Internally coded multicarrier frequency-hopping CDMA communication system and its performance analysis

M.G. Shayesteh and M. Nasiri-Kenari

Abstract: A new internally coded scheme that combines frequency hopping (FH) and multicarrier (MC) code division multiple access (CDMA) techniques using a super-orthogonal encoder is proposed. In this method, the total bandwidth is partitioned into Ns_1 disjoint bands and each band is also segmented into Ns_2 subbands. On the basis of a super-orthogonal encoder output and a pseudorandom sequence, one of the Ns_1 bands is selected. The data bit is then transmitted in the band in the form of the multicarrier FH (MC-FH) CDMA scheme, that is, Ns_2 carriers are transmitted in the Ns_2 subbands of the selected band. At the receiver, both hard and soft detectors are used. The performance of the proposed method in the additive white Gaussian noise (AWGN) and fading channels is evaluated. The results indicate that the proposed method outperforms the previously presented uncoded and coded MC-FH-CDMA systems, where the data are transmitted over the whole bandwidth, keeping the same bandwidth (spectral efficiency). Further, in the new method, the carriers hop in part of the total bandwidth, and hence coherent detection is more feasible.

1 Introduction

In recent years, there has been intensive interest on multicarrier (MC) slow frequency-hopping (FH) multiple access schemes for high-rate multimedia communications [1–10]. In the FH system, the transmit frequency is changed based on the assigned signature code and it can be selected from the entire frequency band. FH can be considered as a serial frequency diversity technique. In the slow FH, the transmission frequency changes at most once over one symbol duration, whereas in the fast FH, there are multiple hops per information symbol [1–3].

An MC modulation (MCM) [4] is a parallel frequency diversity technique that is used to minimise the effects of fading. In this method, the information signal is transmitted simultaneously over several independent carrier frequencies. In this paper, we present a new internally coded MC-FH scheme. Our proposed MC scheme is completely different from those in [5–10] whereas it is similar to those in [1, 2].

In [1], an MC-FH technique has been proposed, in which diversity is obtained via both the MC transmission and the FH. In [2], the performance of the MC-FH system proposed in [1] has been analysed for the multiuser case and the authors have introduced a new coded structure which significantly outperforms the uncoded structure of [1] without requiring any extra bandwidth. Here, we briefly describe the methods introduced in [1, 2] as they are of greater relevance in this work and also we compare our method with those methods. In [1, 2], the total bandwidth is segmented into N_f distinct subbands of equal bandwidth where each subband contains N_h' different frequency carriers. On the basis of the assigned pseudorandom (PN) codes, N_f carriers are selected, one from each subband. In [1], the N_f carriers are modulated with the data bit of the user using binary phase shift keying (BPSK) modulation. That is, for each data bit, N_f carriers with an identical phase (0° or 180° based on user's data) are added and transmitted simultaneously through the channel. However, in [2], instead of sending N_f carriers with the same phase in one bit interval, these carriers are transmitted with different phases, in which their values are determined by the output bits of an encoder. That is, each of the N_f output bits of a super-orthogonal encoder determines the phase of one of the N_f carriers. As a result, the performance of [2] substantially improves without requiring any extra bandwidth in comparison with the uncoded method of [1]. The processing gain of the two methods is $N_f N_h'$ and the free distance of the method of [2] is $N_f(\log_2 N_f + 4)/2$.

In [1, 2], each carrier is selected from one subband, in which it can hop, whereas in the conventional FH, the transmit frequency hops within the whole bandwidth. The systems proposed in [1, 2] makes the carrier frequency hop slower compared with the fast FH and also imposes each carrier to hop solely in a fraction of the total bandwidth in comparison with the conventional FH method. Thus, a coherent reception in [1, 2] will be feasible in a slow fading channel. Note that although FH multiple access schemes such as MC-FH-CDMA do not show performance superiority over the MC-CDMA system [11, 12] (and therefore DS-CDMA system), in some applications, such as military and jamming environments, they are much more plausible.

The internally coded MC-FH-CDMA system considered in this paper is based on the FH and MC techniques. This

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coding scheme, similar to the method of [2], does not increase the bandwidth as opposed to the conventional coding methods. We use a super-orthogonal convolutional encoder as its path-generating function required for the performance evaluation is available. In the proposed method, the entire bandwidth is divided into Ns_1 disjoint bands and each band is also partitioned into Ns_2 distinct subbands. Two signature codes are assigned to each user. The outputs of the super-orthogonal encoder and a first PN code (PN1) determine one of the Ns_1 bands. Then, the data bit is transmitted in that band in the form of the MC-FH scheme introduced in [1] using BPSK modulation and the second PN code (PN2). That is, in each bit interval, Ns_2 carriers with an identical phase 0° or 180° (which is determined by the corresponding data bit of the user) are transmitted simultaneously in the selected band, one in each subband. The carrier in each subband is selected by the PN2.

We consider a downlink transmission. At the receiver, we use hard and soft detectors for the additive white Gausian noise (AWGN) channel, whereas for the fading channel, we consider only the soft detector before we apply the Viterbi decoding algorithm. We evaluate the performance of the proposed method using the Chernoff bound and the Beaulieu series. We compare the results with those of the uncoded and coded MC-FH-CDMA systems presented in [1, 2] for the same processing gain, bandwidth and spectral efficiency. Our results indicate that the proposed method has better performance than the methods introduced in [1, 2]. In addition, the new method has another important advantage as follows: the carriers are transmitted in one of the Ns_1 total bands, with each carrier hopping only within a subband of that band. Thus, for the same bandwidths for the proposed system and the systems of [1, 2], the hopping in our method occurs in $1/Ns_1$ bandwidth of the methods of [1, 2], which makes the proposed method more practical and the coherent detection in the fading channel more feasible. A similar internally coded scheme for the TH-UWB system has been considered in [13].

This paper is organised as follows. In Section 2, we describe the proposed system. In Sections 3 and 4, we evaluate the performance of the system in the AWGN and fading channels, respectively. In Section 5, we provide some numerical results. Finally, Section 6 concludes the paper.

2 System description

Fig. 1 shows the frequency-axis division and the block diagram of the proposed system. The total bandwidth B_T is segmented into Ns_1 bands each with a bandwidth W_b, and similar to the MC-FH system [1], each one of the Ns_1 bands is divided into Ns_2 subbands each with a bandwidth W_{sb}. Each subband also consists of N_h different frequency carriers (channels) spaced apart by W_d. Thus, there are $Ns_1 Ns_2 N_h$ different frequency carriers available for multiple access communications.

Two PN sequences, namely PN1 and PN2, are assigned to each user. In each bit interval, the output of a super-orthogonal encoder [14] with a constraint length of K, which produces 2^{K-2} different symbols, is added to the PN1 in the mode 2^{K-2}, where PN1 takes on integer values between 0 and $2^{K-2}-1$ uniformly. The result determines one of the Ns_1 bands, so we have $Ns_1 = 2^{K-2}$. Then, at the selected band, such as in the conventional MC-FH system, the uncoded data bit is transmitted over the Ns_2 subbands using BPSK modulation, that is, for each data bit, Ns_2 carriers with the same positive or negative phase (0° or 180° based on the data) are sent in the Ns_2 subbands, one in each subband. Actually, the PN2 determines one of the N_h carriers in each of the Ns_2 subbands (totally Ns_2 carriers), in which the data are transmitted. In this way, the equivalent baseband transmitted signal of the user k can be written as

$$s_{\text{tr}}^k(t) = \sqrt{2P}\sum_j (-1)^{D_j^k} \sum_{m=0}^{Ns_2-1} \exp(i2\pi[c_{1,j}^k W_\text{b} + mW_\text{sb} + c_{2,m}^k W_\text{d}](t-jT_s))p_{T_s}(t-jT_s) \quad (1)$$

where P is the power of each carrier, $D_j^k \in \{0,1\}$ the binary sequence of the transmitted symbols of the kth user at the jth bit interval and $c_{1,j}^k$ takes an integer value between 0 and $Ns_1 - 1$ uniformly and specifies one of the Ns_1 bands. It is determined based on the super-orthogonal encoder output and PN1 as

$$c_{1,j}^k = s_j^k + \text{PN1}_j^k \quad \text{mode } Ns_1 \quad (2)$$

where s_j^k and PN1_j^k are the coded symbols and the PN1 component of the user k at the jth bit interval, respectively (Fig. 1). W_b and W_sb are the bandwidths of each band and subband, respectively, and W_d is the frequency space between the two adjacent carriers. $c_{2,m}^k$ is the PN2 of the kth user which takes on the values 0 to $N_h - 1$ uniformly and determines the carrier frequency (one of the N_h channels) in the mth subband, which is used to send the data using BPSK modulation. $p_{T_s}(t)$ is a unit amplitude rectangular pulse with duration T_s, where T_s is the time duration of one bit. We choose $W_\text{d} = 1/T_s$ and with this selection, the carriers are orthogonal. The processing gain is defined as

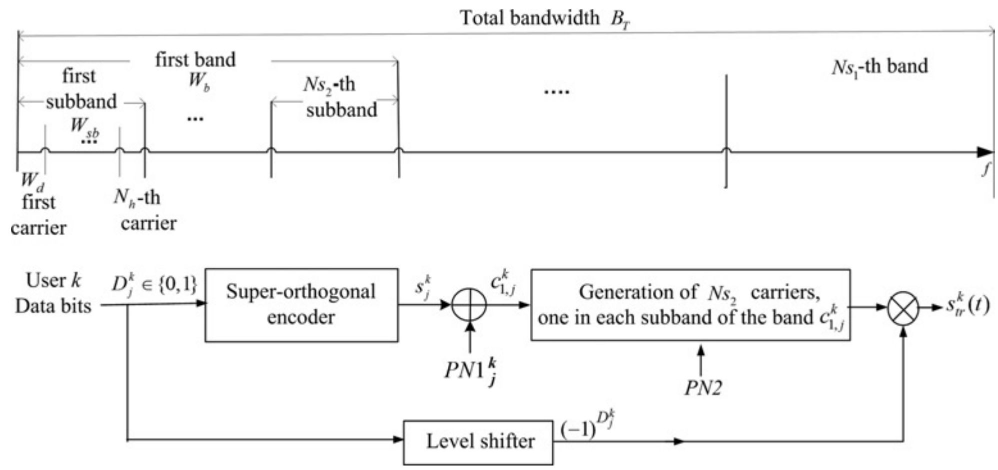

Fig. 1 *Frequency access division and the block diagram of the proposed method*

the ratio of the system bandwidth to the bit rate. So we have

$$\text{Processing gain} = \text{PG} = \frac{B_{\mathrm{T}}}{1/T_s} = Ns_1 Ns_2 N_h \quad (3)$$

We consider a downlink transmission, thus the received signal in a frequency-selective Rayleigh fading channel can be written as

$$\begin{aligned} r(t) &= \sum_{k=1}^{N_{\mathrm{u}}} s_{\mathrm{rec}}^k(t) + n(t) \\ &= \sum_{k=1}^{N_{\mathrm{u}}} \sqrt{2P} \sum_j (-1)^{D_j^k} \sum_{m=0}^{Ns_2-1} g_{j,c_{1,j}^k,m} \\ &\quad \times \exp\Big(i2\pi[c_{1,j}^k W_{\mathrm{b}} + mW_{\mathrm{sb}} + c_{2,m}^k W_{\mathrm{d}}](t - jT_s)\Big) \\ &\quad \times p_{T_s}(t - jT_s) + n(t) \end{aligned} \quad (4)$$

where N_{u} is the number of active users and $g_{j,c_{1,j}^k,m} = \beta_{j,c_{1,j}^k,m} \mathrm{e}^{i\theta_{j,c_{1,j}^k,m}}$ is the fading complex coefficient observed by the mth carrier of the kth user at the jth bit interval which depends on the selected band and the selected carrier of each subband. Its amplitude, $\beta_{j,c_{1,j}^k,m}$, has a Rayleigh distribution and its phase, $\theta_{j,c_{1,j}^k,m}$, has a uniform distribution, that is

$$\begin{aligned} p_{\beta_{j,c_{1,j}^k,m}}(\beta) &= \frac{2\beta}{\sigma^2} \exp\left(-\frac{\beta^2}{\sigma^2}\right) \ \beta > 0, \\ E(\beta^2_{j,c_{1,j}^{(k)},m}) &= \sigma^2, \\ p_{\theta_{j,c_{1,j}^k,m}}(\theta) &= \frac{1}{2\pi} \ \ 0 \le \theta \le 2\pi \end{aligned} \quad (5)$$

Note that since we consider a downlink transmission, all users have the same fading coefficients in the same channel of the same subband of a specific band. Since the adjacent carriers are located in the different subbands, the distance among them is large with high probability, and therefore we can assume that the assigned carriers experience independent frequency non-selective fading. The channel is also assumed to be slowly fading, that is, the complex channel gain corresponding to each carrier does not change significantly during several bit intervals. For the AWGN channel, the coefficients $g_{j,c_{1,j}^k,m}$ can be set equal to 1.

In (4), $n(t)$ is the baseband equivalent AWGN with zero mean and two-sided power spectral density of $N_0/2$.

At the receiver front end, in each bit interval, we use a sliding correlator which moves over the Ns_2 subbands of each of the Ns_1 bands. Assuming that the receiver knows the PN2, the correlator base signal is

$$v_{j,h,m}^k(t) = \begin{cases} \exp[i2\pi(hW_{\mathrm{b}} + mW_{\mathrm{sb}} + c_{2,m}^k W_{\mathrm{d}})(t - jT_s)], & jT_s < t < (j+1)T_s \\ 0 & \text{else} \end{cases}$$

$$h = 0, 1, \ldots, Ns_1 - 1, \quad m = 0, 1, \ldots, Ns_2 - 1. \quad (6)$$

Note that $v_{j,h,m}^k(t)$ depends only on PN2. Therefore the receiver output of the kth user in the band h at the jth bit interval is obtained as

$$\begin{aligned} R_{j,h}^k &= \sum_{m=0}^{Ns_2-1} \mathrm{Re}\left\{\frac{1}{T_s} g_{j,h,m}^* \int_{jT_s}^{(j+1)T_s} r(t) v_{j,h,m}^{*k}(t)\, \mathrm{d}t\right\} \\ &= S_{j,h} + I_{j,h} + n_{j,h} \end{aligned} \quad (7)$$

where $S_{j,h}$, $I_{j,h}$ and $n_{j,h}$ are the receiver outputs due to the desired user, interference and noise, respectively. These values, that is, $R_{j,h}^k$, are then used for the branch metric calculations of the trellis diagram of the underlying convolutional code, as will be explained later.

2.1 Receiver output

In the following, we obtain the correlator output due to the first user (which is assumed to be the desired user), interference and noise. Without loss of generality, we consider the interval $(0, T_s)$ and for simplicity, we drop the index $j = 0$ in the rest of the paper, that is, $g_{h,m} \triangleq g_{0,h,m}$. We first obtain the output for the fading channel, then by setting $g_{j,h,m} = 1 \Rightarrow \beta_{j,h,m} = 1\ \forall j, h, m$, the results can be applied to the AWGN channel.

2.1.1 Output due to the desired user: The correlator output due to the desired (first) user in the interval $(0, T_s)$ is obtained by replacing $s_{\mathrm{rec}}^1(t)$ (4) in the place of $r(t)$ in (7), which results in

$$\begin{aligned} S_h &= \sum_{m=0}^{Ns_2-1} S_{h,m} \\ &= \sum_{m=0}^{Ns_2-1} \mathrm{Re}\left\{\frac{1}{T_s} g_{h,m}^* \int_0^{T_s} \sqrt{2P} p_{T_s}(t) (-1)^{D_0^1} \sum_{m'=0}^{Ns_2-1} g_{c_{1,0}^1,m'} \right. \\ &\quad \times \exp\big(i2\pi[c_{1,0}^1 W_{\mathrm{b}} + mW_{\mathrm{sb}} + c_{2,m}^1 W_{\mathrm{d}}]t\big) \\ &\quad \left. \times \exp\big(-i2\pi[hW_{\mathrm{b}} + m'W_{\mathrm{sb}} + c_{2,m'}^1 W_{\mathrm{d}}]t\big) \mathrm{d}t \right\} \\ &= \begin{cases} \sqrt{2P} D_0^1 \sum_{m=0}^{Ns_2-1} \beta_{h,m}^2 & \text{if } h = c_{1,0}^1, \\ 0 & \text{if } h \ne c_{1,0}^1, \end{cases} \\ &\qquad h = 0, 1, \ldots, Ns_1 - 1 \end{aligned} \quad (8)$$

where we have used $W_{\mathrm{d}} = 1/T_s$.

2.1.2 Effect of multiple access interference: The correlator output due to the interfering user k in the hth band is the summation of the effects of the interference in the Ns_2 subbands, $I_{h,m}^k$, that is

$$I_h^k = \sum_{m=0}^{Ns_2-1} I_{h,m}^k \quad (9)$$

where

$$\begin{aligned} I_{h,m}^k &= \mathrm{Re}\left\{\frac{1}{T_s} g_{h,m}^* \int_0^{T_s} \sqrt{2P} (-1)^{D_0^k} \sum_{m'=0}^{Ns_2-1} g_{c_{1,0}^k,m'} \right. \\ &\quad \times \exp\Big(i2\pi[c_{1,j}^k W_{\mathrm{b}} + mW_{\mathrm{sb}} + c_{2,m}^k W_{\mathrm{d}}]t\Big) \\ &\quad \left. \times \exp\big(-i2\pi[hW_{\mathrm{b}} + m'W_{\mathrm{sb}} + c_{2,m'}^1 W_{\mathrm{d}}]t\big) p_{T_s}(t) \right\} \end{aligned}$$

$$= \begin{cases} \frac{\sqrt{2P}}{T_s}(-1)^{D_0^k}\beta_{h,m}^2 \int_0^{T_s} \exp\{(c_{2,m}^k - c_{2,m}^1)W_d t\}\mathrm{d}t & \text{if } h = c_{1,0}^k \\ 0 & \text{if } h \neq c_{1,0}^k. \end{cases} \quad (10)$$

The result of the above integral is T_s if $c_{2,m}^k = c_{2,m}^1$ and zero if $c_{2,m}^k \neq c_{2,m}^1$. The probability that the two users have the same PN2, that is, they send their data over the same carrier of a specific subband, is

$$P(c_{2,m}^k = c_{2,m}^1) \triangleq \alpha = \frac{1}{N_h} \quad (11)$$

So, $I_{h,m}^k$ can take on the values 0 or $\pm\sqrt{2P}\beta_{h,m}^2$ in the band $h = c_{1,0}^k$. In this way, the probability density function (PDF) and the characteristic function of the interference term due to the user k in the mth subband of the band $h = c_{1,0}^k$, that is, $I_{h,m}^k$, conditioned on the data and fading coefficients, are obtained as follows

$$p_{I_{h,m}^k|\beta_{h,m},D_0^k=0or1}(x) = (1-\alpha)\delta(x) + \alpha\delta\left(x \mp \sqrt{2P}\beta_{h,m}^2\right) \Rightarrow$$
$$\varphi_{I_{h,m}^k|\beta_{h,m},D_0^k=0or1}(s) = (1-\alpha) + \alpha\exp\left(\pm s\sqrt{2P}\beta_{h,m}^2\right) \quad (12)$$

where the characteristic function is obtained by taking the Laplace transform of the PDF.

The interference components due to the kth user in the Ns_2 different subbands conditioned on the data and fading coefficients are independent. Consequently, its characteristic function in the band $h = c_{1,0}^k$ conditioned on the fading coefficients can be computed as

$$\begin{aligned}\varphi_{I_h^k|\beta_{h,\{m\}}}(s) &= P(D_0^k = 0)\varphi_{I_h^k|D_0^k=0,\beta_{h,\{m\}}}(s) \\ &\quad + P(D_0^k = 1)\varphi_{I_h^k|D_0^k=1,\beta_{h,\{m\}}}(s) \\ &= \frac{1}{2}\prod_{m=0}^{Ns_2-1}\left((1-\alpha) + \alpha\exp\left(s\sqrt{2P}\beta_{h,m}^2\right)\right) \\ &\quad + \frac{1}{2}\prod_{m=0}^{Ns_2-1}\left((1-\alpha) + \alpha\exp\left(-s\sqrt{2P}\beta_{h,m}^2\right)\right)\end{aligned} \quad (13)$$

where $\beta_{h,\{m\}} \triangleq \beta_{h,0}, \ldots, \beta_{h,Ns_2-1}$.

Since the interference terms due to the different users are independent, the characteristic function of the total interference due to the U interfering users conditioned on the fading coefficients can be computed as

$$\varphi_{I_h|\beta_{h,\{m\}}}(s) = \prod_{k=1}^{U}\varphi_{I_h^k|\beta_{h,\{m\}}}(s) = \left(\varphi_{I_h^k|\beta_{h,\{m\}}}(s)\right)^U \quad (14)$$

2.1.3 Output noise:

The output noise in the mth subband of each band is obtained as

$$n_{h,m} = \mathrm{Re}\left\{\frac{g_{h,m}^*}{T_s}\int_0^{T_s} n(t)v_{h,m}^{*1}(t)\,\mathrm{d}t\right\}$$
$$h = 0, 1, \ldots, Ns_1 - 1, \quad m = 0, 1, \ldots, Ns_2 - 1 \quad (15)$$

We can easily show that the output noise has zero mean and its components are uncorrelated in different subbands, that is, $E(n_{h,m}n_{h,m'}) = 0$, $m \neq m'$. The variances of the noise in each subband and each band are calculated as

$$\sigma_{n_{h,m}|\beta_{h,m}}^2 = \frac{N_0}{2T_s}\beta_{h,m}^2 \Rightarrow$$
$$\sigma_{n_h|\beta_{h,\{m\}}}^2 = \sum_{m=0}^{Ns_2-1}\sigma_{n_{h,m}|\beta_{h,m}}^2 = \frac{N_0}{2T_s}\sum_{m=0}^{Ns_2-1}\beta_{h,m}^2 \quad (16)$$

So the PDF and the characteristic function of the output noise in each band are as follows

$$p_{n_h|\beta_{h,\{m\}}}(n) = \frac{1}{\sqrt{2\pi(N_0/2T_s)\beta_{h,m}^2}}\exp\left\{\frac{-n^2}{2(N_0/2T_s)\beta_{h,m}^2}\right\} \Rightarrow$$
$$\varphi_{n_h|\beta_{h,\{m\}}}(s) = \exp\left\{s^2(N_0/4T_s)\sum_{m=0}^{Ns_2-1}\beta_{h,m}^2\right\} \quad (17)$$

2.2 Upper bound on the bit error rate

As stated, a super-orthogonal encoder, which is a near-optimal convolutional code, is used to determine the band number, in which the data bit is transmitted. For a convolutional code, only the lower and upper bounds on the bit error rate (BER) are analytically available. As mentioned, in our method, the output of the super-orthogonal encoder is considered as one of the 2^{K-2} symbols, not as a sequence of bits. Hence, the path-generating function in our application differs from that obtained in [14, 15], in which the encoder output is considered as a sequence of bits. The path-generating function in our application is computed as [13]

$$T(D, N) = \frac{N(1-D)D^K}{1 - D(1 + N(1 + D^{K-3} - 2D^{K-2}))} \quad (18)$$

where in the series expansion of the above equation, the power of D denotes the (symbol) Hamming weight of the encoder output and the power of N represents the Hamming weight of the input bit. The number of bit errors due to an error event with weight d, that is, c_d, can be calculated from the path generating function as

$$\left.\frac{\partial T(D, N)}{\partial N}\right|_{N=1} = \sum_{d=d_{\mathrm{free}}}^{\infty} c_d D^d \quad (19)$$

where d_{free} is the free distance [14, 15] of the super-orthogonal code, which in our application equals

$$d_{\mathrm{free}} = K = \log_2 Ns_1 + 2 \quad (20)$$

Using the union bound, the upper bound [14] on the BER can be obtained as

$$P_b < \sum_{d=d_{\mathrm{free}}}^{\infty} c_d P_d \quad (21)$$

where P_d is the probability of an error event with Hamming distance of d [14, 15]. Note that since the outputs of the super-orthogonal encoder are considered as symbols, not as a sequence of bits, the distance considered here is the symbol distance of the two paths (not the bit distance). Without loss of generality, we assume that the desired user sends an all-zero sequence. Thus, P_d is the probability that the metric of a non-zero path with symbol weight d is larger than that of the all-zero path. So

$$P_d = \Pr\left\{\sum_{n=1}^{d} M_{h_n\neq 0} > \sum_{n=1}^{d} M_{h_n=0}\right\}$$
$$= \Pr\left\{\sum_{n=1}^{d}\left(M_{h_n\neq 0} - M_{h_n=0}\right)\right\} = \Pr\left\{Z = \sum_{n=1}^{d} Z_n > 0\right\} \tag{22}$$

where $M_{h_n\neq 0}$ and $M_{h_n=0}$ denote the metrics of the branches corresponding to the non-zero and all-zero paths, at the instant of the nth different branches of the two paths, respectively, and

$$Z_n \triangleq M_{h_n\neq 0} - M_{h_n=0} \tag{23}$$

indicates the difference of these branches. Note that the two paths may have a length larger than d, but they differ in only the d branches. We will explain how to compute the branch metrics using the correlator outputs given in (7) later.

To compute P_d, we use the Chernoff bound [14, 15] for the hard detector and the Beaulieu series [16] for the soft detector. To this end, we need to calculate the characteristic function of the decision variable $Z = \sum_{n=1}^{d} Z_n$ in (22), that is, $\varphi_Z(s)$. We note that the variables Z_n s in (23) are independent because they belong to different bit intervals and they also have similar characteristic function. So it suffices to find the characteristic function of one of them. Hence, we can obtain

$$p(Z = \sum_{n=1}^{d} Z_n) = p(Z_1) * \cdots * p(Z_d)$$
$$\varphi_Z(s) = \left(\varphi_{Z_n}(s)\right)^d \tag{24}$$

2.2.1 Chernoff bound: According to the Chernoff bound [14, 15], for the random variable Z, we have

$$P(Z > 0) < \min_{s>0}\{\varphi_z(s)\} \tag{25}$$

So, from (22) and (24), we obtain an upper bound for P_d as

$$P_d \leq \min_{s>0}\{\varphi_Z(s)\} = \left\{\min_{s>0}\left[\varphi_{Z_n}(s)\right]\right\}^d \tag{26}$$

The upper bound [14] on the BER noting (19), (21) and (26) is obtained as

$$P_b < \sum_{d=K}^{\infty} c_d P_d < \sum_{d=K}^{\infty} c_d\left[\min_s \varphi_{Z_n}(s)\right]^d$$
$$= \left.\frac{\partial T(D, N)}{\partial N}\right|_{N=1, D=\min_s \varphi_{Z_n}(s)} \tag{27}$$
$$= \left.\frac{D^K}{(1-2D)^2}\left(\frac{1-D}{1-D^{K-2}}\right)^2\right|_{D=\min_s \varphi_{Z_n}(s)}$$

2.2.2 Beaulieu series: Another method for the computation of P_d is to use the Beaulieu series [16]. It is an infinite series for the computation of the cumulative distribution function (CDF) of a random variable using the samples of its characteristic function. We have [16]

$$P_d = P(Z > 0) = \frac{1}{2} + \sum_{m\in N_{\text{odd}}} \frac{2\,\text{Im}\{\varphi_Z(jmw_0)\}}{m\pi},$$
$$\varphi_Z(s = jmw_0) = \left(\varphi_{Z_n}(jmw_0)\right)^d \tag{28}$$

where N_{odd} is the set of all odd natural numbers, $\text{Im}\{\cdot\}$ stands for the imaginary part, $\varphi_Z(jmw_0)$ is the characteristic function of Z sampled at jmw_0 and ω_0 is the sampling rate. Note that with a suitable value of ω_0, the series converges rapidly to an acceptable accurate value. Thus, it is enough to consider only the first few terms.

After calculating P_d for different values of d, an upper bound for the BER can be evaluated using (21), that is, $P_b < \sum_{d=K}^{\infty} c_d P_d$ where c_d's are the coefficients of the Taylor series expansion of $\partial T(D, N)/\partial N|_{N=1}$ in (19). Note that P_d decays exponentially with d. So, we need to consider only the first few terms of the above series.

3 Performance evaluation in AWGN channel

As mentioned before, the correlator output in an AWGN channel is computed by setting $\beta_{j,h,m} = 1$ in the results of the fading channel. Hence, considering (8), (13) and (17), we obtain

$$S_h = \begin{cases} D_0^1\sqrt{2P}\,Ns_2 & \text{if } h = c_{1,0}^1, \quad h = 0, 1, \ldots, Ns_1 - 1 \\ 0 & \text{if } h \neq c_{1,0}^1, \end{cases} \tag{29}$$

$$\varphi_{I_h|U}(z) = \left[\frac{1}{2}\left((1-\alpha) + \alpha\exp\left(s\sqrt{2P}\right)\right)^{Ns_2}.\right.$$
$$\left. + \frac{1}{2}\left((1-\alpha) + \alpha\exp\left(-s\sqrt{2P}\right)\right)^{Ns_2}\right]^U \tag{30}$$

$$\sigma_{n_h}^2 = \frac{Ns_2 N_0}{2T_s}, p_{n_h}(n) = \frac{1}{\sqrt{2\pi\sigma_{n_h}^2}}\exp\left\{\frac{-n^2}{2\sigma_{n_h}^2}\right\}$$
$$\Rightarrow \varphi_{n_h}(s) = \exp\left\{s^2\frac{Ns_2 N_0}{4T_s}\right\} \tag{31}$$

Using the binomial expansion of (30), a change of variable and taking the inverse Laplace transform, we can show that the PDF of the interference is obtained as

$$p_{I_h|U}(I) = \sum_{l=-Ns_2 U}^{Ns_2 U} P_{\text{dis}}^U(l)\,\delta\left(I - l\sqrt{2P}\right) \tag{32}$$

where

$$P_{\text{dis}}^U(l) \triangleq \left(\frac{\alpha^{Ns_2}}{2}\right)^U \sum_{k=0}^{U}\sum_{m=0}^{Ns_2 k}\binom{U}{k}$$
$$\times \binom{Ns_2 k}{m}\binom{Ns_2(U-k)}{m-l}\left(\frac{1-\alpha}{\alpha}\right)^{2m-l} \tag{33}$$

We define the signal-to-noise ratio as the energy of the desired signal to twice that of the variance of the output noise. So, from (29) and (31), we obtain

$$\text{SNR} = \frac{Ns_2 T_s P}{N_0} \tag{34}$$

PDF of the correlator output. Without loss of generality, we assume that the desired user sends its data in the band $h = (c_1^1 \Delta = 0)$. Hence, in the band $h \neq 0$, in which we assume that there are U_1 independent interferers, the output of the correlator is $R_{h\neq 0} \Delta R_{h\neq 0}^1 = I_{h|U_1} + n_h$. Noting that the noise and interference are independent, the PDF of the correlator output considering (31) and (32) is

$$p_{(R_{h\neq 0}=I+n)|U_1}(R) = p_{I|U_1}(R) * p_n(R), \\ = \sum_{l=-Ns_2U_1}^{Ns_2U_1} P_{\text{dis}}^{U_1}(l) \\ \times p_n\left(R - l\sqrt{2P}\right), h \neq 0 \tag{35}$$

where $*$ represents the convolution, $p_{n_h}(\cdot) \sim N(0, \sigma_{n_h}^2)$ denotes the PDF of the noise component, which is Gaussian with a zero mean and variance $\sigma_{n_h}^2$ given in (31) and $P_{\text{dis}}^{U_1}(l)$ is given in (33).

In the band $h = (c_1^1 \triangleq 0)$, in which the desired user sends its data, if we assume that there are U_0 interferers, the correlator output is $R_{h=0} \triangleq R_{h=0}^1 = S_{h=0|D_0^1=0} + I_{h|U_0} + n_h$. Thus the PDF of the output is obtained as (35) shifted by $Ns_2\sqrt{2P}$ (the effect of the desired user)

$$p_{(R_{h=0}=S+I+n)|U_0}(R) = \sum_{l=-Ns_2U_0}^{Ns_2U_0} P_{\text{dis}}^{U_0}(l)p_n \\ \times \left(R - (l + Ns_2\sqrt{2P})\right), \quad h = (c_1^1 \triangleq 0) \tag{36}$$

Next, we consider two kinds of decoding before the Viterbi algorithm, that is, hard and soft detectors.

3.1 Performance of the hard detector

In this decoding, the correlator output is compared with two threshold levels and then one of the values -1, 0 or 1 is assigned as

$$r_{j,h} = \begin{cases} +1 & R_{j,h} \geq \text{thr}, \\ 0 & -\text{thr} \leq R_{j,h} \leq \text{thr}, \quad \text{thr} = \dfrac{Ns_2\sqrt{2P}}{2} \\ -1 & R_{j,h} \leq -\text{thr}, \end{cases} \tag{37}$$

where the value thr has been chosen based on the correlator output due to the desired user (29). The above values are then used to calculate the branch metrics of the hard input decoder (Viterbi algorithm) of the underlying convolutional code as follows. In the trellis diagram, at the jth bit interval, for the branches with the output symbol h and input bit 0, we use the value of $r_{j,h}$ as the metric, whereas for the branches with the output symbol h and input bit 1, we use the negative of $r_{j,h}$ as the metric, that is

$$M_h = \begin{cases} r_{j,h} & \text{if } D_j^1 = 0, \quad \text{output symbol} = h \\ -r_{j,h} & \text{if } D_j^1 = 1, \quad \text{output symbol} = h \end{cases} \tag{38}$$

We observe from (37) and (38) that $Z_n = M_{h\neq 0} - M_{h=0}$ defined in (23) takes on one of the values -2, -1, 0, 1 or 2. So, its PDF and characteristic function are as follows

$$P_{Z_n}(x) = P_{-2}\delta(x+2) + P_{-1}\delta(x+1) + P_0\delta(x) + P_1\delta(x-1) \\ + P_2\delta(x-2) \tag{39}$$

$$\varphi_{Z_n}(s) = P_{-2}e^{-2s} + P_{-1}e^{-s} + P_0 + P_1e^s + P_2e^{2s} \tag{40}$$

where $P_i \triangleq P(Z_n = i)$

We define $P(M_{h\neq 0}, M_{h=0}|0, 1)$ as the joint probability of the branch metrics of the non-zero path ($M_{h\neq 0}$) and zero path ($M_{h=0}$) with the condition that in the band corresponding to the non-zero path, the data are not sent and in the band corresponding to the zero path, the data bit is sent. Note that 0 and 1 in $P(M_{h\neq 0}, M_{h=0}|0, 1)$ stand for not sending data in the band $h \neq 0$ and sending data in the band $h = 0$, respectively. Then, we can compute $P_i = P(M_{h\neq 0} - M_{h=0} = i|0, 1)$ as follows

$$P_{-2} = P(M_{h\neq 0} - M_{h=0} = -2|0, 1) = P(M_{h\neq 0} \\ = -1, M_{h=0} = 1|0, 1) \triangleq P(-1, 1|0, 1) \\ P_{-1} = P(-1, 0|0, 1) + P(0, 1|0, 1), \quad P_0 = P(1, 1|0, 1) \\ + P(-1, -1|0, 1) + P(0, 0|0, 1) \\ P_1 = P(1, 0|0, 1) + P(0, -1|0, 1), P_2 = P(1, -1|0, 1) \tag{41}$$

Note that $P(M_{h\neq 0}, M_{h=0}|0, 1) \neq P(M_{h\neq 0}|0)P(M_{h=0}|1)$, that is, the variables $M_{h\neq 0}|0$ (the branch metric of the non-zero path conditioned on not sending data in the band $h \neq 0$) and $M_{h=0}|1$ (the branch metric of the zero path conditioned on sending data in the band $h = 0$) are not independent. Because if there are U_0 and U_1 interfering users in the bands corresponding to the zero and non-zero branches, respectively, then U_0 and U_1 must satisfy $U_0 + U_1 \leq N_u - 1$ (N_u is the number of users), which makes the above two variables dependent. But, conditioned on U_0 and U_1, they will be independent, that is, $P(M_{h\neq 0}, M_{h=0}|0, 1, U_1, U_0) = P(M_{h\neq 0}|0, \quad U_1)P(M_{h=0}|1, U_0)$. So, we can use the above conditional probability to find $P_i = P(Z_n = i)$. For example, for P_{-2}, we have

$$P_{-2} = \sum_{U_1=0}^{N_u-1} \sum_{U_0=0}^{N_u-1-U_1} P(M_{h\neq 0} = -1|0, U_1) \\ P(M_{h=0} = 1|1, U_0)P(U_1, U_0) \tag{42}$$

where $P(U_1, U_0)$ is the probability of having U_1 and U_0 interfering users in the bands $h \neq 0$ and $h = 0$, respectively. It can be obtained as

$$P(U_1, U_0) = \binom{N_u - 1}{U_1}\binom{N_u - 1 - U_1}{U_0}\left(\frac{1}{Ns_1}\right)^{U_1} \\ \times \left(\frac{1}{Ns_1}\right)^{U_0}\left(1 - \frac{2}{Ns_1}\right)^{N_u-1-U_1-U_0} \tag{43}$$

where $1/Ns_1$ is the probability that two users send their data in the same band

$$p(c_1^k = c_1^1) = \frac{1}{Ns_1} \tag{44}$$

Considering (37) and (38), the conditional probability $P(M_{h\neq 0} = -1|0, U_1)$ is computed as

$$P(M_{h\neq 0} = -1|0, U_1) = P\{\text{input bit of non-zero branch} \\ = 0 \text{ and } (R_{h\neq 0} < -\text{thr} \Rightarrow r_{h\neq 0} = -1)\} \\ + P\{\text{input bit of non-zero branch} \\ = 1 \text{ and } (R_{h\neq 0} > \text{thr} \Rightarrow r_{h\neq 0} = 1)\} \\ = \frac{1}{2}\int_{-\infty}^{-\text{thr}} f_{(R_{h\neq 0}=I+n)|U_1}(R)\,\mathrm{d}R \\ + \frac{1}{2}\int_{\text{thr}}^{\infty} f_{(R_{h\neq 0}=I+n)|U_1}(R)\,\mathrm{d}R \tag{45}$$

However, for $P(M_{h=0} = 1|1, U_0)$, since we know that the input bit is zero in the zero path,

$$P(M_{h=0} = 1|1, U_0) = P\{R_{h=0} > \text{thr} \Rightarrow r_{h=0} = 1\} = \int_{\text{thr}}^{\infty} f_{(R_{h=0}=S+I+n)|U_0}(R)\, \mathrm{d}R \quad (46)$$

Hence, we can compute P_{-2} noting (36), (45) and (46) as

$$P_{-2} = \sum_{U_1=0}^{N_u-1} \sum_{U_0=0}^{N_u-1-U_1} \left[\sum_{l=-Ns_2U_1}^{Ns_2U_1} P_{\text{dis}}^{U_1}(l) \times \left\{ 1 - Q\left(\left(-\frac{1}{2} + \frac{l}{Ns_2} \right) \sqrt{2\text{SNR}} \right) + Q\left(\left(\frac{1}{2} + \frac{l}{Ns_2} \right) \sqrt{2\text{SNR}} \right) \right\} \right] \times \left[\sum_{l=-Ns_2U_0}^{Ns_2U_0} P_{\text{dis}}^{U_0}(l) \left\{ 1 - Q\left(\left(\frac{1}{2} + \frac{l}{Ns_2} \right) \times \sqrt{2\text{SNR}} \right) \right\} \right] P(U_1, U_0) \quad (47)$$

where $Q(u) = (1/\sqrt{2\pi}) \int_u^{\infty} \exp(-x^2/2)\, \mathrm{d}x$

Similarly, we can compute P_{-1}, P_0, P_1 and P_2. Then, $\varphi_{Z_n}(s)$ is obtained from (40). Next, we compute $\min_s(\varphi_{Z_n}(s))$ numerically. Finally, P_b is calculated from (27).

3.2 Performance of the soft detector

In this case, the output of the correlator is directly applied to the Viterbi algorithm. That is, the metric assigned at the jth bit interval to the branches with the output symbol of h in the trellis diagram, that is, $M_{j,h}$, is equal to

$$M_{j,h} = R_{j,h} \quad (48)$$

The characteristic function of the correlator output in the band $h \neq 0$ noting (30) and (31), and considering the fact that the noise and interference terms are independent, is obtained as

$$\varphi_{R_{h\neq 0}|U_1}(s) = \varphi_{I_h|U_1}(s)\varphi_{n_h}(s) = \left[\frac{1}{2}\left((1-\alpha) + \alpha \exp(s\sqrt{2P}) \right)^{Ns_2} + \frac{1}{2}\left((1-\alpha) + \alpha \exp(-s\sqrt{2P}) \right)^{Ns_2} \right]^{U_1} \exp\left\{ s^2 \frac{Ns_2 N_0}{4T_s} \right\} \quad (49)$$

where, as mentioned before, U_1 is the number of interfering users in the band $h \neq 0$. In the band $h = 0$, in which the desired user sends its data, the characteristic function will be

$$\varphi_{R_{h=0}|U_0}(s) = \left[\frac{1}{2}\left((1-\alpha) + \alpha \exp(s\sqrt{2P}) \right)^{Ns_2} + \frac{1}{2}\left((1-\alpha) + \alpha \exp(-s\sqrt{2P}) \right)^{Ns_2} \right]^{U_0} \times \exp\left\{ s^2 \frac{Ns_2 N_0}{4T_s} \right\} . \exp\left\{ sNs_2\sqrt{2P} \right\} \quad (50)$$

where the third term in the above equation is the effect of the desired user's signal and U_0, as before, denotes the number of interfering users in the band $h = 0$.

Considering $Z_n = M_{h\neq 0} - M_{h=0} = R_{h\neq 0} - R_{h=0}$ and the discussions of the previous section, the conditional and unconditional characteristic functions Z_n are obtained as

$$p_{Z_n|U_0,U_1}(r) = p_{R_{h\neq 0}|U_1}(r) * p_{R_{h=0}|U_0}(-r)$$

$$\varphi_{Z_n|U_0,U_1}(s) = \varphi_{R_{h\neq 0}|U_1}(s)\varphi_{R_{h=0}|U_0}(-s) \quad (51)$$

$$\varphi_{Z_n}(s) = \sum_{U_1=0}^{N_u-1} \sum_{U_0=0}^{N_u-1-U_1} \varphi_{R_{h\neq 0}|U_1}(s)\varphi_{R_{h=0}|U_0}(-s) P(U_1, U_0) \quad (52)$$

Next, we obtain $\varphi_Z(s)$ from (24) as $\varphi_Z(s) = (\varphi_{Z_n}(s))^d$. Then, we can compute the upper bound on the BER using the Beaulieu series, that is, (28) and (21).

4 Performance analysis in fading channel

In the fading channel, we consider a soft decision before the Viterbi decoding. The characteristic function of the receiver output in the band $h \neq 0$ noting (13), (14) and (17) is computed as

$$\varphi_{R_{h\neq 0}|U_1,\beta_{h\neq 0,\{m\}}}(s) = \varphi_{I_h|U_1,\beta_{h\neq 0,\{m\}}}(s)\varphi_{n_h|\beta_{h\neq 0,\{m\}}}(s) = \left(\frac{1}{2} \prod_{m=0}^{Ns_2-1} \left[(1-\alpha) + \alpha \exp(-s\sqrt{2P}\beta_{h\neq 0,m}^2) \right] + \frac{1}{2} \prod_{m=0}^{Ns_2-1} \left[(1-\alpha) + \alpha \exp(s\sqrt{2P}\beta_{h\neq 0,m}^2) \right] \right)^{U_1} \times \exp\left\{ s^2 \frac{N_0}{4T_s} \sum_{m=0}^{Ns_2-1} \beta_{h\neq 0,m}^2 \right\} \quad (53)$$

where $\beta_{h,\{m\}} \triangleq \beta_{h,0}, \ldots, \beta_{h,Ns_2-1}$. In the band $h = 0$, in which the desired user sends its data, the conditional characteristic function is easily computed as

$$\varphi_{R_{h=0}|U_0,\beta_{h=0,\{m\}}}(s) = \left(\frac{1}{2} \prod_{m=0}^{Ns_2-1} \left[(1-\alpha) + \alpha \exp(-s\sqrt{2P}\beta_{h=0,m}^2) \right] + \frac{1}{2} \prod_{m=0}^{Ns_2-1} \left[(1-\alpha) + \alpha \exp(s\sqrt{2P}\beta_{h=0,m}^2) \right] \right)^{U_0} \times \exp\left\{ s^2 \frac{N_0}{4T_s} \sum_{m=0}^{Ns_2-1} \beta_{h=0,m}^2 \right\} \exp\left\{ s\sqrt{2P} \sum_{m=0}^{Ns_2-1} \beta_{h=0,m}^2 \right\} \quad (54)$$

Considering the discussions of the previous section and (51), the unconditional characteristic function is obtained as

$$\varphi_{Z_n}(s) = \sum_{U_1=0}^{N_u-1} \sum_{U_0=0}^{N_u-1-U_1} \left(\int \varphi_{R_{h\neq 0}|U_1,\beta_{h\neq 0,m}}(s) \times p\left(\beta_{h\neq 0,\{m\}}\right) \mathrm{d}\left(\beta_{h\neq 0,\{m\}}\right) \right) \times \left(\int \varphi_{R_{h=0}|U_0,\beta_{h=0,\{m\}}}(-s) \times p\left(\beta_{h=0,\{m\}}\right) \mathrm{d}\left(\beta_{h=0,\{m\}}\right) \right) P(U_1, U_0) \quad (55)$$

In the appendix, we have shown that the result of the second integral in (55) is

$$\int \varphi_{R_{h=0}|U_0,\beta_{h=0,\{m\}}}(s)\, p\left(\beta_{h=0,\{m\}}\right) \mathrm{d}\left(\beta_{h=0,\{m\}}\right)$$
$$= \left(\frac{1}{2}\right)^{U_0} \sum_{k=0}^{U_0} \binom{U_0}{k} \left(\begin{array}{c} \sum_{l=0}^{k} \sum_{l'=0}^{U_0-k} \binom{k}{l} \binom{U_0-k}{l'} \\ \times (1-\alpha)^{l+l'} \alpha^{U_0-l-l'} \\ \times \dfrac{1}{\{-s^2(N_0\sigma^2/4T_s) + s\sigma^2\sqrt{2P} \times (U_0 - 2k + l - l' + 1) + 1\}} \end{array} \right)^{Ns_2} \quad (56)$$

Similarly, the first integral in (55) is computed as

$$\left(\int \varphi_{R_{h\neq 0}|U_1,\beta_{h\neq 0,\{m\}}}(s)\, p\left(\beta_{h\neq 0,\{m\}}\right) d\left(\beta_{h\neq 0,\{m\}}\right) \right)$$
$$= \left(\frac{1}{2}\right)^{U_1} \sum_{k=0}^{U_1} \binom{U_1}{k} \left[\begin{array}{c} \sum_{l=0}^{k} \sum_{l'=0}^{U_1-k} \binom{k}{l} \binom{U_1-k}{l'} \\ \times (1-\alpha)^{l+l'} \alpha^{U_1-l-l'} \\ \times \dfrac{1}{(-s^2(N_0\sigma^2/4T_s) + s\sigma^2\sqrt{2P} \times (U_1 - 2k + l - l') + 1)} \end{array} \right]^{Ns_2} \quad (57)$$

Therefore by replacing (56) and (57) in (55), $\varphi_{Z_n}(s)$ can be found. Noting that Z_ns are independent and identical variables, $\varphi_Z(s) = (\varphi_{Z_n}(s))^d$. Next, we can compute P_d from the Beaulieu series, that is, (28), and then the upper bound on the BER can be computed from (21).

For the fading channel, we define the average SNR from (8) and (16) as

$$\gamma_b = E_{\beta_{\{m\}}}\left(\frac{S^2}{2\,\mathrm{Var}(n_h)}\right) = \frac{PNs_2 T_s \sigma^2}{N_0} \quad (58)$$

where $E_y(x)$ denotes the expectation of x over the variable y.

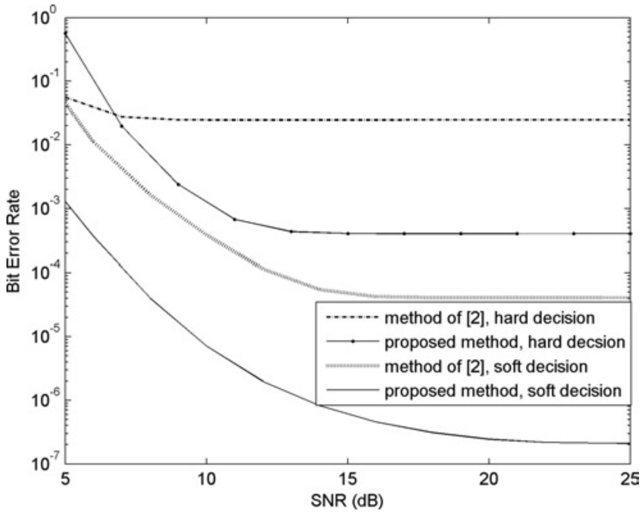

Fig. 3 *Performance of the proposed method and the coded MC-FH-CDMA system [2] against the SNR in AWGN channel, $Ns_1 = 8$, $Ns_2 = 2$, $N_h = 16$, and $N_u = 15$*

5 Numerical results

In this section, we present some numerical results based on the analytical evaluations derived in the previous sections and computer simulations. For comparison, we have also evaluated the performance of the uncoded MC-FH system [1] and the coded scheme introduced in [2].

For fair comparison, the three systems must have the same processing gain, signal-to-noise ratio and coding gain (for the coded schemes). Therefore we use the parameters $Ns_1 = 8$, $Ns_2 = 2$ and $N_h = 16$ which result in the processing gain 256 and coding gain $d_{\text{free}} = 5$.

Fig. 2 shows the performance of the proposed method, the uncoded MC-FH system [1] and the coded scheme of [2] against the number of users in an AWGN channel for the same processing gain of 256 and SNR = 12 dB. It is observed that the new method significantly outperforms the uncoded MC-FH method proposed in [1] and the coded method of [2]. As an example, to reach the BER of 10^{-4}, the number of users is 3 for the uncoded MC-FH [1] and 15 for the coded method of [2], whereas it is

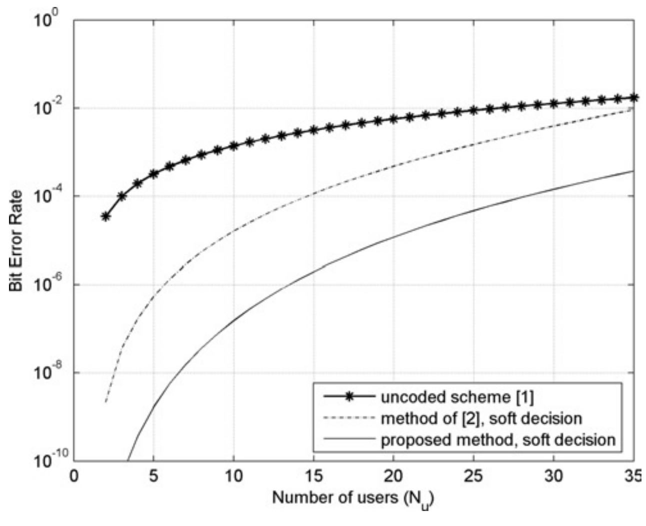

Fig. 2 *Performance of the proposed method, the uncoded MC-FH [1] and the coded system of [2] against the number of users in AWGN channel, $Ns_1 = 8$, $Ns_2 = 2$, $N_h = 16$, and SNR = 12 dB*

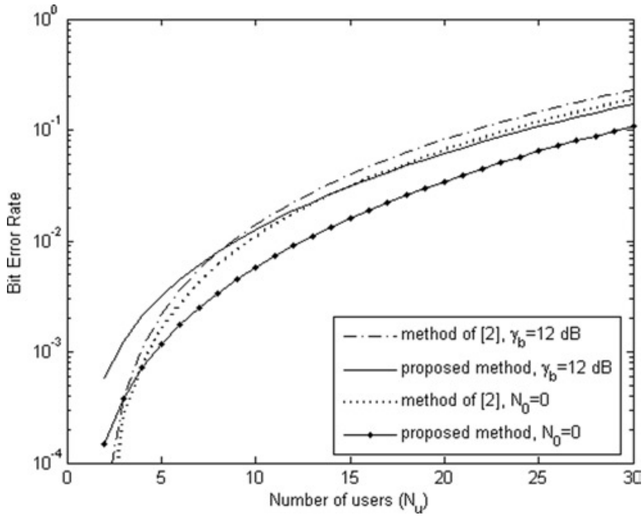

Fig. 4 *Performance of the proposed method and the coded MC-FH-CDMA system [2] in fading channel $Ns_1 = 8$, $Ns_2 = 2$, $N_h = 16$*

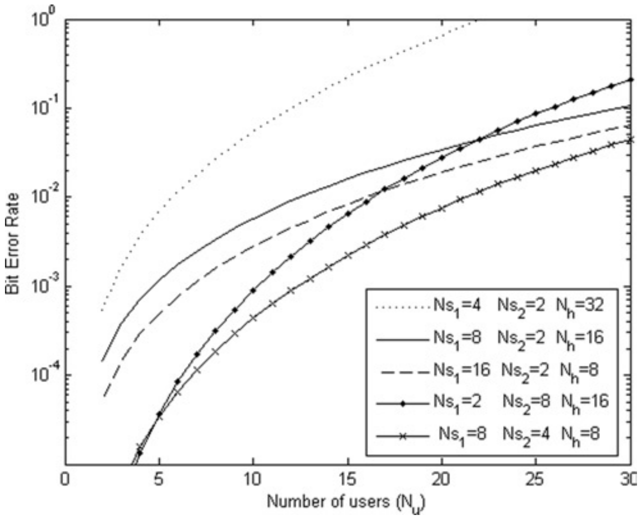

Fig. 5 *Performance of the proposed method against the number of users for the constant processing gain 256, and different values of Ns_1, Ns_2, and N_h in fading channel, $N_0 = 0$*

about 28 for the proposed method, which shows a great improvement.

In Fig. 3, we have plotted the performance of the proposed method and the method of [2] against the SNR in an AWGN channel for both hard and soft decisions, where the number of users is set equal to 15. It is observed that the proposed method with the soft decision works the best. We also note that in the two coded methods, the soft decision outperforms the hard decision.

Fig. 4 shows the effect of the fading channel on the performance of the proposed system and the method of [2] in noiseless ($N_0 = 0$) and noisy ($\gamma_b = 12$ dB) channels. We again observe that our method outperforms the coded method introduced in [2].

In Fig. 5, we have demonstrated the effect of the parameters Ns_1, Ns_2 and N_h on the performance of the proposed system in a fading channel for the constant processing gain 256. It is realised that for constant $Ns_2 = 2$, the performance improves by increasing Ns_1 and decreasing N_h. The reason is that the coding gain ($K = \log_2 Ns_1 + 2$) increases. Also, for constant $Ns_1 = 8$, the system works better by increasing Ns_2 and decreasing N_h. As stated before, in our method, the carriers hop in a bandwidth smaller than those of [1, 2], which is also another advantage.

Fig. 6. compares the analytical and simulation results for the proposed method. It is observed that the simulation results confirm the analytical evaluations. Note that the analytical results show the upper bound on the BER while the simulation results demonstrate the exact BER.

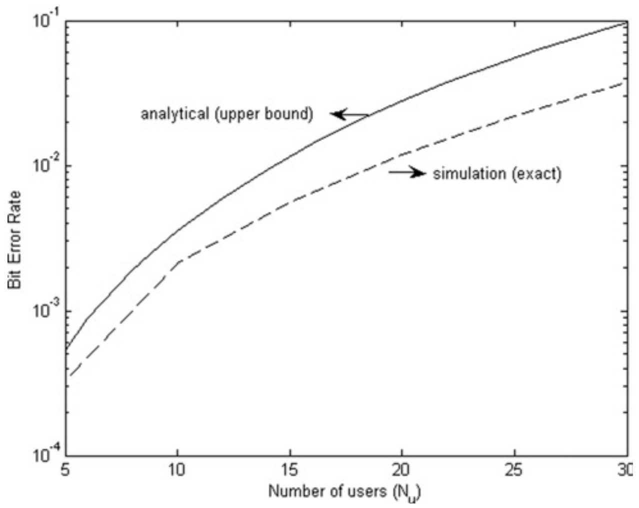

Fig. 6 *Performance of the proposed method using the analytical and simulation evaluations for fading channel, processing gain 256, $Ns_1 = 4$, $Ns_2 = 4$, and $N_h = 16$*

6 Conclusions

We have considered an internally coded multiple access MC-FH-CDMA technique. We have analysed the performance of the correlator receiver followed by the Viterbi decoder with hard and soft inputs in an AWGN channel and with a soft input in a fading channel. We have obtained the upper bound on the BER using the Chernoff bound and the Beaulieu series. It has been shown that the proposed method significantly outperforms the conventional uncoded MC-FH-CDMA and its coded scheme and it supports a larger number of users for a specific BER, whereas the new method does not require any extra bandwidth compared with the uncoded and coded MC-FH-CDMA systems. We have observed that the soft decision has better performance than the hard decision. It has also been realised that for the constant processing gain, the performance of the proposed method improves for higher values of Ns_1 because of the increment of the coding gain by Ns_1. An important advantage of the proposed method is that the carriers hop in a bandwidth which is Ns_1 times smaller than that of the previously presented coded and uncoded MC-FH systems, which makes our system more practical and coherent detection more feasible. We have also shown that the simulation results confirm the analytical evaluations.

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9 Appendix

9.1 Calculation of the integrals in (55)

Here, we compute the integrals in (55). Since the effect of the fading coefficients appears as $\beta_{h,m}^2$ in the equations, by a change of variable, we conclude that $\lambda_{h,m} = \beta_{h,m}^2$ has exponential distribution as

$$p_{\lambda_{j,m}}(\lambda) = \frac{1}{\sigma^2}\exp\left(-\frac{\lambda}{\sigma^2}\right); \quad \lambda_{h,m} = \beta_{h,m}^2; \quad E(\beta_{h,m}^2) = \sigma^2 \tag{59}$$

We first consider the second integral in (55)

$$\begin{aligned}&\left(\int \varphi_{R_{h=0}|U_1,\beta_{h=0,\{m\}}}(-s)p\left(\beta_{h=0,\{m\}}\right)\mathrm{d}\left(\beta_{h=0,\{m\}}\right)\right) = \\ &\int_0^\infty \cdots \int_0^\infty \left(\frac{1}{2}\prod_{m=0}^{Ns_2-1}(1-\alpha)+\alpha\exp\left(-s\sqrt{2P}\lambda_{h=0,m}\right)\right. \\ &\left.+\frac{1}{2}\prod_{m=0}^{Ns_2-1}(1-\alpha)+\alpha\exp\left(s\sqrt{2P}\lambda_{h=0,m}\right)\right)^{U_0} \\ &\cdot\exp\left\{s^2\frac{N_0}{4T_s}\sum_{m=0}^{Ns_2-1}\lambda_{h=0,m}\right\}\exp\left\{-s\sqrt{2P}\sum_{m=0}^{Ns_2-1}\lambda_{h=0,m}\right\} \\ &\times\frac{1}{(\sigma^2)^{Ns_2}}\exp\left\{-\frac{1}{\sigma^2}\sum_{m=0}^{Ns_2-1}\lambda_{h=0,m}\right\}\mathrm{d}\lambda_{h=0,0}\cdots\mathrm{d}\lambda_{h=0,m} \\ &\cdots\mathrm{d}\lambda_{h=0,Ns_2-1}\end{aligned} \tag{60}$$

We use the binomial expansion of the first term of the integral, which results in

$$\begin{aligned}&\left(\frac{1}{2}\prod_{m=0}^{Ns_2-1}\left[(1-\alpha)+\alpha\exp\left(s\sqrt{2P}\lambda_{h=0,m}\right)\right]\right. \\ &\left.+\frac{1}{2}\left[\prod_{m=0}^{Ns_2-1}(1-\alpha)+\alpha\exp\left(-s\sqrt{2P}\lambda_{h=0,m}\right)\right]\right)^{U_0} \\ &=\left(\frac{1}{2}\right)^{U_0}\sum_{k=0}^{U_0}\binom{U_0}{k}\left\{\prod_{m=0}^{Ns_2-1}\left[(1-\alpha)\right.\right. \\ &\left.\left.+\alpha\exp\left(s\sqrt{2P}\lambda_{h=0,m}\right)\right]\right\}^k \\ &\times\left\{\prod_{m=0}^{Ns_2-1}\left[(1-\alpha)+\alpha\exp\left(-s\sqrt{2P}\lambda_{h=0,m}\right)\right]\right\}^{U_0-k} \\ &=\left(\frac{1}{2}\right)^{U_0}\sum_{k=0}^{U_0}\binom{U_0}{k}\prod_{m=0}^{Ns_2-1}\left[(1-\alpha)+\alpha\exp\right. \\ &\left.\times\left(s\sqrt{2P}\lambda_{h=0,m}\right)\right]^k\left[(1-\alpha)+\alpha\exp\left(-s\sqrt{2P}\lambda_{h=0,m}\right)\right]^{U_0-k} \\ &=\left(\frac{1}{2}\right)^{U_0}\sum_{k=0}^{U_0}\binom{U_0}{k}\prod_{m=0}^{Ns_2-1}\left\{\sum_{l=0}^{k}\binom{k}{l}(1-\alpha)^l\alpha^{k-l}\right. \\ &\left.\times\exp\left(s(k-l)\sqrt{2P}\lambda_{h=0,m}\right)\right\} \\ &\times\left\{\sum_{l'=0}^{U_0-k}\binom{U_0-k}{l'}(1-\alpha)^{l'}\alpha^{U_0-k-l'}\right. \\ &\left.\times\exp\left(-s(U_0-k-l')\sqrt{2P}\lambda_{h=0,m}\right)\right\} \\ &=\left(\frac{1}{2}\right)^{U_0}\sum_{k=0}^{U_0}\binom{U_0}{k}\prod_{m=0}^{Ns_2-1}\left\{\sum_{l=0}^{k}\sum_{l'=0}^{U_0-k}\binom{k}{l}\binom{U_0-k}{l'}\right. \\ &\left.(1-\alpha)^{l+l'}\alpha^{U_0-l-l'}\exp\left(-s(U_0-2k+l-l')\sqrt{2P}\lambda_{h=0,m}\right)\right\}\end{aligned} \tag{61}$$

Replacing the above term in (60) gives

$$\begin{aligned}&\int \varphi_{R_{h=0}|U_0,\beta_{h=0,\{m\}}}(-s)p(\beta_{h=0,\{m\}})\mathrm{d}(\beta_{h=0,\{m\}}) = \\ &=\frac{1}{(\sigma^2)^{Ns_2}}\int_0^\infty\cdots\int_0^\infty\left(\frac{1}{2}\right)^{U_0}\sum_{k=0}^{U_0}\binom{U_0}{k} \\ &\times\prod_{m=0}^{Ns_2-1}\left\{\sum_{l=0}^{k}\sum_{l'=0}^{U_0-k}\binom{k}{l}\binom{U_0-k}{l'}(1-\alpha)^{l+l'}\alpha^{U_0-l-l'}\right. \\ &\left.\times\exp\left(-s(U_0-2k+l-l')\sqrt{2P}\lambda_{h=0,m}\right)\right\} \\ &\times\prod_{m=0}^{Ns_2-1}\left\{\exp\left(s^2\frac{N_0}{4T_s}\lambda_{h=0,m}\right)\exp\left(-s\sqrt{2P}\lambda_{h=0,m}\right)\right. \\ &\left.\cdot\exp\left(\frac{-1}{\sigma^2}\lambda_{h=0,m}\right)\mathrm{d}\lambda_{h=0,0}\cdots\mathrm{d}\lambda_{h=0,m}\cdots\mathrm{d}\lambda_{h=0,Ns_2-1}\right\} \\ &=\frac{1}{(\sigma^2)^{Ns_2}}\left(\frac{1}{2}\right)^{U_0}\sum_{k=0}^{U_0}\binom{U_0}{k}\int_0^\infty\cdots\int_0^\infty \\ &\cdot\prod_{m=0}^{Ns_2-1}\left\{\sum_{l=0}^{k}\sum_{l'=0}^{U_0-k}\binom{k}{l}\binom{U_0-k}{l'}(1-\alpha)^{l+l'}\alpha^{U_0-l-l'}\right. \\ &\times\exp\left[-\left(-s^2\frac{N_0}{4T_s}+s(U_0-2k+l-l'+1)\sqrt{2P}+\frac{1}{\sigma^2}\right)\right. \\ &\left.\left.\times\lambda_{h=0,m}\right]\right\}\mathrm{d}\lambda_{h=0,0}\ldots\mathrm{d}\lambda_{h=0,m}\ldots\mathrm{d}\lambda_{h=0,Ns_2-1}\end{aligned} \tag{62}$$

Since the above Ns_2 integrals are independent and have the same forms, it suffices to calculate one of them. Then, we obtain

$$\int \varphi_{R_{h=0}|U_0,\beta_{h=0,\{m\}}}(s)p(\beta_{h=0,\{m\}})d(\beta_{h=0,\{m\}})$$
$$= \alpha^{U_0}\left(\frac{1}{2}\right)^{U_0}\sum_{k=0}^{U_0}\binom{U_0}{k}\left(\left\{\sum_{l=0}^{k}\sum_{l'=0}^{U_0-k}\binom{k}{l}\binom{U_0-k}{l'}\right.\right.$$
$$\times(1-\alpha)^{l+l'}\alpha^{U_0-l-l'}\int_0^{\infty}\frac{1}{\sigma^2}\exp\left[-\left(-s^2\frac{N_0}{4T_s}\right.\right.$$
$$\left.\left.\left.\left.+s(U_0-2k+l-l'+1)\sqrt{2P}+\frac{1}{\sigma^2}\right)\lambda_{h=0,m}\right]\right\}\mathrm{d}\lambda_{h=0,m}\right)^{Ns_2}$$
$$= \left(\frac{1}{2}\right)^{U_0}\sum_{k=0}^{U_0}\binom{U_0}{k}\left(\sum_{l=0}^{k}\sum_{l'=0}^{U_0-k}\binom{k}{l}\binom{U_0-k}{l'}\right.$$
$$\left.\times(1-\alpha)^{l+l'}\alpha^{U_0-l-l'}\frac{1}{(-s^2(N_0\sigma^2/4T_s)+s\sigma^2\sqrt{2P}(U_0-2k+l-l'+1)+1)}\right)^{Ns_2} \quad (63)$$

The first integral in (55) has the same form as the second integral except that the term $\exp\{-s(2P)^2\sum_{m=0}^{Ns_2-1}\lambda_{h=0,m}\}$, because the desired signal does not appear. Thus, using the same procedure, the result of the first integral is obtained as in (57).