Statistical Characterization and Bit-Error Rate Analysis of Lightwave Systems With Optical Amplification Using Two-Photon-Absorption Receiver Structures

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***Abstract*—In this paper, the authors study the performance of a two-photon absorption (TPA) receiver in a typical communication system employing an optical amplifier. For the study, the authors develop a comprehensive performance analysis, taking into account noises due to an optical amplifier, shot noise, and thermal noise, using Gaussian approximation technique. In evaluating the performance of the system using this technique, the mean and the variance of the received signal will be derived. Comparing the results of receivers using the TPA process with that of a typical single-photon absorption (SPA) structure leads the authors to conclude that TPA receivers can be used when the speed of the electronic parts of the receiver structures is not too high, as expected.**

***Index Terms*—Bit error rate (BER) analysis, optical amplifier, statistical characterization, two-photon absorption (TPA) receiver.**

I. Introduction

OPTICAL domain is well suited for transmitting extremely high data rate information in communications. However, one of the main bottlenecks that limit the use of this potential capacity is the need to change the domain from high-speed optical-signal processing to low-speed electrical-signal processing for detection purposes. To remedy this bottleneck, a nonlinear-detection scheme that uses a two-photon absorption (TPA) process [1]–[5] has been introduced recently. This technique is used both as a single signal beam for detecting high-speed spectrally encoded optical code division multiple access (CDMA) pulses [2], [3] and as a clock sampling signal for optical time division multiple access (TDMA) pulse [4], [5]. However, the results of this paper are well suited in considering the performance of a spectrally encoded optical CDMA [3], [6]. In light detectors using the TPA process, a band gap frequency of the photodetector is chosen to be between the values of once and twice the frequency of the transmitted light [7], [8]. Thus, an electron will be liberated from a photodetector if two photons are incident on the photodetector simultaneously [1]. On the other hand, different approaches were introduced to obtain the bit error rate (BER) of a system using optical amplifiers. Various noise models due to an optical amplifier in a direct detection receiver using conventional photodetectors were studied in the past decade [9]–[15].

In this paper, we will examine for the first time the statistical properties of an output signal from a TPA detector using an optical amplifier. Using these properties, the BER of both the typical single-photon absorption (SPA) communication system and that of the TPA detector with an optical amplifier will be derived. The Gaussian approximation technique is used for computing the BER of such a system due to the high complexity of a TPA detector, which makes an exact mathematical modeling using the characteristic function of an output-signal probability density function intractable.

The rest of the paper is organized as follows. In Section II, the statistical behavior of a typical TPA detector will be analyzed. Using the analysis of Section II, the BER of an optical system using a TPA photodetector is derived in Section III. Moreover, in this section, we discuss numerical results obtained for a TPA detector and compare these results with those using SPA optical detectors. We conclude the paper in Section IV.

II. Statistical Characteristics of a TPA Detector

In this section, we develop a statistical model for a TPA detector using a semiclassic analysis. In a TPA detector, the rate of the photoelectrons generated in a photodetector (dN/dt) is proportional to the square of the intensity of the optical signal, i.e., $I^2(t)$ in addition to the intensity itself [1]; hence, we have

$$\left(\frac{dN}{dt}\right) = \frac{\alpha}{hf} I(t) + \frac{\gamma}{2hf} I^2(t) \tag{1}$$

where α and γ are constants, h is the Planck's constant, and f is the frequency of light. It should be noted that the statistical nature of the incident optical pulse could effect the transition probability in a TPA process [16] and thus highlighting the importance of the statistical characterization of TPA detectors. Due to a direct relation between power and intensity (see Appendix A), the mean of the generated photoelectrons is proportional to the received power and the square of this parameter. By defining two new parameters k_1 and k_2 and by assuming that the complex envelope of the received signal is $A(t)$ and the baseband equivalent of the received noise is $Q(t)$, we can express the average number of generated photoelectrons from a

Manuscript received July 31, 2005; revised November 22, 2005. This work was supported in part by the Hi-Technology Industries Center of Iran.

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Digital Object Identifier 10.1109/JLT.2005.863229

TPA detector over T (in seconds), which is the time duration of one bit of information, as (see Appendix A)

$$Y_{\mathrm{TPA}} = \int_0^T \left(k_1 \left|(A(t)+Q(t))\right|^2 + k_2 \left|(A(t)+Q(t))\right|^4 \right) dt \tag{2}$$

where the values of k_1 and k_2 can be expressed in terms of α and γ as described in Appendix A. In this paper, we examine the properties of a pure TPA receiver, which implies that $k_1 = 0$. A further analysis is required for assuming both the SPA and TPA detection simultaneously; however, as the number of transmitted photons increases (i.e., the intensity of the optical signal increases), the TPA process becomes more dominant (before reaching the saturation at high power levels) and the TPA term dominates the analysis [7].

Using this assumption, the TPA receiver can be shown as in Fig. 1. As specified in Appendix A, for convenience, $A(t)$ and $Q(t)$ are normalized so that $|A(t)|^2$ represents the photon intensity and $\overline{m}$ defined as [15]

$$\overline{m} = \frac{1}{hf}\int_0^T P(t)dt = \frac{1}{2}\int_0^T |A(t)|^2\, dt \tag{3}$$

is the average number of received photons in the receiver. In this equation, $P(t)$ is the power of the received optical signal. An optical bandpass filter that is placed before the detector would only allow the signal components in the passband Fourier domain of the filter to pass through. The bandwidth of the filter is assumed to be equal to the bandwidth of the signal, i.e., B_0. Without loss of generality and for the sake of mathematical simplicity, we assume that $M = B_0 T = 2L + 1$ is an odd integer, which is the total number of modes in the received signal [15]. An optical baseband representation of $A(t)$ can be expressed by its Fourier series expansion as [15]

$$A(t) = \sum_{n=-L}^{L} a_n \mathrm{e}^{jn\Omega t} \tag{4}$$

where a_n is the nth Fourier harmonic coefficient of $A(t)$, and $\Omega = (2\pi/T)$. Similarly, the equivalent filtered baseband representation of noise $Q(t)$ at the input of the receiver should be compatible with the normalization of $A(t)$ and can be expressed by its Fourier series expansion as [15]

$$Q(t) = \sum_{n=-L}^{L} q_n \mathrm{e}^{jn\Omega t} \tag{5}$$

where q_ns are independent identically distributed complex-valued Gaussian random variables and can be expressed as [15]

$$q_n = \frac{1}{T}\int_0^T Q(t)\mathrm{e}^{-jn\Omega t} dt. \tag{6}$$

The averaged power density of $Q(t)$ is assumed to be N_T, and it is expressed in photons per second due to the normalization of $Q(t)$. Its value for a single amplifier configuration is equal to $N_T = n_{\mathrm{sp}}(G-1)$, where G is the gain of the optical preamplifier, and n_{sp} (which is greater than or equal to 1) is the spontaneous emission parameter. It should be noted that the average number of received photons, which is expressed in (3), is equal to the average number of transmitted photons multiplied by the gain of the amplifier, i.e., G. By substituting (4) and (5) in (2) and by assuming that $k_1 = 0$, the decision variable Y_{TPA} is simplified to

$$\begin{aligned} Y_{\mathrm{TPA}} &= k_2 \int_0^T |A(t)+Q(t)|^4\, dt \\ &= k_2 \int_0^T \left| \sum_{n=-L}^{L} (a_n + q_n)\mathrm{e}^{jn\Omega t} \right|^4 dt. \end{aligned} \tag{7}$$

The above expression can be rewritten as

$$\begin{aligned} Y_{\mathrm{TPA}} = k_2 \int_0^T \sum_{m=-L}^{L}\sum_{n=-L}^{L}\sum_{p=-L}^{L}\sum_{q=-L}^{L} &((a_m+q_m)(a_n+q_n) \\ \times (a_p+q_p)^*(a_q+q_q)^* &\exp\left(j\Omega t(m+n-p-q)\right))\, dt \end{aligned} \tag{8}$$

where x^* is the complex conjugate of x. The above expression simplifies to the following equation due to the integration over period T (see Appendix B):

$$\begin{aligned} Y_{\mathrm{TPA}} = k_2 T \sum_{\substack{m,n,p,q=-L \\ m+n-p-q=0}}^{L} &((a_m+q_m)(a_n+q_n) \\ &\times (a_p+q_p)^*(a_q+q_q)^*). \end{aligned} \tag{9}$$

One can further rewrite the above equation as

$$\begin{aligned} Y_{\mathrm{TPA}} = k_2 T \sum_{\substack{m,n,p,q=-L \\ m+n-p-q=0}}^{L} &(a_m + q_{cm} + jq_{sm})(a_n + q_{cn} + jq_{sn}) \\ &\times (a_p + q_{cp} - jq_{\mathrm{sp}})(a_q + q_{cq} - jq_{sq}) \end{aligned} \tag{10}$$

where q_{cn} and q_{sn} are the real and imaginary parts of each noise coefficient, i.e., $q_n = q_{cn} + jq_{sn}$, and they are independent with identical variances equal to N_T/T [15]. As can be observed from (10), in obtaining the moment generating function of the received signal Y_{TPA}, one needs to obtain the moment generating function of each term in (10), which will prove to be an intractable task. For the analysis of the performance of such systems, we obtain the mean and the variance of the received signal Y_{TPA}, and the BER is obtained using a Gaussian approximation. At first, we will obtain these parameters for the simple case where the number of received modes is equal to 1, i.e., the bandwidth of the electrical system is equal to the bandwidth of the optical signal. Henceforth, we obtain the BER of the system with number of modes greater than 1. In this case, as the number of modes increases, the repetition rate of the optical pulse decreases. This leads to less bit rates in the transmission, and the "integrate and dump" part of the receiver

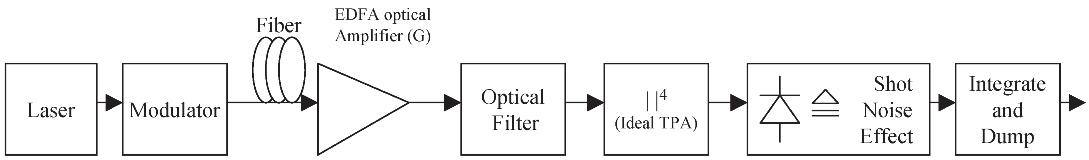

Fig. 1. Schematic of a lightwave system with an optical amplifier using a TPA receiver.

(see Fig. 1) can operate slower so less speed in the electronics of the receiver is needed for processing the incoming signal.

1) Case 1—$M = 2L + 1 = 1$ $(L = 0)$: For this case, we obtain the mean and the variance of the received signal $Y_{\rm TPA}$ that is equal to the following expression (see Appendix C):

$$Y_{\rm TPA} = k_2 T \left((a_0 + q_{c0})^4 + q_{s0}^4 + 2(a_0 + q_{c0})^2 q_{s0}^2\right). \quad (11)$$

The above random variable $Y_{\rm TPA}$ is a function of two Gaussian random variables namely, $\sqrt[4]{k_2 T}(a_0 + q_{c0})$ and $\sqrt[4]{k_2 T} q_{s0}$. The mean of the first Gaussian random variable $\sqrt[4]{k_2 T}(a_0 + q_{c0})$ is equal to $\sqrt[4]{k_2 T}\sqrt{2\overline{m}/T}$, and the other Gaussian variable $\sqrt[4]{k_2 T} q_{s0}$ has a mean equal to 0. However, the variances for both Gaussian terms are equal and can be expressed as $\sqrt{k_2 T}(N_T/T)$. To simplify the mathematical notation, we define a new parameter k_3 as $(4k_2/T)$, so the mean of $a_0 + q_{c0}$ can be expressed as $\sqrt[4]{k_3}\sqrt{\overline{m}}$ and the variance for both Gaussian terms can be expressed as $\sqrt{k_3}(N_T/2)$.

In obtaining the mean and the variance of the received signal, we first obtain the expected value of the received signal and its absolute square value. By using the expressions in Appendices D and E, we can write

$$E(Y_{\rm TPA}) = k_3 \left(\overline{m}^2 + 4\overline{m} N_T + 2N_T^2\right) \quad (12)$$

$$E\left(Y_{\rm TPA}^2\right) = k_3^2 \left(\overline{m}^4 + 16\overline{m}^3 N_T + 72\overline{m}^2 N_T^2 + 96\overline{m} N_T^3 + 24 N_T^4\right). \quad (13)$$

2) Case 2—$M = 2L + 1 > 1$: Similarly, in this case, we first obtain the mean and the variance of each Fourier coefficient of the received signal, assuming that the corresponding normalized energy for each Fourier coefficient of the signal is $\overline{m}_n$, i.e., $(T/2)a_n^2 = \overline{m}_n$ and $\sum_{n=-L}^{L} \overline{m}_n = \overline{m}$.

As can be seen from (10), obtaining the mean and the variance for an arbitrary pulse shape could prove to be difficult. In obtaining these parameters, note that they are related to the values of the Fourier coefficient of the received signal, i.e., its temporal shape.

In the following discussion, we will choose two methods to obtain the performance of the system. In the first method, it is assumed that the entire signal is confined to a short time period T_c, where $B_o T_c = 1$ and that, for the rest of the time period, it contains noise only. In the second method, a special pulse shape is considered to be received at the detector, e.g., a sinc-pulse shape. The Fourier coefficients of a sinc-pulse shape are equal, which simplifies obtaining the mean and the variance of the received signal significantly.

a) Simplified approach: In this approach, we will divide the integration interval into $2L + 1$ subintervals such that only one of the subintervals contains the main energy of the pulse plus noise, while others are comprised of the noise only. Using these assumptions, we can express the received signal as the sum of $2L + 1$ modes, i.e.,

$$Y_{\rm TPA} = k_2 T_c \Bigg[\left((a_0 + q_{c0})^4 + q_{s0}^4 + 2(a_0 + q_{c0})^2 q_{s0}^2\right) + \sum_{\substack{n=-L \\ n \neq 0}}^{L} \left(q_{cn}^4 + q_{sn}^4 + 2q_{cn}^2 q_{sn}^2\right) \Bigg] \quad (14)$$

where the Fourier coefficients $a_0 = \sqrt{2\overline{m}/T_c}$ and $a_n = 0$ for $n \neq 0$. The mean and the variance of the received signal can be written as

$$E(Y_{\rm TPA}) = k_3 \left(\overline{m}^2 + 4\overline{m} N_T + 2(2L+1) N_T^2\right) \quad (15)$$

$$E\left(Y_{\rm TPA}^2\right) = k_3^2 \left(\overline{m}^4 + 16\overline{m}^3 N_T + 72\overline{m}^2 N_T^2 + 96\overline{m} N_T^3 + 24(2L+1) N_T^4\right). \quad (16)$$

b) Direct approach (Sinc-pulse shape): In this section, we will obtain the statistical characterization of a system using a sinc-pulse shape. In obtaining the BER of the system, we need to obtain the mean and the variance of the received signal. These values are derived by calculating the mean and the variance of each Fourier coefficient of the received signal in (10). Using these values, we can obtain the BER of the system through a Gaussian approximation.

By using the relation between $A(t)$ and $\overline{m}$ as in (3), one can write the amplitude $A(t)$ of a sinc pulse of a laser as

$$A(t) = \sqrt{\frac{2\overline{m}}{T_c}} \operatorname{sinc}\left(\frac{\pi t}{T_c}\right) \quad (17)$$

where $\operatorname{sinc}(u) = \sin(u)/u$. The Fourier coefficients of this signal that is periodic with period T can be written as

$$a_n = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} A(t) \mathrm{e}^{-jn\Omega t} dt = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \sqrt{\frac{2\overline{m}}{T_c}} \frac{\sin\left(\frac{\pi t}{T_c}\right)}{\frac{\pi t}{T_c}} \mathrm{e}^{-jn\Omega t} dt. \quad (18)$$

Since $T = (2L+1)T_c$ and $(2L+1) \gg 1$, one can find the Fourier coefficients to be equal to

$$a_n = \begin{cases} \sqrt{\frac{2\overline{m}}{T_c}} \frac{T_c}{T} = \frac{1}{2L+1} \sqrt{\frac{2\overline{m}}{T_c}}, & n = 1, 2, \ldots, 2L+1 = M \\ 0, & \text{otherwise.} \end{cases} \quad (19)$$

As it can be seen, the amplitudes for all Fourier coefficients are equal and are equal to $(1/(2L+1))\sqrt{2\overline{m}/T_c}$.

In this approach, we will use the expansion of the received signal on T as in (10) for analyzing the performance of the system. In this equation, there are 15 types of products specified in [17] of which the six types mentioned include $m \neq n \neq p \neq q$, $m = n \neq p \neq q$, $m \neq n \neq p = q$, $m = p \neq n = q$, $m = q \neq n = p$, and $m = n = p = q$ types that can meet the condition $m + n = p + q$; see Appendix B.

For the computation of the mean and the variance of the received signal, one should carefully calculate the fraction of the energy in each Fourier coefficient and then obtain the mean and the variance of the received signal. However, for a sinc pulse, the amounts of energy specified to each Fourier series coefficient $\overline{m}_n$ are equal and can be expressed as $\overline{m}_n = (\overline{m}/(2L+1))$, thus significantly simplifying the problem of finding the mean and the variance of the received signal.

For each type of indices, due to the independence between the different modes of the noise (different indices), one can obtain the mean of the ith received signal $Y_{\mathrm{TPA},i}$ and the mean of its square as in the following (see Appendix E).

1) $m \neq n \neq p \neq q$

$$\begin{aligned} E[Y_{\mathrm{TPA},1}] &= E\left(k_2 T\left((a_m+q_m)(a_n+q_n)(a_p+q_p)^*(a_q+q_q)^*\right)\right) \\ &= k_2 T E\left[(a_m+q_m)\right] E\left[(a_n+q_n)\right] \\ &\quad \times E\left[(a_p+q_p)^*\right] E\left[(a_q+q_q)^*\right] \\ &= k_3 \overline{m}_n^2 \end{aligned} \tag{20}$$

$$\begin{aligned} E\left[Y_{\mathrm{TPA},1}^2\right] &= E\Big(k_2^2 T^2 \Big((a_m+q_m)^2(a_n+q_n)^2 \\ &\qquad \times \left((a_p+q_p)^*\right)^2 \left((a_q+q_q)^*\right)^2\Big)\Big) \\ &= k_2^2 T^2 E\left[(a_m+q_m)^2\right] E\left[(a_n+q_n)^2\right] \\ &\quad \times E\left[\left((a_p+q_p)^*\right)^2\right] E\left[\left((a_q+q_q)^*\right)^2\right] \\ &= k_3^2 \overline{m}_n^4 \end{aligned} \tag{21}$$

2) $m = n \neq p \neq q$

$$\begin{aligned} E[Y_{\mathrm{TPA},2}] &= E\left(k_2 T\left((a_m+q_m)(a_n+q_n)(a_p+q_p)^*(a_q+q_q)^*\right)\right) \\ &= k_2 T E\left[(a_m+q_m)^2\right] E\left[(a_p+q_p)^*\right] E\left[(a_q+q_q)^*\right] \\ &= k_3 \overline{m}_n^2 \end{aligned} \tag{22}$$

$$\begin{aligned} E\left[Y_{\mathrm{TPA},2}^2\right] &= E\Big(k_2^2 T^2 \Big((a_m+q_m)^2(a_n+q_n)^2 \\ &\qquad \times \left((a_p+q_p)^*\right)^2 \left((a_q+q_q)^*\right)^2\Big)\Big) \\ &= k_2^2 T^2 E\left[(a_m+q_m)^4\right] \\ &\quad \times E\left[\left((a_p+q_p)^*\right)^2\right] E\left[\left((a_q+q_q)^*\right)^2\right] \\ &= k_3^2 \overline{m}_n^4 \end{aligned} \tag{23}$$

3) $m \neq n \neq p = q$

$$\begin{aligned} E[Y_{\mathrm{TPA},3}] &= E\left(k_2 T\left((a_m+q_m)(a_n+q_n)(a_p+q_p)^*(a_q+q_q)^*\right)\right) \\ &= k_2 T E\left[(a_m+q_m)\right] E\left[(a_m+q_n)\right] E\left[\left((a_p+q_p)^*\right)^2\right] \\ &= k_3 \overline{m}_n^2 \end{aligned} \tag{24}$$

$$\begin{aligned} E\left[Y_{\mathrm{TPA},3}^2\right] &= E\Big(k_2^2 T^2 \Big((a_m+q_m)^2(a_n+q_n)^2 \\ &\qquad \times \left((a_p+q_p)^*\right)^2 \left((a_q+q_q)^*\right)^2\Big)\Big) \\ &= k_2^2 T^2 E\left[(a_m+q_m)^2\right] \\ &\quad \times E\left[\left((a_n+q_n)\right)^2\right] E\left[\left((a_p+q_p)^*\right)^4\right] \\ &= k_3^2 \overline{m}_n^4 \end{aligned} \tag{25}$$

4) $m = p \neq n = q$

$$\begin{aligned} E[Y_{\mathrm{TPA},4}] &= E\left(k_2 T\left((a_m+q_m)(a_n+q_n)(a_p+q_p)^*(a_q+q_q)^*\right)\right) \\ &= k_2 T E\left[|a_m+q_m|^2\right] E\left[|a_n+q_n|^2\right] \\ &= k_3\left(\overline{m}_n^2 + 2\overline{m}_n N_T + N_T^2\right) \end{aligned} \tag{26}$$

$$\begin{aligned} E\left[Y_{\mathrm{TPA},4}^2\right] &= E\Big(k_2^2 T^2 \Big((a_m+q_m)^2(a_n+q_n)^2 \\ &\qquad \times \left((a_p+q_p)^*\right)^2 \left((a_q+q_q)^*\right)^2\Big)\Big) \\ &= k_2^2 T^2 E\left[|a_m+q_m|^4\right] E\left[|a_n+q_n|^4\right] \\ &= k_3^2\left(\overline{m}_n^2 + 4\overline{m}_n N_T + 2N_T^2\right)^2 \end{aligned} \tag{27}$$

5) $m = q \neq n = p$

$$\begin{aligned} E[Y_{\mathrm{TPA},5}] &= E\left(k_2 T\left((a_m+q_m)(a_n+q_n)(a_p+q_p)^*(a_q+q_q)^*\right)\right) \\ &= k_2 T E\left[|a_m+q_m|^2\right] E\left[|a_n+q_n|^2\right] \\ &= k_3\left(\overline{m}_n^2 + 2\overline{m}_n N_T + N_T^2\right) \end{aligned} \tag{28}$$

$$\begin{aligned} E\left[Y_{\mathrm{TPA},5}^2\right] &= E\Big(k_2^2 T^2 \Big((a_m+q_m)^2(a_n+q_n)^2 \\ &\qquad \times \left((a_p+q_p)^*\right)^2 \left((a_q+q_q)^*\right)^2\Big)\Big) \\ &= k_2^2 T^2 E\left[|a_m+q_m|^4\right] E\left[|a_n+q_n|^4\right] \\ &= k_3^2\left(\overline{m}_n^2 + 4\overline{m}_n N_T + 2N_T^2\right)^2 \end{aligned} \tag{29}$$

6) $m = n = p = q$

$$\begin{aligned} E[Y_{\mathrm{TPA},6}] &= E\left(k_2 T\left((a_m+q_m)(a_n+q_n)(a_p+q_p)^*(a_q+q_q)^*\right)\right) \\ &= k_2 T E\left[|a_m+q_m|^4\right] \\ &= k_3\left(\overline{m}_n^2 + 4\overline{m}_n N_T + 2N_T^2\right) \end{aligned} \quad (30)$$

$$\begin{aligned} E\left[Y_{\mathrm{TPA},6}^2\right] &= E\Big(k_2^2 T^2\Big((a_m+q_m)^2(a_n+q_n)^2 \\ &\qquad \times\left((a_p+q_p)^*\right)^2\left((a_q+q_q)^*\right)^2\Big)\Big) \\ &= k_2^2 T^2 E\left[|a_m+q_m|^8\right] \\ &= k_3^2\left[\overline{m}_n^4 + 16\overline{m}_n^3 N_T + 72\overline{m}_n^2 N_T^2 + 96\overline{m}_n N_T^3 + 24N_T^2\right]. \end{aligned} \quad (31)$$

To address the problem of finding the mean and the variance of the received signal, after computing the mean and variance of each type, one should count the number of each desired type that meets the condition $m + n = p + q$.

In counting the number of each specified type with the above condition, we assume that $m + n = p + q = u$, where $-2L \le u \le 2L$ due to the fact that $-L \le m, n, p, q \le L$. The total number of possible values for m, n is $2L - u + 1$ for $0 \le u \le 2L$ and $2L + u + 1$ for $-2L \le u \le -1$. The same number of values is possible for p and q, so the total possible value for all indices Num is equal to

$$\begin{aligned} \mathrm{Num} &= 2\sum_{u=1}^{2L}(2L-u+1)^2 + (2L+1)^2 \\ &= 2\sum_{u=1}^{2L} w^2 + (2L+1)^2 \\ &= \frac{2L(2L+1)(4L+1)}{3} + (2L+1)^2 \\ &= \frac{(2L+1)\left(2(2L+1)^2+1\right)}{3} \end{aligned} \quad (32)$$

where w is defined as $2L - u + 1$ in the first equality. We define $\mathrm{Num}(i)$ as the number of possible values for ith type of indices, where the numbering of the types are specified as previously. We will count the number of modes in $0 \le u \le 2L$. The same number of values as $1 \le u \le 2L$ can be obtained for $-2L \le u \le -1$. The even and odd values of u will be considered separately. We should note that $m + n = u$ and $p + q = u$ and carefully count the values that satisfy both equations simultaneously.

As discussed above, the total number of possible values for m and n is $2L - u + 1$ for $0 \le u \le 2L$, which satisfy the equation $m + n = u$. For the even values of u, there is one case, i.e., $m = n = u/2$, such that $m = n$; for the rest of possible cases ($2L - u$ values), we have $m \ne n$. This is also the case for p and q. Therefore, there is only one term that corresponds to type 6 ($m = n = p = q = u/2$), $2L - u$ terms that correspond to type 2 ($m = n \ne p \ne q$), and $2L - u$ terms that correspond to type 3 ($m \ne n \ne p = q$). The rest of the terms correspond to types 1, 4, and 5. For each of the $2L - u$ values that belong to the $m \ne n$ case, there is only one corresponding term in p and q that results in $m = p$ and $n = q$; similarly, there is only one corresponding term that results in $m = q$ and $n = p$. Due to the fact that the number of values such that $m \ne n$ is equal to $2L - u$ and the number of values that $m = p$ and $n = q$, with respect to the fact that $m + n = p + q = u$, is equal to one, the total number of terms for the fourth type ($m = p \ne n = q$) is equal to $2L - u$. The same reasoning is valid for the fifth type ($m = q \ne n = p$). Therefore, the total number of terms for the first type ($m \ne n \ne p \ne q$) is equal to $(2L - u)(2L - u - 2)$. It must be mentioned here that the total number of values is equal to $(2L - u)(2L - u - 2) + 4(2L - u) + 1 = (2L - u + 1)^2$, which was obtained before.

For the odd values of u, there is not a value for m and n where $m = n$. Therefore, for odd values of u, types 2, 3, and 6 do not exist. The total number of cases that satisfy equation $m + n = u$ is equal to $2L - u + 1$. The total number of values that satisfy equations $m = p$, $n = q$, and $p + q = u$ is equal to one, so the total number of terms for type 4 is equal to $2L - u + 1$. The same reasoning is true for the fifth type ($m = q \ne n = p$). Therefore, the total number of terms for the first type ($m \ne n \ne p \ne q$) is equal to $(2L - u + 1)(2L - u - 1)$. This is in accordance with the total number of values obtained before so that $(2L - u + 1)(2L - u - 1) + 2(2L - u + 1) = (2L - u + 1)^2$. Assuming that the number of modes is an odd number, one can find the number of modes in each type as

$$\begin{aligned} \mathrm{Num}(1) &= 2\left(\sum_{\substack{u=2 \\ u=\text{even integers}}}^{2L}(2L-u)(2L-u-2) + \sum_{\substack{u=1 \\ u=\text{odd integers}}}^{2L-1}(2L-u+1)(2L-u-1)\right) \\ &\quad + (2L)(2L-2) \\ &= 2\left(\sum_{\substack{w=0 \\ w=\text{even integers}}}^{2L-2} w(w-2) + \sum_{\substack{w=1 \\ w=\text{odd integers}}}^{2L-1}(w+1)(w-1)\right) \\ &\quad + (2L)(2L-2) \\ &= 2\left(\frac{2}{3}L(L-1)(2L-4) + \frac{2}{3}L(L+1)(2L-2)\right) \\ &\quad + (2L)(2L-2) \\ &= \frac{(2L)(2L-2)(4L+1)}{3} \end{aligned} \quad (33)$$

$$\begin{aligned} \mathrm{Num}(2) &= 2\left(\sum_{\substack{u=2 \\ u=\text{even integers}}}^{2L}(2L-u)\right) + (2L) \\ &= 2\left(\sum_{\substack{w=0 \\ w=\text{even integers}}}^{2L-2} w\right) + (2L) = 2L^2 \end{aligned} \quad (34)$$

$$\begin{aligned}
&\text{Num}(3)\\
&= 2\left(\sum_{\substack{u=2\\u=\text{even integers}}}^{2L}(2L-u)\right)+(2L)\\
&= 2\left(\sum_{\substack{w=0\\w=\text{even integers}}}^{2L-2} w\right)+(2L)=2L^2 \quad (35)\\
&\text{Num}(4)\\
&= 2\left(\sum_{\substack{u=2\\u=\text{even integers}}}^{2L}(2L-u)+\sum_{\substack{u=1\\u=\text{odd integers}}}^{2L-1}(2L-u+1)\right)+(2L)\\
&= 2\left(\sum_{\substack{w=0\\w=\text{even integers}}}^{2L-2}(w)+\sum_{\substack{w=1\\w=\text{odd integers}}}^{2L-1}(w+1)\right)+(2L)\\
&= (2L+1)^2-(2L+1) \quad (36)\\
&\text{Num}(5)\\
&= 2\left(\sum_{\substack{u=2\\u=\text{even integers}}}^{2L}(2L-u)+\sum_{\substack{u=1\\u=\text{odd integers}}}^{2L-1}(2L-u+1)\right)+(2L)\\
&= 2\left(\sum_{\substack{w=0\\w=\text{even integers}}}^{2L-2}(w)+\sum_{\substack{w=1\\w=\text{odd integers}}}^{2L-1}(w+1)\right)+(2L)\\
&= (2L+1)^2-(2L+1) \quad (37)\\
&\text{Num}(6)\\
&= 2\left(\sum_{\substack{u=1\\u=\text{even integers}}}^{2L} 1\right)+1=2L+1 \quad (38)
\end{aligned}$$

where $2L-u$ was substituted by w in the above equations whenever it was necessary. As can be seen in this approach, the sum of different terms is equal to $[2L+1][2(2L+1)^2+1]/3$, which is in line with the result obtained in (32). By combining (20)–(31) and (33)–(38) and substituting in (10), we can find the mean and the variance of the received signal as

$$E[Y_{\text{TPA}}]=\sum_{i=1}^{6}E[Y_{\text{TPA},i}]\text{Num}(i) \quad (39)$$

$$\text{Var}[Y_{\text{TPA}}]=\sum_{i=1}^{6}\left(E\left[Y_{\text{TPA},i}^2\right]-E^2\left[Y_{\text{TPA},i}\right]\right)\text{Num}(i). \quad (40)$$

In considering the effect of a shot noise, the mean of the received signal must be added to the variance to obtain the variance of the received signal (see Appendix F).

III. Numerical Results

In our numerical analysis, we use the parameters shown in Table I for a typical optical transmission system. After obtaining the mean and the variance of the received signal, one obtains the BER of the system using these values. If a threshold value Th is considered at the detector in which the received signal greater than Th is considered as 1 and the received signal less than Th is considered as 0, we can write the BER of the transmission system as [15]

TABLE I
Typical Values of Parameters Used in This Paper

n_{sp}	*Spontaneous Emission Factor*	1.1
T_r	*Receiver Temperature*	300° *K*
R_L	*Load Resistance*	1000 Ω
T_c	*width of ultrashort pulse*	400*fsec*
η	*Quantum Efficiency*	0.8
k_3	*Detection Efficiency in TPA receiver*	5.0 *e* -11 for T_c

$$\text{BER}=\frac{1}{2}Q\left(\frac{E(Y_1)-\text{Th}}{\text{Var}(Y_1)}\right)+\frac{1}{2}Q\left(\frac{\text{Th}-E(Y_0)}{\text{Var}(Y_0)}\right) \quad (41)$$

where Y_1 and Y_0 are received signals for transmitting 1 and 0 at the transmitter, respectively. To obtaining these values, the dark current is neglected in the received signal. The effect of the thermal noise is considered as an increase in the variance of the received signal specified in (40) by an amount of $\sigma_{\text{th}}^2=(2.k_{\text{B}}.\text{T}_{\text{r}}.T_c/R_{\text{L}}.\text{e}^2)$. In this relation, σ_{th}^2 is the power of thermal noise, k_{B} is Boltzmann's constant, T_{r} is the equivalent temperature of the receiver, e is the charge of electron, and R_{L} is the resistance of the load, which is seen from the receiver input.

The BERs for different conditions are sketched in Figs. 2–10. For a fair comparison among different conditions, the transmitted number of photons, i.e., the horizontal axis of the figures, is kept the same for all conditions. For the cases where the number of modes is greater than 1, the simplified approach is used to compute the BER of the system, unless it is mentioned that the direct approach is used.

The BER of a system using TPA and SPA detectors where the receiver is assumed to have a simple integrate and dump structure with a speed equivalent to the bit rate of the corresponding data source [15] is shown in Fig. 2. The values of the average number of photons have been increased up to a point that the TPA occurs for a better comparison. As can be seen in Fig. 2, in the system that does not use an amplifier and hence does not posses any noise mixed with the signal, the TPA-detector performance is much worse than that of the SPA detector. This is due to the less sensitivity of a TPA detector. To increase the performance of a TPA detector, we will use an optical amplifier. In our performance analysis using an optical amplifier, we assume that we can neglect the effect of the amplifier saturation.

The BERs of the single-mode systems where the processing speed is equal to the bandwidth of the optical signal, which is equal to 2.5 Tb/s, are shown in Figs. 3–5. The BER of SPA and TPA detectors with the number of modes equal to 1, using different values for the gain of the optical amplifier, are plotted in Figs. 3 and 4, respectively. As can be observed from Fig. 3, when SPA detector is used, the performance of the system using

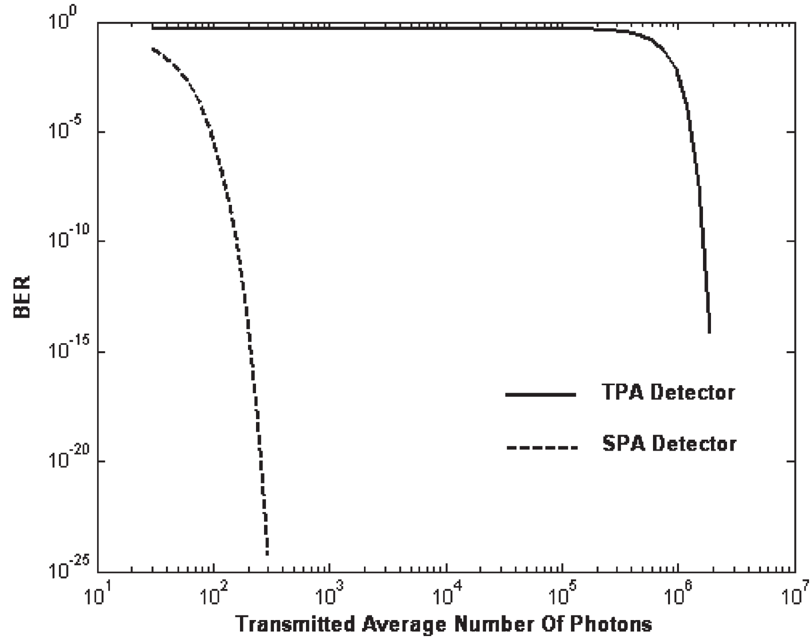

Fig. 2. BER versus the mean number of photons for TPA and SPA detectors without optical amplification and the number of modes = 1, i.e., processing speed = 2.5 Tb/s.

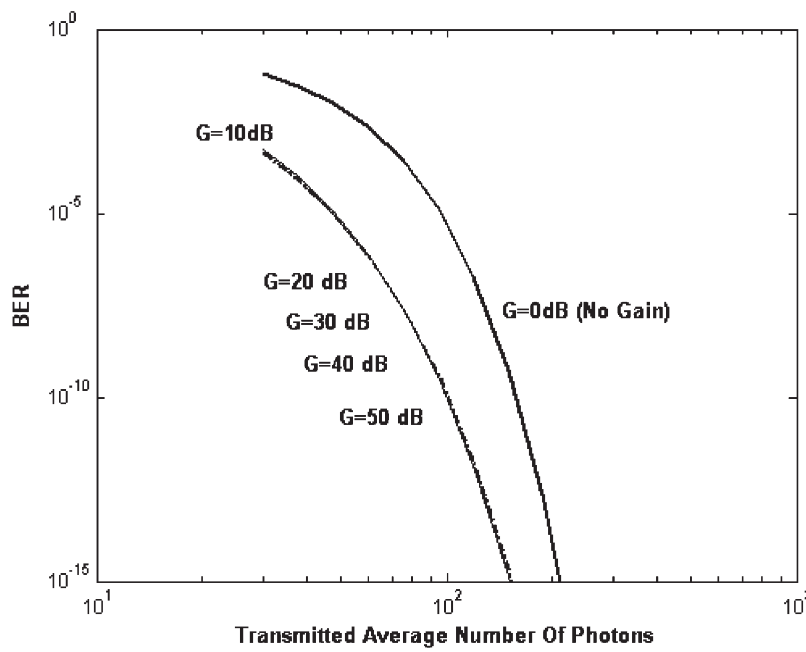

Fig. 3. BER versus the mean number of photons for an SPA detector with an optical amplifier and the number of modes = 1, i.e., processing speed = 2.5 Tb/s.

an optical amplifier is better than that of the system that does not use optical amplification. However, increasing the value of the amplification gain beyond a limit does not change the performance of the system significantly, which is 10 dB for our example. A detailed discussion on this issue is provided in [9] and [10]. However, as it is seen in Fig. 4, the performance of the system using a TPA detector becomes better even for gain values of the optical amplifier beyond 20 dB. It is seen in this figure that the usage of an optical amplifier improves the BER of a TPA detector. This is not the case with an SPA detector in the regions of the transmitted photons that is considered in the figures.

The BERs of both SPA and TPA detectors using an optical amplifier with a gain equal to 50 dB (which is the best gain value in Fig. 4 for a TPA detector) and with the number of modes equal to 1 is sketched in Fig. 5. As can be observed from this figure, the BER of the system with a TPA detector, when using an optical amplifier, never reaches that of an SPA detector. However, by comparing Fig. 2 with Fig. 5, it is observed that the difference between the BERs of the above two detectors decreases significantly when an optical amplifier is used in the system.

Figs. 6–10 plot the BER versus the number of transmitted photons for the case where the number of modes is equal to

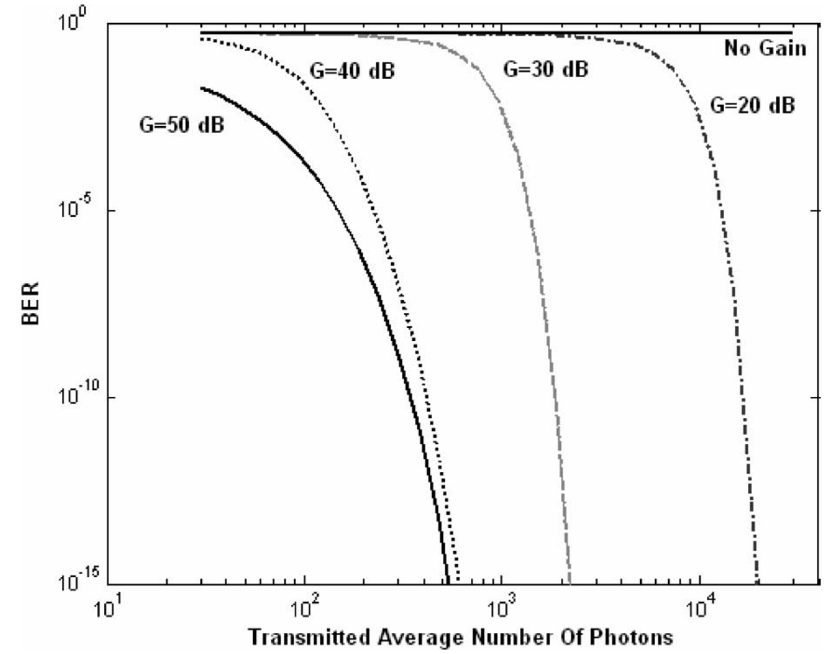

Fig. 4. BER versus the mean number of photons for a TPA detector with an optical amplifier and the number of modes = 1, i.e., processing speed = 2.5 Tb/s.

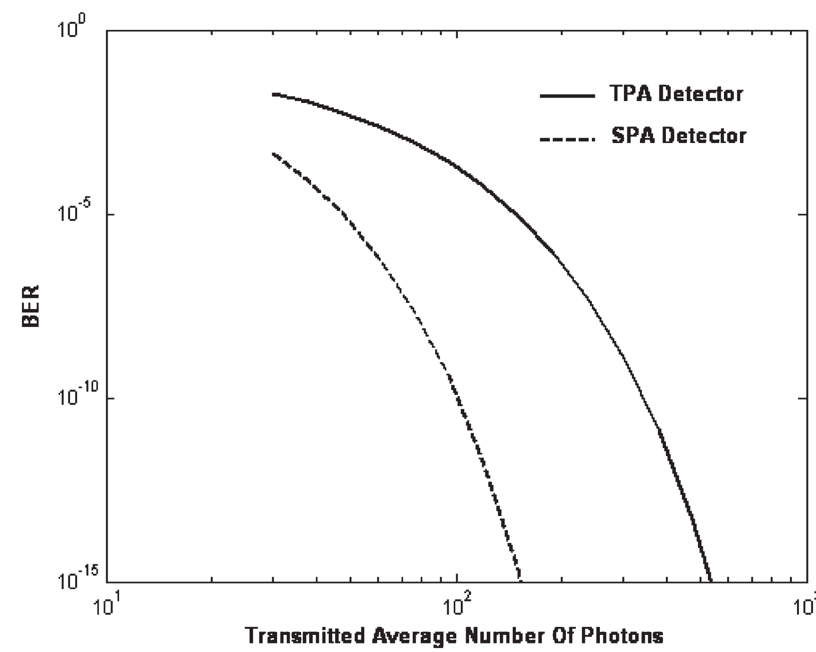

Fig. 5. BER versus the mean number of photons for TPA and SPA detectors with an optical amplifier with a gain of 50 dB and the number of modes = 1, i.e., processing speed = 2.5 Tb/s.

1000, i.e., the processing speed is 1000 times less than the bandwidth of the optical signal and is equal to 2.5 Gb/s. The BERs of the SPA and TPA detectors with the number of modes equal to 1000, using different values for the gain of the optical amplifier, is plotted in Figs. 6 and 7, respectively. It can be observed from Fig. 6, as in Fig. 3, that increasing the gain of the optical amplifier beyond a certain limit, e.g., 20 dB, does not necessarily improve the BER of the system using an SPA detector. However, the BER of the TPA detector improves significantly as the gain of the optical amplifier increases as can be seen from Fig. 7.

The BERs of both the SPA and the TPA detectors using an optical amplifier with a gain equal to 50 dB and with the number of modes equal to 1000 is sketched in Fig. 8. As can be observed from this figure, a TPA detector performs better than an SPA detector.

The BER of the system using a TPA detector with the number of modes equal to 1000 for a sinc-pulse shape is plotted in Fig. 9 for different gain values. As can be seen in this figure, the performance of the system improves as the gain increases.

The BER of the system using a TPA detector with the number of modes equal to 1000 for a sinc-pulse shape is plotted in

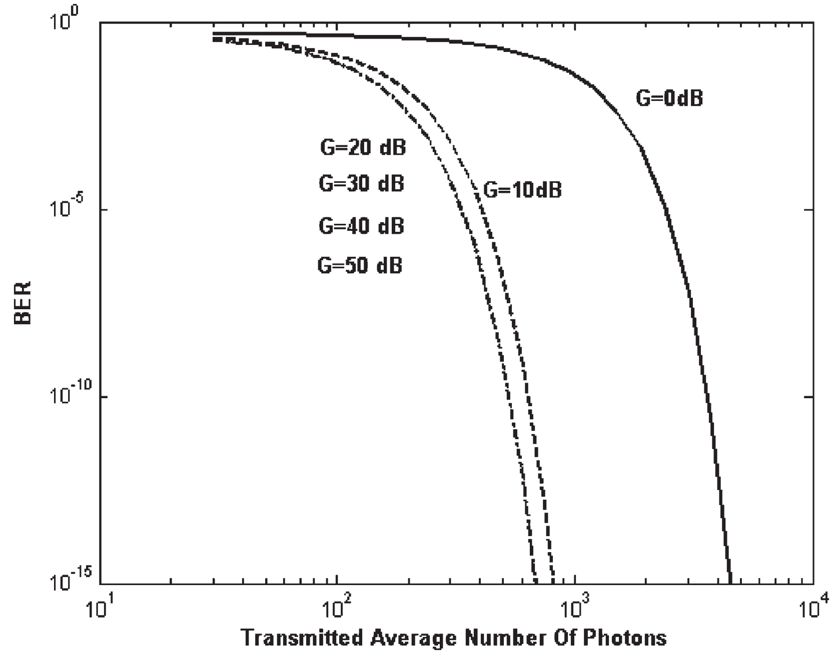

Fig. 6. BER versus the mean number of photons for an SPA detector with an optical amplifier and the number of modes $= 1000$, i.e., processing speed $= 2.5$ Gb/s.

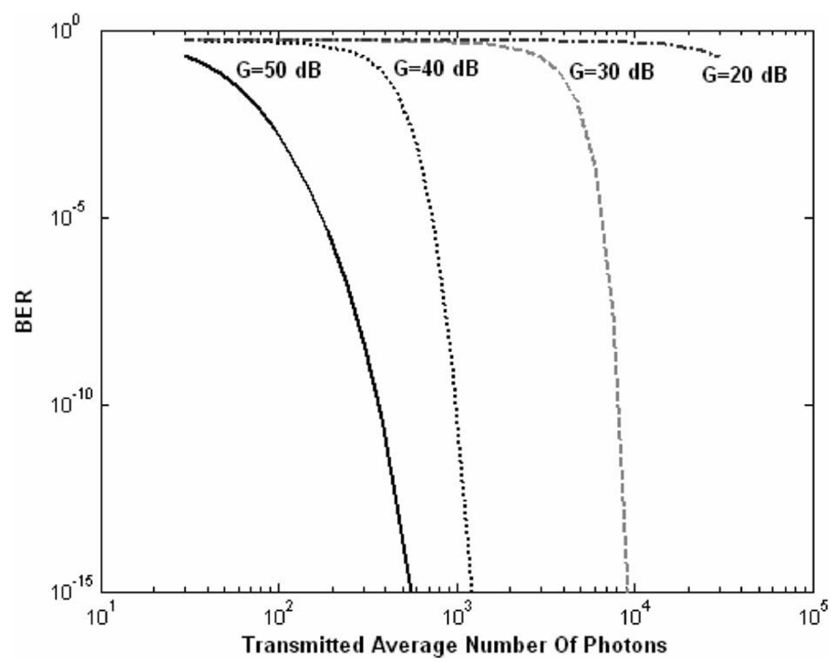

Fig. 7. BER versus the mean number of photons for a TPA detector with an optical amplifier and the number of modes $= 1000$, i.e., processing speed $= 2.5$ Gb/s.

Fig. 10 using the simplified and the direct approaches. As can be seen in this figure, the performances of both systems are close for gain values lower than 50 dB. However, this is not valid for high gain values, e.g., as high as 50 dB. This is due to the fact that the effect of the thermal noise becomes less important when the gain value increases; hence, we can compare the results of both approximations using (15), (16), (20), and (21). Equations (20) and (21) are considered for the computation of the mean and the variance of the direct approach. In this case, the number of modes in $m \neq n \neq p \neq q$ case is larger than the other terms. In fact, it can be shown that the number of this term is on the order of the number of modes raised by the power of 3, which dominates the number of modes. Assuming that the mean number of photons is large enough such that $\overline{m} \gg n_{\rm sp}$, then by comparing (15) and (20), we observe that the mean of the received signal in the simplified approach is $1 + (4N_T/\overline{m})$ times that of the direct approach. By comparing the variances of both terms [using, (15), (16), (20), (21), and (40)], we will find that the variance of the simplified approach is $8\overline{m}N_T$ times that of the direct approach, so we expect the performance of the direct approach to be better than that of the simplified approach. However, the effect of the increase in

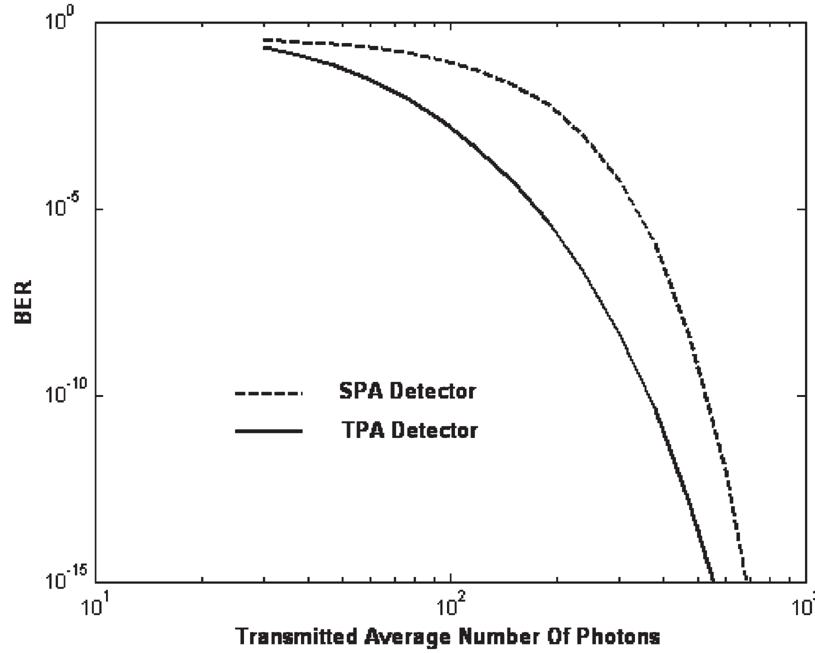

Fig. 8. BER versus the mean number of photons for TPA and SPA detectors with an optical amplifier with a gain of 50 dB and the number of modes = 1000, i.e., processing speed = 2.5 Gb/s.

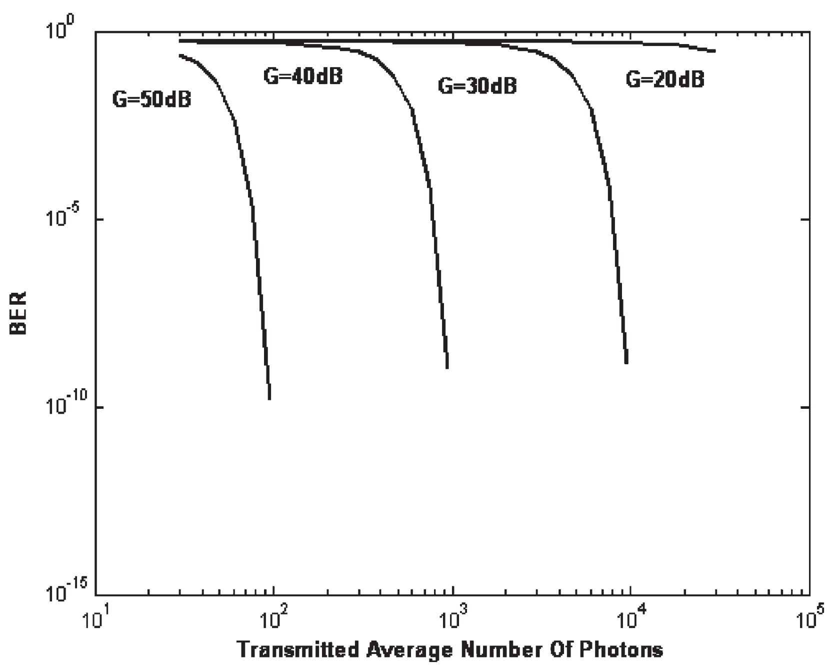

Fig. 9. BER versus the mean number of photons for a TPA detector with an optical amplifier for a sinc-pulse shape using the direct method and the number of modes = 1000, i.e., processing speed = 2.5 Gb/s.

the variance does not emerge, as the thermal noise is the dominant term in the variance of the received signal for gain values of less than 50 dB. For gain value of 50 dB, the other terms in the variance (except thermal noise) become comparable with that of a thermal noise in the simplified approach, and its performance degrades slightly than in the direct approach where the variance of the received signal (except thermal noise) is $1/8\overline{m}N_T$ times that of the simplified approach and is negligible in comparison with the variance of the thermal noise. The above discussion leads us to conclude that, in the high regime of amplification, the increase in variance is more effective than the increase in the mean, while the opposite is true when the value of the amplification gain is not so high (e.g., below 40 dB in this case) due to the dominance of the thermal noise in the variance of the received signal.

To justify the above discussion, the BER of the system using a TPA detector with the number of modes equal to 1000 for a sinc-pulse shape with a thermal noise equal to 0 for both the

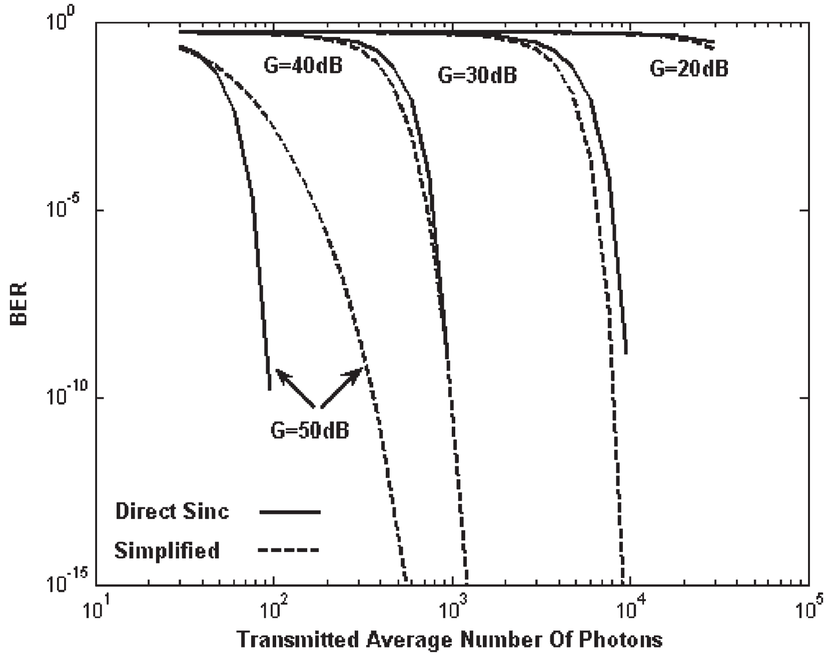

Fig. 10. BER versus the mean number of photons for a TPA detector with an optical amplifier for a sinc-pulse shape using the simplified and the direct methods and the number of modes $= 1000$, i.e., processing speed $= 2.5$ Gb/s.

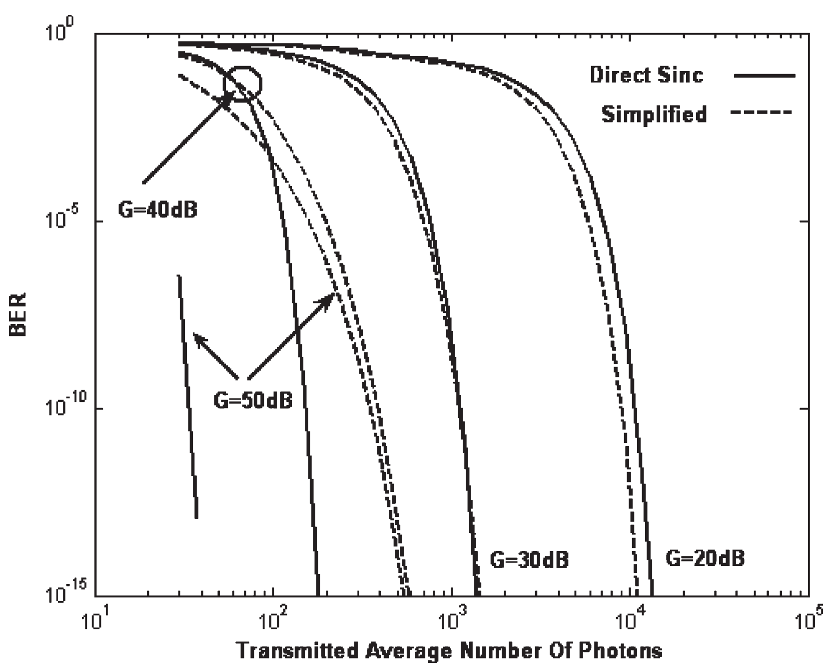

Fig. 11. BER versus the mean number of photons for a TPA detector with an optical amplifier for a sinc-pulse shape using the simplified and the direct methods and the number of modes $= 1000$, i.e., processing speed $= 2.5$ Gb/s with zero thermal noise.

simplified and the direct approaches is plotted in Fig. 11. As can be seen in this figure, the performance of the simplified approach is slightly better than that of the direct approach in low gain values due to the low values of the received signal. In the low values of the gain, the mean of the signal, i.e., $\sqrt[4]{k_3}\sqrt{m_n}$, is lower than 1, so raising its value to the power of 3 decreases its value in comparison with squaring. So, the variance of the received signal in the low gain regime for the simplified approach is lower than that of the direct approach. This leads to the improvement of the performance of the simplified approach in the low gain regime. However, the performance of the direct approach becomes much better than that of the simplified approach in the high gain regime, where the variance of the signal increases rapidly for the simplified approach as the gain value increases. As can be seen from Fig. 11, increasing the gain value beyond 30 dB improves the performance of the direct approach significantly versus the performance of the simplified approach.

IV. Conclusion

In this paper, we discussed and analyzed the performance of the system using a nonlinear detector with a TPA mechanism with and without an optical amplifier. The comparison between the TPA detector and the SPA detector, when there is no optical amplifier in the path, shows a significant difference between the performances of the two receivers. When optical amplifiers are introduced along the path between the transmitter and the receiver, the performance of the system using a TPA detector depends on the pulse shape of the received signal, i.e., its Fourier coefficients. It is shown that the BER of the system using a TPA detector can reach that of an SPA detector with low-speed processing times when low-speed detectors and optical amplification are used.

Appendix A
Obtaining Equation (2)

To obtain (2) between the number of released photoelectrons versus the power of the received optical field, we assume that the intensity is uniform over the area of the photodetector S so that we can substitute the equation $P(t) = \int_S I(t)dr = SI(t)$ into (1), which yields the following expression:

$$\left(\frac{dN}{dt}\right) = \frac{\alpha}{Shf}P(t) + \frac{\gamma}{2S^2hf}P^2(t). \tag{A1}$$

Let the received signal's field power be normalized such that $|f(t)|^2 = P(t)/hf$, where $f(t)$ is the normalized received field. Thus, (A1) can be expressed as

$$\left(\frac{dN}{dt}\right) = \frac{\alpha}{S}|f(t)|^2 + \frac{hf\gamma}{2S^2}|f(t)|^4. \tag{A2}$$

If the generation of the photoelectrons is assumed to be uniform in the volume of the detector, the average number of generated photoelectrons over T (the arbitrary time duration) Y_{TPA} can be written as

$$Y_{\text{TPA}} = V\int_0^T \left(\frac{\alpha}{S}|f(t)|^2 + \frac{hf\gamma}{2S^2}|f(t)|^4\right)dt \tag{A3}$$

where $V = SL$ is the volume in which the photoelectrons are generated, i.e., the volume of the detector, and L is the depth of the photodetector medium. The resulting generated photoelectrons Y_{TPA} can be expressed as the sum of two terms as

$$Y_{\text{TPA}} = \frac{V\alpha}{S}Y_{\text{TPA}}^{(1)} + \frac{Vhf\gamma}{2S^2}Y_{\text{TPA}}^{(2)} \tag{A4}$$

where $Y_{\text{TPA}}^{(1)}$ is the number of photoelectrons that have been generated due to one photon absorption mechanism, and $Y_{\text{TPA}}^{(2)}$ is the number of photoelectrons that have been generated due to the TPA process.

Assuming that the received signal's field $f(t)$ can be expressed as the multiplication of a slowly varying term $F(t)$ and a cosine term with the frequency of light as

$$f(t) = F(t)\cos(2\pi ft + \phi) \tag{A5}$$

where ϕ is the phase of the received signal. For the first term in (A2), since the intensity is the square of the field of the received signal, one can write

$$\begin{aligned} Y_{\text{TPA}}^{(1)} &= \int_0^T |f(t)|^2\,dt \\ &= \frac{1}{2}\int_0^T |F(t)|^2\,dt. \end{aligned} \tag{A6}$$

For the second term $Y_{\text{TPA}}^{(2)}$, since the number of generated photoelectrons is equal to the square value of the intensity and the intensity is the square of the absolute value of the received field signal, we have

$$\begin{aligned} Y_{\text{TPA}}^{(2)} &= \int_0^T |f(t)|^4\,dt \\ &= \frac{3}{8}\int_0^T |F(t)|^4\,dt. \end{aligned} \tag{A7}$$

So the total number of received signal can be expressed as

$$Y_{\text{TPA}} = \int_0^T \left(k_1|F(t)|^2 + k_2|F(t)|^4\right)dt \tag{A8}$$

where k_1 is equal to $(V\alpha/2S)$, and k_2 is equal to $(3Vhf\gamma/16S^2)$. To obtain (2), we should note that the total received signal field is the sum of the signal plus the noise, i.e., $F(t)$ can be expressed as $A(t) + Q(t)$. By substituting this quantity in (A8), we derive (2).

Appendix B
Obtaining Equation (9)

By changing the order of integration and summation, (8) can be written as

$$\begin{aligned} Y_{\text{TPA}} = k_2 \sum_{m=-L}^{L}\sum_{n=-L}^{L}\sum_{p=-L}^{L}\sum_{q=-L}^{L}\int_0^T &((a_m+q_m)(a_n+q_n) \\ &\times(a_p+q_p)^*(a_q+q_q)^*\exp(j\Omega t(m+n-p-q)))\,dt. \end{aligned} \tag{B1}$$

Also, the Fourier coefficients of the signal are time independent so that (B1) can be simplified to

$$\begin{aligned} Y_{\text{TPA}} = k_2 \sum_{m=-L}^{L}\sum_{n=-L}^{L}\sum_{p=-L}^{L}\sum_{q=-L}^{L} &(a_m+q_m)(a_n+q_n) \\ \times(a_p+q_p)^*(a_q+q_q)^* &\int_0^T \exp\left(j\Omega t(m+n-p-q)\right)dt \end{aligned} \tag{B2}$$

where $\int_0^T \exp(j\Omega t(m+n-p-q))dt$ equals 0 when $m+n-p-q \neq 0$ and equals T when $m+n-p-q=0$. Thus, we can rewrite (B2) as in (9).

Appendix C
Obtaining Equation (11)

For the number of modes equal to 1, (10) can be written as

$$Y_{\text{TPA}} = k_2 T(a_0+q_{c0}+jq_{s0})(a_0+q_{c0}+jq_{s0}) \times(a_0+q_{c0}-jq_{s0})(a_0+q_{c0}-jq_{s0}). \quad \text{(C1)}$$

This equation can be written as

$$Y_{\text{TPA}} = k_2 T\left((a_0+q_{c0}+jq_{s0})(a_0+q_{c0}-jq_{s0})\right)^2 \quad \text{(C2)}$$

and can be simplified to

$$Y_{\text{TPA}} = k_2 T\left((a_0+q_{c0})^2+q_{s0}^2\right)^2 \quad \text{(C3)}$$

which can be easily expressed as (11).

Appendix D

In this Appendix, we will review some of the higher order even moments of Gaussian random variables that are used in the text or other in Appendixes. Assuming a Gaussian random variable (x) with a mean equal to m and a variance equal to σ^2, we then have

$$E(x^2) = m^2+\sigma^2 \quad \text{(D1)}$$
$$E(x^4) = m^4+6m^2\sigma^2+3\sigma^4 \quad \text{(D2)}$$
$$E(x^6) = m^6+15m^4\sigma^2+45m^2\sigma^4+15\sigma^6 \quad \text{(D3)}$$
$$E(x^8) = m^8+28m^6\sigma^2+210m^4\sigma^4 +420m^2\sigma^6+105\sigma^8. \quad \text{(D4)}$$

Appendix E
Obtaining the Mean Value of the Powers of Term (a_n+q_n)

Four new parameters are defined as follows to simplify the computations:

$$b_n = \sqrt[4]{k_2 T}a_n \quad \text{(E1)}$$
$$p_n = \sqrt[4]{k_2 T}q_n \quad \text{(E2)}$$
$$p_{c_n} = \sqrt[4]{k_2 T}q_{cn} \quad \text{(E3)}$$
$$p_{s_n} = \sqrt[4]{k_2 T}q_{sn} \quad \text{(E4)}$$

which are normalized versions of their counterparts. Since the mean of each b_n is equal to $\sqrt[4]{k_2 T}\sqrt{2\overline{m}_n/T}$ and the variances of the random variables p_{cn} and p_{sn} are equal to $\sqrt{k_2 T}(N_T/T)$, by substituting k_2 as $k_3 T/4$, the mean of b_n can be expressed as $\sqrt[4]{k_3}\sqrt{\overline{m}_n}$ and variances of Gaussian random variables p_{cn} and p_{sn} will be expressed as $\sqrt{k_3}(N_T/2)$. We can find the mean of (b_n+p_n) and the square of this term as

$$\begin{aligned} E(b_n+p_n) &= E(b_n+p_{cn}+jp_{sn}) \\ &= E(b_n+p_{cn})+jE(p_{sn}) \\ &= \sqrt[4]{k_3}\sqrt{\overline{m}_n} \end{aligned} \quad \text{(E5)}$$

$$\begin{aligned} E\left((b_n+p_n)^2\right) &= E\left((b_n+p_{cn}+jp_{sn})^2\right) \\ &= E\left((b_n+p_{cn})^2\right)-E\left(p_{sn}^2\right) \\ &\quad +2jE\left(p_{sn}(b_n+p_{cn})\right) \\ &= \sqrt{k_3}\left(\overline{m}_n+\frac{N_T}{2}\right)-\sqrt{k_3}\frac{N_T}{2} \\ &= \sqrt{k_3}\overline{m}_n. \end{aligned} \quad \text{(E6)}$$

It should be noted that the mean of the square of the absolute of (b_n+p_n) is equal to

$$\begin{aligned} E\left(|b_n+p_n|^2\right) &= E\left((b_n+p_{cn})^2+p_{sn}^2\right) \\ &= \sqrt{k_3}(\overline{m}_n+N_T). \end{aligned} \quad \text{(E7)}$$

To obtain the mean of the absolute fourth power of the amplitude of (b_n+p_n), we first expand it to the summation of three terms as

$$\begin{aligned} E\left(|b_n+p_n|^4\right) &= E\left(|b_n+p_{cn}+jp_{sn}|^4\right) \\ &= E\left(\left((b_n+p_{cn})^2+p_{sn}^2\right)^2\right) \\ &= E\left((b_n+p_{cn})^4+2(b_n+p_{cn})^2p_{sn}^2+p_{sn}^4\right). \end{aligned} \quad \text{(E8)}$$

Thus, we will derive the mean of each term using Appendix D as follows:

$$E(p_{sn}^4) = k_3\frac{3N_T^2}{4} \quad \text{(E9)}$$

$$\begin{aligned} E\left((b_n+p_{cn})^4\right) &= k_3\left(\overline{m}_n^2+6\overline{m}_n\frac{N_T}{2}+3\frac{N_T^2}{4}\right) \\ &= k_3\left(\overline{m}_n^2+3\overline{m}_nN_T+3\frac{N_T^2}{4}\right) \end{aligned} \quad \text{(E10)}$$

$$\begin{aligned} E\left(2(b_n+p_{cn})^2p_{sn}^2\right) &= 2k_3\left(\left(\overline{m}_n+\frac{N_T}{2}\right)\frac{N_T}{2}\right) \\ &= k_3\left(\overline{m}_nN_T+\frac{N_T^2}{2}\right). \end{aligned} \quad \text{(E11)}$$

From (E8)–(E11), we can conclude that

$$E\left(|b_n+p_n|^4\right) = k_3\left(\overline{m}_n^2+4\overline{m}_nN_T+2N_T^2\right). \quad \text{(E12)}$$

To obtain the expected value of the eighth power of the amplitude of (b_n+p_n), we first expand it to the summation of three terms as

$$\begin{aligned} &E\left(|b_n+p_n|^8\right) \\ &= E\left[\left(|b_n+p_n|^4\right)^2\right] \\ &= E\left((b_n+p_{cn})^4+2(b_n+p_{cn})^2p_{sn}^2+p_{sn}^4\right)^2 \\ &= E\left((b_n+p_{cn})^8+4(b_n+p_{cn})^6p_{sn}^2\right. \\ &\quad \left.+6(b_n+p_{cn})^4p_{sn}^4+4(b_n+p_{cn})^2p_{sn}^6+p_{sn}^8\right). \end{aligned} \quad \text{(E13)}$$

Thus, we can write the mean of each coefficient derived in the expansion as follows:

$$\begin{aligned}
&E\left((b_n+p_{cn})^8\right)\\
&\quad= k_3^2\left(\overline{m}_n^4+28\overline{m}_n^3\frac{N_T}{2}+210\overline{m}_n^2\frac{N_T^2}{4}+420\overline{m}_n\frac{N_T^3}{8}+105\frac{N_T^4}{16}\right)\\
&\quad= k_3^2\left(\overline{m}_n^4+14\overline{m}_n^3N_T+\frac{105}{2}\overline{m}_n^2N_T^2+\frac{105}{2}\overline{m}_nN_T^3+\frac{105}{16}N_T^4\right) \qquad \text{(E14)}
\end{aligned}$$

$$\begin{aligned}
&E\left(4(b_n+p_{cn})^6p_{sn}^2\right)\\
&\quad= 4k_3^2\left(\overline{m}_n^3+15\overline{m}_n^2\frac{N_T}{2}+45\overline{m}_n\frac{N_T^2}{4}+15\frac{N_T^3}{8}\right)\left(\frac{N_T}{2}\right)\\
&\quad= k_3^2\left(2\overline{m}_n^3N_T+15\overline{m}_n^2N_T^2+\frac{45}{2}\overline{m}_nN_T^3+\frac{15}{4}N_T^4\right) \qquad \text{(E15)}
\end{aligned}$$

$$\begin{aligned}
&E\left(6(b_n+p_{cn})^4p_{sn}^4\right)\\
&\quad= 6k_3^2\left(\overline{m}_n^2+6\overline{m}_n\frac{N_T}{2}+3\frac{N_T^2}{4}\right)\left(3\frac{N_T^2}{4}\right)\\
&\quad= k_3^2\left(\frac{9}{2}\overline{m}_n^2N_T^2+\frac{27}{2}\overline{m}_nN_T^3+\frac{27}{8}N_T^4\right) \qquad \text{(E16)}
\end{aligned}$$

$$\begin{aligned}
&E\left(4(b_n+p_{cn})^2p_{sn}^6\right)\\
&\quad= 4k_3^2\left(\overline{m}_n+\frac{N_T}{2}\right)\left(15\frac{N_T^3}{8}\right)\\
&\quad= k_3^2\left(\frac{15}{2}\overline{m}_nN_T^3+\frac{15}{4}N_T^4\right) \qquad \text{(E17)}
\end{aligned}$$

$$\begin{aligned}
&E\left(p_{sn}^8\right)\\
&\quad= k_3^2\left(105\frac{N_T^4}{16}\right). \qquad \text{(E18)}
\end{aligned}$$

By adding (E14)–(E18) and using (E13), one can easily find the following equation:

$$E\left(|b_n+p_n|^8\right)=k_3^2\left(\overline{m}_n^4+16\overline{m}_n^3N_T+72\overline{m}_n^2N_T^2+96\overline{m}_nN_T^3+24N_T^4\right). \qquad \text{(E19)}$$

To obtain the mean of the fourth power of (b_n+p_n), we first expand it to the summation of five terms as

$$\begin{aligned}
&E\left((b_n+p_n)^4\right)\\
&\quad= E\left((b_n+p_{cn}+jp_{sn})^4\right)\\
&\quad= E\left((b_n+p_{cn})^4+4(jp_{sn})(b_n+p_{cn})^3\right)\\
&\qquad+E\left(6(jp_{sn})^2(b_n+p_{cn})^2+4(jp_{sn})^3(b_n+p_{cn})+(jp_{sn})^4\right)\\
&\quad= E\left((b_n+p_{cn})^4-6(b_n+p_{cn})^2p_{sn}^2+p_{sn}^4\right). \qquad \text{(E20)}
\end{aligned}$$

From the above equation and (E9)–(E11), we can conclude that

$$E\left((b_n+p_n)^4\right)=k_3\overline{m}_n^2. \qquad \text{(E21)}$$

To obtain the mean of the eighth power of (b_n+p_n), we first expand it to the summation of nine terms as

$$\begin{aligned}
&E\left((b_n+p_n)^8\right)\\
&\quad= E\left((b_n+p_{cn}+jp_{sn})^8\right)\\
&\quad= E\left((b_n+p_{cn})^8+8(jp_{sn})(b_n+p_{cn})^7\right)\\
&\qquad+E\left(28(jp_{sn})^2(b_n+p_{cn})^6+56(jp_{sn})^3(b_n+p_{cn})^5\right)\\
&\qquad+E\left(70(jp_{sn})^4(b_n+p_{cn})^4+56(jp_{sn})^5(b_n+p_{cn})^3\right)\\
&\qquad+E\left(+28(jp_{sn})^6(b_n+p_{cn})^2+8(jp_{sn})^7(b_n+p_{cn})+(jp_{sn})^8\right)\\
&\quad= E\left((b_n+p_{cn})^8-28(b_n+p_{cn})^6p_{sn}^2+70(b_n+p_{cn})^4p_{sn}^4\right)\\
&\qquad+E\left(-28(b_n+p_{cn})^2p_{sn}^6+p_{sn}^8\right). \qquad \text{(E22)}
\end{aligned}$$

From the above equation and (E14)–(E18), we can conclude that

$$E\left((b_n+p_n)^8\right)=k_3^2\overline{m}_n^4. \qquad \text{(E23)}$$

It should be noted that the mean of the conjugate of the different powers of random variable (b_n+p_n), i.e., $(b_n+p_n)^*$ is equal to the mean of the same power of the random variable (b_n+p_n) itself. This equality is mainly due to the fact that the mean of the odd powers of the expansions of (E5), (E6), (E20), and (E22), which are sensitive to the sign of j, are equal to 0.

APPENDIX F
EFFECT OF A SHOT NOISE IN THE SEMICLASSIC MODEL

As can be seen in Fig. 1, before the integrate and dump subsystem, there is a quantization subsystem. This subsystem converts the incoming field to photoelectrons that are liberated from this subsystem. The input/output relation of this module can be expressed as [15]

$$\Psi(z)=\Phi(\mathrm{e}^s-1) \qquad \text{(F1)}$$

where $\Psi(z)$ is the moment generating function of the liberated photoelectrons, and $\Phi(s)$ is the characteristic function of the received field in the photodetector. To obtain the mean and the variance of the liberated photoelectrons, we first need to obtain the first and the second derivatives of the moment generating function of the liberated photoelectrons. Using (F1), we can write

$$\Psi'(z)=\mathrm{e}^s\Phi'(\mathrm{e}^s-1) \qquad \text{(F2)}$$

$$\Psi''(z)=\mathrm{e}^s\Phi'(\mathrm{e}^s-1)+\mathrm{e}^{2s}\Phi''(\mathrm{e}^s-1). \qquad \text{(F3)}$$

To obtain the mean and the variance of the liberated photoelectrons Y_a versus that of the incoming field Y_b, we obtain the values of the moment generating function at $z = 1$ or, equivalently, $s = 0$ as,

$$\begin{aligned} E[Y_a] &= \Psi'(z)|_{z=1} \\ &= \mathrm{e}^s \Phi'(\mathrm{e}^s - 1)|_{s=0} \\ &= E[Y_b] \end{aligned} \tag{F4}$$

$$\begin{aligned} \mathrm{var}[Y_a] &= \left[\Psi''(z) - (\Psi'(z))^2\right]_{z=1} \\ &= \Big[\mathrm{e}^s \Phi'(\mathrm{e}^s - 1) + \mathrm{e}^{2s} \Phi''(\mathrm{e}^s - 1) \\ &\qquad - \left(\mathrm{e}^s \Phi'(\mathrm{e}^s - 1)\right)^2 \Big]_{s=0} \\ &= \left(E[Y_b] + E\left[Y_b^2\right] - E[Y_b]^2\right) \\ &= \mathrm{var}[Y_b] + E[Y_b]. \end{aligned} \tag{F5}$$

This means that to consider the effect of a shot noise, the mean of the received signal must be added to the variance to result in the variance of the received signal, but the mean of the received signal remains as before.

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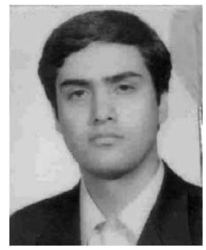

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