Sparsity-based Microwave Medical Imaging techniques

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Microwave tomography attempts to detect the tumors inside a tissue by reconstructing the distribution of the dielectric properties. In this invited keynote speech, we propose a new microwave imaging method which increases the resolution of the recovered images. The electromagnetic inverse scattering problem is solved using the Distorted Born Iterative Method (DBIM). The ill-posed inverse problem at each iteration of DBIM is solved using a novel sparsity-based recovery scheme. Recently, we have suggested two CS-based microwave medical imaging techniques, L2-IMATCS (M. Azghani, P. Kosmas, and F.Marvasti, "Microwave Medical Imaging Based on Sparsity and an Iterative Method with Adaptive Thresholding" TMI) and L2-ISATCS (M. Azghani, P. Kosmas, and F.Marvasti, "Fast Microwave Medical Imaging Based on Iterative Smoothed Adaptive Thresholding", AWPL). Our proposed imaging technique in this paper, the L2-regularized Iterative Reweighted Least Squares (L2-IRLS), offers higher quality reconstructions compared to the two previous schemes. In L2-IRLS, we aim to minimize an L2- regularized L_1 minimization problem with the aid of reweighted L_2 norm regularizer. The cost function to be minimized is as follows:

$$\min \ \|\mathbf{x}\|_1 + \lambda \|\mathbf{x}\|_2^2 \quad \text{subject to} \quad \mathbf{y}=\mathbf{A}\mathbf{x} \qquad (1)$$

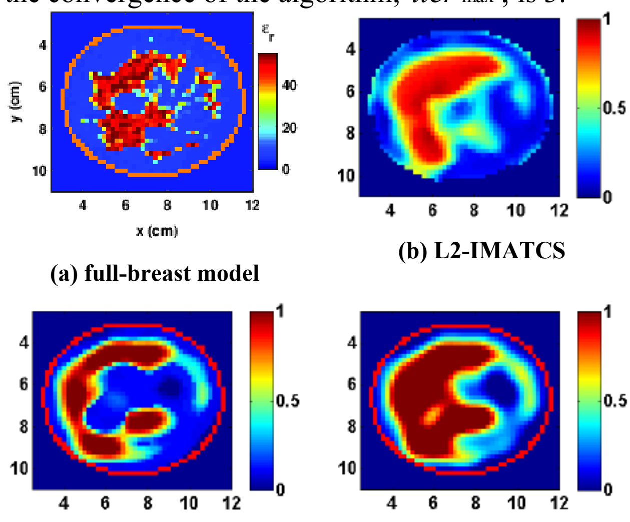
We define an upper bound for (1) as given:

$$J(x,\boldsymbol{\omega},\varepsilon) = \frac{1}{2}\left(\sum_{j=1}^{N} x_j^2 \left(\boldsymbol{\omega}_j + 2\lambda\right) + \sum_{j=1}^{N}\left(\varepsilon^2 \boldsymbol{\omega}_j + \boldsymbol{\omega}_j^{-1}\right)\right) \qquad (2)$$

where the reweighting vector $\boldsymbol{\omega}$ is updated using the recovered vector of the previous iteration, x_j. Minimizing the above upper bound, we obtain the L2-IRLS technique illustrated in Algorithm 1. The maximum number of iterations required for the convergence of the algorithm, $iter_{\max}$, is 3.

Algorithm 1 L2-regularized Iterative Reweighted Least Squares algorithm (L2-IRLS)

input:
A measurement matrix $\mathbf{A} \in \mathbb{R}^{m \times n}$.
A measurement vector $\mathbf{y} \in \mathbb{R}^{m}$.
The maximum number of iterations $iter_{max}$.
output:
A recovered estimate $\hat{\mathbf{x}} \in \mathbb{R}^{n}$ of the original signal.
procedure L2-IRLS($\mathbf{y}, \mathbf{x}$)
  $\mathbf{x}^0 \leftarrow 0$
  $\epsilon \leftarrow 1$
  $\mathbf{W} \leftarrow \mathbf{I}$
  for k=1:$iter_{max}$ **do**
    $\mathbf{x}^{n+1} \leftarrow \left(\mathbf{W}^n + \lambda_2 \mathbf{I} + \mathbf{\Phi}^H \mathbf{\Phi}\right)^{-1} \mathbf{\Phi}^H \mathbf{y}$
    $w_i^{n+1} = \dfrac{1}{\left(\mathbf{x}_i^{n+1^2} + \epsilon^2\right)^{1/2}}$
    $\mathbf{W}^n = \mathrm{diag}(w_1^n, w_2^n, \cdots, w_N^n)$
  end for
  return $\hat{\mathbf{x}} \leftarrow \mathbf{x}^{iter_{max}}$
end procedure



(a) full-breast model (b) L2-IMATCS (c) L2-ISATCS (d) L2-IRLS

Figure 1. The map of dielectric constant (a), reconstructed dielectric constant using L2-IMATCS (b), L2-ISATCS (c), and L2-IRLS (d) calculated at 1 GHz.

It should be noted that we have also proved the convergence of the algorithm. The reconstructed dielectric constants using L2-IMATCS, L2-ISATCS and the proposed L2-IRLS method are depicted in Figure 1. (b) ,(c), and (d), for the reconstruction of a full-breast model shown in Figure 1 (a). Our extensive simulations confirm that the proposed L2-IRLS scheme outperforms its counterparts in the resolution of the recovered image using only 3 iterations of the algorithm.