots \\ k_c \end{bmatrix} \end{cases} \tag{15}$$

So, we have

$$y_{ss}(k)y_{ss}^{\mathrm{H}}(k) = B_1^{(\psi)} y(k) y^{\mathrm{H}}(k) (B_1^{(\psi)})^{\mathrm{H}} \tag{16}$$

$$y_{nn}(k)y_{nn}^{\mathrm{H}}(k) = B_2^{(\psi)} y(k) y^{\mathrm{H}}(k) (B_2^{(\psi)})^{\mathrm{H}} \tag{17}$$

where $y_{ss}(k)y_{ss}^{\mathrm{H}}(k)$ and $y_{nn}(k)y_{nn}^{\mathrm{H}}(k)$ consist of those elements of $\boldsymbol{y}(k)y^{\mathrm{H}}(k)$ that correspond to the signal and noise subspaces, respectively, and $(\cdot)^{\mathrm{H}}$ denotes the Hermitian operation. To make it more obvious in the above example, if we consider $B_1^{(\psi)}$ as $Q \times M$ and $B_2^{(\psi)}$as $(M-Q) \times M$ matrices, then the dimensions of the matrices in (16) and

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(17) are as follows

$$\underbrace{y_{ss}(k)}_{Q\times 1}\underbrace{y_{ss}^{\rm H}(k)}_{1\times Q} = \underbrace{B_1^{(\psi)}}_{Q\times M}\underbrace{y(k)}_{M\times 1}\underbrace{y^{\rm H}(k)}_{1\times M}\underbrace{(B_1^{(\psi)})^{\rm H}}_{M\times Q} \tag{18}$$

$$\underbrace{y_{nn}(k)}_{(M-Q)\times 1}\underbrace{y_{nn}^{\rm H}(k)}_{1\times (M-Q)} = \underbrace{B_2^{(\psi)}}_{(M-Q)\times M}\underbrace{y(k)}_{M\times 1}\underbrace{y^{\rm H}(k)}_{1\times M}\underbrace{(B_2^{(\psi)})^{\rm H}}_{M\times (M-Q)} \tag{19}$$

Since we assume that the noise and primary users' signals are independent and noise in each subchannel is independent from the noises of other subchannels, it follows that $y_{ss}(k)$ is independent of $y_{nn}(k)$. Therefore, using the joint probability density function, we have

$$p(y_{ss}(k)|\boldsymbol{\Psi}, \boldsymbol{\Phi}) = \frac{1}{(2\pi)^{Q/2}\left(\det(R_{ss})\right)^{1/2}} \times \exp\left(-\frac{1}{2}y_{ss}^{\rm H}(k)R_{ss}^{-1}y_{ss}(k)\right) \tag{20}$$

$$p(y_{nn}(k)|\boldsymbol{\Psi}, \boldsymbol{\Phi}) = \frac{1}{(2\pi)^{(M-Q)/2}\left(\det(R_{nn})\right)^{1/2}} \times \exp\left(-\frac{1}{2}y_{nn}^{\rm H}(k)R_{nn}^{-1}y_{nn}(k)\right) \tag{21}$$

where $\boldsymbol{\Phi}$ represents the parameters of GARCH model ($\Phi_G = [\Phi_{G,1};\ \Phi_{G,2};\ \ldots;\ \Phi_{G,M}]$) and cross-correlation values among subchannels. $\boldsymbol{\Psi}$ contains the locations of the occupied subchannels as defined in (8). R_{ss} and R_{nn} are the covariance matrices of the signal and noise subspaces, respectively. R_{ss} contains those elements of R_{yy} that correspond to the occupied subchannels and the elements of R_{nn} are the variances of unoccupied subchannels. Thus, R_{ss} and R_{nn} can be expressed as

$$R_{ss} = B_1^{(\psi)}R_{yy}(B_1^{(\psi)})^{\rm H} \tag{22}$$

$$R_{nn} = B_2^{(\psi)}\mathrm{diag}(\sigma_1^2, \sigma_2^2, \ldots, \sigma_{(M-Q)}^2)(B_2^{(\psi)})^{\rm H} \tag{23}$$

5 ML formulation for spectrum-sensing problem

To localise the occupied subchannels in the spectrum, we know that the log-likelihood function of the received signal (L_y) can be obtained as

$$L_y = L_{\rm s} + L_{\rm n} \tag{24}$$

where $L_{\rm s}$ and $L_{\rm n}$ are the log-likelihood functions of the signal subspace and noise subspace, respectively, which can be derived from (20) and (21).

Now, we consider the ML problem as

$$\hat{\Psi} = \arg\max_{\boldsymbol{\Psi},\boldsymbol{\Phi}}(P_y(\boldsymbol{\Psi}, \boldsymbol{\Phi})) \equiv \arg\min_{\boldsymbol{\Psi},\boldsymbol{\Phi}}(L_y(\boldsymbol{\Psi}, \boldsymbol{\Phi})) \tag{25}$$

where $\hat{\Psi}$ is the estimated values of the locations of occupied subchannels (8). We denote the probability density function of $\boldsymbol{y}(k)$ by P_y; considering the likelihood function of P_y, we can obtain L_y which is written as $L_y(\boldsymbol{\Psi}, \boldsymbol{\Phi})$ to emphasise its dependence on $\boldsymbol{\Psi}$ and $\boldsymbol{\Phi}$.

The parameters of $\boldsymbol{\Phi}$, that is, $\Phi_G = [\Phi_{G,1};\Phi_{G,2};\ldots;\Phi_{G,M}]$ and the cross-correlation values, are ML estimated before the estimation of $\boldsymbol{\Psi}$. The cross-correlation values among the subchannels are assumed to be constant during the estimation. So, as shown in [5], the ML estimation of the cross-correlations among the subchannels is the sample cross-correlation. From [28], if we consider K realisations of $\varepsilon(k)$ as $\varepsilon(1)$, $\varepsilon(2)$, …, $\varepsilon(K)$, then the ML estimation of GARCH parameters for the jth subchannel can be derived as

$$\hat{\Phi}_{G,j} = \arg\max_{\Phi_{G,j}}\sum_{k=1}^{K}\frac{1}{\sqrt{2\pi\sigma_j^2(k)}}\exp\left(-\frac{\varepsilon_j^2(k)}{2\sigma_j^2(k)}\right) = \arg\min_{\Phi_{G,j}}\frac{1}{K}\sum_{k=1}^{K}\left(\frac{\varepsilon_j^2(k)}{\sigma_j^2(k)} + \log\sigma_j^2(k)\right) \tag{26}$$

By substituting the ML estimation of $\boldsymbol{\Phi}$ (i.e. $\hat{\Phi}$) in (25), we

1. For $Q = 1 : M$
2. Obtain B_1 and B_2,
3. Obtain $y_{ss}y_{ss}^H$ and $y_{nn}y_{nn}^H$ using B_1, B_2, and y as noted in (16) and (17),
4. Obtain sample cross-correlations between subchannels received signal,
5. Obtain ML estimate of GARCH model parameters using (26),
6. Obtain estimate of R_{ss} using B_1, the sample cross-correlations, and the GARCH model according to (22),
7. Obtain estimate of occupancy vector using (27)
8. End
9. Obtain $\hat{Q}$ using $(\hat{\Psi}^{(1)}, \hat{\Psi}^{(2)}, \ldots, \hat{\Psi}^{(M)})$ and (28),
10. Estimate the occupancy vector $\Psi^{(\hat{Q})}$.

Fig. 3 *Proposed spectrum-sensing algorithm*

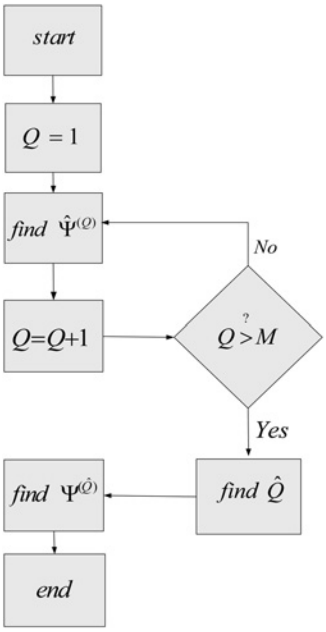

Fig. 4 *Flowgraph of the proposed spectrum-sensing method*

have

$$\hat{\Psi} = \arg\min_{\boldsymbol{\Psi}}(L_y(\boldsymbol{\Psi}, \hat{\Phi})) \tag{27}$$

To estimate the locations of the occupied subchannels in the spectrum, we estimate the number of those subchannels (Q). To obtain a better estimate of Q, instead of using (27), its modified version is utilised. The modification of (27) is performed by adding a penalty term to the ML problem, which is shown below.

$$\hat{Q} = \arg\min_{Q}(L_y(\Psi^{(Q)}, \hat{\Phi}) + \text{penalty term}) \tag{28}$$

In the above equation, $\boldsymbol{\Psi}$ is written as $\Psi^{(Q)}$ to emphasise its dependence on Q. The mentioned penalty term is derived from the minimum description length principle defined in [30] and is equal to $2Q(2M - Q)\log K$. To estimate the number of occupied subchannels, we estimate their locations for all possible values of Q. After obtaining the estimation ($\hat{Q}$), we compute the final estimation of the locations of the occupied subchannels in the spectrum. As mentioned, the exact value of Q is assumed to be unknown. The proposed algorithm is presented in Fig. 3, and the flowchart of the proposed optimisation algorithm to sense the spectrum is depicted in Fig. 4. In this figure, Q is defined as the hypothesised number of occupied subchannels, $\Psi^{(\hat{Q})}$ is defined as the estimation of occupancy vector with the estimated $\hat{Q}$ occupied subchannels and M is the number of subchannels.

6 Results

In this section, we present the simulation results to evaluate the performance of the proposed algorithm and compare it with the performances of two other methods. The first method is energy detector denoted (ED) [31] with the noise uncertainty values (shown by 'uc') equal to 0.5 and 1 dB. The second method is the one proposed in [23] denoted by ES-MDL, which stands for energy sorting and minimum

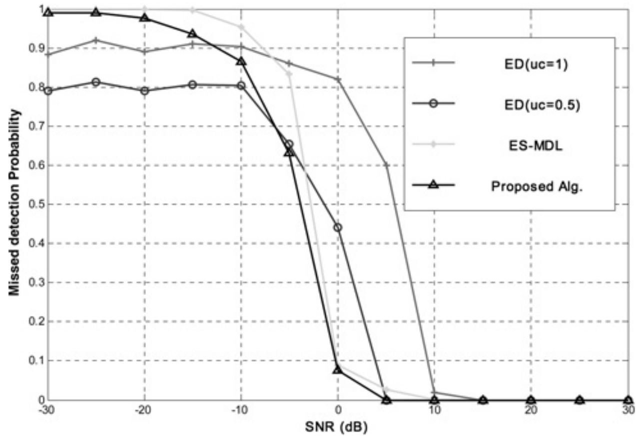

Fig. 5 *Missed detection probability against SNR, M = 6 and Q = 3*

description length criteria. In ES-MDL, at first, the estimated number of occupied subchannels ($\hat{Q}$) is found by eigenvalue decomposition and ITC. Then, the locations of occupied subchannels in the spectrum are estimated by choosing $\hat{Q}$ subchannels with the largest energies. It is notable that in ES-MDL, the correlations among the signals of subchannels are considered only in estimating the number of occupied subchannels but not in estimating their locations in the spectrum. To solve the optimisation problems of (26) and (28), we apply quasi-Newton method [32]. All the curves are obtained by Monte-Carlo simulation with 10^5 runs. The GARCH model used here is GARCH(1, 1).

To estimate the threshold of the energy detector, we should have an estimation of noise variance. This estimation is performed by sample energy. The threshold λ_{fa} is found as follows [33]

$$\lambda_{\text{fa}} = \sigma_n^2\left(1 + \frac{Q^{-1}(P_{\text{fa}})}{\sqrt{K/2}}\right) \tag{29}$$

where σ_n^2 is the estimated noise variance using sample covariance estimation method, $Q(\cdot)$ represents the Q-function, P_{fa} represents the false alarm probability and K is the number of samples. Equation (29) is derived with the assumption of having no noise uncertainty in which P_{fa} is set to 10^{-3}, but as can be seen from Figs. 6 and 9, by

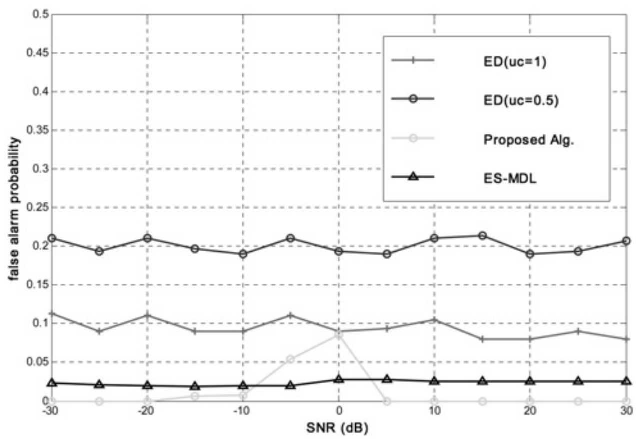

Fig. 6 *False alarm probability against different SNR values, M = 6 and Q = 3*

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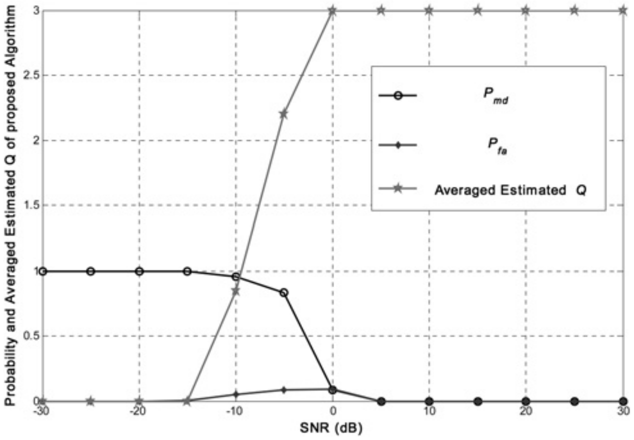

Fig. 7 *Missed detection probability, false alarm probability and average estimated Q against different SNR values in the proposed algorithm, $M = 6$ and $Q = 3$*

determining the threshold from (29) and for the case of having noise uncertainty, P_{fa} has values higher than 10^{-3}. Also, considering the values obtained in [10, 23] for P_{fa}, the obtained P_{fa} is contained in a range of practical interest for the spectrum sensing.

6.1 Effect of signal to noise ratio (SNR)

In this subsection, the performance of the proposed algorithm is investigated for different values of SNR. P_{md} (missed detection probability) and P_{fa} for the proposed algorithm are compared with those of ES-MDL and ED in Figs. 5 and 6, respectively. To compute P_{md}, the number of occupied subchannels that are detected as unoccupied ones is computed. And, in order to compute P_{fa}, the number of unoccupied subchannels that are detected as occupied ones is computed. We only compare the algorithms for the cases $P_{md} < 0.5$, where there is a possibility to have a reliable performance. We consider six subchannels ($M = 6$), where half of them are occupied ($Q = 3$). Since we consider primary users operating in a wideband spectrum, the signals received in adjacent subchannels can be correlated.

It should be noted that in our method the additive noise is modelled by the GARCH model whereas the ED and ES-MDL methods consider Gaussian distribution for noise.

From Fig. 5 we observe that for all acceptable values of P_{md} and for all values of SNR, our algorithm has better performance than the ES-MDL. As seen, the missed detection probability of the proposed method is less than the energy detector for SNRs between −14 and 15 dB in the case of uc = 0.5 and SNRs between −7 and 5 dB for uc = 1. For high values of SNR (higher than 5 dB), the proposed algorithm can almost precisely sense the spectrum.

From Fig. 6, for SNR values less than −10 dB, the performance of the proposed algorithm is close to ES-MDL. We can also observe that for SNR values higher than 5 dB, the false alarm probability of the proposed algorithm can be approximately zero, which relates to the accuracy of the estimation and number of simulation runs. In other words, in these SNR values, the maximum spectrum utilisation is achievable using the proposed algorithm. Moreover, for almost all SNR values, the proposed algorithm achieves less false alarm probability than the ES-MDL and energy detector. Considering the results, we find out that at SNR values around 0 dB, the number of occupied channels Q is estimated correctly, but with erroneous estimation of their locations. So, a missed detection leads to a false alarm and as a result, $P_{md} = P_{fa}$.

Fig. 7 shows the missed detection probability, false alarm probability and average estimated Q for the proposed algorithm. From the figure, we can see that for SNR values less than −15 dB, the number of occupied subchannels is estimated to be zero. Hence, in these SNRs, P_{fa} is zero and P_{md} is one. For SNR values between −15 to −5 dB, the estimated number of occupied subchannels increases which results in decrease in P_{md}, and increase in P_{fa}. In other words, in these SNR values, the location of $\hat{Q}$ occupied subchannels is estimated erroneously and as a result, the change rate of probability of detection is almost the same as the change rate of probability of false alarm. For SNR values higher than 0 dB, by having a correct estimate of Q, the probabilities of missed detection and false alarm are equal (i.e. $P_{md} = P_{fa}$).

6.2 Effect of sensing time

In this subsection, we investigate the effect of sensing time (or received data length) on the performance of the proposed algorithm, ED and ES-MDL. To this aim, P_{md} and P_{fa} of the mentioned algorithms are plotted in terms of the received data length (K) in Figs. 8 and 9, respectively. We

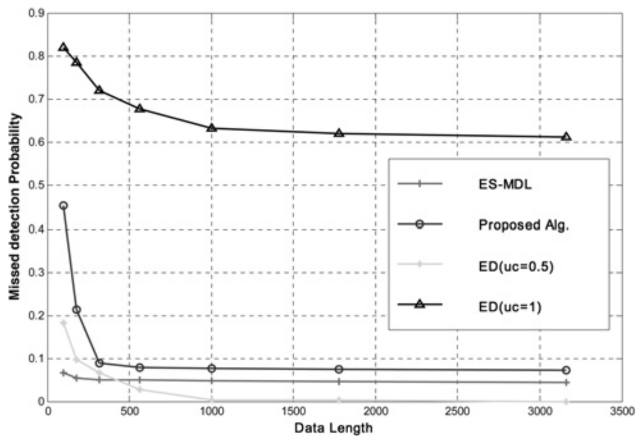

Fig. 8 *Missed detection probability against different data lengths (sensing time), $M = 6$, $Q = 3$ and SNR = 5 dB*

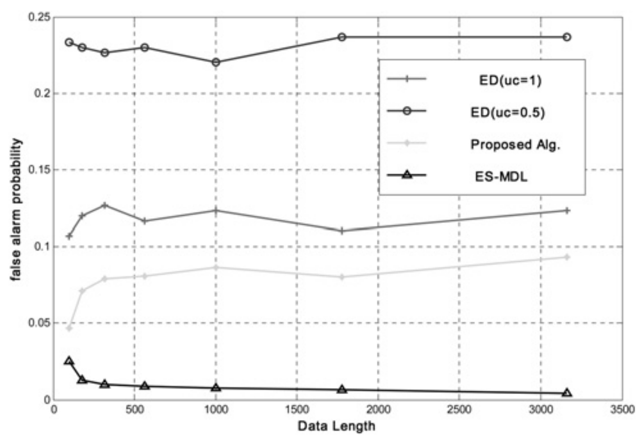

Fig. 9 *False alarm probability against different data lengths (sensing time), $M = 6$, $Q = 3$ and SNR = 5 dB*

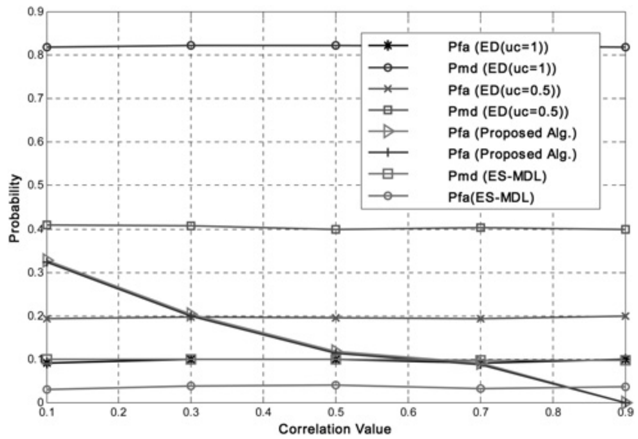

Fig. 10 *Missed detection and false alarm probabilities against different correlation values, M = 6, Q = 3 and SNR = 0 dB*

consider SNR = 5 dB. As can be seen from Fig. 8 by increasing the sensing time, P_{md} decreases at first but remains constant after a specific time, which shows the time for achieving a reliable spectrum sensing. Moreover, considering Fig. 9, the false alarm probability remains rather constant after a specific data length in all methods.

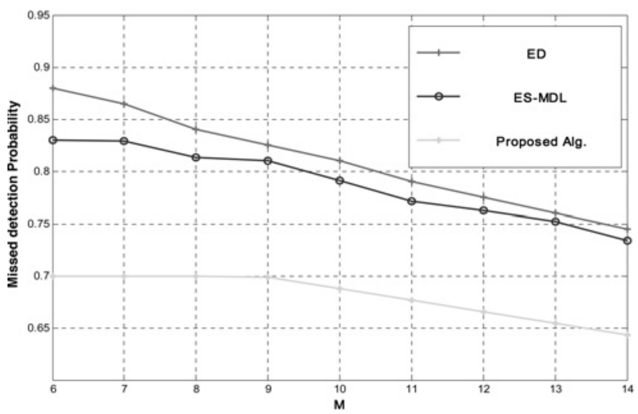

Fig. 11 *Missed detection probability against different values of M, Q = 4, SNR = −5 dB and uc = 1*

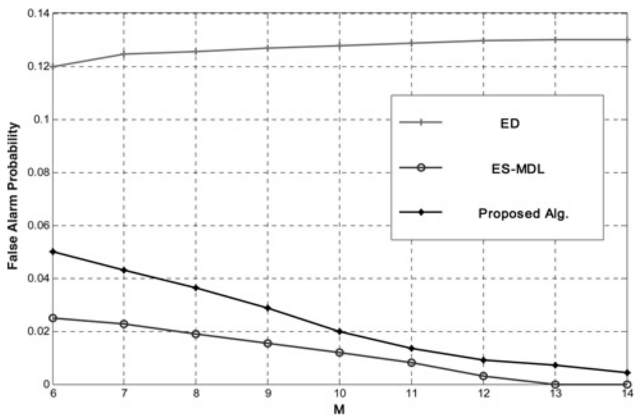

Fig. 12 *False alarm probability against different values of M, Q = 4, SNR = −5 dB and uc = 1*

6.3 *Effect of correlations among occupied subchannels' signals*

In Fig. 10, the performances of the three methods for different values of correlation among subchannels are presented. As the proposed method is wideband, which considers correlation among subchannels, we observe that both the false alarm probability and the missed detection probability decrease by increasing the correlation among subchannels. However, the performance of energy detector and the ES-MDL are not affected by the correlation among subchannels.

6.4 *Effect of number of subchannels (M)*

In this section, we illustrate the effect of number of subchannels (M) on the performance. For this purpose, we examine P_{md} and P_{fa} for different values of M. The SNR value is set to −5 dB to have non-zero values for P_{md} and P_{fa}. In Figs. 11 and 12, the performances of the proposed algorithm, energy detector and ES-MDL are shown for different values of M where Q is set to 4. It is observed that by increasing M, the missed detection and false alarm probabilities decrease.

To evaluate the complexity of the proposed method, we consider the number of operations used in the cost functions which are defined in (26) and (28). Considering (26), to estimate GARCH model parameters, the number of additions is equal to K. Besides, noting (14) and (15) used in (28), and $Q \leq M$, the computational complexity can be shown to be of order M^3.

7 Conclusion

In this study we proposed a new approach for estimating the primary users' signals in non-Gaussian and non-stationary noise environment for CR networks. We used DOA estimation model for spectrum sensing. Because of the non-stationarity characteristics of GARCH model, it was used for noise modelling. Signal and noise subspaces are utilised to decrease the ML complexity. ML estimation was used for estimating the locations of the occupied channels; and ML estimation and ITC were utilised to estimate the number of occupied subchannels. The efficiency of the proposed scheme was examined by simulations which indicate its higher performance in comparison with the energy detector and the ES-MDL detector.

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