Energy Allocation for Parameter Estimation in Block CS-Based Distributed MIMO Systems

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Abstract—**Exploiting Compressive Sensing (CS) in MIMO radars, we can remove the need of the high rate A/D converters and send much less samples to the fusion center. In distributed MIMO radars, the received signal can be modeled as a block sparse signal in a basis. Thus, block CS methods can be used instead of classical CS ones to achieve more accurate target parameter estimation. In this paper a new method of energy allocation to the transmitters is proposed to improve the performance of the block CS-based distributed MIMO radars. This method is based on the minimization of an upper bound of the sensing matrix block-coherence. Simulation results show a significant increase in the accuracy of multiple targets parameter estimation.**

I. Introduction

MULIPLE-INPUT multiple-output (MIMO) radar [1] is a radar that consists of multiple transmitters and multiple receivers. In these radars, the received signals are sent to a common processing center that is called fusion center. MIMO radars are divided into two groups: distributed MIMO radars and co-located MIMO radars. In co-located type [2], transmitters and receivers are located close to each other relative to their distance to the target; thus all transmitter-receiver pairs view the target from the same angle. In distributed MIMO radars [3], the transmitters are located far apart from each other relative to their distance to the target. In this type of MIMO radars, the target is viewed from different angles. Thus, if the received signal from a particular transmitter and receiver is weak, it can be compensated by the received signals from other transmitter-receiver pairs. This type of MIMO radars is shown to offer superior target detection, more accurate target parameter estimation, and higher resolution [1],[3].

Using Compressive Sensing (CS) methods in MIMO radars, the sampling rate and as a result the cost of the receivers can be reduced, and because of the existence of the multiple receivers, this reduction is very significant. Exploiting CS, we can remove the need of the high rate A/D converters and send much less samples to the fusion center. Compressive sensing [4] is a new signal processing method that allows us to accurately reconstruct sparse or compressible signals from a number of samples which is much smaller than that is necessary according to the Shannon-Nyquist sampling theory.

Let us consider $\boldsymbol{\Psi}$ as a basis matrix with basis vectors as columns, we can express $\boldsymbol{x}$ as

$$\boldsymbol{x} = \boldsymbol{\Psi}\boldsymbol{s} \tag{1}$$

where $\boldsymbol{s}$ is the weighting coefficient vector with length of N in the basis $\boldsymbol{\Psi}$. If only K elements of $\boldsymbol{s}$ are nonzero, $\boldsymbol{x}$ is called K-sparse. Compressive sensing is more valuable when $K \ll N$. $\boldsymbol{x}$ is compressible if it has just a few large coefficients and many small coefficients [4].

We can improve the CS-based MIMO radars using different methods like: optimal design of measurement matrix [5],[6], and optimal design of transmitted waveforms [7],[8]. It is shown that these systems can estimate target parameters better than MIMO radars that are using some other estimation methods with higher sampling rates [7],[8].

Let consider $\boldsymbol{s}$ as a concatenation of blocks with length d, i.e.,

$$\boldsymbol{s} = [\underbrace{s_1, \dots, s_d}_{\boldsymbol{s}[1]}, \underbrace{s_{d+1}, \dots, s_{2d}}_{\boldsymbol{s}[2]}, \dots, \underbrace{s_{N-d+1}, \dots, s_N}_{\boldsymbol{s}[N/d]}]^T \tag{2}$$

where $(.)^T$ denotes transpose of a matrix. $\boldsymbol{x}$ is called block K-sparse in the basis $\boldsymbol{\Psi}$, if at most K blocks of $\boldsymbol{s}$ have nonzero Euclidean norms [9]. For block-sparse signals it is better to use block CS methods instead of usual CS methods. References [8] and [10] use block CS methods in distributed MIMO radars and show the advantages of these methods over using the classical CS ones. However, so far, no methods have been proposed for improving the performance of the block CS-based distributed MIMO radars.

In this paper, we proposed a transmitted energy allocation method to minimize an upper bound of the sensing matrix block-coherence. Block- coherence is a measure that should be small enough to have a perfect recovery in block CS methods [9]. We show that block CS-based distributed MIMO radar can be more accurate in target parameters estimation using this optimization method when the total transmitted energy is constant.

The paper is organized as follows. In section II, we provide the received signal model of CS-based distributed MIMO radar system. In section III, two important block recovery algorithms are introduced. A new method of transmitted energy allocation based on sensing matrix block-coherence is introduced in section IV. Section V is allocated to simulation results, and finally, we make some concluding remarks in section VI.

II. Received Signal Model for CS-based Distributed MIMO Radar

We consider a distributed MIMO radar system consisting of M_t transmitters and N_r receivers. The locations of the i^{th} transmitter and the l^{th} receiver are $\boldsymbol{t}_i = [t_{x_i}, t_{y_i}]$ and $\boldsymbol{r}_l = [r_{x_l}, r_{y_l}]$ on a Cartesian coordinate system, respectively. We assume that there are K targets that are moving in a two dimensional plane. However, without loss of generality, this modeling can be extended to the three dimensional case. The location and the velocity of the k^{th} target are $\boldsymbol{p}_k = [p_x^k, p_y^k]$ and $\boldsymbol{v}_k = [\tilde{v}_x^k, \tilde{v}_y^k]$, respectively. The transmitters transmit orthogonal waveforms of duration T_P, and Pulse Repetition Interval (PRI) is T. $x_i(t)$ is a complex baseband waveform with energy equal to 1, and $p_i x_i(t)$ is the waveform transmitted from the i^{th} transmitter. Let us assume the total transmitted energy is M_t (i. e. $\sum_{i=1}^{M_t}(p_i)^2 = M_t$). Now, we model the received signal in four stages as follows:

Stage 1: Under a narrow band assumption on the waveforms, the baseband signal arriving at the l^{th} receiver from the i^{th} transmitter can be expressed as

$$\boldsymbol{z}_{il}(t) = \sum_{k=1}^{K} \beta_k^{il} p_i x_i(t - \beta_k^{il})\, e^{j2\pi(f_k^{il}(t-\tau_k^{il}) - f_c\tau_k^{il})} + n_{il}(t) \quad (3)$$

where β_k^{il} denotes the attenuation coefficient corresponding to the k^{th} target between the i^{th} transmitter and the l^{th} receiver, $n_{il}(t)$ denotes the corresponding received noise, f_c is the carrier frequency, and f_k^{il} and τ_k^{il} are respectively the corresponding k^{th} target Doppler shift and delay that can be expressed as [8]:

$$f_k^{il} = \frac{f_c}{c}\ (\boldsymbol{v}_k.\boldsymbol{u}_{r_l}^k - \boldsymbol{v}_k.\boldsymbol{u}_{t_i}^k) \quad (4)$$

$$\tau_k^{il} = \frac{1}{c}\ (\|\boldsymbol{p}_k - \boldsymbol{t}_i\| + \|\boldsymbol{p}_k - \boldsymbol{r}_i\|) \quad (5)$$

where c is the speed of light and $\boldsymbol{u}_{t_i}^k$ and $\boldsymbol{u}_{r_l}^k$ denote the unit vector from the i^{th} transmitter to the k^{th} target and unit vector from the k^{th} target to the l^{th} receiver, respectively.

Like [8], after down converting the received bandpass signal from the radio frequency, it is passed through a bank of M_t matched filters corresponding to M_t transmitters. Let us assume β_k^{il} does not vary within the estimation process duration and the Doppler shift is small (the velocity of targets is much smaller than c). Hence, $\beta_k^{il} p_i e^{j2\pi f_k^{il} t}$ can be taken outside of the integral in the matched filter operation. Therefore, if T_s denotes the sampling period time, the sampled output of the i^{th} matched filter at the l^{th} receiver in the m^{th} pulse of the estimation process from the k^{th} target can be expressed as

$$z_{il,k}^m(n) = \beta_k^{il} p_i e^{j2\pi\left(f_k^{il}\left((m-1)T + nT_s - \tau_k^{il}\right) - f_c\tau_k^{il}\right)} + n_{il}^m(nT_s) = \beta_k^{il}\Psi_{il,k}^m(n) + n_{il}^m(nT_s) \quad (6)$$

where

$$\Psi_{il,k}^m(n) = p_i e^{j2\pi\left(f_k^{il}\left((m-1)T + nT_s - \tau_k^{il}\right) - f_c\tau_k^{il}\right)} \quad (7)$$

Stage 2: Let us define:

$$\boldsymbol{e}^m(n) = [(\boldsymbol{e}_1^m(n))^T, \dots, (\boldsymbol{e}_{N_r}^m(n))^T]^T, \quad (8)$$

$$\boldsymbol{\Psi}_k^{\boldsymbol{m}}(n) = diag\{\boldsymbol{\Psi}_{1,k}^{\boldsymbol{m}}(n), \dots, \boldsymbol{\Psi}_{N_r,k}^{\boldsymbol{m}}(n)\}, \quad (9)$$

$$\boldsymbol{\beta}_k = [(\boldsymbol{\beta}_{1,k})^T, \dots, (\boldsymbol{\beta}_{N_r,k})^T\ ]^T \quad (10)$$

where

$$\boldsymbol{e}_l^{\boldsymbol{m}}(n) = [n_{1l}^m(nT_s), \dots, n_{M_t l}^m(nT_s)]^T, \quad (11)$$

$$\boldsymbol{\Psi}_{l,k}^{\boldsymbol{m}}(n) = diag\{\Psi_{1l,k}^m(n), \dots, \Psi_{M_t l,k}^m(n)\}, \quad (12)$$

$$\boldsymbol{\beta}_{l,k} = [\beta_k^{1l}, \dots, \beta_k^{M_t l}\ ]^T \quad (13)$$

Putting together the output of all the matched filters of a receiver, and then, putting together the output of all the receivers at a same time in a vector, we have

$$\boldsymbol{z}^m(n) = \boldsymbol{\Psi}^m(n)\boldsymbol{\beta} + \boldsymbol{e}^m(n) \quad (14)$$

where

$$\boldsymbol{\Psi}^m(n) = [\boldsymbol{\Psi}_1^{\boldsymbol{m}}(n), \dots, \boldsymbol{\Psi}_K^{\boldsymbol{m}}(n)], \quad (15)$$

$$\boldsymbol{\beta} = [(\boldsymbol{\beta}_1)^T, \dots, (\boldsymbol{\beta}_K)^T\ ]^T \quad (16)$$

Stage 3: We discretize the estimation space and consider it as a four-dimensional space including: the position in the direction x, the position in the direction y, the velocity in the direction x, and the velocity in the direction y, which are denoted by x, y, v_x, v_y, respectively. The points of this discretized space have the following form.

$$\left(x_h, y_h, v_x^h, v_y^h\right) \quad , h = 1, \dots, L \quad (17)$$

If we define

$$\overline{\boldsymbol{\beta}}_h = \begin{cases} \boldsymbol{\beta}_k, & \text{if the } k^{\text{th}} \text{ target is at } \left(x_h, y_h, v_x^h, v_y^h\right) \\ \boldsymbol{0}_{(M_t N_r)\times 1}, & otherwise \end{cases} \quad (18)$$

$$\boldsymbol{s} = [(\overline{\boldsymbol{\beta}}_1)^T, \dots, (\overline{\boldsymbol{\beta}}_L)^T]^T \quad (19)$$

we can rewrite (16) as

$$\boldsymbol{z}^m(n) = \widetilde{\boldsymbol{\Psi}}^m(n)\boldsymbol{s} + \boldsymbol{e}^m(n) \quad (20)$$

where

$$\widetilde{\boldsymbol{\Psi}}^m(n) = [\widetilde{\boldsymbol{\Psi}}_1^{\boldsymbol{m}}(n), \dots, \widetilde{\boldsymbol{\Psi}}_L^{\boldsymbol{m}}(n)], \quad (21)$$

$$\widetilde{\boldsymbol{\Psi}}_h^{\boldsymbol{m}}(n) = \boldsymbol{\Psi}_k^{\boldsymbol{m}}(n)|_{(p_x^k, p_y^k, \tilde{v}_x^k, \tilde{v}_y^k) = (x_h, y_h, v_x^h, v_y^h)} \quad (22)$$

Stage 4: If N_p pulses are used in the estimation process and N_s denotes the number of the achieved samples in each PRI, we have $N_p \times N_s$ samples at the output of each matched filter. Finally, we stack $\{\{\boldsymbol{z}^m(n)\}_{n=1}^{N_s}\}_{m=1}^{N_p}$ into

$$\boldsymbol{z}_{(N_p\times N_s\times M_t\times N_r)\times(1)} = [(\boldsymbol{z}^1(0))^T, \dots, (\boldsymbol{z}^1(N_s-1))^T, \dots, (\boldsymbol{z}^{N_p}(0))^T, \dots, (\boldsymbol{z}^{N_p}(N_s-1))^T]^T \quad = \boldsymbol{\Psi s} + \boldsymbol{e} \tag{23}$$

where

$$\boldsymbol{\Psi}_{(N_p\times N_s\times M_t\times N_r)\times(L\times M_t\times N_r)} = [(\widetilde{\boldsymbol{\Psi}}^1(0))^T, \dots, (\widetilde{\boldsymbol{\Psi}}^1(N_s-1))^T, \dots, (\widetilde{\boldsymbol{\Psi}}^{N_p}(0))^T, \dots, (\widetilde{\boldsymbol{\Psi}}^{N_p}(N_s-1))^T]^T \tag{24}$$

$$\boldsymbol{e}_{(N_p\times N_s\times M_t\times N_r)\times(\mathbf{1})} = [(\boldsymbol{e}^1(0))^T, \dots, (\boldsymbol{e}^1(N_s-1))^T, \dots, (\boldsymbol{e}^{N_p}(0))^T, \dots, (\boldsymbol{e}^{N_p}(N_s-1))^T]^T \tag{25}$$

The number of targets is usually much smaller than the length of $\boldsymbol{s}$. Hence, $\boldsymbol{s}$ is a block K-sparse vector with the block-length of $d = M_t \times N_r$. Using CS we can reconstruct $\boldsymbol{s}$ from far fewer samples (measurements) that are obtained by multiplying $\boldsymbol{\varphi}_{(M)\times(N_p\times N_s\times M_t\times N_r)}$ with the received signal in each receiver [11]. Thus, at the fusion center we have

$$\boldsymbol{y} = \boldsymbol{\varphi z} = \boldsymbol{\theta s} + \boldsymbol{E} \tag{26}$$

where

$$\boldsymbol{E} = \boldsymbol{\varphi e} \tag{27}$$

$$\boldsymbol{\theta} = \boldsymbol{\varphi \Psi} \tag{28}$$

$\boldsymbol{\varphi}$ is called the measurement matrix, and $\boldsymbol{\theta}$ is called the sensing matrix. The measurement matrix must be suitable and create small coherence for sensing matrix in classical CS. The coherence of the sensing matrix is the maximum absolute value of the correlation between the sensing matrix columns. It has been shown that zero-mean Gaussian random matrix can be used as a suitable measurement matrix [4].

III. The Block Recovery Algorithms

There are several block recovery algorithms that can be used for recovering $\boldsymbol{s}$ in (26). In this paper, the extensions of the Matching Pursuit (MP) and Orthogonal Matching Pursuit (OMP) algorithms to the block-sparse case are used. These algorithms are named Block Matching Pursuit (BMP) and Block Orthogonal Matching Pursuit (BOMP) [9], respectively.

Let we divide the sensing matrix into L blocks as

$$\boldsymbol{\theta} = [\underbrace{\boldsymbol{\theta}_1, \dots, \boldsymbol{\theta}_d}_{\boldsymbol{\theta}[1]}, \dots, \underbrace{\boldsymbol{\theta}_{L\times(d-1)+1}, \dots, \boldsymbol{\theta}_{L\times d}}_{\boldsymbol{\theta}[L]}] \tag{29}$$

where $\boldsymbol{\theta}_l$ is the l^{th} column of $\boldsymbol{\theta}$. Both BMP and BOMP algorithms start by initializing $\boldsymbol{s}_0 = \boldsymbol{0}$ and the residual as $\boldsymbol{r}_0 = \boldsymbol{y}$. At the i^{th} stage ($i \geq 1$), according to

$$l_i = \arg\max_l \|\boldsymbol{\theta}^H[l]\boldsymbol{r}_{i-1}\|_2, \tag{30}$$

the block that is the best matched to $\boldsymbol{r}_{i-1}$ is chosen. Superscript $(.)^H$ denotes the Hermitian of a matrix. After choosing the index l_i, $\boldsymbol{s}$ and the residual are updated.

IV. The Proposed Method for Energy Allocation

Block-coherence is defined as [9]:

$$\mu_B = \max_{l,r\neq l} \tfrac{1}{d}\rho(\boldsymbol{\theta}^H[l]\boldsymbol{\theta}[r]) \tag{31}$$

where $\rho(.)$ is the spectral norm which is denoted by $\rho(\boldsymbol{A}) = \sqrt{\lambda_{max}(\boldsymbol{A}^H\boldsymbol{A})}$. $\lambda_{max}(\boldsymbol{B})$ is the largest eigenvalue of the positive-semidefinite matrix $\boldsymbol{B}$.

According to section III, for the ideal performance of BMP and BOMP algorithms, $\|\boldsymbol{\theta}^H[r]\boldsymbol{\theta}[l]\boldsymbol{\theta}^H[l]\boldsymbol{\theta}[r]\|_1$ ($\|\boldsymbol{B}\|_1$ is the sum of the absolute values of the elements of $\boldsymbol{B}$) should merge to zero for $l \neq r$. Because if a block of $\boldsymbol{s}$ (for example $\boldsymbol{s}[l]$) is equal to zero, $\boldsymbol{y}$ is not the combination of its corresponding block columns in $\boldsymbol{\theta}$. If $\|\boldsymbol{\theta}^H[r]\boldsymbol{\theta}[l]\boldsymbol{\theta}^H[l]\boldsymbol{\theta}[r]\|_1$ for $l \neq r$ is small enough, we will not choose this block as a candidate of the nonzero-norm blocks. It is shown that for matrix $\boldsymbol{A}_{w\times d}$ with arbitrary w, we have [12]:

$$\lambda_{max}(\boldsymbol{A}^H\boldsymbol{A}) \leq \max_i \sum_{k=1}^{d} |(\boldsymbol{A}^H\boldsymbol{A})_{i,k}| \tag{32}$$

Also, we know that

$$\max_i \sum_{k=1}^{d} |(\boldsymbol{A}^H\boldsymbol{A})_{i,k}| \leq \sum_{k=1}^{d}\sum_{i=1}^{d} |(\boldsymbol{A}^H\boldsymbol{A})_{i,k}| = \|\boldsymbol{A}^H\boldsymbol{A}\|_1 \tag{33}$$

Thus, we have

$$\lambda_{max}(\boldsymbol{\theta}^H[r]\boldsymbol{\theta}[l]\boldsymbol{\theta}^H[l]\boldsymbol{\theta}[r]) \leq \|\boldsymbol{\theta}^H[r]\boldsymbol{\theta}[l]\boldsymbol{\theta}^H[l]\boldsymbol{\theta}[r]\|_1 \tag{34}$$

In order to merge $\|\boldsymbol{\theta}^H[r]\boldsymbol{\theta}[l]\boldsymbol{\theta}^H[l]\boldsymbol{\theta}[r]\|_1$ to zero, it is necessary that $\lambda_{max}(\boldsymbol{\theta}^H[r]\boldsymbol{\theta}[l]\boldsymbol{\theta}^H[l]\boldsymbol{\theta}[r])$ merges to zero. Hence, it is clear that for a good performance of BMP or BOMP, the block-coherence of the sensing matrix should merge to zero. Reference [9] has shown that if this measure is small enough, the BMP and BOMP algorithms choose the correct block in each stage. Therefore, we want to allocate optimal energy to the transmitters in order to reduce this value.

Now, we want to achieve an upper bound for the block-coherence of the sensing matrix. According to [13], we can write

$$\mu_B = \max_{l,r\neq l} \tfrac{1}{d}\rho(\boldsymbol{\theta}^H[l]\boldsymbol{\theta}[r]) \leq \max_{l,r\neq l} \tfrac{1}{d}\rho(\boldsymbol{\theta}^H[l])\rho(\boldsymbol{\theta}[r]) \tag{35}$$

By using (34) and (35), we have

$$\mu_B \leq \max_{l,r\neq l} \tfrac{1}{d}\sqrt{\|\boldsymbol{\theta}^H[l]\boldsymbol{\theta}[l]\|_1}\sqrt{\|\boldsymbol{\theta}^H[r]\boldsymbol{\theta}[r]\|_1} \tag{36}$$

It is clear that if the square of a positive expression is minimized, that expression is also minimized. Besides, we know

$$\max_{l,r\neq l} \|\boldsymbol{\theta}^H[l]\boldsymbol{\theta}[l]\|_1 \|\boldsymbol{\theta}^H[r]\boldsymbol{\theta}[r]\|_1 \leq \sum_{l=1}^{L}\sum_{\substack{r=1\\,r\neq l}}^{L} \|\boldsymbol{\theta}^H[l]\boldsymbol{\theta}[l]\|_1 \|\boldsymbol{\theta}^H[r]\boldsymbol{\theta}[r]\|_1 \leq \left(\sum_{l=1}^{L} \|\boldsymbol{\theta}^H[l]\boldsymbol{\theta}[l]\|_1\right)^2 \tag{37}$$

Therefore, the cost function for minimization problem in order to improve the performance of the mentioned block-CS methods can be considered as fallows

$$\sum_{l=1}^{L} \|\boldsymbol{\theta}^H[l]\boldsymbol{\theta}[l]\|_1 \tag{38}$$

If we define

$$\boldsymbol{H}_1 = diag\{\underbrace{\underbrace{p_1, \dots, p_{M_t}}, \dots, \underbrace{p_1, \dots, p_{M_t}}}_{N_r}\} \tag{39}$$

$$\bar{\bar{\boldsymbol{\Psi}}} = \boldsymbol{\Psi}|_{(p_1,\dots,p_{M_t})=(1,\dots,1)} \tag{40}$$

the l^{th} block of $\boldsymbol{\Psi}$ can be written as:

$$\boldsymbol{\Psi}[l] = \bar{\bar{\boldsymbol{\Psi}}}[l]\boldsymbol{H}_1, \tag{41}$$

and the l^{th} block of $\boldsymbol{\theta}$ can be shown as

$$\boldsymbol{\theta}[l] = \boldsymbol{\varphi}\bar{\bar{\boldsymbol{\Psi}}}[l]\boldsymbol{H}_1, \tag{42}$$

Considering (42), our optimization problem can be expressed as:

$$\begin{aligned} &\min_{p_1,\dots,p_{M_t}} \sum_{l=1}^{L} \left\| {\boldsymbol{H}_1}^H \bar{\bar{\boldsymbol{\Psi}}}^H[l]\boldsymbol{\varphi}^H\boldsymbol{\varphi}\bar{\bar{\boldsymbol{\Psi}}}[l]\boldsymbol{H}_1 \right\|_1 \\ &\text{s.t.} \quad \sum_{i=1}^{M_t}(p_i)^2 = M_t \\ &, p_i \geq 0 \quad \text{for i} = 1,\dots,M_t \end{aligned} \tag{43}$$

We should change the problem into a standard form. We do this in three stages as follows.

Stage 1: We know

$$\begin{aligned} &\sum_{l=1}^{L} \left\| {\boldsymbol{H}_1}^H \bar{\bar{\boldsymbol{\Psi}}}^H[l]\boldsymbol{\varphi}^H\boldsymbol{\varphi}\bar{\bar{\boldsymbol{\Psi}}}[l]\boldsymbol{H}_1 \right\|_1 \\ &= \sum_{l=1}^{L} \left\| abs({\boldsymbol{H}_1}^H \bar{\bar{\boldsymbol{\Psi}}}^H[l]\boldsymbol{\varphi}^H\boldsymbol{\varphi}\bar{\bar{\boldsymbol{\Psi}}}[l]\boldsymbol{H}_1) \right\|_1 \\ &= \left\| abs({\boldsymbol{H}_1}^H \textstyle\sum_{l=1}^{L}(\bar{\bar{\boldsymbol{\Psi}}}^H[l]\boldsymbol{\varphi}^H\boldsymbol{\varphi}\bar{\bar{\boldsymbol{\Psi}}}[l])\,\boldsymbol{H}_1) \right\|_1 \end{aligned} \tag{44}$$

$\boldsymbol{H}_1$ is a matrix with real and positive elements. Thus, we can rewrite the cost function of (43) as

$$\left\| \underbrace{{\boldsymbol{H}_1}^H abs(\sum_{l=1}^{L}(\bar{\bar{\boldsymbol{\Psi}}}^H[l]\boldsymbol{\varphi}^H\boldsymbol{\varphi}\bar{\bar{\boldsymbol{\Psi}}}[l]))\boldsymbol{H}_1}_{\boldsymbol{G}} \right\|_1 \tag{45}$$

Elements of $\boldsymbol{G}$ are real and positive. Therefore, (45) is equal to

$$\mathbf{1}_{1\times d}{\boldsymbol{H}_1}^H\bar{\boldsymbol{A}}\boldsymbol{H}_1\mathbf{1}_{d\times 1} \tag{46}$$

where $\mathbf{1}_{m\times n}$ is an $m\times n$ matrix with elements that all are equal to 1, and

$$\bar{\boldsymbol{A}} = abs\left(\sum_{l=1}^{L}\left(\bar{\bar{\boldsymbol{\Psi}}}^H[l]\boldsymbol{\varphi}^H\boldsymbol{\varphi}\bar{\bar{\boldsymbol{\Psi}}}[l]\right)\right) \tag{47}$$

Stage 2: Let us define:

$$\bar{\boldsymbol{p}} = \boldsymbol{H}_1 = [\underbrace{\underbrace{p_1,\dots,p_{M_t}},\dots,\underbrace{p_1,\dots,p_{M_t}}}_{N_r}]^T \tag{48}$$

Therefore, the cost function can be expressed as:

$$\bar{\boldsymbol{p}}^H\bar{\boldsymbol{A}}\bar{\boldsymbol{p}} \tag{49}$$

Stage 3: We can define:

$$\boldsymbol{p} = \left[\tilde{p}_1,\dots,\tilde{p}_{M_tN_r}\right]^T, \tag{50}$$

and have the following cost function.

$$\boldsymbol{p}^H\bar{\boldsymbol{A}}\boldsymbol{p} \tag{51}$$

Then, by considering the following conditions we have an optimization problem that is equal to (43).

$$\begin{aligned} &\boldsymbol{J}_{i,l}\boldsymbol{p} = 1 \quad \text{for } i = 1,\dots,M_t \\ &\qquad\qquad\qquad\; l = 1,\dots,N_r-1 \\ &,\boldsymbol{J}_{i,l} = [\bar{\boldsymbol{J}}_i,\underbrace{\mathbf{0}_{1\times M_t},\dots,\mathbf{0}_{1\times M_t}}_{l-1},-\bar{\boldsymbol{J}}_i,\underbrace{\mathbf{0}_{1\times M_t},\dots,\mathbf{0}_{1\times M_t}}_{N_r-l-1}] \\ &,\bar{\boldsymbol{J}}_i = [\underbrace{0,\dots,0}_{i-1},1,\underbrace{0,\dots,0}_{M_t-i}] \end{aligned} \tag{52}$$

$$\|\boldsymbol{p}\| = M_tN_r \tag{53}$$

$$\tilde{p}_i \geq 0 \quad \text{for i} = 1,\dots,M_tN_r \tag{54}$$

It is clear that the conditions in (52) are used for equalizing $\boldsymbol{p}$ to $\bar{\boldsymbol{p}}$. We can use the CVX software to solve this optimization problem [14].

V. Simulation Results

In this section, the performance of block CS-based distributed MIMO radar system using the proposed method is evaluated. Let us consider a distributed MIMO radar system with 2 transmitters and 2 receivers. The locations of the transmitters and receivers are as follows (the distance unit is meter):

$$\boldsymbol{t}_1 = [100,0], \boldsymbol{t}_2 = [200,0], \boldsymbol{r}_1 = [0,200], \boldsymbol{r}_2 = [0,100]$$

The carrier frequency of the transmitted waveforms is $\mathrm{f_c} =$ 1 GHz. We choose T= 200 ms, $\mathrm{T_s} = 0.2$ ms, $\mathrm{N_s} = 10$, and $\mathrm{N_p} = 4$. The estimation space is divided into L = 144 points. The position and velocity of these points are determined as (the velocity unit is m/s):

$$x_h \in \{80,90,100\}, \qquad h = 1,\dots,144$$

$$y_h \in \{260,270,280\}, \qquad h = 1,\dots,144$$

$$v_x^h \in \{100,110,120,130\}, \qquad h = 1,\dots,144$$

$$v_y^h \in \{100,110,120,130\}, \qquad h = 1,\dots,14$$

There are 2 targets, and their positions and velocities have been chosen as

$$\boldsymbol{p}_1 = [100,260], \boldsymbol{p}_2 = [80,280] \tag{55}$$

$$\boldsymbol{v}_1 = [120,100], \boldsymbol{v}_2 = [110,120] \tag{56}$$

The elements of the noise vector ($\boldsymbol{e}$) are i.i.d. and zero-mean Gaussian random variables, and the target attenuation coefficients are Rayleigh random variables with means of 0.8147 and variances of 0.1813.

We introduce a new measure: total transmitted Energy to the Noise energy Ratio (ENR) defined as the ratio of the total transmitted Energy from the $\mathrm{M_t}$ transmitter to $\mathrm{E}\{\|\mathrm{e}\|^2\}$:

$$\begin{aligned} \text{ENR} &= (\mathrm{M_t}\times\mathrm{N_s}\times\mathrm{N_p})/(\mathrm{N_s}\times\mathrm{N_p}\times \mathrm{d}\times\mathrm{var_n}) \\ &= \frac{\mathrm{M_t}}{\mathrm{d}\times\mathrm{var_n}} = \frac{1}{\mathrm{N_r}\times\mathrm{var_n}} \end{aligned}$$

where $\mathrm{var_n}$ is the noise variance. The measurement matrix entries are i.i.d. and zero-mean Gaussian random variables.

Let us use BOMP#1 and BMP#1, respectively, for denoting the BOMP and BMP methods when the transmitted energy is optimally allocated to the transmitters according to the proposed method; Notations BOMP and BMP are used for the

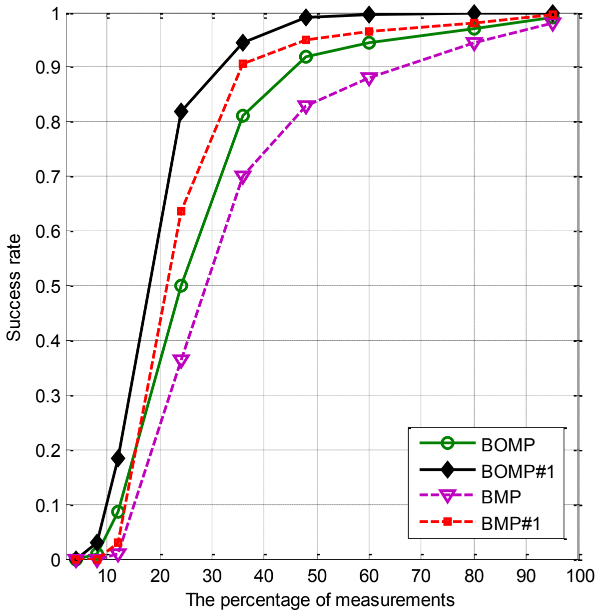

Fig. 1. Success rate versus the percentage of measurements. Here ENR is 0 dB.

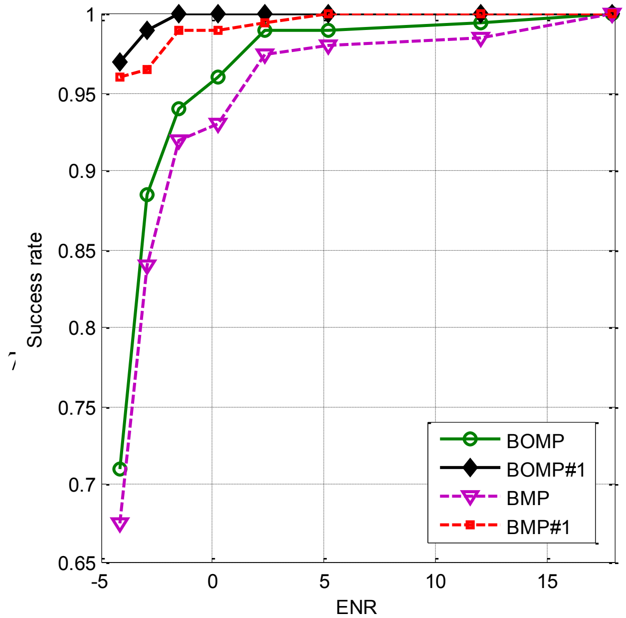

Fig. 2. Success rate versus ENR. Here the percentage of measurement s is 50%.

mentioned methods when the uniform energy allocation is exploited. Fig. 1 compares the success rate of BOMP, BMP, BOMP#1, and BMP#1 for different percentages of measurements. The success rate is the number of the correct estimations of the two targets parameters to the number of total runs, and the percentage of measurements is calculated as

$$\frac{M}{M_t \times N_r \times N_s \times N_p} \times 100\% \quad (57)$$

We have used 200 independent runs to generate these results. ENR in this figure is equal to 0 dB. We can see that using the proposed energy allocation scheme, both BMP and BOMP methods are improved for all different measurement numbers.

Now we fix the percentage of measurements to 50%. Fig. 2 shows the success rate of BOMP, BMP, BOMP#1, and BMP#1 versus the ENR. It is clear that the improved systems perform better than the unimproved ones. It can easily be seen from Fig. 2 that using the proposed energy allocation in the block CS-based distributed MIMO radar, more than 95% of the estimations are correct even when ENR is equal to -4 dB.

VI. Conclusion

We have proposed a transmitted energy allocation scheme for block CS-based distributed MIMO radar systems. This energy allocation scheme is based on the minimization of an upper bound of the sensing matrix block-coherence. Exploiting the proposed energy allocation, Distributed MIMO radars can correctly estimate the multiple targets parameters with the probability of more than 0.95 even when the probability of the correct estimation is less than 0.75 in the non-improved ones.

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