BER Analysis of Cooperative Systems in Free-Space Optical Networks

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***Abstract*—Free-space optical (FSO) communication suffers from several challenges in practical deployment; the major of them is fading or scintillation. To overcome such limitations, spatial diversity based on MIMO techniques has been proposed for FSO systems. User Cooperation diversity is a new form of spatial diversity which is introduced to overcome some limitations of MIMO structures. Although the promising effects of cooperative transmission in RF communications have greatly been considered so far, there have been a few notable research on cooperative diversity in FSO. In this paper, we first consider a 3-way FSO communication setup, in which the cooperative protocol can be applied to achieve the spatial diversity without much increase in hardware, compared to their regular transmission. Then, we introduce different cooperative strategies and investigate their bit error rate (BER) performance in the presence of shot noise, using the photon-count method. We compare the results with those of the direct path link (non-cooperative scheme) and the two transmitters case, which are the upper and lower bounds on the BER of the cooperative scheme, respectively. The results illustrate the advantages of cooperation under a number of different scenarios for realistic SNRs.**

***Index Terms*—BER analysis, cooperative systems, free-space optics, spatial diversity, shot noise.**

I. Introduction

FREE-SPACE OPTICAL (FSO) communication, which has attracted significant attentions recently, provides the essential combination of qualities required to bring the traffic to the optical fiber backbone [1]–[4]. Virtually unlimited bandwidth, low cost, ease and speed of deployment, and excellent security are among the most attractive features of a FSO communication [3], [4]. It is also a promising solution for the "last mile" problem. FSO is becomming a good solution for point to point communications between fixed locations on land, and is also used for communications between moving platforms on land, the surface of the sea, and in air.

The advantages of FSO do not come without some costs. Aerosol scattering, caused by rain, snow, and fog, results in performance degradation [2]. Another possible impairment of FSO links is building-sway as a result of wind loads, thermal expansion, and weak earthquakes [1]. But the major problem is the need to combat link fading due to scattering and scintillation. Links may experience fading due to inhomogeneities of the index of refraction in the optical beam. This phenomenon, which leads to stochastic amplitude (and power) fluctuations at the receiver, degrades the link performance particularly for distances of 1 km and above [4]–[6].

To overcome such limitations, both error control coding [7], [8] and multiple input multiple output (MIMO) techniques [9]–[11] have been proposed for FSO systems. The latter have been shown to significantly improve the system performance in spatially uncorrelated channels by creating additional degrees of freedom via spatial diversity [9]–[11]. The spatial diversity of MIMO systems overcomes the degradation of the performance caused by fading.

Due to size, cost, and hardware limitations, a wireless device may not always be able to support multiple transmit antennas. To overcome this limitation, a new form of space diversity, known as user cooperation diversity, has been recently proposed [12]–[14]. This new technique takes the advantage of the broadcast nature of the radio channel allowing (single-antenna) terminals in a multiuser environment to share their physical resources in order to create a virtual transmit and/or receive array through distributed transmission and signal processing. Cooperative transmission can dramatically improve the performance by creating diversity using the antennas available at the other nodes of the network. It has been shown that node cooperation is an effective way of providing diversity in wireless fading networks [12]–[14].

Although the promising effects of cooperative transmission in RF communications have greatly been considered so far, there have been a few notable research on cooperative diversity in FSO [15], [16]. The authors, Safari-Uysal, in [15] have investigated the performance of cooperative diversity schemes over log-normal fading channels, but they have not considered the FSO links and their constraints in special. In [16], both serial (multi-hop transmission) and parallel (i.e., cooperative diversity) relaying encoupled with amplify-and-forward and decode-and-forward modes have been studied. The main problem with the cooperative transmission in FSO is that the laser propagation is line of sight. That is while in RF transmission, the propagation angle of the antenna is wide enough that both the relay and destination nodes can receive the signal, but in FSO, the relay node can not see the transmitter and consequently can not cooperate in the transmission. So, there must be another laser for sending the signal from the transmitter to the relay which means extra hardware compared to the RF case. In this paper, we consider a 3-way FSO communication setup in which there is a mutual communication between nodes A and B and a further communication link between nodes A and B to C. [1] (see Fig. 1). For

[1]There are many proposals, such as [17] and [18] for example, that considered complex FSO networks for future use, in which the nodes act as both a router, and a transceiver.

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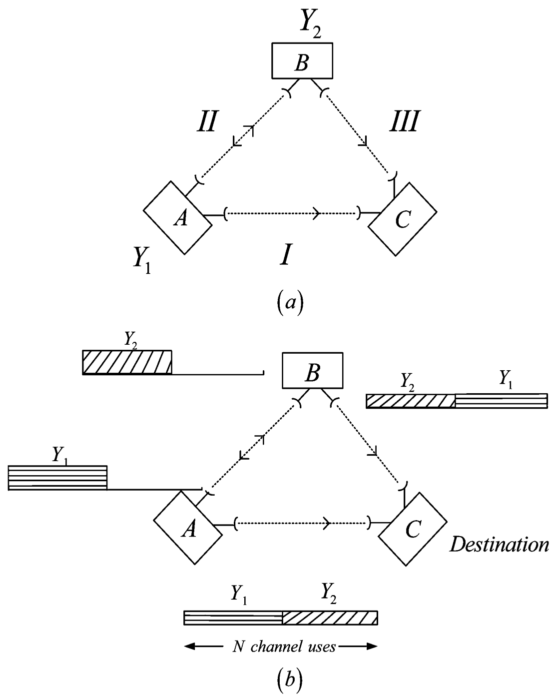

Fig. 1. (a) The block diagram of the considered 3-way communication system. (b) Data frame transmission of cooperative nodes A and B to the destination node C.

this senario, we propose a cooperative communication between nodes A and B to achieve the spatial diversity without much change in hardware since telescopes of the link A-B have existed previously in the system.

We assume a full-duplex mode; that is both of the endpoints transmit and receive at the same time. Note that practical FSO transmitter-reciever strctures work in full-duplex mode; so it is a reasonable assumption. We also assume that the transmitted signals are convolutionally coded, but the framework is flexible and can be used with any channel error correcting code. We introduce different cooperative strategies and derive their bit error rates (BER) in the presence of shot noise, using the photon-count method. We compare the results with those of the direct path link (non-cooperative scheme) and the two transmitters case. The results illustrate the advantages of cooperation under a number of different scenarios for realistic SNRs.

The paper is organized as follows. Section II develops the proposed system and its cooperative structure and presents the assumptions regarding the link fading. In Section III, we introduce the cooperative protocols. In Section IV, we derive the BER performance analysis for the strategies introduced in Section III. We also provide the BER for the direct path link for comparison. In Section V, we present some numerical results to compare the cooperative strategies with each other and with the non-cooperative communication, and finally in Section VI, we conclude the paper.

II. System Model

Fig. 1(a) depicts the block diagram of the considered 3-way FSO setup. Assume that three separate full-duplex FSO links I, II, and III are set up between three buildings A, B, and C. From now on, we consider the buildings as nodes. There are two telescopes on the top of each building, one for each FSO link. Both of the telescopes of the nodes A and B are connected to the same source. That is, as can be seen from the figure, node A (B) sends the same data on the links connected to it towards the nodes B (A) and C.

For the 3-way FSO communication setup considered, a cooperative protocol can be applied to achieve the spatial diversity without much increase in hardware. To this end, we consider a cooperative transmission of two nodes A and B to the destination node C [2]. The nodes A and B change their regular transmission to each other and to node C to benefit from cooperative strategies. In our cooperative protocol, the source nodes (A and B) allocate half of their transmission times to each other for the cooperation as shown in Fig. 1(b). To this end, their transmission times to the node C are divided into two parts, each called as a frame. The nodes A and B send their own data in the first part of their transmission time to each other and the destination node C (see Fig. 1(b)). We emphasize again that node A (B) transmits the same information to the nodes B (A) and C. In the second transmission frame, the node A (or B) sends the received data from its partner B (or A) in the first frame to the node C. The way that A and B send their partner's data in the second frame is specified by the cooperative strategy. The considered cooperative strategies will be introduced in Section III.

Consider the source node A that its data is shown with Y_1 and horizontal lines in Fig. 1(b). We assume the same power constraint P_s for all transmitters (or links). The link A-B is the source-relay link for both A and B. As can be seen in the figure, we assume that A sends the same coded block to both B and C in the first frame. But, the signal is sent to B with the doubled power to reach the power constraint P_s of the transmitter [3]. The node B also uses the same strategy as A. So these nodes act like a relay for each other.

Assume that the bit stream of the user is applied to a convolutional encoder. For cooperation, this coded stream is sent to both the destination and the relay. Under weak turbulence conditions, the system performance can be improved considerably by using channel coding. However, in the cases of moderate to strong turbulence, channel coding alone is not sufficient to mitigate channel fading efficiently [19] [4]. As stated above, the relay can transmit the cooperative signal in different manners, which will be discussed later in Section III.

Several models exist for the aggregate amplitude distribution, but none is universally accepted. The most prominent among the models is the log-normal model [20], [21], and we adopt it here. Let h be the fading coefficient, then we have $|h| = e^X$, where X is normal. In this case, the optical intensity, proportional to $|h|^2 = e^{2X}$, is also log-normally distributed. Since we

[2]For the case that these nodes are not to send their data to each other, they can simply direct their second telescopes to the destination node C; and as a result by using two-transmitter can achieve a higher performance improvement.

[3]Note that the data is sent in the link A-B only in the first frame (the half time of the other link).

[4]Performance bounds for coded FSO communications by interleaver are derived in [7]. We do not use interleaver, so no diversity is gained through coding. This allows us to consider only the cooperative diversity.

use photon counting method for detection, the intensity of the received pulse is important.

We use binary pulse position modulation (PPM). That is the bit duration is divided into 2 chips, and the pulse is transmitted in the first or the second chip if the bit is "0" or "1", respectively. The photo-detector analysis used is based on a semiclassical treatment of photodetection, where the incident field is viewed as a wave, and this wave produces a modulated Poisson point process of photo-electrons that contribute to the detector current at the output. The number of photoelectrons at the output of the photo detectors is a poission process. The mean of this process is proportional to the mean of optical intensity received by the photodetector [22]. We assume that all the nodes use the same powers and, in the absence of fading, the mean of the received photon counts per chip due to the signal is m_s. In addition to the desired signal, we presume the presence of a background optical field, and the mean of the received photon counts per chip due to this field is denoted as m_b.

Throughout the paper, we need the statistical properties of X where $|h| = e^X$. X, as considered, is a normal random variable with mean μ_X and variance σ_X^2. The variance of X shows the intensity of fading and can be written as a function of the distance as [21]

$$\sigma_X^2 = \min\left\{0.124k^{\frac{7}{6}}C_n^2 l^{\frac{11}{6}}, 0.5\right\} \quad (1)$$

where k is the wave number, l is the length of the path, and C_n is the Refractive index structure constant. σ_X^2 is typically between 0.01 (weak fading) and 0.4 (strong fading) [21]. In FSO links, the data rate is high enough that bit duration is much smaller than the coherence time of the channel. So we can well assume that $|h|$ is constant during a code block transmission. By this assumption, if the channel falls in deep fade, all the coded bits of the block may corrupt. Several methods have been proposed to overcome this problem, such as interleaving, but in this paper we do not use these methods in order to focus only on the cooperative diversity. To consider only the fading effects, we assume that the fading coefficients conserve the energy. To this end, we must have $E[|h|^2] = 1$. It can easily be observed that this assumption results in the following equation, which we use through out the paper

$$\mu_X = -\sigma_X^2. \quad (2)$$

In the following sections, the fading coefficients for the paths A-B, A-C, and B-C are indicated by h_{ab}, h_{ac}, and h_{bc}, respectively. Here, we assume that all the coefficients are independent log-normally distributed and we have

$$|h_{ac}| = e^{X_{ac}} |h_{ab}| = e^{X_{ab}} |h_{bc}| = e^{X_{bc}}$$
$$X_{ij} \sim N\left(\mu_{X_{ij}}, \sigma_{X_{ij}}^2\right). \quad (3)$$

III. Cooperative Strategies

In this section, we introduce different cooperative strategies for the structure described in Section II. Different cooperative protocols have been proposed for RF communications such as Amplify and Forward (AF), Decode and Forward (DF) [14], and Coded Cooperation [23]. The Protocols considered here are mainly the modified versions of DF strategy. To introduce the cooperative strategies, as stated before, we assume that C is the destination node and the source nodes A and B send their signals to the C with cooperation as shown in Fig. 1(b).

A. Bit Detect and Forward (BDF)

In this strategy, the relay (partner) node detects each code bit of the cooperative signal individually and forwards it to the destination, regardless of the underlying channel coding. In other words, the relay node (A or B) detects each code bit to "0" or "1" and sends the bit with the new power to the destination node C without applying the error correcting decoder to correct the possible channel errors in the received block.

B. Adaptive Bit Detect and Forward (ABDF)

As we might expect, fixed detect and forward is limited by the quality of the link between the source and relay [14]. So, if the partners adapt their transmission format according to the realized quality of the source-relay link, they achieve much better performance.

This observation suggests the following adaptive algorithm, named as ABDF. The cooperative strategy is similar to the BDF, but if the link quality, i.e., $|h_{ab}|^2$, falls below a certain threshold th, the nodes A and B do not participate in cooperation and simply repeat their own transmissions to the destination node C. It means that if the relay can not decode the partner's message, it repeats its own coded block, transmitted in the first frame, in the second transmission frame to C again. From the optimum detector point of view described in Appendix A, sending the coded block twice is similar to sending it once with the doubled power. As stated above, the performance of the detect and forward strategy is limited by the quality of the source-relay link. So in ABDF strategy, the source nodes A and B participate in cooperation only if the source-relay link (A-B) is in an acceptable quality.

C. Adaptive Decode and Forward (ADF)

In the two previous mentioned strategies, the relay nodes do not benefit from the underlying channel code applied to the received signal. But in the ADF, the relay node decodes the bit stream and transmits the cooperative signal according to the decoded data. It is expected that the ADF strategy performs better compared to BDF, because some of the incorrectly detected bits can be corrected by the channel decoder.

This strategy is like the coded cooperation scheme introduced in [23]. The relay encodes and transmits the date only if it can correctly decode the received message, determined by a cyclic redundancy check (CRC) code applied to each code block. If the relay cannot correctly decode the message, like the ABDF strategy, it simply sends its own data transmitted in its first frame. The users act independently in the second frame, with no knowledge of whether their own first frames were correctly decoded by their partners. Each of the nodes A and B may decode the data correctly or not. As a result, there are four possible cooperative cases for the transmission of the second frame. In *Case 1*, both users A and B successfully decode their partners' data, so they encode and transmit the partners' data in the second frame, resulting in the fully cooperative scenario. In *Case 2*, neither user successfully decodes its partners data, and the system reverts to the non-cooperative case and A and B repeat their own messages sent in their first frames. In *Case 3*, B successfully decodes A, but A does not successfully decode the signal of B. *Case 4* is identical to Case 3 with the roles of node A and node B being reversed. We parameterize the four cases by $\phi \in \{1,2,3,4\}$ with probability P_ϕ.

Clearly the destination must know which of these four cases has occurred in order to correctly decode the received bits. We assume that totally two additional bits are transmitted by partner nodes (one bit by each partner) to indicate their states to the destination node C [23].

IV. Performance Analysis

In this section, we first derive the performance analysis for the direct path link to establish the baseline performance and then derive the BER of the cooperative protocols.

For a convolutional code, only the lower and upper bounds on the bit error rate are analytically available. Let the path generating function of the code be $T(X,D)$. The number of error events with weight of d, i.e., A_d, and the number of bit errors due to an error event with weight d, i.e., B_d, can be calculated from the path generating function as follows [24]:

$$T(X,D)|_{D=1} = \sum_{d=d_{\text{free}}}^{\infty} A_d X^d$$
$$\frac{\partial T(X,D)}{\partial D}|_{D=1} = \sum_{d=d_{\text{free}}}^{\infty} B_d X^d \tag{4}$$

where d_{free} is the free distance of the code. Using union bound, it can be easily shown that for a convolutional code, the error event probability and the upper bound on the bit error rate are as follows [24]:

$$P(E) < \sum_{d=d_{\text{free}}}^{\infty} A_d P_d$$
$$P_b < \frac{1}{k}\sum_{d=d_{\text{free}}}^{\infty} B_d P_d \tag{5}$$

where k is the number of information bits per unit time, B_d is given in (4), and P_d, an error event with Hamming distance of d, is the probability that the metric of a nonzero path with output Hamming weight of d is larger than that of the all zero path conditioned on all-zero bit being transmitted. Assuming a hard input decoder[5] (a binary symmetric channel (BSC)), P_d can be calculated from the error probability of each code bit (channel bit error probability), denoted by p. For small p, the bounds in (5) are dominated by the first term (the free distance term), and can be approximated as [24]

$$P(E) \approx A_{d_{\text{free}}} 2^{d_{\text{free}}} p^{d_{\text{free}}/2}$$
$$P_b \approx \frac{1}{k} B_{d_{\text{free}}} 2^{d_{\text{free}}} p^{d_{\text{free}}/2} \tag{6}$$

where k, A_d, and B_d are given in (4). We assume that the SNR is high enough and so p is low enough that the approximations of (6) are appropriate. So, for the performance evaluation, we need to compute p. As it was stated previously, we use the binary PPM modulation. In the receiver, the detector input for each bit is the number of photo electrons collected in the two corresponding chips. For the bit error rate evaluation, without loss of generality, we consider node A as source and node B as its relay (the same results hold when node B is considered as the source and node A as its relay).

First, we compute the channel bit error probability, i.e., p, for the direct transmission link (non-cooperative link) A-C. We show the number of received photoelectrons in the first and second chips by y_0 and y_1, respectively. In Appendix A, it has been shown that the detector simply compares y_0 and y_1, and if the photoelectrons collected in the first chip duration is greater than those collected in the second chip duration $(y_0 > y_1)$), "0" is detected, otherwise "1" is declared. So, conditioned on $|h_{ac}|$, the channel bit error probability, i.e., $p_{D||h_{ac}|}$, is as follows:

$$p_{D||h_{ac}|} = P\{y_0 > y_1|1\} = P\{A = y_0 - y_1 > 0|1\}. \tag{7}$$

To evaluate (7), based on using the saddle point approximation, we must first compute the moment generating function (MGF) of A. Conditioned on $|h_{ac}|, y_0$ and y_1 are poission random variables. Let α be a constant and y be a poission random variable with mean m, then the MGF of αy is as follows:

$$\Psi_{\alpha y}(s) = \exp[m(e^{\alpha s} - 1)] \tag{8}$$

Then, the MGF of A, defined in (7), conditioned on $|h_{ac}|$, is equal to

$$\begin{aligned}\Psi_{A||h_{ac}|}(s) &= \Psi_{y_0||h_{ac}|}(s)\Psi_{y_1||h_{ac}|}(-s)\\ &= \exp[m_b(e^s - 1) - (m_b + |h_{ac}|^2 m_s).\\ &\quad \times (e^{-s} - 1)]\end{aligned} \tag{9}$$

Using (9), we can compute the conditional channel bit error probability by saddle point approximation as [22]

$$p_{D||h_{ac}|} = \frac{\exp[\psi_{A||h_{ac}|}(s_0)]}{\sqrt{2\pi\psi''_{A||h_{ac}|}(s_0)}}$$
$$\psi_{A||h_{ac}|}(s) \triangleq \ln[\Psi_{A||h_{ac}|}(s)] - \ln|s|. \tag{10}$$

s_0 is the positive root of $\psi'_A(s_0) = 0$, where ψ'_A is the first derivative of ψ_A. s_0 is computed numerically in this paper. Now, by using (10) and (6), the bit error rate for the direct link is computed as

$$P_{bD||h_{ac}|} = \frac{1}{k} B_{d_{\text{free}}} 2^{d_{\text{free}}} p_{D||h_{ac}|}^{d_{\text{free}}/2}. \tag{11}$$

To derive the unconditional bit error rate for the direct link, we must average (11) over $|h_{ac}|$ as follows:

$$\begin{aligned}P_{bD} &= \int_0^\infty P_{bD||h_{ac}|} p_{|h_{ac}|}(x)dx\\ &= \int_0^\infty P_{bD||h_{ac}|} \frac{1}{x\sigma_{X_{ac}}\sqrt{2\pi}} e^{-\frac{(\ln(x)-\mu_{X_{ac}})^2}{2\sigma^2_{X_{ac}}}} dx.\end{aligned} \tag{12}$$

[5]Here, we assume a hard input decoder. A soft input decoder performs much better than the hard input decoder, especially in fading channel. Since the concentration of the paper is not on the coding gain, but on the cooperative gain, we only considere hard decoding, as the appropriate equation of (6) for the performance evaluation of this decoder has been already available.

In the following, we derive the BER probabilities for the cooperative transmission strategies described in Section III. As stated before, we consider the link A-C as the direct link, and A-B-C as the relay path. For the performance evaluation, we first compute the conditional BER for specific values of fading coefficients, we indicate them by a vector defined as $H = \{|h_{ac}|, |h_{ab}|, |h_{bc}|\}$. The unconditional BER is calculated by averaging over H.

A. Bit Detect and Forward (BDF)

As stated previously, the relay node detects each code bit and forward it to the destination, regardless of the channel coding. We derive the BER of the node A transmission, when the node B is the relay node. The destination node C receives two pulses for each bit, one from A through the link A-C, and the other from B through the link A-B-C. There are two possible choices for the pulse received by C from the relay node B; the pulse may be detected correctly in B or not. Because of the possible detection error at node B, the optimum detector is very complicated. So we consider the detector described in Appendix A, which assumes that all the bits received from the relay path A-B-C are detected correctly at B and are resended to C. Here, in the detector, we have two received signals, so the parameter N defined in the Appendix A is 2. From Appendix A, the detector makes decision according to the following rule:

$$(y_{11} - y_{01}) \ln\left(\frac{m_{11}}{m_{01}}\right) + (y_{12} - y_{02}) \ln\left(\frac{m_{12}}{m_{02}}\right) \gtrless_0^1 0. \quad (13)$$

In (13) each parameter is indexed by ij, where the first number shows the index of the chip and the second one shows the index of the path. The same as the direct path link, we use the indexes of 0 and 1 for differentiating the first and second chips of each bit interval. We also set the indexes of 1 and 2 for the signal received by C from the source node A and the relay node B, respectively. For example y_{11} is the number of photoelectrons collected in the second chip of the signal received by C from the node A. Conditioned on H, y_{ij} is a poission random variable. We show the mean of this variable by m_{ij}. If the bit "0" is sent from A to C (the pulse is transmitted in the first chip) we have $m_{01} = m_b + |h_{ac}|^2 m_s$, and if the bit "1" is sent (the pulse is transmitted in the second chip) we have $m_{01} = m_b$. Similar equations can be written for the bits sent from B to C. The bits "0" and "1" are equally probable, so without loss of generality, we assume that "1" is sent. We have the following equations:

$$\begin{aligned} \frac{m_{11}}{m_{01}} &= \frac{m_b + |h_{ac}|^2 m_s}{m_b} = 1 + \gamma |h_{ac}|^2 \triangleq \gamma_{ac} \\ \frac{m_{12}}{m_{02}} &= \frac{m_b + |h_{bc}|^2 m_s}{m_b} \\ &= 1 + \gamma |h_{bc}|^2 \triangleq \gamma_{bc} \ \gamma \triangleq \frac{m_s}{m_b}. \end{aligned} \quad (14)$$

So, by using (13) and (14), the conditional channel bit error probability is as follows:

$$\begin{aligned} p_{\mathrm{BDF}|H} = P\{A = &\ln(1 + \gamma |h_{ac}|^2)(y_{01} - y_{11}) \\ &+ \ln(1 + \gamma |h_{bc}|^2)(y_{02} - y_{12}) > 0 | 1, H\}. \end{aligned} \quad (15)$$

Like the procedure followed for the performance evaluation of direct link, we first find the MGF of A conditioned on H. The MGF of A has two terms, one for the case of correct detection in node B (with the probability p_r), and the other for the case of incorrect detection in node B (with the probability $1 - p_r$). p_r is related to the fading coefficient of the source-relay link (A-B). In fact, p_r is the channel (link A-B) bit error probability, and conditioned on H, this probability is computed by the same procedures as taken in (7)–(10). Assume that the bit "1" is transmitted. If the bit is detected correctly in the relay, the mean of y_{02} and y_{12} will be m_b and $m_b(1 + \gamma |h_{bc}|^2)$,respectively, and the means are replaced otherwise (incorrect detection). Conditioned on H, y_{ij}'s in (15) are independent, so, from (6) and (8), we can write the MGF of A as

$$\begin{aligned} &\Psi_{\mathrm{A|H}} \\ &= \exp\left[k m_b \left[e^{s \ln \gamma_{ac}} - 1 + \gamma_{ac}\left(e^{-s \ln \gamma_{ac}} - 1\right)\right]\right] \\ &\quad \times \left\{(1 - p_{r|H}) \exp\left[k m_b \left[e^{s \ln \gamma_{bc}} - 1 + \gamma_{bc}\right.\right.\right. \\ &\quad \left.\left.\times \left(e^{-s \ln \gamma_{bc}} - 1\right)\right]\right] \\ &\quad \left. + p_{r|H} \exp\left[k m_b \left[\gamma_{bc} e^{s \ln \gamma_{bc}} - 1 + \left(e^{-s \ln \gamma_{bc}} - 1\right)\right]\right]\right\} \end{aligned} \quad (16)$$

where γ_{ac} and γ_{bc} are defined in (14). Then using (15) and (16), the channel bit error probability is computed as

$$\begin{aligned} p_{\mathrm{BDF}|H} &= \frac{\exp[\psi_{A|H}(s_0)]}{\sqrt{2\pi \psi''_{A|H}(s_0)}} \\ \psi_{A|H}(s) &\triangleq \ln[\Psi_{A|H}(s)] - \ln|s|. \end{aligned} \quad (17)$$

Then, from (6) and (17), we have

$$P_{b\mathrm{BDF}|H} = \frac{1}{k} B_{d_{\mathrm{free}}} 2^{d_{\mathrm{free}}} p_{\mathrm{BDF}|H}^{d_{\mathrm{free}}/2}. \quad (18)$$

To find the unconditional bit error rate, i.e., $P_{b\mathrm{BDF}}$, we must average (18) over H like (12).

B. Adaptive Bit Detect and Forward (ABDF)

As stated in Section III, in this strategy, if the measured $|h_{ab}|^2$ falls below a certain value, the relay node B simply repeats its own signal, transmitted in the first frame, into the second frame. Otherwise, when $|h_{ab}|^2$ is above the threshold, the relay forwards the received data from the source, as described in subsection A. We denote the threshold by th and compute it as

$$P\{T = |h_{ab}|^2 < \mathrm{th}\} = \int_0^{\mathrm{th}} p_{|h_{ac}|^2}(t) dt = 1 - \alpha \quad (19)$$

where α is an arbitrary chosen constant that can be considered as the level of cooperation. The variable T in (19) has Lognormal distribution. Then, its cumulative distribution function (CDF) is as follows:

$$F_T(t) = 1 - Q\left(\frac{\ln(t) - \mu_T}{\sigma_T}\right). \quad (20)$$

From (19) and (20), we easily have

$$Q\left(\frac{\ln(\mathrm{th}) - \mu_T}{\sigma_T}\right) = \alpha. \quad (21)$$

Then th is obtained by solving (21) for a given α. For the performance evaluation, Note that for $|h_{ab}|^2 > \text{th}$ and $|h_{ab}|^2 < \text{th}$, we can use (18) and (11), respectively. So the conditional BER for the ABDF strategy is as follows:

$$P_{b\text{ABDF}|H} = \mathbf{1}(|h_{ab}|^2 > \text{th})P_{b\text{BDF}|H} + (1-\alpha)P_{bD||h_{ac}|} \quad (22)$$

where $P_{bD||h_{ac}|}$ and $P_{b\text{BDF}|H}$ are derived in (11) and (18), respectively. $\mathbf{1}(y)$ is 1 if y is true and 0 otherwise. To find the unconditional bit error rate, we must average (22) over H like (12).

C. Adaptive Decode and Forward (ADF)

As mentioned in Section III, in the ADF strategy the relay encodes and transmits the data only if it can correctly decode the received message from the partner in the first frame, which can be determined by a cyclic redundancy check (CRC) code. We denote the block decoding failure probabilities of nodes A and B by $P_{\text{block}A}$ and $P_{\text{block}B}$, respectively. The exact derivations of $P_{\text{block}A}$ and $P_{\text{block}B}$ are very complicated. In [25], an upper bound for the block decoding failure has been derived as follows:

$$P_{\text{block}|H} \leq 1 - (1 - P(E|H))^B \quad (23)$$

where B is the number of trellis branches in the code word, and $P(E|H)$ is the probability of an error event conditioned on H. To make the bound tighter, the authors in [26] have introduced the limit-before-average technique and have shown that this bound is good for fading channels. So by using the method of [26], conditioned on $H, P_{\text{block}A}$ and $P_{\text{block}B}$ are independently computed from (5) as follows:

$$P_{\text{block}A|H} = P_{\text{block}B|H} = 1 - \left(1 - \min\left(1, \sum_{d=d_{\text{free}}}^{\infty} A_d P_{\text{d|H}}\right)\right)^B \quad (24)$$

where A_d is given in (5). For the performance evaluation, the same procedure as taken in subsection A for BDF is used. In the detector, we have two received signals, so the N defined in the Appendix A is 2. Again we denote the signals received by the destination node C from the source node A and the relay node B by the indexes of 1 and 2, respectively, and use the same notations as those utilized in subsection A. So, by using (13) and (14), the conditional channel bit error probability is computed as follows:

$$p_{\text{ADF}|H} = P\{A = \ln(1+\gamma|h_{ac}|^2)(y_{01} - y_{11}) + \ln(1+\gamma|h_{bc}|^2)(y_{02} - y_{12}) > 0|1, H\} \quad (25)$$

where y_{ij}'s are defined in (13). Like the previous strategies, we first derive the MGF of A conditioned on H. The MGF of A conditioned on H computed by averaging over 4 terms, each term is due to one of the 4 cases defined in Section III-C for the ADF strategy. That is

$$\Psi_{A|H} = \sum_{\phi=1}^{4} P_{\phi|H}\Psi_{A|H,\phi} \quad (26)$$

where P_ϕ's are defined in Section III-C and can be calculated easily by using (24). For instance, for $\phi = 3$ (which means that the node B decodes the node A data successfully, but the node A does not), since the block decoding failures independently occur in A and B, we have

$$P_{\phi=3|H} = P_{\text{block}A|H}(1 - P_{\text{block}B|H}) \quad (27)$$

where $P_{\text{block}A|H}$ and $P_{\text{block}A|H}$ are derived in (24). $P_{\phi|H}$ for other 3 cases can be computed the same way as (27).

Now, from (26), $\Psi_{A|H,\phi}$ must be computed. we compute $\Psi_{A|H,\phi}$ for $\phi = 3$. The three others can be evaluated similarly. Since we set A-C as the direct path, B acts as the relay node that have decoded the message from the source node A correctly, so B relays the node A message to the destination node C in its second transmission frame. On the other hand, since A has not decoded the message from B correctly, A repeats its own data, transmitted in the first frame, in the second transmission frame to C as well. So both A and B send the signal of node A in their second transmission frames to C. For this case, the conditional MGF of A, defined in (25), is easily computed as follows:

$$\begin{aligned}\Psi_{A|H,\phi=3} &= \exp\left[2km_b\left[e^{s\ln\gamma_{ac}} - 1 + \gamma_{ac}\left(e^{-s\ln\gamma_{ac}} - 1\right)\right]\right] \\ &\times \exp\left[km_b\left[e^{s\ln\gamma_{bc}} - 1 + \gamma_{bc}\left(e^{-s\ln\gamma_{bc}} - 1\right)\right]\right]. \end{aligned} \quad (28)$$

The coefficient 2 in the exponent of the first term is due to the fact that node A does not participate in cooperation and repeats its own data. The MGF of A in other 3 cases can be easily computed the same way as (28). The results are as follows:

$$\begin{aligned}\Psi_{A|H,\phi=1} &= \exp\left[km_b\left[e^{s\ln\gamma_{ac}} - 1 + \gamma_{ac}\left(e^{-s\ln\gamma_{ac}} - 1\right)\right]\right] \\ &\quad\times \exp\left[km_b\left[e^{s\ln\gamma_{bc}} - 1 + \gamma_{bc}\left(e^{-s\ln\gamma_{bc}} - 1\right)\right]\right] \\ \Psi_{A|H,\phi=2} &= \exp\left[2km_b\left[e^{s\ln\gamma_{ac}} - 1 + \gamma_{ac}\left(e^{-s\ln\gamma_{ac}} - 1\right)\right]\right] \\ \Psi_{A|H,\phi=4} &= \exp\left[km_b\left[e^{s\ln\gamma_{ac}} - 1 + \gamma_{ac}\left(e^{-s\ln\gamma_{ac}} - 1\right)\right]\right]. \end{aligned} \quad (29)$$

Then, by substituting (27), (28) and (29) in (26), $\Psi_{A|H}$ is computed and finaly $P_{b\text{ADF}|H}$ is derived by placing (26) in (17).

V. Numerical Results

In this section, we evaluate the performance of the proposed cooperative strategies, using analytical results obtained in Section IV. For comparison with the direct path link, we assume that in the non-cooperative case, the source node uses the same code as the cooperative strategies with the doubled power to reach the same power constraint. So, the channel coding gain has no effect on the performance improvement of cooperative schemes over non-cooperative one, and we can focus only on the possible gain obtained by the cooperation. For the performance evaluation of the non-cooperative two transmitters case, we can set $p_{r|H} = 0$ in (16) and easily use the results derived for the BDF strategy. In all the figures, the parameter α defined in (17) for ABDF and the parameter B defined in (23) for ABD are set to 0.98 and 20, respectiveley. For evaluation of the channel bit error probability, we use the approximate equation of (6). The convolutional code used is the best (3, 2, 2) code (with maximal free distance) [24]. The

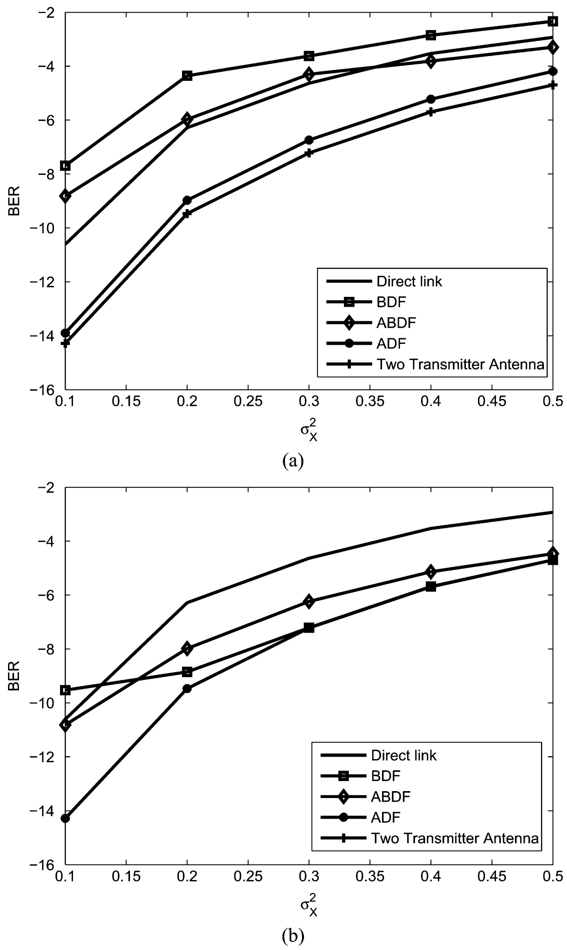

Fig. 2. Plots of BER versus σ^2_X where $\sigma^2_{X_{ac}} = \sigma^2_{X_{bc}} = \sigma^2_X$. (a) All the three links in Fig. 1 have the same statistics; $\sigma^2_{X_{ab}} = \sigma^2_X$ (b) $\sigma^2_{X_{ab}}$ is kept constant and equal to 0.05. $m_s = 40, m_b = 3, \alpha = 0.98, B = 20$.

convolutional code used is a (3,2,2) code with rate 2/3, free distance of 4 ($d_{\rm free} = 4$), and following transfer matrix [24]:

$$G(D) = \begin{pmatrix} 1 & D & 1+D \\ D^2 & 1 & 1+D+D^2 \end{pmatrix}. \tag{30}$$

Assume that the links A-C and B-C in Fig. 1 have the same statistics; it means that $\sigma^2_{X_{ac}} = \sigma^2_{X_{bc}} = \sigma^2_X$. Fig. 2 shows the plots of BER versus σ^2_X for two cases. In Fig. 2(a), all the three links of Fig. 1 have the same statistics and the parameters $\sigma^2_{X_{ab}}$ related to the source-relay link quality is equal to σ^2_X. In Fig. 2(b), $\sigma^2_{X_{ab}}$ is kept constant and equal to 0.05. As can be realized, there is a considerable performance improvement in all cooperative strategies over non-cooperative case when the source relay link is in good quality (Fig. 2(b)). This improvment is much more observable when the fading in the links B-C and A-C are strong. In both cases, ADF has the best performance among the cooperative strategies. From Fig. 2(a), when the link quality between the partners is the same as the links between the sources and the destinatin, the BDF and ABDF strategies perform worse than non-cooperative one at moderate SNRs, but ADF still performs well, the same as the two transmitters case.

Since source-relay link quality is critical in the decode-and-forward strategy, we consider an example in which the parameters $\sigma^2_{X_{ac}}$ and $\sigma^2_{X_{bc}}$ related to the source-destination links quality

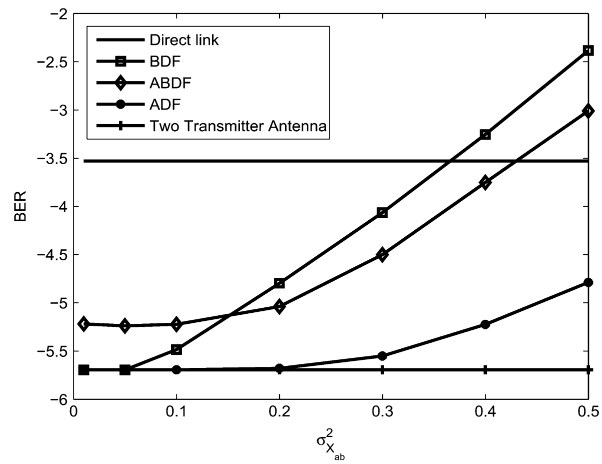

Fig. 3. Plots of BER versus $\sigma^2_{X_{ab}}$ in the case that the parameters $\sigma^2_{X_{ac}}$ and $\sigma^2_{X_{bc}}$ are kept constant and equal to 0.4. $m_s = 40, m_b = 3, \alpha = 0.98, B = 20$.

are kept constant and equal to 0.4. Fig. 3 shows the plots of BER versus the source-relay link quality, i.e., $\sigma^2_{X_{ab}}$. As can be realized, when the source-relay link is in good quality, both BDF and ADF have the same performance as the two transmitters case, and the adaptive strategy ABDF has the worst performance. When the quality of the source-relay link degrades, the performance of ABDF surpasses that of BDF. On the other hand, when the source-relay link fading is very strong, both BDF and ABDF performances are worse than that of the non-cooperative case, but ADF still has a good performance. At high SNRs, cooperative strategies work quite well and their performances approach to that of the two transmitters case. The reason is that the source-relay link performance is improved and the relay node can receive the message with a lower bit error probability. Fig. 4 shows the plots of BER versus the mean photon count per bit (m_s) that is proportional to the optical intensity. In this figure, we have $\sigma^2_{X_{bc}} = \sigma^2_{X_{ac}} = 0.4$, but the parameter $\sigma^2_{X_{ab}}$ is kept constant and equal to 0.4 and 0.1 in Fig. 4(a) and (b), respectively. m_s is proportional to the power of the received signal, so the slope of the plots at high SNRs can be considered as the order of diversity. As can be seen from this figure, the ADF has the best performance among the cooperative strategies. On the other hand, when the source-relay link A-B is in good quality (Fig. 4(b)), ADF and BDF have the same BER as that of the two transmitters case, and all the strategies have the same BER plot slope. But, when the link quality between the partners is the same as the links between the sources and the destinatin (Fig. 4(a)), BDF has the worst performance and gives no diversity, but ADF and ABDF still have the same BER plot slopes as that of two transmitters case at high SNRs. In fact, ADF presents the same performance as that of two transmitters case at high SNRs.

VI. Conclusion

In this paper, we have considered the cooperative strategies for FSO networks. We have first considered a 3-way FSO communication setup and showed that the cooperative protocol can be applied to this setup to achieve the spatial diversity without

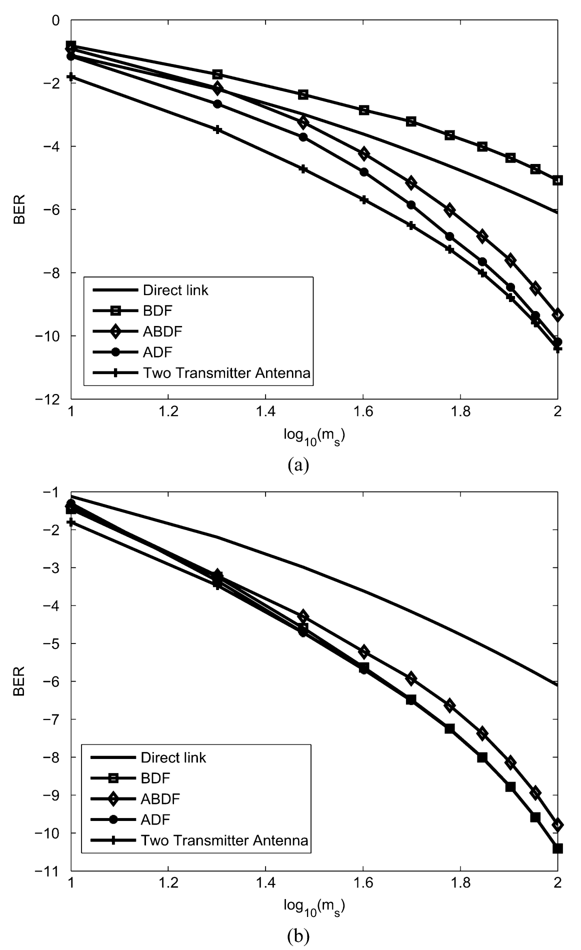

Fig. 4. Plots of BER versus the mean photon count per bit (m_s) in the case that $\sigma^2_{X_{bc}} = \sigma^2_{X_{ac}} = 0.4$. (a) $\sigma^2_{X_{ab}} = 0.4$ (b) $\sigma^2_{X_{ab}} = 0.1$. $m_b = 3, \alpha = 0.98, B = 20$.

any demand for extra hardware. Then, we have introduced different cooperative strategies for this structure, and investigated their bit error rate (BER) performance in the presence of shot noise, using the photon-count method. We have compared the results with those of the direct path link and the two transmitters case, which are the upper and lower bounds on the BER of the cooperative scheme, respectively.

We have shown that when the quality of the source-relay link is good, all the cooperative strategies performance is much better than that of the non-cooperative case. We have also shown that when the link quality between the partners is the same as the link between the sources and the destination, BDF and ABDF have no considerable gains over non-cooperative one at moderate SNRs, but ADF approximately has the same performance as that of the two transmitters case. Our numerical results have indicated that the ADF has the best performance.

Appendix A

In this appendix, we derive the optimum detector for the case that N binary PPM modulated signals, containing the same message, are received in the destination. Let's show the number of photoelectrons collected in the first and the second chip durations of the ith received BPPM signal by y_{0i} and y_{1i}, respectively. We assume that y_{0i}s and y_{1i}s are independent poission distributed variables. let m_{1i} and m_{0i} be the means of the poission processes related to the chips with and without the ith pulse, respectively. The optimum detector is easily derived as follows:

$$\frac{P(y_{0i}, y_{1i}, i=1,\ldots,N|1)}{P(y_{0i}, y_{1i}, i=1,\ldots,N|0)} \gtrless_0^1 1$$
$$\Rightarrow \frac{\prod_{i=1}^N \frac{e^{-m_{0i}} m_{0i}^{y_{0i}}}{y_{0i}!} \frac{e^{-m_{1i}} m_{1i}^{y_{1i}}}{y_{1i}!}}{\prod_{i=1}^N \frac{e^{-m_{1i}} m_{1i}^{y_{0i}}}{y_{0i}!} \frac{e^{-m_{0i}} m_{0i}^{y_{1i}}}{y_{1i}!}} \gtrless_0^1 1$$
$$\Rightarrow \prod_{i=1}^N m_{1i}^{y_{1i}-y_{0i}} m_{0i}^{y_{0i}-y_{1i}} \gtrless_0^1 1$$
$$\Rightarrow \prod_{i=1}^N \left(\frac{m_{1i}}{m_{0i}}\right)^{y_{1i}-y_{0i}} \gtrless_0^1 1$$
$$\Rightarrow \sum_{i=1}^N (y_{1i} - y_{0i}) \ln\left(\frac{m_{1i}}{m_{0i}}\right) \gtrless_0^1 0. \qquad (31)$$

When $(m_{1i})/(m_{0i})$s are equal for all is, (31) simplifies as follows:

$$\sum_{i=1}^N y_{1i} \gtrless_0^1 \sum_{i=1}^N y_{0i}. \qquad (32)$$

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