Cooperative Spectrum Sensing Based on a Low-Complexity Cyclostationary Detection Method for Cognitive Radio Networks

Hamid Arezumand, Paeiz Azmi, and Hamed Sadeghi
Department of Electrical and Computer Engineering,
Tarbiat Modares University, Tehran, Iran
Emails: {h.arezumand, pazmi, hamed.sadeghi}@modares.ac.ir

***Abstract*—Fast reliable spectrum sensing (SS) is a crucial problem in the cognitive radio systems. To address this issue, cyclostationarity-based detection methods, which are generally more complex but more reliable than energy detection methods, have been proposed. This paper presents a new method to detect the presence of the second-order cyclostationarity in the OFDM-based primary user (PU) signals. The proposed method has a low computational complexity, while it presents a close performance as compared to the well-known GLRT-based Dandawaté-Giannakis's method. Moreover, since the proposed low-complexity method is robust against the noise uncertainty, it can be a good alternative to the energy detection method. In order to be able to perform the hypothesis test, we derive the asymptotic distribution of the test statistic under the null hypothesis. We further propose some cooperative spectrum sensing methods to improve the detection performance. An approximate threshold selection method is proposed for performing statistical test at the fusion center (FC). Extensive simulation results confirms the superiority of the proposed schemes.**

I. INTRODUCTION

Traditional non-beneficial usage of available bandwidths, along with the increase in demand for higher data rates, leads to the development of novel techniques for flexible and efficient access to the licensed frequency bands. To avoid causing a destructive interference, secondary users (SUs) should reliably detect the presence of primary users (PUs). Therefore, the ability to perform reliable spectrum sensing is crucial to SUs . However, the signals received by cognitive radios usually are effected by the fading impairments of radio-frequency channels, and consequently, SUs should be able to detect very week signals in very low SNR situations [1].

A common detector that needs no prior knowledge about the PU signal is the well-known energy detector (ED). But, because of the so-called noise uncertainty drawback in EDs [1], they cannot perform well in low-SNR conditions. However, in wireless networks there is usually some information about the modulation properties of the primary signal [2]. These properties could be exploited in the design of detectors that have acceptable performance in very low SNRs. One popular approach is the cyclostationary (CS) detection method [2], [3], which operates much better than energy detection, but is generally more complex [4]. These detectors can inherently distinguish PUs from SUs as well as interferers, if they exhibit dissimilar cyclic features. This important requirement could not be satisfied by conventional energy detectors [5], [6].

One of the key challenges in CS-based spectrum sensing is the computational complexity associated with CS detection algorithms. In fact, there is a trade-off among the complexity of CS detector, the required sensing-time and the probability of detection. The CS detector that recently has been proposed in the literature is the multi-cycle detector [7], [8], which is based on Dandawaté-Giannakis' algorithm [9]. But the main drawback of this approach is its complexity of implementation. To meet the sensing-time and complexity requirements, in this paper, we propose a novel multi-cycle sensing scheme by exploiting the second-order cyclostationarity of the OFDM-based primary signals. The proposed method drastically reduces the implementation complexity, while maintaining a comparable detection performance to multi-cycle method of [7]. In addition, we obtain the asymptotic distribution of the proposed decision statistic under null hypothesis and then establish a hypothesis test.

In order to further alleviate the impact of shadowing and fading impairments, cooperative spectrum sensing (CSS) methods are proposed in the literature [10], [7]. In this paper, we propose a novel cooperative sensing method for fusing the test statistics is suggested. Also this method requires more communication bandwidth, but has better detection performance compared with existing methods. In addition, we propose a method for approximating the null distribution of cooperative detector.

This paper is organized as follows. In Section II, the system model is briefly introduced. Section III presents the proposed and GLRT-based spectrum sensing methods. In addition, computational complexities of the proposed, GLRT-based and energy detection methods are compared. In section IV, cooperative spectrum sensing methods are proposed. Extensive simulation results are conducted in Section V. Finally, the conclusions are drawn in Section VI. Furthermore, the null distributions of some proposed cooperative detectors are derived in Appendices A and B.

II. SYSTEM MODEL

In this paper, we assume that CRs receive the primary signal through independent Rayleigh fading channels. We accept the

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baseband discrete-time model given by ($i = 1, 2, \ldots, S$):

$$x_i[n] = \eta h_i s[n] + w_i[n], \quad n = 1, \ldots, M, \tag{1}$$

where $h_i \sim \mathcal{N}_c(0, \sigma_{h_i}^2)$ and $w_i[n] \sim \mathcal{N}_c(0, \sigma_{w_i}^2)$ are circularly symmetric complex Gaussian (CSCG) random processes, representing time-invariant frequency-nonselective Rayleigh fading exposed by the channel between PU and CR, and AWGN channel, respectively. In addition, $\tilde{x}[n]$ and $y_i[n]$ are respectively the PU signal and the received signal at ith CR. Moreover, $\eta = 0$ and $\eta = 1$ correspond to H_0 and H_1 hypotheses, respectively.

III. Non-Cooperative Cyclostationarity Detection

In this section, we derive some decision statistics for the hypothesis testing problem at hand. The discrete-time unbiased and consistent estimation of the CAF of a random process $x[n]$ is given as [9]:

$$\hat{R}_{xx^*}^{\alpha}(\nu) \triangleq \frac{1}{M} \sum_{i=1}^{M} x[i]\, x^*[i+\nu]\, e^{-j2\pi\alpha i} \tag{2}$$

In order to establish the detection statistic, we exploit the properties of estimated CAF.

A. Proposed Spectrum Sensing Method

In this section, we propose a low-complexity cyclostationarity detection statistic based on estimated CAF properties of OFDM signals. As we know, when the lag parameter of estimated CAF sets to $\nu = \pm T_d$, the CAF should reveal local peaks at multiples of fundamental cycle frequency of OFDM signal, that is $k/T_u, k = \pm 1, \pm 2, \ldots, \pm T_u - 1$. At the cycle frequencies other that k/T_u, the magnitude of CAF should be small as compared with local peaks.

Based on mentioned property, we propose the following multi-cycle test statistic:

$$\mathcal{T} = \frac{\sum_{k=-L}^{L} \left| R_{xx^*}^{k/T_u}(T_d) \right|^2}{\sum_{k=-L}^{L} \left| R_{xx^*}^{(k+\varepsilon_k)/T_u}(T_d) \right|^2} \tag{3}$$

where ε_k is chosen so that the $(k+\varepsilon_k)/T_u$ does not belong to the set of cycle frequencies k/T_u. For example, it can be an arbitrary (non-integer) number in the interval $(0.25, 0.75)$. When the signal is present, the denominator approaches zero, while the numerator increases. Hence, the statistic will be increased, and detection will decide H_1.

In order to perform binary hypothesis testing, we require the distribution of the test statistic under null hypothesis testing. Under H_0, we have $x[n] = w[n]$, where $w[n] \sim \mathcal{N}_c\left(0, \sigma^2\right)$. Using the central limit theorem [11], The asymptotic distribution of CAF under H_0 is as follow [12]

$$\hat{R}_{ww^*}^{\alpha}(\nu) \sim \mathcal{N}_c\left(0, \frac{\sigma^4}{M}\right) \tag{4}$$

Therefore,

$$\frac{2M}{\sigma^4} \left| \hat{R}_{ww^*}^{\alpha}(\nu) \right|^2 \sim \chi_2^2 \tag{5}$$

Since the estimated CAFs, $\hat{R}_{ww^*}^{\alpha_k}(\nu)$, $k = 0, \pm 1, \pm 2, \ldots,, \pm L$, are asymptotically independent under null hypothesis testing [7], distribution of $\frac{2M}{\sigma^4}\left| R_{xx^*}^{k/T_u}(T_d) \right|^2$ and $\frac{2M}{\sigma^4}\left| R_{xx^*}^{(k+\varepsilon_k)/T_u}(T_d) \right|^2$ are chi-square with 2 degrees of freedom. Due to the fact that the sum of the independent chi-square random variable is also a chi-square random variable whose degrees of freedom is the sum of the degrees of freedom of independent random variables, $\frac{2M}{\sigma^4}\sum_{k=-L}^{L}\left| R_{xx^*}^{k/T_u}(T_d) \right|^2$ and $\frac{2M}{\sigma^4}\sum_{k=-L}^{L}\left| R_{xx^*}^{(k+\varepsilon_k)/T_u}(T_d) \right|^2$ are chi-square with $2(2L+1)$ degrees of freedom under the null hypothesis testing. Then the distribution of the test statistic under the null hypothesis testing is

$$\mathcal{T} \sim F(2(2L+1), 2(2L+1)), \text{ under } \mathrm{H}_0 \tag{6}$$

where $F(d_1, d_2)$ denotes the F distribution, d_1 and d_2 are the numerator and denominator degrees of freedom, respectively.

B. GLRT-based Cyclostationarity Sensing Method

This subsection, briefly introduces the well-known Dandawaté-Giannakis's method [9] which is recently modified in [7]. This statistical test relies upon the asymptotic normality and consistency of second-order cyclic statistics, and detects the presence of cycles in second-order cyclic cumulates, without assuming any specific distribution on the transmitted data [9]. For more details, the reader is referred to [9], [7], [13].

Let we define a matrix consisting of CAF estimates at the cycle frequency α for different time lags:

$$\begin{aligned} \underline{\hat{r}}_{xx^*}^{\alpha} \triangleq \Big[& \Re\left\{\hat{R}_{xx*}^{\alpha}(\nu_1)\right\}, \ldots, \Re\left\{\hat{R}_{xx*}^{\alpha}(\nu_N)\right\}, \\ & \Im\left\{\hat{R}_{xx*}^{\alpha}(\nu_1)\right\}, \ldots, \Im\left\{\hat{R}_{xx*}^{\alpha}(\nu_N)\right\} \Big], \end{aligned} \tag{7}$$

where $\Re\{\}$ and $\Im\{\}$ denote the real and imaginary parts, respectively. It has been shown that $\lim_{M\to\infty} \sqrt{M}\, \underline{\hat{r}}_{xx^*}^{\alpha} \overset{D}{=} \mathcal{N}(\underline{r}_{xx^*}^{\alpha}, \Sigma_{xx^*})$, where $\overset{D}{=}$ denote the convergence in distribution and $\mathcal{N}(\underline{\mu}, \Sigma_{xx^*})$ is a multivariate normal distribution with mean vector $\underline{\mu}$ and covariance matrix Σ_{xx^*} [9]. Noted that $\underline{r}_{xx^*}^{\alpha}$ is non-random, so the distribution of $\underline{\hat{r}}_{xx^*}^{\alpha}$ under H_0 and H_1 differs only in the mean.

The asymptotic complex normality of $\underline{\hat{r}}_{xx^*}^{\alpha}$ allows proposing the following generalized likelihood ratio, [7]

$$\Lambda_{GLR} = \frac{f\left(\underline{\hat{r}}_{xx^*}^{\alpha} | H_1\right)}{f\left(\underline{\hat{r}}_{xx^*}^{\alpha} | H_0\right)} = \exp\left(\frac{1}{2} M \underline{\hat{r}}_{xx^*}^{\alpha} \hat{\Sigma}_{xx^*}^{-1} \left(\underline{\hat{r}}_{xx^*}^{\alpha}\right)^t\right), \tag{8}$$

where $\hat{\Sigma}_{xx^*}$ is an estimation of the asymptotic covariance matrix of $\underline{\hat{r}}_{xx^*}^{\alpha}$, defined as:

$$\hat{\Sigma}_{xx^*} = \begin{bmatrix} \mathrm{Re}\left\{\frac{\boldsymbol{Q}+\boldsymbol{P}}{2}\right\} & \mathrm{Im}\left\{\frac{\boldsymbol{Q}-\boldsymbol{P}}{2}\right\} \\ \mathrm{Im}\left\{\frac{\boldsymbol{Q}+\boldsymbol{P}}{2}\right\} & \mathrm{Re}\left\{\frac{\boldsymbol{P}-\boldsymbol{Q}}{2}\right\} \end{bmatrix}. \tag{9}$$

In the above equation, the two covariance matrices $\boldsymbol{Q}$ and $\boldsymbol{P}$ are by:

$$\begin{aligned}\boldsymbol{Q} &= \frac{1}{MT}\sum_{s=-(T-1)/2}^{(T-1)/2} W(s)F_\tau(\alpha-\tfrac{2\pi s}{M})F_\tau(\alpha+\tfrac{2\pi s}{M})\\ \boldsymbol{P} &= \frac{1}{MT}\sum_{s=-(T-1)/2}^{(T-1)/2} W(s)F^*_\tau(\alpha+\tfrac{2\pi s}{M})F_\tau(\alpha+\tfrac{2\pi s}{M}),\end{aligned} \tag{10}$$

where W is a normalized spectral window of odd length T and $F_\tau(w) = \frac{1}{M}\sum_{k=1}^{M} x[k]\, x^*[k+\tau]\, e^{-jwk}$. Finally, the generalized log-likelihood test statistic for the binary hypothesis testing corresponding to signal $x[n]$ is:

$$\mathcal{Z}_x^\alpha = 2\ln(\Lambda_{GLR}) = M\hat{\underline{r}}^\alpha_{xx^*}.\hat{\Sigma}^{-1}_{xx^*}.(\hat{\underline{r}}^\alpha_{xx^*})^t, \tag{11}$$

where t denotes the conjugate transpose of a matrix. To set a threshold for hypothesis testing, we need the asymptotic distribution of $\mathcal{Z}_x^\alpha$. In [9], it is shown that, regardless of the distribution of the input data, the asymptotic distribution of the $\mathcal{Z}_x^\alpha$ under the hypothesis H_0 is central chi-squared with $2M$ degrees of freedom (i.e. $\lim_{M\to\infty}\mathcal{Z}_x^\alpha \overset{D}{=} \chi^2_{2N}$).

C. Discussion on Computational Complexity

In this section, we compare the computational complexities of the proposed, GLRT-based and energy detection methods. The most time-consuming step in the GLRT-based algorithm is estimating the covariance matrix (9). Since this step is omitted in the proposed algorithm, it causes a significant reduction in computational complexity. In fact, the computational complexity of the proposed method can be expressed as $M(1+\frac{1}{2}\log_2 M)$, where M denotes the number of observation samples. In the above expression, it is assumed that the fast-Fourier transform (FFT) algorithm is used for computing the CAF and $F_\tau(\omega)$. The term $\frac{M}{2}\log_2 M$ is responsible for the computation of FFT and M denotes the number of multiplications presented in CAF.

However, the computational complexity of GLRT-based method is given as $M(1+\frac{1}{2}\log_2 M) + 4LT$. The term $M(1+\frac{1}{2}\log_2 M)$ is responsible for computation of CAF and the term $4LT$ is due to the estimation of the covariance matrix. Note that the parameter T is the length of the normalized window used in estimation of the covariance matrix. As done in [7] the value of this parameter is usually set to 2049. The parameter L is the number of cycle frequency that used in the test statistic.

It should be noted that in the proposed method, increasing the number of cycle frequencies employed in the decision statistic does not significantly increase the computational complexity.

As a final remark, the computational complexity of the energy detection method can be expressed by M. In energy detection method, simply the autocorrelation of the signal is considered. Although this method has very low complexity, however, it has some challenging drawbacks (see Section V-B). As we will show in Section V, the proposed method can be considered as a tradeoff between the GLRT-based and energy detection methods.

IV. Proposed Cooperative Spectrum Sensing Methods

One of the most challenging issues in the secondary access of CRs to the licensed bands is the reliable detection primary users in low SNR conditions. To overcome this problem, cooperative spectrum sensing (CSS) methods are proposed in the literature. In this section, some CSS methods based on the proposed low-complexity cyclostationary detection is presented. It is assumed that each CR observes M samples and then forms the decision statistic (3).

A. Hard Decision-Based Cooperative Spectrum Sensing

In hard decision-based CSS, after that CRs form their decision statistics based on (3), they make final binary decisions and send them to a fusion center. Although there are different methods for combining the received binary decisions, however, the most popular methods are AND, OR and majority. The probability of false alarm at the fusion center for the above-mentioned schemes can be obtained as:

$$\text{AND}: \quad P_{fa} = \prod_{n=1}^{S} P_{fa_n} \tag{12}$$

$$\text{OR}: \quad P_{fa} = 1 - \prod_{n=1}^{S}(1-P_{fa_n}) \tag{13}$$

$$\text{Majority}: \quad P_{fa} = \sum_{n=\lceil S/2\rceil}^{S}\begin{pmatrix} S \\ n\end{pmatrix} P^n_{fa_n}(1-P_{fa_n})^{(S-n)} \tag{14}$$

In the above equations, P_{fa_n} and S are the false alarm probability of nth CR and the number of CRs in secondary network, respectively. One of the most advantages of the hard decision-based CSS is the low-overhead communications needed for reporting local results to the fusion center. However, we will show that the hard decision-based CSS can be outperformed by the soft decision-based CSS methods.

B. Soft Decision-Based Cooperative Spectrum Sensing

In centralized cooperative spectrum sensing, the fusion center decides about the presence/absence of a PU based on the decision statistics received from CRs. Different methods are proposed in literature for fusing the decisions statistics transmitted from CRs. A simple fusion rule is the summation of test statistics:

$$\mathcal{T}_s = \sum_{n=1}^{S}\mathcal{T}_n \tag{15}$$

The CDF of $\mathcal{T}_s$ can be obtained by numerical simulations or convolving the pdfs of different test statistics $\mathcal{T}_n$. However, we propose a simple but efficient threshold selection method in Appendix A.

Another well-known fusion rule is the MAX rule:

$$\mathcal{T}_m = \max_{n=1,\ldots,S}\mathcal{T}_n \tag{16}$$

The null distribution of the above test statistic is computed in Appendix B.

In what follow, we propose a new method for the cooperative sensing. Suppose that each CR calculates the numerator and denominator of (3), separately. That is, for nth CR we have

$$\mathcal{T}_{nom,n} = \sum_{k=-L}^{L} \left| R_{xx^*}^{k/T_u}(T_d) \right|^2, \tag{17}$$

and

$$\mathcal{T}_{den,n} = \sum_{k=-L}^{L} \left| R_{xx^*}^{(k+\varepsilon_k)/T_u}(T_d) \right|^2. \tag{18}$$

After that (17) and (18) are computed at each CR, they are transmitted to the fusion center. Then, we constitute the following decision statistic at the fusion center:

$$\mathcal{T}_{proposed} = \frac{\sum\limits_{n=1}^{S} \mathcal{T}_{nom,n}}{\sum\limits_{n=1}^{S} \mathcal{T}_{den,n}} \tag{19}$$

The null distributions of $\mathcal{T}_{nom,n}$ and $\mathcal{T}_{den,n}$ is chi-square with $2(2L+1)$ degrees of freedom. Thus, $\sum_{n=1}^{S} \mathcal{T}_{nom,n}$ and $\sum_{n=1}^{S} \mathcal{T}_{den,n}$ are distributed as $\chi^2_{2(2L+1)S}$. Therefore, the null distribution of the proposed test statistic can be obtained as:

$$\mathcal{T}_{proposed} \sim F(2(2L+1)S, 2(2L+1)S). \tag{20}$$

Although the overhead of this method is twice the (15) and (16), but it will be shown that it has better performance than the others. Furthermore, since its null distribution has close-form expression, the threshold selection at the FC is straightforward.

V. Simulation Results

The primary network is an IEEE 802.11 WLAN network. The subcarrier modulation of OFDM signal is QPSK. The symbol rate and sampling rate are 250 KSym/Sec and R_s=20 MHz, respectively. All simulations employ false-alarm rate of 0.01 and time-lag of $\nu = T_u$ where T_u is duration of FFT length of WLAN OFDM signal. Cyclostationary features of an OFDM signal occurs in multiples of the symbol rate $\alpha = k/T_s$, $k = 0, \pm 1, \pm 2, \ldots$. Prior knowledge of T_g (cyclic prefix duration) and T_u is assumed. The local peaks of the estimated cyclic autocorrelation function $\hat{R}_{xx^*}^{\alpha}(\nu)$ are occurred in $\nu = \pm T_u$ and $\alpha = k/(T_g + T_u)$. In this paper, a kaiser window length and β parameter are set to 2049 and 10, respectively. The parameters employed for approximating the CDF are $H = 1000$ and $\eta = 0.5$.

A. Investigating the Asymptotic Distribution of the Proposed Decision Statistic

The accuracy of the asymptotic distribution (6) under the null hypothesis is evaluated in Fig. 1. The theoretical CDF curve is obtained from equation (3) with $L = 5$. The simulated CDF are obtained by 50.000 independent Monte-Carlo runs, in which the PU signals are assumed to be white Gaussian random variables. It is evident that if the detection time is sufficiently long, the simulation results should confirm the theoretical asymptotic distribution curves.

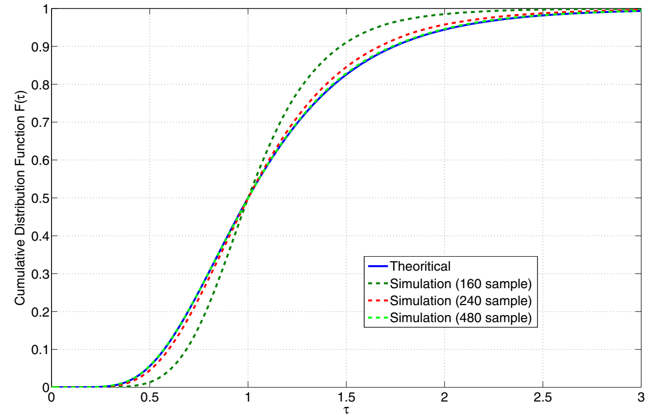

Fig. 1. Theoretical versus simulated CDF of the proposed decision statistic under null hypothesis. Increasing the detection time improves the accuracy of the asymptotic distribution.

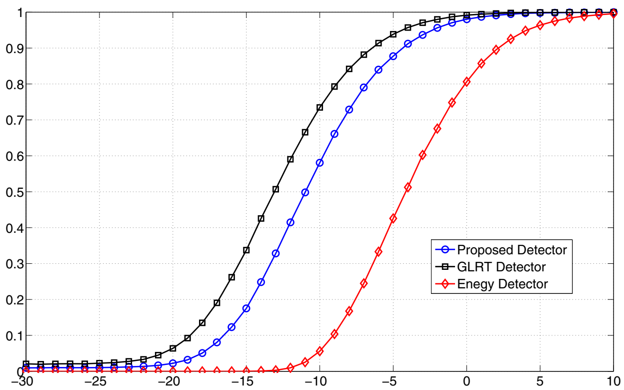

Fig. 2. Probability of detection for the proposed, GLRT-based and energy detection methods in a Rayleigh flat fading channel.

B. Detection Performance Comparisons

In Fig. 2 we compare the detection performance of the three methods as a function of channel SNR. The energy detector is assumed to experiences 1 dB noise uncertainty. As mentioned in section III-C, increasing the number of cycle frequencies does not significantly heighten the computational complexity of the proposed method. Therefore, we use 9 cycle frequencies in detection process of the proposed and GLRT-based methods. The results reveal that while our proposed method has much lower complexity than the GLRT-based method, it provides a comparable performance. Moreover, it has better detection performance than the ED method.

The performance of different hard cooperative methods are illustrated in Fig. 3. Each of the CR user employed proposed detector with 2 cycle frequencies. It can be seen that the OR fusion rule has lower probability of interference between primary and secondary user than the other hard fusion rules. However this method decreases throughput of CR network and has the most conservative behavior.

Fig. 4 depicts the performances of the cooperative sensing methods. Each employs 2 cycle frequencies for the detector

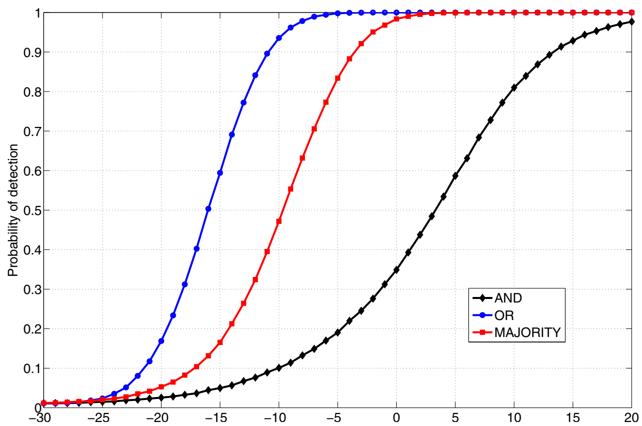

Fig. 3. Probability of detection versus channel average SNR for different hard cooperative methods over Rayleigh flat fading channels. The vector of local channel SNRs (in dB) at secondary users is defined as $\gamma = [\bar{\gamma}-5, \bar{\gamma}-4, \bar{\gamma}-3, \bar{\gamma}-2, \bar{\gamma}-1, \bar{\gamma}+1, \bar{\gamma}+2, \bar{\gamma}+3, \bar{\gamma}+4, \bar{\gamma}+5]$ where $\bar{\gamma}$ is average SNR.

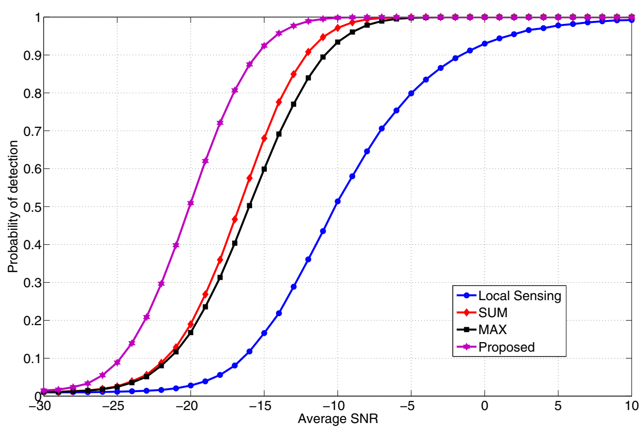

Fig. 4. Probability of detection versus channel average SNR for different proposed soft cooperative methods over Rayleigh flat fading channels. The vector of local channel SNRs (in dB) at secondary users is defined as $\gamma = [\bar{\gamma}-5, \bar{\gamma}-4, \bar{\gamma}-3, \bar{\gamma}-2, \bar{\gamma}-1, \bar{\gamma}+1, \bar{\gamma}+2, \bar{\gamma}+3, \bar{\gamma}+4, \bar{\gamma}+5]$ where $\bar{\gamma}$ is average SNR.

proposed in (3). As we can see, the proposed cooperative method will have better detection performance compared to the other methods, at the expense of an increase in communication overhead of the secondary network. Also, the SUM method has better performance than the MAX method.

The accuracy of the approximated CDF proposed in (22), approximation with normal distribution, and simulated CDF is investigated for decision statistic (15) in Fig. 5. It is assumed that each CR calculates the decision statistic with 9 cycle frequencies. As it can be seen, the numerical inversion method is accurate, while the Gaussian approximation performs with lower accuracy.

VI. Conclusions

In this paper, a fast reduced-complexity multi-cycle cyclostationariy detector has been proposed and then its performance and computational complexity has been compared

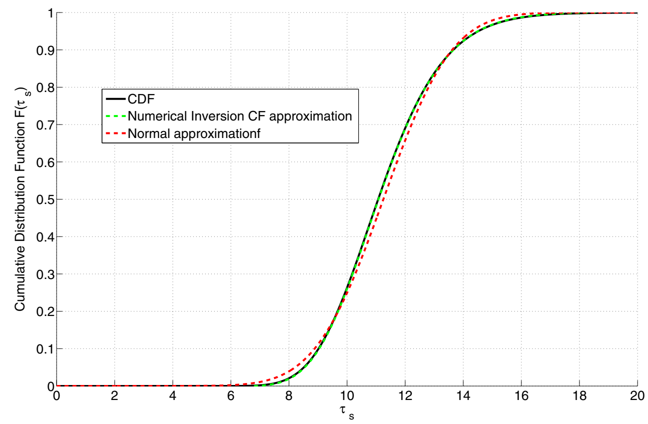

Fig. 5. Accuracy of proposed Gaussian approximation as well as numerical inversion method, are compared to the simulated CDF (10 CRs in network).

with well-known multi-cycle GLRT-based detector as well as energy detector. The analytical and simulation results have been shown that the computational complexity has been significantly reduced, while a slight degradation in detection performance is occurred, compared to the GLRT-based scheme. Also, we have shown that the proposed method is robust against the noise uncertainty problem, in contrast to the energy detectors. Consequently, the proposed detector can be considered as a tradeoff between the complicated GLRT-based cyclostationary detector and simple energy detectors.

Different cooperative spectrum sensing methods has been analyzed in this paper. We further propose a cooperative detector that has better detection performance than the existing methods, at a cost of a little increase in communication overhead of the cognitive radio network. Finally, we have proposed a straightforward method for threshold selection at the fusion center. Particularly, we have formulated a general approach for calculating the null distribution of the decision statistic of the cooperative detectors.

Appendix A
Estimating the Null Distribution of SUM fusion rule

We propose two methods for approximating the null distribution of (15). With the assumption of 10 or more CRs in network, we first propose to approximate the distribution of (15) with a Gaussian pdf (according to central limit theorem [14]):

$$\mathcal{T}_s \sim \mathcal{N}\left(\sum_{k=1}^{S}\mu_k, \sum_{k=1}^{S}\sigma_k^2\right), \tag{21}$$

where μ_k and σ_k^2 are mean and variance of kth SU's test statistic, respectively.

Alternatively, we propose the following more complex, but more accurate, method for approximating the null distribution. In fact, we propose to numerically invert the resulting characteristic function based on the method presented in [15]. The following derivations follow the same lines as in [15]. The cumulative distribution function $F(y)$ of random variable Y with zero mean and unit variance can be approximated by:

$$F(y) \approx \frac{1}{2} + \frac{\eta y}{2\pi} - \sum_{\substack{\nu=1-H \\ \nu\neq 0}}^{H-1} \frac{\Phi_Y(\eta\nu)}{2\pi j\nu} e^{-j\eta\nu y} \tag{22}$$

where $\Phi_Y(.)$ is the characteristic function of Y. η is a constant chosen such that the full range of distribution is represented. Furthermore, the characteristic function of a normalized variable $Z = (Y-\mu)/\sigma$ is given by $\Phi_Z(w) = \Phi_Y(\frac{w}{\sigma})\exp(\frac{-jw\mu}{\sigma})$.

Since (22) is defined for normalized random variable with zero mean and unit variance, the test statistic has to be normalized as well. We employ the following expressions for computing the approximate CDF (22).

Mean and variance of an F random variable with d_1 and d_2 degrees of freedoms can be computed as [14]:

$$\mu = \frac{d_2}{d_2-2} \qquad d_2 > 2 \tag{23}$$

$$\sigma^2 = \frac{2d_2^2(d_1+d_2-2)}{d_1(d_2-2)^2(d_2-4)} \quad d_2 > 4 \tag{24}$$

On the other hand, the characteristic function of an F random variable with b_1 and b_2 degrees of freedoms is defined as:

$$\Phi_{\mathcal{T}_n}(w) = \frac{\Gamma(b_2)}{\Gamma(b_1)\Gamma(b_2-b_1)} \int_0^1 t^{b_1-1}(1-t)^{b_2-b_1-1} e^{xt}\, dt \tag{25}$$

Finally, the characteristic function of $\mathcal{T}_s$ can be obtained as:

$$\Phi_{\mathcal{T}_s}(w) = \prod_{n=1}^{S} \Phi_{\mathcal{T}_n}(w) \tag{26}$$

APPENDIX B
THE DISTRIBUTION OF THE MAXIMUM OF S INDEPENDENT F RANDOM VARIABLE

We assumed that the F random variables have $d_{1,k}$ and $d_{2,k}$ degrees of freedom for their nominators and denominators, respectively. The cumulative distribution function of an F random variable with d_1 and d_2 degrees of freedom is given by:

$$F_{F(2(2L+1),2(2L+1))}(\mathcal{T}_k, d_1, d_2) = I_{\frac{d_1\mathcal{T}_k}{d_1\mathcal{T}_k+d_2}}(d_1/2, d_2/2), \tag{27}$$

where $I_x(a,b)$ is a regularized incomplete Beta function defined as $I_x(a,b) = \frac{B(x;a,b)}{B(a,b)}$. In this equation, $B(x;a,b)$ is an incomplete Beta function with $B(x;a,b) = \int_0^x t^{a-1}(1-t)^{b-1}\, dt$. The cumulative distribution function of the maximum of S independent random variable is the product of the CDFs of the individual random variables.

$$p(\max_k \mathcal{T}_k < a) = p(\mathcal{T}_1 < a, ..., \mathcal{T}_S < a) = \prod_{k=1}^{S} p(\mathcal{T}_k < a) \tag{28}$$

Hence, CDF of the maximum of S random variables with $d_{1,k}$ and $d_{2,k}$ numerator and denominator degrees of freedom for the kth random variable, is given by

$$\begin{aligned}\mathcal{F}_{F(2(2L+1),2(2L+1))}(\mathcal{T}_m, S, \{d_{1,k}, d_{2,k}\}_{k=1}^{S}) = \\ \prod_{k=1}^{S}\left(I_{\frac{d_{1,k}\mathcal{T}_k}{d_{1,k}\mathcal{T}_k+d_{2,k}}}(d_{1,k}/2, d_{2,k}/2)\right).\end{aligned} \tag{29}$$

We use the above equation for computing the intended CDF.

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