VOICE AND IMAGE RECOVERY FROM LOST SAMPLES

Farokh Marvasti, Gilbert Adams, Ut Goenchanart and C. Liu

Dept. of ECE, Ill. Inst. of Tech.
Chicago, IL 60616
Tel: (312) 567-3414

ABSTRACT

In a digital communication system, a common problem is a loss of samples due to channel fading, interfering signal and additive noise. In this paper we introduce new schemes for the interpolation of the missing samples, called division method. From nonuniform sampling theorem we know that as long as the sampling rate of a signal satisfies the Nyquist rate, the signal is uniquely determined by the set of the nonuniform samples. We consider our distorted signals as a set of nonuniform samples and derive the methods based on the theorem. A number of simulations have been performed for both distorted voice and image signals. The advantage of this method is that it is very simple to implement in real time.

I. INTRODUCTION

In the transmission of digital signals, there is a possibility of loss of samples. The missing samples have to be interpolated somehow if the quality of the signal is not to be sacrificed. Various techniques have been employed for this purpose; they range from simple low pass filtering to more sophisticated estimation of the missing samples from the previous and future sample. A different but common approach is to add redundancy bits for erasure correction, this method has the advantage of detecting and correcting errors in the samples that would otherwise be assumed to be correct.

What we are proposing here is a different approach. It consists of a nonlinear system which can be implemented in real time on a simple, low-cost microprocessor system to recover the lost samples.

Simulations for this method is performed both for voice and image signals. The results of the simulations are quite impressive. Besides that, several other simulations are done and the results are also provided for a comparison.

II. INTERPOLATION FROM MISSING SAMPLES

Signal recovery from nonuniform samples (or missing samples) is well documented in the literature[1] [2] [3] [4] [5] [6]. For a review of the theory and practical reconstruction techniques, see the author's monograph[7] [8] and the two contributed chapters in[9]. Other recent related works are listed in the reference[10] [11] [12] [13] [14] [15]. A simple technique that works quite well when the nonuniform samples are above the Nyquist rate is described below and depicted in Fig. 1. The analysis for one dimension can be found in [16] [17]. We will now extend the analysis for two dimensions. Nonuniform samples, f_s, are low pass filtered, $f_{s_{lp}}$. At the same time f_s is hard limited and rectified, and then low pass filtered, $f_{p_{lp}}$. A division of $f_{s_{lp}}$ by $f_{p_{lp}}$ yields a

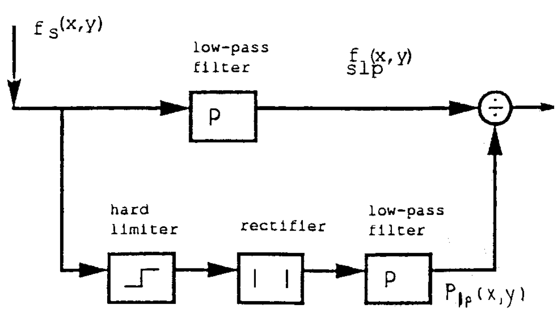

Figure 1. Diagram of the division method

good approximation of the original analog signal f(t). The analysis of the division method is as follows:

From Fig. 1, we have

$$f_s(x,y) = f(x,y)p(x,y),$$

where the comb function, p(x,y), is defined as

$$p(x,y) = \sum_m \sum_n \delta(x-x_{nm}, y-y_{nm}),$$

we now define the following functions

$$h_1(x,y) = x-nT_1-\theta_1(x,y)$$
$$h_2(x,y) = x-mT_2-\theta_2(x,y)$$

where T_1, T_2 are average sample densities at each dimension and $\theta_1(x,y), \theta_2(x,y)$ are the diviation of the non-uniform samples. Where for any integer values of n and m,

$$h_1(x_{nm}, y_{nm}) = h_2(x_{nm}, y_{nm}) = 0.$$

An impulse in two dimensional space can be written as

$$\delta(x-x_{nm}, y-y_{nm}) = |J|\,\delta[h_1(x,y), h_2(x,y)],$$

where J is the Jacobian and defined as

$$J = \begin{vmatrix} \frac{\partial h_1}{dx} & \frac{\partial h_2}{dx} \\ \frac{\partial h_1}{dy} & \frac{\partial h_2}{dy} \end{vmatrix} = \left| \frac{\partial h_1}{dx}\frac{\partial h_2}{dy} - \frac{\partial h_1}{dy}\frac{\partial h_2}{dx} \right|$$

or

$$|J| = \left| \left[1-\frac{\partial \theta_1}{dx}\right]\left[1-\frac{\partial \theta_2}{dy}\right] - \left[1-\frac{\partial \theta_1}{dy}\right]\left[1-\frac{\partial \theta_2}{dx}\right] \right|.$$

Therefore,

$$p(x,y) = \sum_m \sum_n |J|\,\delta\left(x-nT_1-\theta_1(x,y), y-mT_2-\theta_2(x,y)\right)$$

Using Fourier series expansion, we get

$$p(x,y) = \frac{|J|}{T_1 T_2} \sum_m \sum_n e^{j\left(\frac{2n\pi\phi_1}{T_1}+\frac{2m\pi\phi_2}{T_2}\right)},$$

where

$$\phi_1 = x-\theta_1(x,y)$$
$$\phi_2 = y-\theta_2(x,y).$$

Finally, we can write p(x,y) as

$$p(x,y) = \frac{|J|}{T_1 T_2}\left\{1+2\sum_{m=1}^{\infty}\cos\left[\frac{2\pi m}{T_1}\left(x-\theta_1(x,y)\right)\right]\right\} \left\{1+2\sum_{n=1}^{\infty}\cos\left[\frac{2\pi n}{T_2}\left(x-\theta_2(x,y)\right)\right]\right\},$$

which resembles two dimensional phase modulation.

In other words, The Fourier spectrum of the comb function composes of many components: (1) a low-pass component $\frac{|J|}{T_1 T_2}$, and (2) band/high pass components around carrier frequencies $\frac{2\pi m}{T_1}$ and $\frac{2\pi n}{T_2}$ as shown in Figure 2.

Consequently, the non-uniform samples $f_s(x,y)$ can be written as

$$f_s(x,y) = f(x,y)\frac{|J|}{T_1 T_2}\left\{1+2\sum_{m=1}^{\infty}\cos\left[\frac{2\pi m}{T_1}\left(x-\theta_1(x,y)\right)\right]\right\} \left\{1+2\sum_{n=1}^{\infty}\cos\left[\frac{2\pi n}{T_2}\left(x-\theta_2(x,y)\right)\right]\right\}.$$

The low-pass version of $f_s(x,y)$ is

$$f_{s_{lp}}(x,y) \approx f(x,y)\frac{|J|}{T_1 T_2}.$$

The low-pass version of p(x,y) is

$$p_{lp}(x,y) = \frac{|J|}{T_1 T_2}.$$

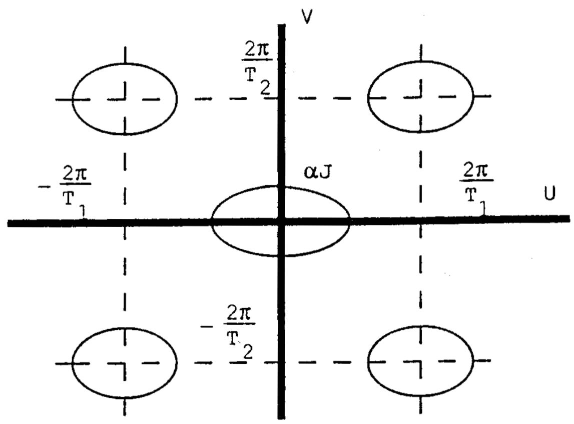

Figure 2. Spectrum of Comb Function

Hence,

$$\frac{f_{s_{lp}}(x,y)}{p_{lp}(x,y)} \approx f(x,y).$$

This division is possible if the Jacobian function is not equal to zero. A sufficient condition is satisfied when non-uniform samples do not deviate by more than $\frac{T}{2}$ for the (x,y) point distribution[7].

III. SIMULATIONS

The simulation results for both speech and image data are shown in Figure 3 and Figure 4. Result from the image data at 30 percent lost is illustrated in Figure 7 , whereas Figure 5 and Figure 6 show the original and the corrupted image respectively. The speech data was oversampled at twice the Nyquist rate. The image data was band limitted using low-pass filter. The result was compared with the linear interpolation and iterative method [12][2][3]. From figure 3, we can see that the division technique yields very good result when the sample lost is less than 30 percent. At higher percent lost, the division method suffers from division by zero due to long erasure or burst error. Figure 4 shows comparision among various techniques including iterative method(for 10 iterations). At 30 percent lost, the division method gives the result equivalent to approximately 20 iterations.

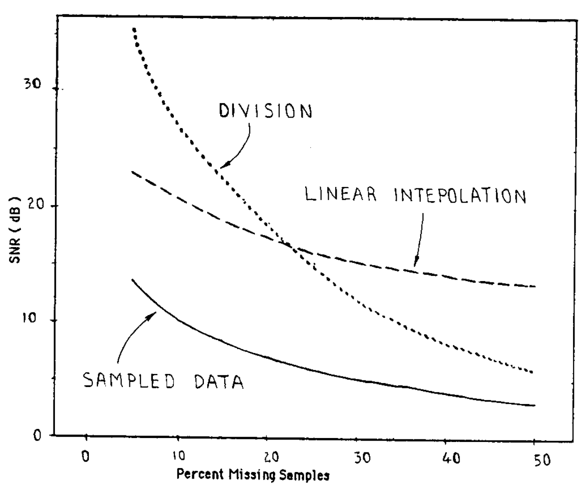

Figure 3. Simulation Results From Speech Data

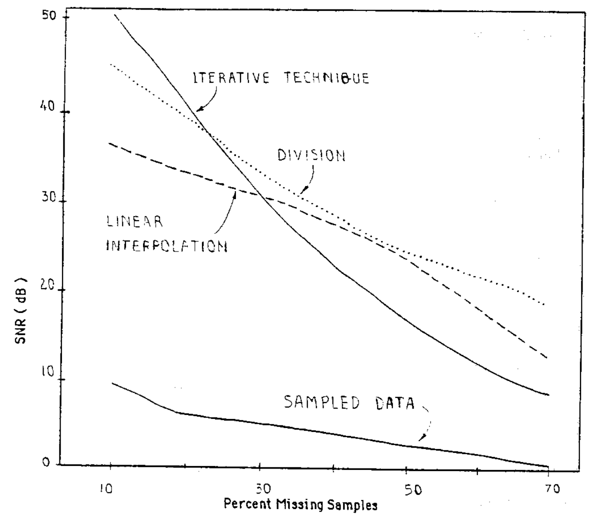

Figure 4. Simulation Result From Image Data

Figure 5 Original Bandlimited Signal

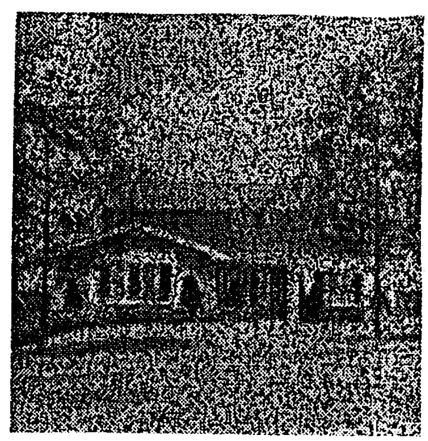

Figure 6 Signal At 30% Missing Samples

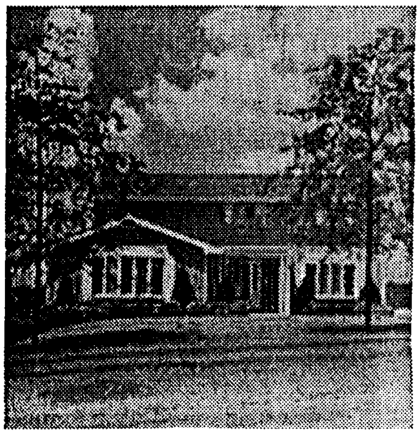

Figure 7. Result After Using Division Method On Data In Fig. 6.

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