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INFORMATION HIDING WITH OPTIMAL DETECTOR FOR HIGHLY CORRELATED SIGNALS

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ABSTRACT

In this paper, a novel scaling based information hiding approach robust against noise and gain attack is presented. The host signal is assumed to be stationary Gaussian modeled with a first-order autoregressive process. For data embedding, the host signal is divided into two parts. One part is manipulated while the other part is kept unchanged for parameter estimation. The decoding scheme using the ratio of samples is suitable for highly correlated signals in which the decoding process is difficult. By calculating the distribution of the ratio, the performance of the maximum likelihood decoder is analytically studied. The proposed algorithm is applied to several artificial Gaussian autoregressive signals to verify the validity of our results.

1. INTRODUCTION

Digital watermarking is a process in which information is embedded within a digital media so that the inserted data becomes part of the media. This technique serves a number of purposes such as broadcast monitoring, data authentication, data indexing and so forth.

In the attempt to match the characteristics of watermark to those of the signal asset, one way is that larger image features bear greater watermark. Two of the simplest ways to implement this principle is by means of either multiplicative [1], or scaling-based watermarking [2]. Since correlation detection is suboptimal for multiplicative watermarking in the transform domain, several alternative optimum and locally optimum decoders have been proposed [1], [3]-[5].

While in some watermarking schemes the aim is to detect whether the received signal contains the embedded logo or not, in some other applications such as covert communications, the decoder should be able to extract the watermark data from the received signal. In this case, the watermark serves as a transmission code and the decoder's task is more complex. In this category, Barni [1], introduces a multi-bit multiplicative watermarking method. Recently, a heuristic multi-bit data hiding method based on the multiband wavelet transform and the empirical mode decomposition is introduced [6]. Both of these approaches suffer from the gain attack which usually happens in the transmission channel.

In this paper, we introduce a multi-bit scaling based data hiding scheme with a blind maximum likelihood decoder. The host signal which represents the low frequency component of natural image or audio signals is considered to be Gaussian modeled with a first-order autoregressive process. For blind detection some parameters are required. To this aim, the host signal is divided into two patches and one of them is remained unchanged for parameter estimation at the decoder side. The other patch is scaled upward or downward by a constant factor depending on the value of the watermark bits. In order to reduce the computational complexity of the proposed algorithm for highly correlated signals and also to be robust against gain attack simultaneously, the detection process is performed on the ratio of samples of these two patches. The exact and approximated distribution of this new variable are calculated. Since the variance of the channel noise is necessary for detection, this value and its effect on the ratio variable is estimated and investigated. The performance of the detector is analytically calculated.

2. SYSTEM MODELING

Since the low frequency components of most natural signals such as audio and image can be modeled as Gausiian with first order markov model, we suppose the host signal to be a first order markov sequence for our model.

Let, $\mathbf{u}$ be a first order markov sequence of normally distributed random variables with mean μ, variance σ^2, and correlation coefficient ρ. If $\mathbf{u}$ contains N variables $u_1, u_2, ..., u_N$, let $\mathbf{x}$ and $\mathbf{y}$ sequences represent the samples of $\mathbf{u}$ in odd and even positions, respectively. That is, $x_i = u_{2i-1}$, $y_i = u_{2i}$, $i = 1, 2, ..., \frac{N}{2}$. As will be discussed in Section 3, we divide the host signal two this two categories to satisfy the blindness of our watermarking approach.

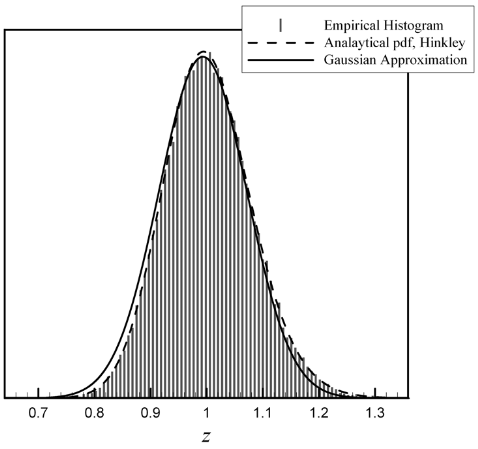

Fig. 1. Hinkley's [7] analytical suggested pdf and our suggested Gaussian approximation versus empirical for an sample case of $\mu = 1$, $\sigma = .2$, and $\rho = .7$.

Now, consider the new sequence $\mathbf{z}$, defined as:

$$z_i = \frac{x_i}{y_i} = \frac{u_{2i-1}}{u_{2i}}, \qquad i = 1, ..., \frac{N}{2}. \tag{1}$$

This variable is the ratio of two correlated normal variables $x = \mathcal{N}(\mu_x, \sigma_x^2)$ and $y = \mathcal{N}(\mu_y, \sigma_y^2)$. The distribution of z when $\mu_x = \mu_y = 0$ is Cauchy [7]. However, [7] suggested that the density function of z for the case of non-zero mean in the denominator is not Cauchian and it suggested a closed form solution for the distribution function in this case which is a bit complicated.

However, using some calculations, it can be shown that for the case of non zero μ_x and μ_y, and $\sigma_y \ll \mu_y$, $\sigma_x \ll \mu_x$, the distribution of $z = \frac{x}{y}$ can be well approximated by a Gaussian distribution. If we supposed that $\sigma_x \simeq \sigma_y \simeq \sigma$ and $\mu_x \simeq \mu_y \simeq \mu$, which is a suitable assumption as $\mathbf{x}$ and $\mathbf{y}$ are the subsamples of the same sequence $\mathbf{u}$ and we consider the signal to be stationery, the parameters μ_z and σ_z^2 of the approximated Gaussian distribution can be derived as follows:

$$\mu_z = 1 - \rho\frac{\sigma^2}{\mu^2} = 1 - \rho A \tag{2}$$

$$\sigma_z^2 = (2 - 2\rho)A + (1 + \rho^2)A^2 \tag{3}$$

where $A = \frac{\sigma^2}{\mu^2}$. It can be shown that the small variance-to-mean assumption we made occur easily in the approximation bands of the wavelet decomposition of images.

The correlation coefficient between successive z_i samples, that is correlation at lag 1, normalized by σ_z^2 can be estimated as:

$$\rho_z(1) = \frac{2\rho^4 A^2 - (\rho^3 - 2\rho^2 + \rho)A}{(1 + \rho^2)A^2 + (2 - 2\rho)A} \tag{4}$$

Notice that for all A one has, $\lim_{\rho \to 1} \rho_z(1) = 1$ and that for $A \ll 1$ one has $\rho_z(1) \simeq \frac{1}{2}\rho(\rho - 1)$.

Fig. 1 compares the empirical histogram of z for a sample case of $\mu = 1$, $\sigma = .2$, and $\rho = .7$. We can see that Hinkley's suggested distribution and our Gaussian suggestion match perfectly. Thus, we have a remarkable accuracy in our suggested distribution function.

In the same way we can compute the higher order correlation coefficients $\rho_z(k) = \frac{2\rho^{4k}A - \rho^{2k-1}(\rho-1)^2}{(1+\rho^2)A + (2-2\rho)}$. Again using the fact that $A \ll 1$, the correlation coefficients can be approximated as $\rho_{z_k} \simeq \rho^{(2k-2)}\rho_{z_1}$.

Now, the density function $f(\mathbf{z})$ of the observation vector $\mathbf{z} = \{z(1)z(2)\ldots\}$ is given as:

$$f(\mathbf{z}) = \frac{1}{\sqrt{(2\pi)^{\frac{N}{2}}}|\mathbf{C}|^{\frac{1}{2}}} e^{-\frac{1}{2}[(\mathbf{z}-\mu_z)^T\mathbf{C}^{-1}(\mathbf{z}-\mu_z)]} \tag{5}$$

where $\mathbf{C} = \{c_{ij}\}$ is the covariance matrix and $c_{ij} = E(z_i z_j) - \mu_z^2 = \rho_z(|i - j|)\sigma_z^2$.

As mentioned before, for $A \ll 1$ we can say $\rho_z(1) \simeq \frac{1}{2}\rho(\rho - 1)$. This formula reveals that the $\mathbf{z}$ sequence will have low correlation not only obviously for the low-correlated parent sequence u (i.e., $\rho \ll 1$), but also for highly correlated $\mathbf{u}$ sequences (ρ is near unity). We will exploit this characteristics for watermark detection.

Considering this fact that $\rho_{z_k} \simeq \rho^{(2k-2)}\rho_{z_1}$, we can suppose $\mathbf{z}$ to be a first order markov sequence and approximate its distribution as:

$$f(\mathbf{z}) \simeq f(z_1)f(z_2|z_1)...f(z_{\frac{N}{2}}|z_{\frac{N}{2}-1}) \tag{6}$$

Thus, using the conditional distribution of jointly Gaussian signals and the fact that $\rho_z \ll 1$, the distribution of $\mathbf{z}$ after some simplifications can be written as:

$$f(\mathbf{z}) \simeq \frac{e^{-\frac{1}{2\sigma_z^2}\sum_1^{\frac{N}{2}} \bar{z}_i^2}}{\sqrt{(2\pi\sigma_z^2)^{\frac{N}{2}}}}[1 + \frac{\rho_z}{\sigma_z^2}\sum_{i=2}^{\frac{N}{2}}(\bar{z}_i\bar{z}_{i-1})] \tag{7}$$

where $\bar{z}_i = z_i - \mu_z$.

3. PROPOSED BLIND WATERMARKING METHOD

3.1. Watermark embedding

Here, the watermark is embedded by scaling the amplitude of the host signal depending on the message bit. We suppose that the host signal follows the model for the $\mathbf{u}$ sequence discussed in the previous section. Then, we extract subsample sequences $\mathbf{x}$ and $\mathbf{y}$ in odd and even positions of $\mathbf{u}$, respectively. That is, $x_i = u_{2i-1}$, $y_i = u_{2i}$, $i = 1, 2, ..., \frac{N}{2}$. Now, we embed the data only in the $\mathbf{x}$ sequence based on the following scaling strategy:

$$x_i' = \begin{cases} x_i \cdot \alpha & \text{For embedding 1} \\ x_i \cdot \frac{1}{\alpha} & \text{For embedding 0} \end{cases} \tag{8}$$

where α is called the strength factor and is larger than 1. Repositioning the $\mathbf{x}'$ in the odd positions of $\mathbf{u}$ we obtain the watermarked signal $\mathbf{u}'$.

As we see, we use $\mathbf{x}$ which contains half samples of the host signal for data hiding and leave the other half $\mathbf{y}$ as a reference. Using this unchanged half, way we could better estimate the original signal parameters at decoder side.

3.2. Watermark decoding

At the receiver, we receive the sequence $\mathbf{u}'' = \mathbf{u}' + \mathbf{n}'$ which is the $\mathbf{u}'$ sequence contaminated by zero mean Additive White Gaussian Noise(AWGN) $n' \sim \mathcal{N}(0, \sigma_n^2)$. Let, sequences $\mathbf{x}''$ and $\mathbf{y}''$ represent the samples of $\mathbf{u}''$ in odd and even positions, respectively. That is, $x_i'' = u_{2i-1}''$ and $y_i'' = u_{2i}''$. Since the watermarked sequence and noise are independent, the distribution of $\mathbf{y}''$ is given as $\mathcal{N}(\mu, \sigma^2 + \sigma_n^2)$. Moreover, the distribution of $\mathbf{x}''$ for '1' or '0' embedding can be written as $x_{i|1}'' \sim \mathcal{N}(\alpha\mu, \sigma_{x_{|1}''}^2)$ and $x_{i|0}'' \sim \mathcal{N}(\frac{1}{\alpha}\mu, \sigma_{x_{|0}''}^2)$, where $\sigma_{x_{|1}''}^2 = \alpha^2\sigma^2 + \sigma_n^2$ and $\sigma_{x_{|0}''}^2 = \alpha^{-2}\sigma^2 + \sigma_n^2$.

If we define $\mathbf{z}''$ as $z_i'' = \frac{x_i''}{y_i''} = \frac{x_i' + n_{x_i}'}{y_i + n_{y_i}'}$, where n_{x_i}' and n_{y_i}' denote the odd- and even indexed noise terms, we can use the model introduced in Section 2 to construct our ML decoder.

Considering small noise term we can estimate z_i'' as:

$$\begin{aligned} z_i'' &= \frac{x_i' + n_{x_i}'}{y_i + n_{y_i}'} \simeq \frac{x_i'}{y_i}[(1 + \frac{n_{x_i}'}{x_i'})(1 - \frac{n_{y_i}'}{y_i})] \\ &\simeq \underbrace{\frac{x_i'}{y_i}}_{z_i'} + \underbrace{(\frac{n_{x_i}'}{y_i} - \frac{x_i'}{y_i}\frac{n_{y_i}'}{y_i} - \frac{n_{x_i}'}{y_i}\frac{n_{y_i}'}{y_i})}_{\zeta} \end{aligned} \tag{9}$$

The first term in this equation is simply the watermarked signal term z_i', which resolves $z_i' = \alpha\frac{x_i}{y_i} = \alpha z_i$ for '1' embedding and $z_i' = \frac{1}{\alpha}\frac{x_i}{y_i} = \frac{1}{\alpha}z_i$ for '0' embedding. The rest of the terms are approximation residues and we will call them as such to differentiate them from noise. Using (2) and (3), we can see that the signal term has the normal distribution of $\mathcal{N}(\alpha\mu_z, \alpha^2\sigma_z^2)$ for '1' embedding or $\mathcal{N}(\frac{1}{\alpha}\mu_z, \frac{1}{\alpha^2}\sigma_z^2)$ for '0' embedding. The residue ζ consists of three terms:

$$\zeta = \underbrace{\frac{n_{x_i}'}{y_i}}_{\nu_1} - \underbrace{\frac{x_i'}{y_i}\frac{n_{y_i}'}{y_i}}_{\nu_2} - \underbrace{\frac{n_{x_i}'}{y_i}\frac{n_{y_i}'}{y_i}}_{\nu_3} \tag{10}$$

If we investigate these three terms, by some calculations it can be shown that $\mu_{\nu_1} = \mu_{\nu_2} = \mu_{\nu_3} = 0$. Besides, $\sigma_{\nu_1}^2 = \frac{\sigma_n^2}{\mu^2}(1+A)$, $\sigma_{\nu_2|1}^2 = [1+(2-4\rho)A+(1+2\rho^2)A^2](1+A)\frac{\sigma_n^2}{\mu^2}\alpha^2$, $\sigma_{\nu_2|0}^2 = [1 + (2-4\rho)A + (1+2\rho^2)A^2](1+A)\frac{\sigma_n^2}{\mu^2\alpha^2}$, and $\sigma_{\nu_3}^2 = [\frac{\sigma_n^2}{\mu^2}(1+A)]^2$.

Because of the independency of noise and signal, all these three terms are independent of each other. Therefore, variance of ζ is $\sigma_\zeta^2 = \sigma_{\nu_1}^2 + \sigma_{\nu_2}^2 + \sigma_{\nu_3}^2$. Besides, we have $E(\zeta) = 0$. Thus, we can interpret the received signal z'' as the sum of a signal term z' and a normal distributed noise term $zeta \sim \mathcal{N}(0, \sigma_{zeta}^2)$. We use this model to define our ML decision based decoder.

Implementing the density function defined in (5), the ML decision based decoder is defined as:

$$\begin{aligned} &|\mathbf{C}_{|\mathbf{1}}|^{\frac{-1}{2}} e^{-\frac{1}{2}[(\mathbf{z}''-\mu_{\mathbf{z}_{|1}''})^T \mathbf{C}_{|1}{}^{-1}(\mathbf{z}''-\mu_{\mathbf{z}_{|1}''})]} \\ &\gtrless_0^1 -|\mathbf{C}_{|\mathbf{0}}|^{\frac{-1}{2}} e^{-\frac{1}{2}[(\mathbf{z}''-\mu_{\mathbf{z}_{|0}''})^T \mathbf{C}_{|0}{}^{-1}(\mathbf{z}''-\mu_{\mathbf{z}_{|0}''})]} \end{aligned} \tag{11}$$

where $\mathbf{C}_{|1} = \alpha^2\mathbf{C}+\sigma_\zeta^2\mathbf{I}$, $\mathbf{C}_{|0} = \frac{1}{\alpha^2}\mathbf{C}+\sigma_\zeta^2\mathbf{I}$, and $\mathbf{C}$ is given in Section 2. After some simplification and with considering the fact that $\mathbf{C}_{|\mathbf{1}}$ and $\mathbf{C}_{|\mathbf{0}}$ are symmetric matrixes, we can rewrite (11) as:

$$\begin{aligned} &\mathbf{z}''^T(\mathbf{C}_{|\mathbf{0}}{}^{-1} - \mathbf{C}_{|\mathbf{1}}{}^{-1})\mathbf{z}'' \\ - \;& 2\mathbf{z}''^T(\mathbf{C}_{|\mathbf{0}}{}^{-1}\mu_{\mathbf{z}_{|0}''} - \mathbf{C}_{|\mathbf{1}}{}^{-1}\mu_{\mathbf{z}_{|1}''}) \gtrless_0^1 T_1 \end{aligned} \tag{12}$$

where $T_1 = \log\frac{|\mathbf{C}_{|\mathbf{1}}|}{|\mathbf{C}_{|\mathbf{0}}|} + \mu_{\mathbf{z}_{|1}''}{}^T\mathbf{C}_{|\mathbf{1}}{}^{-1}\mu_{\mathbf{z}_{|1}''} - \mu_{\mathbf{z}_{|0}''}{}^T\mathbf{C}_{|}\mathbf{0}^{-1}\mu_{\mathbf{z}_{|0}''}$. Besides, $\mu_{\mathbf{z}_{|1}''}$ and $\mu_{\mathbf{z}_{|0}''}$ are $\frac{N}{2}\times 1$ size vectors with all their members equal to $\alpha\mu_z$ and $\frac{\mu_z}{\alpha}$, respectively.

We can estimate μ, σ, and ρ parameters which are needed to decode the watermark data from the even part of the received signal $\mathbf{q}$ as follows:

$$\hat{\mu} = \mu_{y''}, \hat{\sigma} = \sqrt{\max(\sigma_{y''}^2 - \sigma_n^2, 0)}, \hat{\rho} = \sqrt{\rho_{y''}\frac{\hat{\sigma}^2 + \sigma_n^2}{\hat{\sigma}^2}} \tag{13}$$

Therefore, we see that the decoder parameters can be estimated from the received signal and thus our watermarking approach is a blind watermarking scheme.

Further simplifications are in order for the ML decoder in (12) that avoid inversion of the two covariance matrices. Consider simpler realization of the two conditional densities in (7). For $\rho_z \ll 1$, we were able to simplify the densities of $\mathbf{z}''$ as:

$$f(\mathbf{z}'') \simeq \frac{e^{-\frac{1}{2\sigma_{z''}^2}\sum_1^{\frac{N}{2}}\bar{z}_i''^2}}{\sqrt{(2\pi\sigma_{z''}^2)^{\frac{N}{2}}}}[1 + \frac{\rho_{z''}}{\sigma_{z''}^2}\sum_{i=2}^{\frac{N}{2}}(\bar{z}_i''\bar{z}_{i-1}'')] \tag{14}$$

Where $\bar{z}_i'' = z_i'' - \mu_z'$. Thus, after some simplification we can write the ML decision strategy as:

$$\begin{aligned} &\Big\{ -\frac{N}{2}\log\frac{\sigma_{z''|1}^2}{\sigma_{z''|0}^2} + (\sigma_{z''|0}^{-2} - \sigma_{z''|1}^{-2})\sum_{i=1}^{\frac{N}{2}} z_i''^2 \\ &-2\mu_z(\frac{\alpha^{-1}}{\sigma_{z''|0}^2} - \frac{\alpha}{\sigma_{z''|1}^2})\sum_{i=1}^{\frac{N}{2}} z_i'' + \frac{N}{2}\mu_z^2(\frac{\alpha^{-2}}{\sigma_{z''|0}^2} - \frac{\alpha^2}{\sigma_{z''|1}^2})\Big\} \\ &\gtrless_0^1 2\sum_{i=2}^{\frac{N}{2}}\log\left[\frac{1+\sigma_{z''|0}^{-4}\alpha^{-2}\rho_z\sigma_z^2(z_i''-\frac{\mu_z}{\alpha})(z_{i-1}''-\frac{\mu_z}{\alpha})}{1+\sigma_{z''|1}^{-4}\alpha^2\rho_z\sigma_z^2(z_i''-\alpha\mu_z)(z_{i-1}''-\alpha\mu_z)}\right] \end{aligned} \tag{15}$$

Where $\bar{z}_i'' = z_i'' - \mu_z'$, $\rho_{z''} = \rho_z \frac{\sigma_{z'}^2}{\sigma_{z'}^2+\sigma_\zeta^2}$, $\sigma_{z''|1}^2 = \alpha^2\sigma_z^2 + \sigma_\zeta^2$ and $\sigma_{z''|0}^2 = \alpha^{-2}\sigma_z^2 + \sigma_\zeta^2$. Again we see that μ_z, σ_z, and ρ_z are needed for data decoding which in turn depend upon μ, σ, and ρ which can be calculated using the estimated host signal parameters as in (13).

In summary, the ML detector is based on the ratio of the interleaved samples of the received signal. We will call this detector as Optimum Ratio Decoding (ORD). If we implement it using (12), then we will call it Gaussian ORD (G-ORD) since the only assumption in (12) is the Gaussian modeling of the host signal as in (5). On the other hand, if we implement it using (15) we will denote it as Markov ORD (M-ORD), since in addition to the Gaussian assumption we use the first order Markov assumption in the ratio signal z.

4. PERFORMANCE ANALYSIS

In this section we are to find the error probability of the proposed watermarking method in the presence of AWGN attack.

As given in Section 3.2, in the decoder we have the decision rule given in (12).

If we define $\mathbf{H} = \mathbf{C}_{|\mathbf{0}}{}^{-1} - \mathbf{C}_{|\mathbf{1}}{}^{-1}$, $\mathbf{k} = \mathbf{C}_{|\mathbf{0}}{}^{-1}\mu_{\mathbf{z}''_{|\mathbf{0}}} - \mathbf{C}_{|\mathbf{1}}{}^{-1}\mu_{\mathbf{z}''_{|\mathbf{1}}}$, and $\mathbf{s} = \mathbf{H^{-1}k}$, after some simplification we can rewrite (12) as:

$$(\mathbf{z}'' - \mathbf{s})^T\mathbf{H}(\mathbf{z}'' - \mathbf{s}) \gtrless_0^1 T_1' \tag{16}$$

where $T_1' = T_1 + \mathbf{s}^T\mathbf{H}\mathbf{s}$. Considering the fact that $\mathbf{C}$ is a symmetric matrixes, we can write its eigendecomposition as $\mathbf{C} = \mathbf{U}\text{diag}(\lambda_i)\mathbf{U}^T$, where $\mathbf{U}$ is the matrix of eigenvectors an is an orthogonal matrix, i.e., $\mathbf{U}\mathbf{U}^T = \mathbf{I}$. Thus, using the definition of $\mathbf{C}_{|\mathbf{1}}$ and $\mathbf{C}_{|\mathbf{0}}$, we can write their eigendecomposition respectively as $\mathbf{C}_{|\mathbf{1}} = \mathbf{U}\text{diag}(\lambda_{i|1} + \sigma_\zeta^2)\mathbf{U}^T$ and $\mathbf{C}_{|\mathbf{0}} = \mathbf{U}\text{diag}(\lambda_{i|0} + \sigma_\zeta^2)\mathbf{U}^T$, where $\lambda_{i|1} = \alpha^2\lambda_i$ and $\lambda_{i|0} = \frac{1}{\alpha^2}\lambda_i$. Therefore, $\mathbf{H}$ can be shown as

$$\mathbf{H} = \mathbf{U}\text{diag}\left(\frac{\lambda_{i|1} - \lambda_{i|0}}{(\lambda_{i|0} + \sigma_\zeta^2)(\lambda_{i|1} + \sigma_\zeta^2)}\right)\mathbf{U}^T \tag{17}$$

Substituting (17) in (16), we have:

$$(\mathbf{z}'' - \mathbf{s})^T\mathbf{U}\text{diag}\left(\frac{\lambda_{i|1} - \lambda_{i|0}}{(\lambda_{i|0} + \sigma_\zeta^2)(\lambda_{i|1} + \sigma_\zeta^2)}\right)\mathbf{U}^T(\mathbf{z}'' - \mathbf{s}) \gtrless_0^1 T_1'$$

If we define $\mathbf{d} = \text{diag}(g_i)\mathbf{U}^T(\mathbf{z}'' - \mathbf{s})$, where $g_i = \sqrt{\frac{\lambda_{i|1} - \lambda_{i|0}}{(\lambda_{i|0}+\sigma_{n_z}^2)(\lambda_{i|1}+\sigma_{n_z}^2)}}$, we can express the detector in terms of its sufficient statistics, D:

$$D = \sum_{i=1}^{\frac{N}{2}} {d_i}^2 \gtrless_0^1 T_1' \tag{18}$$

The characteristic function of d_i^2 which is the square of a Gaussian variable is given as $\frac{1}{\sqrt{(1-jw\sigma_{d_i}^2)}} \exp\{-\frac{jw\mu_{d_i}^2}{1-jw\sigma_{d_i}^2}\}$.

Moreover, since d_i^2's are independent, the characteristic function of D can be expressed in the product form:

$$\Psi_D(jw) = \prod_{i=1}^{\frac{N}{2}} \frac{1}{\sqrt{(1 - jw\sigma_{d_i}^2)}} \exp\{-\frac{jw\mu_{d_i}^2}{1 - jw\sigma_{d_i}^2}\} \tag{19}$$

In this expression, μ_{d_i} represents the mean of d_i, which for '1' or '0' embedding becomes $\mu_{d_i|1} = g_i\big(\mathbf{U}^T(\mu_{\mathbf{z}''_{|\mathbf{1}}} - \mathbf{s})\big)_i$ and $\mu_{d_i|0} = g_i\big(\mathbf{U}^T(\mu_{\mathbf{z}''_{|\mathbf{0}}} - \mathbf{s})\big)_i$, respectively.

Besides, $\sigma_{d_i}^2$, the variance of d_i, can be computed from the covariance matrix of $\mathbf{d}$ which can be shown to be diag(λ_i''). Thus, we have $\sigma_{d_i|1}^2 = g_i^2(\lambda_{i|1} + \sigma_n^2), \sigma_{d_i|0}^2 = g_i^2(\lambda_{i|0} + \sigma_n^2)$.

Now to find the error probability, we must find the distribution of D. Using the CLT, for a suitable large N, we can suppose D to have a Gaussian distribution. Using the characteristic function of D_i (19), the mean and variance of this distribution are calculated as:

$$\mu_D = \sum_{i=1}^{\frac{N}{2}}(\sigma_{d_i}^2 + \mu_{d_i}^2), \qquad \sigma_D^2 = \sum_{i=1}^{\frac{N}{2}}(2\sigma_{d_i}^4 + 4\sigma_{d_i}^2\mu_{d_i}^2)$$

The error occurs when we send '1' and decode '0'. Therefore, the error probability is given as:

$$\begin{aligned} P_e &= \frac{1}{2}P(D_{|1} < T_1') + \frac{1}{2}P(D_{|0} > T_1') \\ &= \frac{1}{2}[1 + Q(\frac{T_1' - \mu_{D|0}}{\sigma_{D|0}}) - Q(\frac{T_1' - \mu_{D|1}}{\sigma_{D|1}})] \end{aligned} \tag{20}$$

where $Q(x)$ is the error function.

Thus, we obtained a closed form solution for the error probability of our decoder.

5. PERFORMANCE EVALUATION

In this section, we investigate the performance of our watermarking method under AWGN attack with different noise powers. The experiments are given for different Document-to-Watermark Ratios (DWR). We fix DWR for each experiments and test our method for different Watermark-to-Noise Ratios (WNR). The experiments are conducted for two test cases of first order markov sequence of normally distributed random variables. For the first test case, we have $\mu = 1$, $\sigma = 0.2$, $\rho = 0.9$, and for the other one we have $\mu = 1$, $\sigma = 0.1$, $\rho = 0.7$. We test our detector with different DWRs of 25 fB, 16 dB and 14 dB. The results of the introduced detector for WNRs in the range of $[-6, 10]$ are given in Fig. 2. We can see our decoder is highly robust even against high noisy conditions with low WNR values. Besides, we see that for lower values of $A = \frac{\sigma^2}{\mu^2}$, the performance is better, which was expected as the approximations derived in the model is better hold for lower A values.

Finally, we compare our method with AQIM method [8] in the same DWR of 19 dB and bit rate of $1/32$. The results

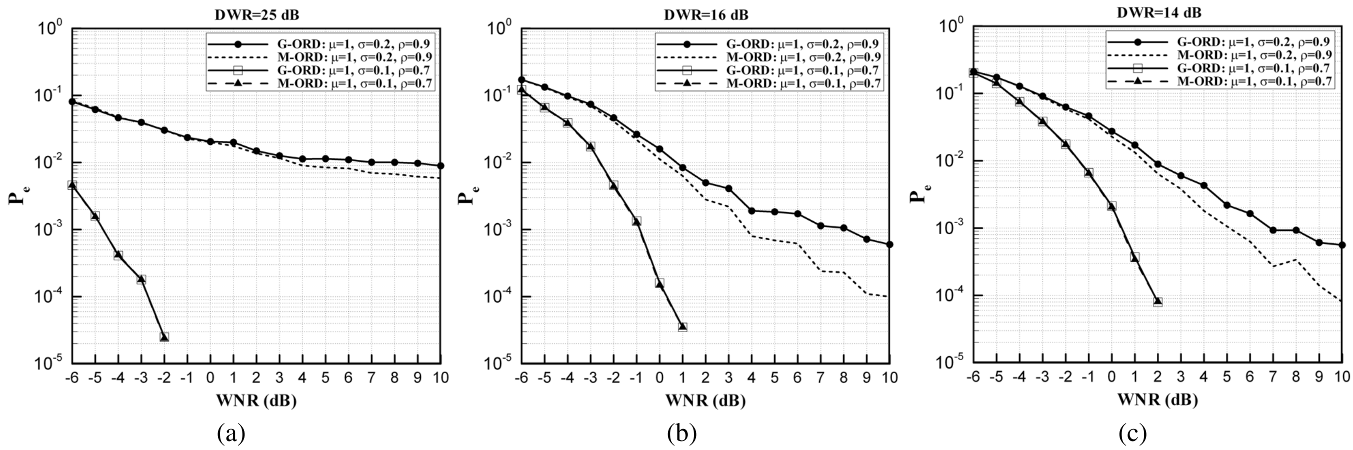

Fig. 2. Probability of error of different version of the ORD decoder for two test cases 1) $\mu = 1$, $\sigma = 0.2$, $\rho = 0.9$, 1) $\mu = 1$, $\sigma = 0.1$, $\rho = 0.7$. (a) DWR=25 dB, (b) DWR=16 dB, (c) DWR=14 dB

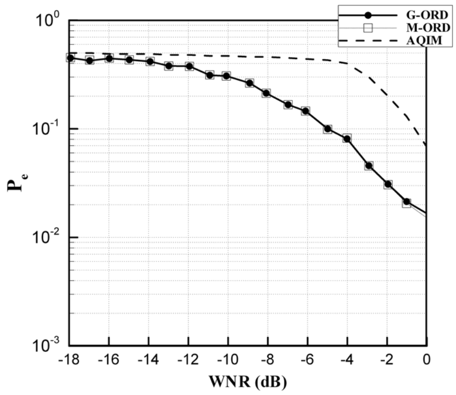

Fig. 3. Comparing the probability of error with DWR=$19dB$ between our method and the AQIM method [8]

are given in Fig. 3. We can see that our technique outperforms AQIM method.

6. CONCLUSION

A new scaling based data hiding approach suitable for different signals such as image and audio signals was introduced. For blind detection, using stationary characteristics, the algorithm is applied on a subset of the each frame of the host signal while the rest is kept unchanged. The host signal is supposed to be Gaussian with autoregressive model of order one. It can be shown that this model is useful for representing the low frequency components of the natural images. The distribution and spectral characteristics of the ratio samples in this model are analytically derived. The main characteristic of the proposed decoder is that it can be easily implemented for highly correlated signals. The error probability of the ML detector at the presence of AWGN is analytically calculated. Simulation results show a great robustness of the proposed method for low WNRs. The performance similarity between G-ORD and M-ORD confirms that our approximation for decreasing the computational cost is efficient.

7. REFERENCES

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