VII. Conclusions

The TSA-MIMO radar architecture offers a greatly reduced complexity at the transmitter. It replaces the computation for determining the optimum transmit signal covariance with the optimization over a small number of transmit beamformer angles. Furthermore, the hardware requirements of the transmitter are also greatly simplified. The performance is generally suboptimal to that of the fully diverse covariance matrix, but only small reductions in performance can be expected. The iterative beamspace algorithm used to select the transmit steering vectors greatly reduces the complexity with respect to the convex iteration procedures which could be used to enforce the rank constraints. It is robust to initial conditions, and target distributions, and so achieves or outperforms the performance of the convex iteration technique.

The subarray geometry for the TSA-MIMO radar should be chosen to produce a filled aperture to avoid ambiguities in the beampattern and high sidelobes. Necessary conditions to achieve this for regular subarray geometries were given.

Acknowledgment

The authors would like to thank Dr. W. Dawber for his helpful comments.

References

[1] G. Buck and J. Gustincic, "Resolution limitations of a finite aperture," *IEEE Trans. Antennas Propag.*, vol. 15, no. 5, pp. 376–381, May 1967.

[2] F. C. Robey, S. Coutts, D. Weikle, J. C. McHarg, and K. Cuomo, "MIMO radar theory and experimental results," in *Proc. 38th Asilomar Conf. Signals Syst. Comput.*, Asilomar, CA, Nov. 2004, vol. 1, pp. 300–304.

[3] B. Donnet and I. Longstaff, "MIMO radar techniques and opportunities," in *Proc. 3rd Eur. Radar Conf.*, Manchester, U.K., Sep. 2006, pp. 112–115.

[4] I. Bekkerman and J. Tabrikian, "Target detection and localization using MIMO radars and sonars," *IEEE Trans. Signal Process.*, vol. 54, no. 10, pp. 3873–3883, Oct. 2006.

[5] A. Hassanien and S. Vorobyov, "Phased-MIMO radar: A tradeoff between phased-array and MIMO radars," *IEEE Trans. Signal Process.*, vol. 58, no. 6, pp. 3137–3151, Jun. 2010.

[6] H. Li and B. Himed, "Transmit subaperturing for MIMO radars with collocated antennas," *IEEE J. Sel. Topics Signal Process.*, vol. 4, Feb. 2010.

[7] D. R. Fuhrmann, J. P. Browning, and M. Rangaswamy, "Signaling strategies for the hybrid MIMO phased-array radar," *IEEE J. Sel. Topics Signal Process.*, vol. 4, Feb. 2010.

[8] J. Li, P. Stoica, K. Forsythe, and D. Bliss, "Range compression and waveform optimization for MIMO radar: A Cramér-Rao Bound based study," *IEEE Trans. Signal Process.*, vol. 56, no. 1, pp. 218–232, Jan. 2008.

[9] L. Vandenberghe, S. Boyd, and S. Wu, "Determinant maximisation with linear matrix inequality constraints," *SIAM J. Matrix Anal. Appl.*, vol. 19, pp. 499–533, Apr. 1998.

[10] B. Tibken, M. Albani, and T. Fischer, "A new optimization problem for the solution of decision problems with applications to passenger occupation detection in cars," in *Proc. IEEE Int. Conf. Contr. Appl.*, Munich, Germany, Oct. 2006, pp. 726–727.

[11] A. d'Aspremont and S. Boyd, Relaxations and Randomized Methods for Nonconvex QCQPs 2003 [Online]. Available: http://www.stanford.edu/class/ee392o/relaxations.pdf

[12] R. Orsi, U. Helmke, and J. Moore, "A newton-like method for solving rank constrained linear matrix inequalities," in *Proc. IEEE 43rd Conf. Decision Contr.*, Atlantis, Bahamas, Dec. 2004, pp. 3138–3144.

[13] D. Wilcox, "MIMO Radar Direction Finding," Ph.D. dissertation, Electron. Electr. Eng. Comput. Sci., Queen's University Belfast, Ireland, 2011.

Optimal HDA Schemes for Transmission of a Gaussian Source Over a Gaussian Channel With Bandwidth Compression in the Presence of an Interference

Morteza Varasteh and Hamid Behroozi

***Abstract*—We consider transmission of a Gaussian source over a Gaussian channel under bandwidth compression in the presence of an interference known only to the transmitter. We study hybrid digital-analog (HDA) joint source-channel coding schemes and propose two novel layered coding schemes that achieve the optimal mean-squared error (MSE) distortion. This can be viewed as the extension of results by Wilson *et al.* ["Joint Source Channel Coding With Side Information Using Hybrid Digital Analog Codes," *IEEE Trans. Inf. Theory*, vol. 56, no. 10, pp. 4922–2940, Oct. 2010], originally proposed for sending a Gaussian source over a Gaussian channel in two cases: 1) Matched bandwidth with known interference only at the transmitter and 2) bandwidth compression where there is no interference in the channel. As the main contribution of this work, we provide optimal power allocation strategies for the proposed HDA schemes which enable us to cancel the channel interference and obtain the "optimum performance theoretically attainable" (OPTA) of additive white Gaussian noise (AWGN) channel with no interference in the case of bandwidth compression. We also provide performance analysis in the presence of signal-to-noise ratio (SNR) mismatch where we expect that HDA schemes perform better than strictly digital schemes.**

***Index Terms*—AWGN channel, dirty paper coding (DPC), hybrid digital-analog (HDA) coding, joint source-channel coding (JSCC).**

I. Introduction

We consider the problem of sending an analog Gaussian source over an average-power-limited Gaussian channel in the presence of an interference known only to the transmitter but when the channel bandwidth is smaller than the source bandwidth (i.e., bandwidth compression case). The problem setup is shown in Fig. 1.

For the point-to-point transmission of a single Gaussian source through an additive white Gaussian noise (AWGN) channel without interference, if the channel and source bandwidths are equal, the optimal performance theoretically attainable (OPTA) [2] can be achieved by one of the following schemes: 1) separate source-channel coding scheme, known as the digital tandem source-channel coding scheme (due to Shannon) [3], [4] and 2) uncoded (or analog) transmission (due to Goblick) [5]. In fact, it is shown [6], [7] that there are infinite schemes that can achieve OPTA for this problem.

There are two inherent problems associated with the digital tandem scheme: the "leveling-off effect" and the "threshold effect" [8], [9]. Since the system typically performs well at a certain designed channel signal-to-noise ratio (CSNR), the system performance does not improve with increased CSNR (leveling-off effect) where the distortion is limited by the quantization. In addition, the system performance

Manuscript received September 11, 2011; accepted December 13, 2011. Date of publication January 02, 2012; date of current version March 06, 2012. The associate editor coordinating the review of this manuscript and approving it for publication was Dr. Maja Bystrom. This work has been supported in part by the Iran NSF under Grant 88114.46 and by the Iran Telecommunication Research Center (ITRC). The material in this paper was presented in part at the IEEE Information Theory Workshop (ITW), Paraty, Brazil, October 2011.

The authors are with the Electrical Engineering Department, Sharif University of Technology, Tehran 11365-9363, Iran (e-mail: varasteh@ee.sharif.edu; behroozi@sharif.edu).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TSP.2011.2182350

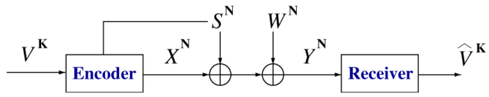

Fig. 1. Sending K samples of a Gaussian source V^K in N uses of a power-limited AWGN channel corrupted by an interference known only to the transmitter.

degrades drastically when the true CSNR is worse than the designed CSNR (threshold effect) since the index cannot be decoded. In order to exploit the advantages of both analog transmission and digital techniques, a family of HDA coding schemes were introduced in the literature; see, e.g., [9]–[17]. These methods usually offer better distortion performance than the purely analog or digital schemes; they do not suffer from the leveling-off effect, have a less severe threshold effect [12] compared with digital tandem source-channel coding schemes, and they can asymptotically achieve Shannon's OPTA limit at the designed CSNR.

With the presence of an interference known only to the transmitter, uncoded transmission is no longer an optimal scheme, while separate source-channel coding maintains optimality by applying dirty paper coding, also known as Costa coding [18]. Wilson, Narayanan and Caire [1] also proposed a new HDA code that can achieve OPTA in the presence of an interference known only to the transmitter. But all these schemes are designed for the matched-bandwidth case. In [1] some HDA schemes are also proposed for transmitting a Gaussian source with bandwidth compression but the channel has no interference.

In this paper, we propose two HDA layered-coding schemes which also transmit superimposed coded and uncoded information, but we fully exploit the potential of the dirty paper coding auxiliary random variable. Our schemes can be considered as generalization results of Costa [18] and Wilson *et al.* [1]. Our main contribution in this paper is to provide optimal power allocation strategies for the proposed layered schemes and thus obtain the OPTA of AWGN channel with no interference in the case of bandwidth compression.

The paper is organized as follows. We present the system model and problem statement in Section II. Then we introduce the proposed optimal HDA schemes that can achieve OPTA in Section III. Conclusions are given in Section IV.

II. Problem-Statement

Let $V \sim \mathcal{N}(0, \sigma_v^2)$, $S \sim \mathcal{N}(0, Q)$ and $W \sim \mathcal{N}(0, \sigma^2)$ be jointly independent Gaussian random variables (RVs). V^N, S^N, and W^N denote N-dimensional random vectors with i.i.d. components distributed as V, S, and W, respectively.

We consider the problem of transmitting an i.i.d. Gaussian source sequence, $V^K \in \mathbb{R}^K$, in N uses of an AWGN channel with i.i.d. Gaussian noise sequence $W^N \in \mathbb{R}^N$. We transmit in the presence of an interference, S^N, which is known to the transmitter but unknown to the receiver. First, the source sequence V^K is encoded to $X^N = \varphi(V^K)$, where the encoder function is of the form $\varphi : \mathbb{R}^K \to \mathbb{R}^N$. The transmitted sequence X^N is average-power limited to $P > 0$, i.e.,

$$\frac{1}{N} \sum_{i=1}^{N} E\left[|X[i]|^2\right] \leq P. \tag{1}$$

Based on the channel output $Y^N \in \mathbb{R}^N$, the receiver makes an estimate of the source sequence V^K as $\widehat{V}^K = \psi(Y^N)$ where $\psi : \mathbb{R}^N \to \mathbb{R}^K$ is the decoding function. The quality of the estimate is measured by the average mean-square error (MSE) distortion, $\Delta = \frac{1}{K} E[\sum_{i=1}^{K} d(V[i], \widehat{V}[i])]$ where $d(V[i], \widehat{V}[i]) = (V[i] - \widehat{V}[i])^2$.

A joint source-channel code with input power constraint P, distortion D, and bandwidth ratio $\lambda = \frac{N}{K}$ consists of a pair of mappings $\varphi : V^K \in \mathbb{R}^K \to X^N \in \mathbb{R}^N$ and $\psi : Y^N \in \mathbb{R}^N \to \widehat{V}^K \in \mathbb{R}^K$ such that both of the following equations are satisfied: $\frac{1}{N} E[\|X^N\|^2] \leq P$, and $E[d(V^K, \psi(Y^N))] \leq D$. Let $\mathcal{F}^{(K,N)}(P)$ denote all encoder and decoder functions (φ, ψ) defined as above. For any target distortion $D \geq 0$, the power-distortion region $\mathcal{P}(D)$ is defined as the convex closure of the set of all achievable power-distortion pairs (P, D), where a power-distortion pair (P, D) is achievable if for any $\delta > 0$, there exists $n_0(\delta)$ such that for any $n \geq n_0(\delta)$ there exists $(\varphi, \psi) \in \mathcal{F}^{(K,N)}(P)$ with distortion $\Delta \leq D + \delta$. Our aim is to show that the OPTA, which is defined for a fixed P as

$$D_{\mathrm{OPTA}}(P) = \inf\{D | (P, D) \in \mathcal{P}(D)\} \tag{2}$$

i.e., the minimum achievable distortion for a fixed P, is achievable using the proposed HDA schemes.

Based on the Shannon's joint source-channel coding (JSCC) theorem [3], [4], the performance of a general JSCC system for transmitting an i.i.d. Gaussian source over an AWGN channel is always lower bounded by the OPTA, which is

$$D_{\mathrm{OPTA}}(P) = \frac{\sigma_v^2}{\left(1 + \frac{P}{\sigma^2}\right)^{\lambda}}.$$

When $\lambda = 1$, the separate source and channel coding (which can be realized as the concatenation of an optimal quantizer with an optimal capacity-achieving code) is optimal. This optimal distortion can also be achieved by uncoded transmission [5], [19]. In [6], it is shown that there is a family of infinitely many schemes that are optimal, which contains the separation scheme and uncoded transmission as special cases.

The optimal distortion can be obtained even in the presence of an interference by using the separate source and channel coding scheme. In [1], an HDA coding scheme (known as HDA Costa) is proposed that can achieve this MSE distortion. As the extension of Bross *et al.*'s [6] result in the presence of an interference known only to the transmitter, it is shown that [1] there is a family of infinitely many schemes that are optimal for this problem where the separation and the HDA Costa schemes are special cases.

III. Proposed Schemes for Transmitting a Gaussian Source With Bandwidth Compression

We now consider the problem of transmitting K samples of an i.i.d. Gaussian source to a single user in $N = \lambda K$ ($\lambda < 1$) uses of an AWGN channel in the presence of an interference fully known only to the transmitter. In this Section, we propose two layered HDA schemes and show that they can achieve the optimal distortion. In each layer, we use HDA schemes, introduced in [1], for transmitting $K = N$ samples of a Gaussian source in N uses of a power-limited Gaussian channel. Here we only give a high-level description and analysis of the schemes without detailed proofs. In particular, in many steps of the analysis we treat finite-blocklength coding schemes as idealized systems with asymptotically large blocklengths.

Theorem 1: For the point-to-point transmission of a single Gaussian source through an AWGN channel with bandwidth compression, and in the presence of an interference known only to the transmitter, the OPTA can be achieved by both of the following two-layer coding schemes:

1) a layered coding scheme, consisting of "Costa" and "HDA Costa" layers;
2) a layered coding scheme, consisting of "superimposed Costa and HDA Costa" layers.

A. Scheme-I: Layering With "Costa" and "HDA Costa" Coding

In this scheme, the source sequence V^K is split in two parts: the first N samples of the source sequence, indicated by V_1^N, is encoded by the

HDA Costa encoder that treats S^N as interference and uses average-power $P_1 = a$. Let U_1 be an auxiliary random variable given by

$$U_1 = X_1 + \alpha_1 S + k_1 V_1 \tag{3}$$

where $X_1 \sim \mathcal{N}(0, P_1)$, $S \sim \mathcal{N}(0, Q)$ and V_1 are independent. The scaling factors α_1, and k_1^2 are set to be

$$\alpha_1 = \frac{P_1}{P + \sigma^2}, \quad k_1^2 = \frac{P_1^2}{(P + \sigma^2)\sigma_v^2}. \tag{4}$$

For given V_1^N and S^N, we find a sequence U_1^N such that (U_1^N, S^N, V_1^N) are jointly typical and transmit $X_1^N = U_1^N - \alpha_1 S^N - K_1 V_1^N$. The code construction and encoding procedures are as follows [1]:

Codebook Generation: Generate a codebook $\mathcal{C}_u$ of block length N and with 2^{NR} codewords by independent i.i.d. generation of the codewords components according to the RV in 3. These codewords are revealed to the encoder and the decoder.

Encoding: Given the source sequence V_1^N and known interference sequence S^N, the encoder looks for an N-dimensional vector $U_1^N \in \mathcal{C}_u$ such that (U_1^N, S^N, V_1^N) are jointly typical, and then if this process was successful the transmitter sends $X_1^N = U_1^N - \alpha_1 S^N - K_1 V_1^N$. Otherwise, an encoding failure is declared and a zero-sequence $X_1^N = \mathbf{0}$ is transmitted.

In the second stream, the remaining $K - N$ samples of the source sequence, denoted by V_2^{K-N}, is encoded to X_2^N by Digital Costa encoder [1] that treats S^N and X_1^N as interference and uses average-power $P_2 = P - a$. Let U_2 be an auxiliary random variable given by

$$U_2 = X_2 + \alpha_2(S + X_1) \tag{5}$$

where $X_2 \sim \mathcal{N}(0, P_2)$, X_1 and S are independent and the scaling factor is set to be

$$\alpha_2 = \frac{P_2}{P_2 + \sigma^2}. \tag{6}$$

We first quantize the source sequence, V_2^{K-N}, using an optimal quantizer to produce an index $m \in 1, \ldots, 2^{NR_2}$, where $R_2 = \frac{1}{2}\log\left(1 + \frac{P_2}{\sigma^2}\right) - \epsilon$. Then the index is transmitted using Costa's writing on dirty paper coding (DPC) scheme [18]. We create an N-length i.i.d. Gaussian codebook $\mathcal{U}_2$ with $2^{NI(U;Y)}$ codewords, where each component of the codeword is Gaussian with zero mean and variance $P_2 + \alpha_2^2(Q + P_1)$. Then, evenly distribute these codewords over 2^{NR_2} bins. For each codeword U_2^N, let $i(U_2^N)$ be the index of the bin containing U_2^N. For a given quantization index m, we look for a codeword U_2^N such that $i(U_2^N) = m$ and (U_2^N, S^N, X_1^N) are jointly typical. Therefore, we need to transmit $X_2^N = U_2^N - \alpha_2(S^N + X_1^N)$. We merge and linearly combine both layers and transmit $X^N = X_1^N + X_2^N$.

At the receiver, the digital Costa layer of the received signal $Y^N = X^N + S^N + W^N$ is decoded first, yielding the Costa auxiliary codeword U_2^N. Decoding of the Costa code requires that $R(D) = \frac{\lambda}{1-\lambda} C(P_2)$, where $R(D) = \frac{1}{2}\log\left[\frac{\sigma_v^2}{D}\right]$ is the rate-distortion function of a Gaussian source and $C(P_2)$ is the capacity of the AWGN channel with an SNR equals to $\frac{P_2}{\sigma^2}$, achievable by Costa coding. Therefore, the distortion of Costa layer is given by

$$D_{\text{Costa}} = \sigma_v^2 \left(1 + \frac{P - a}{\sigma^2}\right)^{-\frac{\lambda}{1-\lambda}}.$$

For the HDA stream, we look for an U_1^N that is jointly typical with Y^N, treating X_2^N as additional Gaussian noise. The optimal estimate of V_1^N is obtained based on these two vectors, U_1^N and Y^N. So, the distortion achieved by the HDA Costa layer is given by

$$D_{\text{HDA}} = \sigma_v^2 \left(1 + \frac{a}{P - a + \sigma^2}\right)^{-1}.$$

As a result, the overall distortion at the receiver can be obtained as [15] $D = \lambda D_{\text{HDA}} + (1 - \lambda) D_{\text{Costa}}$, i.e.,

$$D = \frac{1}{K}\sigma_v^2 \left[\frac{K - N}{\left(1 + \frac{P-a}{\sigma^2}\right)^{\frac{N}{K-N}}} + \frac{N}{1 + \frac{a}{P-a+\sigma^2}}\right]. \tag{7}$$

By optimizing the final achievable distortion D of (7) with respect to a we obtain the optimal power allocation as follows:

$$a^* = \sigma^2 \left[1 - \left(1 + \frac{P}{\sigma^2}\right)^{1-\lambda}\right] + P. \tag{8}$$

Substituting (8) in (7) yields the following achievable distortion:

$$D(a^*) = \frac{\sigma_v^2}{\left(1 + \frac{P}{\sigma^2}\right)^{\lambda}}. \tag{9}$$

We observe that the proposed scheme is optimal in the sense of achieving the OPTA.

B. Scheme II: Layering With Superimposed Costa and HDA Costa Coding

We split the source exactly as in the previous case where one stream is a mapping of $\mathbb{R}^{K-N} arrow \mathbb{R}^N$ by the digital Costa encoder, that maps source blocks $V^{K-N} \in \mathbb{R}^{K-N}$ onto channel codewords $X_{1c}^N \in \mathbb{R}^N$. The other stream is also a mapping $\mathbb{R}^N arrow \mathbb{R}^N$ that maps the remaining N samples of the source sequence, denoted by $V^N \in \mathbb{R}^N$ onto channel codewords $X_{2c}^N \in \mathbb{R}^N$ by the superimposed Costa and HDA encoder, introduced in [1].

First Layer: In the first layer, Costa layer, $K - N$ samples of the source sequence will be mapped to the N-dimensional vector. The auxiliary random variable U_{1c} is defined by

$$U_{1c} = X_{1c} + \alpha_{1c} S \tag{10}$$

where $X_{1c} \sim \mathcal{N}(0, P_1)$, $\alpha_{1c} = \frac{P_1}{P+\sigma^2}$ and $P_1 = P - a$.

Second Layer: In the second layer, the transmitted signal is obtained as the superposition of two signals X_{2c} and X_{2hc}, which are the outputs of a digital Costa encoder and an HDA Costa encoder, respectively. The source signal is first quantized at a rate of $R < C$ using an optimal source code. The first stream consists of a digital Costa encoder that encodes the quantization index by treating $S^N + X_{1c}^N$ as known interference and produces the signal X_{2c}^N, with average power P_c. Powers allocated to both HDA Costa and Costa encoder are functions of transmission rate but we should pay attention to satisfaction of channel input power constraint of both paths in the second layer. The auxiliary random variable for the first stream in the second layer will be as follows:

$$U_{2c} = X_{2c} + \alpha_{2c}(X_{1c} + S) \tag{11}$$

where $X_{2c} \sim \mathcal{N}(0, P_c)$, $\alpha_{2c} = \frac{P_c}{a+\sigma^2}$ and the signal power of this stream (digital Costa scheme) is chosen to be $P_c = (a+\sigma^2)(1-2^{-2R})$.

The second stream consists of an HDA Costa encoder that encodes the quantization error, treating $S^N + X_{1c}^N + X_{2c}^N$ as known interference and produces X_{2hc}^N, with average power $P_{hc} = (a + \sigma^2)2^{-2R} - \sigma^2$. The auxiliary random variable is given by

$$U_{2hc} = X_{2hc} + \alpha_{2hc}(S + X_{2c} + X_{1c}) + ke,$$

where $X_{2hc} \sim \mathcal{N}(0, P_{hc}), e \sim \mathcal{N}(0, \sigma_v^2 2^{-2R})$. The coefficients α_{2hc} and k are set as

$$\alpha_{2hc} = \frac{P_{hc}}{P_{hc}+\sigma^2}, \quad k^2 = \frac{P_{hc}^2}{(P_{hc}+\sigma^2)\sigma_v^2 2^{-2R}}. \tag{12}$$

Therefore, the transmitted signal for the second layer is superposition of two signals, i.e., $X_{2c}^N + X_{2hc}^N$ with average power $P_c + P_{hc} = a$.

At the decoder, we first decode the first layer, treating the second layer as additional noise and then we decode the second layer. As we know the first layer is used as side information for the second layer, so decoding of the second layer is independent of the first layer. Note that the overall achievable distortion from the second layer is the distortion in estimating e. By similar analysis of the final minimum mean squared error (MMSE) distortion in Section III-A, the following average distortion is achievable:

$$D = \frac{1}{K}\sigma_v^2 \left[\frac{K-N}{\left(1+\frac{P-a}{a+\sigma^2}\right)^{\frac{N}{K-N}}} + \frac{N 2^{-2R}}{1+\frac{P_{hc}}{\sigma^2}} \right]. \tag{13}$$

Minimizing (13) with respect to parameter "a" yields

$$a^* = \left(\frac{P+\sigma^2}{\sigma^2}\right)^{\lambda-1}(\sigma^2+P) - \sigma^2. \tag{14}$$

By substituting (14) in (13), the distortion achieved by this scheme is as follows

$$D(a^*) = \frac{\sigma_v^2}{\left(1+\frac{P}{\sigma^2}\right)^{\lambda}}. \tag{15}$$

It can be seen that the interference is totally canceled and the distortion of the AWGN channel without interference is achieved.

C. Performance Analysis in the Presence of SNR Mismatch

In this section, we consider the performance of the above JSCC schemes for the case of SNR mismatch where we design the scheme to be optimal for a channel noise variance σ^2 but the actual noise variance is σ_a^2. Separation based digital schemes suffer from the threshold effect. When the channel SNR is worse than the designed SNR, the index cannot be decoded. Also, when the channel SNR is better than the designed SNR, the distortion is limited by the source coding and does not improve. However, the hybrid digital analog schemes offer better performance in this situation (here we consider only the case when the actual SNR is better than the designed SNR).

Scheme I: When SNR is better than the designed SNR in the first scheme, the digital Costa path will be decoded correctly and the achievable distortion for this layer will be as follows:

$$D_d = \sigma_v^2 \left(1+\frac{P-a}{\sigma^2}\right)^{-\frac{N}{K-N}}. \tag{16}$$

Then, we will estimate the first N samples of the source sequence by using the decoded codeword, U_1^N, and the received sequence, $Y^N = X_1^N + X_2^N + S^N + W^N$. We use the following matrices to estimate the source:

$$B_1 = \begin{bmatrix} K_1\sigma_v^2 \\ 0 \end{bmatrix} \tag{17}$$

$$C_1 = \begin{bmatrix} a+\alpha_1^2 Q + K_1^2\sigma_v^2 & a+\alpha_1 Q \\ a+\alpha_1 Q & P+Q+\sigma_a^2 \end{bmatrix}. \tag{18}$$

Finally, the achievable distortion is as follows:

$$D = \frac{1}{K}\left[\frac{(K-N)\sigma_v^2}{\left(1+\frac{P-a}{\sigma^2}\right)^{\frac{N}{(K-N)}}} + N\left(\sigma_v^2 - B_1^T C_1^{-1} B_1\right) \right]. \tag{19}$$

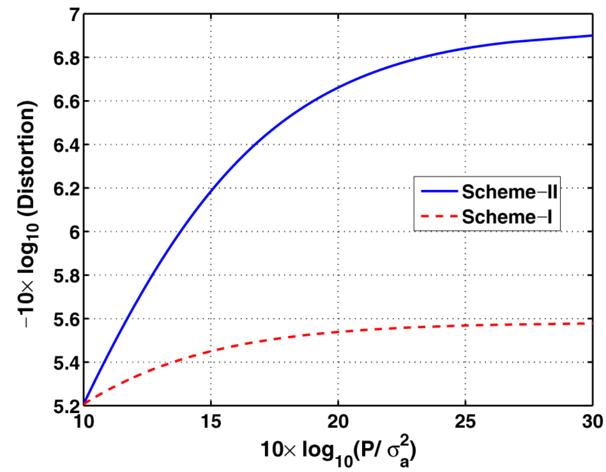

Fig. 2. *SNR mismatch*. Transmission of a Gaussian source V^K over a power-limited AWGN channel corrupted by interference known only to the transmitter in the case of bandwidth compression.

Scheme II: In this scheme, we should pay attention, that unlike the case of SNR matched, the achievable distortion will be affected by the variable rate used in the second layer (superimposed layer). When SNR is better than the designed SNR, the digital Costa layer will be decoded correctly, and we have

$$D_d = \sigma_v^2 \left(1+\frac{P-a}{a+\sigma^2}\right)^{-\frac{N}{K-N}}. \tag{20}$$

For the second layer, the digital Costa stream will be decoded correctly. The distortion of the second layer will be the estimation distortion based on decoded codewords and the received sequence, $Y^N = X_{1c}^N + X_{2c}^N + X_{2hc}^N + S^N + W^N$. The correlation and covariance matrices used for estimating the source will be as follows:

$$B_2 = \begin{bmatrix} k 2^{-2R}\sigma_v^2 \\ 0 \end{bmatrix} \tag{21}$$

$$C_2 = \begin{bmatrix} A_1 & A_2 \\ A_2 & A_3 \end{bmatrix} \tag{22}$$

where

$$A_1 = P_{hc} + \alpha_{hc}^2(Q+P-a+P_c) + k^2 2^{-2R}\sigma_v^2$$
$$A_2 = P_{hc} + \alpha_{hc}(Q+P-a+P_c)$$
$$A_3 = P+Q+\sigma_a^2.$$

Therefore, the distortion achieved by this scheme is as follows:

$$D = \frac{1}{K}\left[\frac{(K-N)\sigma_v^2}{\left(1+\frac{P-a}{a+\sigma^2}\right)^{\frac{N}{K-N}}} + N\left(\sigma_v^2 - B_2^T C_2^{-1} B_2\right) \right]. \tag{23}$$

The performance of both proposed schemes for $\lambda = \frac{N}{K} = 0.5$ are shown in Fig. 2. The designed SNR and the actual SNR in decibels are defined as $10\log_{10}\left(\frac{P}{\sigma^2}\right)$ and $10\log_{10}\left(\frac{P}{\sigma_a^2}\right)$, respectively. In this example, the designed SNR is fixed at 10 dB and the actual SNR is varied from 10 to 30 dB. It can be seen that Scheme II performs better than Scheme I over the entire range of SNRs.

IV. Conclusion

In this work, we considered HDA coding schemes for the transmission of a Gaussian source over a power-limited AWGN channel in the presence of interference known only to the transmitter. In particular, layered JSCC schemes for this problem were analyzed under bandwidth compression assumption. We provided two HDA coding schemes that can achieve OPTA in this problem. The HDA coding schemes have advantages over strictly digital schemes when there is a mismatch in the channel SNR. We also obtained the achievable distortion under SNR mismatch for both proposed schemes when the actual SNR is greater than the designed SNR.

REFERENCES

[1] M. Wilson, K. Narayanan, and G. Caire, "Joint source channel coding with side information using hybrid digital analog codes," *IEEE Trans. Inf. Theory*, vol. 56, no. 10, pp. 4922–4940, Oct. 2010.

[2] T. Berger, *Rate-Distortion Theory: A Mathematical Basis for Data Compression*. Englewood Cliffs, NJ: Prentice-Hall, 1971.

[3] C. E. Shannon, "A mathematical theory of communication (Part 1)," *Bell Syst. Tech. J.*, vol. 27, pp. 379–423, 1948.

[4] C. E. Shannon, "A mathematical theory of communication (Part 2)," *Bell Syst. Tech. J.*, vol. 27, pp. 623–656, 1948.

[5] T. J. Goblick, "Theoretical limitations on the transmission of data from analog sources," *IEEE Trans. Inf. Theory*, vol. IT-11, no. 4, pp. 558–567, Oct. 1965.

[6] S. Bross, A. Lapidoth, and S. Tinguely, "Superimposed coded and uncoded transmissions of a Gaussian source over the Gaussian channel," in *Proc. IEEE Int. Symp. Inf. Theory (ISIT)*, Seattle, WA, Jul. 2006, pp. 2153–2155.

[7] Y. Gao and E. Tuncel, "New hybrid digital/analog schemes for transmission of a Gaussian source over a Gaussian channel," *IEEE Trans. Inf. Theory*, vol. 56, no. 12, pp. 1782–1799, Dec. 2010.

[8] C. E. Shannon, "Communication in the presence of noise," *Proc. IRE*, vol. 37, no. 1, pp. 10–21, Jan. 1949.

[9] M. Skoglund, N. Phamdo, and F. Alajaji, "Hybrid digital-analog source-channel coding for bandwidth compression/expansion," *IEEE Trans. Inf. Theory*, vol. 52, no. 8, pp. 3757–3763, Aug. 2006.

[10] S. Shamai, S. Verdu, and R. Zamir, "Systematic lossy source/channel coding," *IEEE Trans. Inf. Theory*, vol. 44, no. 2, pp. 564–579, Mar. 1998.

[11] B. Chen and G. W. Wornell, "Analog error-correcting codes based on chaotic dynamical systems," *IEEE Trans. Commun.*, vol. 46, no. 7, pp. 881–890, Jul. 1998.

[12] U. Mittal and N. Phamdo, "Hybrid digital-analog (HDA) joint source-channel codes for broadcasting and robust communications," *IEEE Trans. Inf. Theory*, vol. 48, no. 5, pp. 1082–1102, May 2002.

[13] S. Sesia, G. Caire, and G. Vivier, "Lossy transmission over slow-fading AWGN channels: A comparison of progressive, superposition and hybrid approaches," presented at the IEEE Int. Symp. Inf. Theory (ISIT), Adelaide, Australia, Sep. 2005.

[14] Z. Reznic, M. Feder, and R. Zamir, "Distortion bounds for broadcasting with bandwidth expansion," *IEEE Trans. Inf. Theory*, vol. 52, no. 8, pp. 3778–3788, Aug. 2006.

[15] M. P. Wilson, K. Narayanan, and G. Caire, "Joint source channel coding with side information using hybrid digital analog codes," in *Proc. IEEE Inf. Theory and Applications (ITA) Workshop*, La Jolla, CA, Jan. 2007, pp. 299–308.

[16] V. M. Prabhakaran, R. Puri, and K. Ramchandran, "Colored Gaussian source-channel broadcast for heterogeneous (analog/digital) receivers," *IEEE Trans. Inf. Theory*, vol. 54, no. 4, pp. 1807–1814, Apr. 2008.

[17] Y. Wang, F. Alajaji, and T. Linder, "Hybrid digital-analog coding with bandwidth compression for Gaussian source-channel pairs," *IEEE Trans. Commun.*, vol. 57, no. 4, pp. 997–1012, Apr. 2009.

[18] M. Costa, "Writing on dirty paper," *IEEE Trans. Inf. Theory*, vol. 29, no. 3, pp. 439–441, May 1983.

[19] M. Gastpar, B. Rimoldi, and M. Vetterli, "To code, or not to code: Lossy source channel communication revisited," *IEEE Trans. Inf. Theory*, vol. 49, no. 5, pp. 1147–1158, May 2003.

Joint Source-Channel Coding for the MIMO Broadcast Channel

Daniel Persson, Johannes Kron, Mikael Skoglund, and Erik G. Larsson

***Abstract*—We investigate the problem of broadcasting analog sources to several users using short codes, employing several antennas at both the transmitter and the receiver, and channel-optimized quantization. Our main objective is to minimize the sum mean-square error distortion. A joint multi-user encoder, as well as a structured encoder with separate encoders for the different users, are proposed. The first encoder outperforms the latter, which, in turn, offers large improvements compared to state-of-the-art, over a wide range of channel signal-to-noise ratios. Our proposed methods handle bandwidth expansion, i.e., usage of more channel than source dimensions, automatically. We also derive a lower bound on the distortion.**

***Index Terms*—Broadcast, downlink, joint source-channel coding (JSCC), low latency, multiple-input multiple-output (MIMO), multi-user, short codes.**

I. INTRODUCTION

We study the multiple-input multiple-output (MIMO) broadcast problem with short codes, with the end-to-end sum mean square error (MSE) distortion as performance measure. This investigation can be motivated by low-latency applications, where long codes are not suitable. Our study is both of fundamental interest and relevant for applications.

A. Background

MIMO technology improves both capacity and robustness in traditional communications [1]. Recently, Weingarten *et al.* [2] found the capacity region for the MIMO broadcast channel, and it was established that dirty-paper coding (DPC) is optimal in the sense that techniques based on DPC can achieve any rate-tuple in this region. However, DPC relies on infinitely long codewords, and is, therefore, not suitable for a low-delay scenario.

In a scenario with short codes, the source-channel separation theorem in general does not apply, and the optimal solution consists of joint optimization of the source and the channel code. Traditional digital communication systems consist of several separate units: quantizer, channel encoder, precoder, and modulator. Joint source-channel coding (JSCC) strategies, where these units are co-designed, should be considered for the case of short codes.

Many treatments of JSCC exist, e.g., characterization of the distortion regions for the problems of sending a bivariate Gaussian

Manuscript received September 06, 2010; revised February 22, 2011; accepted December 04, 2011. Date of publication December 21, 2011; date of current version March 06, 2012. The associate editor coordinating the review of this manuscript and approving it for publication was Dr. Francesco Verde. This work has been supported in part by the Swedish Research Council (VR), the Swedish Foundation for Strategic Research (SSF), and Security Link. E. G. Larsson is a Royal Swedish Academy of Sciences (KVA) Research Fellow supported by a Grant from the Knut and Alice Wallenberg Foundation.

D. Persson and E. G. Larsson are with the Department of Electrical Engineering, Linköping University, SE-581 83 Linköping, Sweden (e-mail: danielp@isy.liu.se; egl@isy.liu.se).

J. Kron and M. Skoglund are with the School of Electrical Engineering, Royal Institute of Technology, SE-100 44 Stockholm, Sweden (e-mail: johk@ee.kth.se; skoglund@ee.kth.se).

Digital Object Identifier 10.1109/TSP.2011.2180716