# CONTOURLET BASED IMAGE WATERMARKING USING OPTIMUM DETECTOR IN THE NOISY ENVIRONMENT

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## ABSTRACT

In this paper, a new multiplicative image watermarking system is presented. As human visual system is less sensitive to the image edges, watermarking is applied in the contourlet domain, which represents image edges sparsely. In the presented scheme, watermark data is embedded in the most energetic directional subband. By modeling General Gaussian Distribution (GGD) for the contourlet coefficients, the distribution of watermarked noisy coefficients is analytically calculated. At the receiver, based on the Maximum Likelihood (ML) decision rule, the optimal detector is proposed. Experimental results show the imperceptibility and high robustness of the proposed method against Additive White Gaussian Noise (AWGN) and JPEG compression attacks.

***Index Terms***— Multiplicative image watermarking, contourlet transform, maximum likelihood detector

## 1. INTRODUCTION

Digital watermarking is progressively applied to several purposes such as broadcast monitoring, data authentication, data indexing, and secret communication [1]. A digital watermark must have special features to guarantee desired functionalities. Perceptual transparency, data rate and robustness against attacks are three major requirements of any watermarking system. However, depending on the application, the importance of these features varies. For example, for secret communication the robustness against the noise and data rate are the most important features while for data authentication, imperceptiblity and robustness against different processing attacks are the most significant ones.

Multiplicative watermarking is one of the most popular approaches for copy right protection where the watermark is served as a verification code. Since this technique is image content dependent, higher robustness is achieved than other techniques such as additive watermarking methods. The correlation detector is used for multiplicative watermarking in [2]; however, this type of detection is not suitable when the watermarking is performed in the transform domain. Hence, for the Barnie's multiplicative watermarking method in the transform domain [1], several optimum and locally optimum decoders have been proposed so far [3],[4]. In [3], a robust optimum detector for the multiplicative rule $y_i = x_i(1 + \alpha_i w_i)$ in the DCT, DWT and DFT domains is proposed.

In this paper, in order to achieve better robustness specially against AWGN and JPEG compression attacks, the multiplicative watermarking approach in the contourlet transform domain is used. It is noteworthy that since in our application, the watermark serves as a transmission code not a verification one, we adopt a new multiplicative watermarking scheme to embed message bits. The contourlet coefficients are multiplied by two special functions depending on the value of the watermark bits. For data extraction, similar to [1],[5], the ML detector has been used. To this aim, the density function of the noisy contourlet coefficients is analytically computed. In order to decrease the complexity of the receiver, the distribution of these coefficients are approximated with a suitable function. Under this estimation, the optimum threshold of the proposed multiplicative watermarking method is evaluated.

## 2. SYSTEM MODELING

As natural images are not simply stacks of 1-D piecewise smooth scan-lines and have many discontinuity points along smooth curves and contours, many directional image representations have been proposed such as curvelet and contourlet [6]. Implementing the idea of combining subband decomposition with a directional transform, Do and Vetterli [6] introduced a multi-directional and multi-scale transform, known as the contourlet transform, which consists of two major stages: the subband decomposition and the directional transform.

As studied in [7], contourlet coefficients are well modeled by i.i.d. random variables with Generalized Gaussian Distribution (GGD):

$$GG_{\sigma_x,\beta}(x) = C(\sigma_x,\beta)e^{-[\alpha(\sigma_x,\beta)|x|]^\beta}$$
$$, -\infty < x < \infty, \sigma_x > 0, \beta > 0 \quad (1)$$

where $\alpha(\sigma_x,\beta) = \sigma_x^{-1}[\frac{\Gamma(\frac{3}{\beta})}{\Gamma(\frac{1}{\beta})}]^{\frac{1}{2}}$, $C(\sigma_x,\beta) = \frac{\beta\alpha(\sigma_x,\beta)}{2\Gamma(\frac{1}{\beta})}$, and

$\sigma_x$ is the standard deviation of $x$, $\beta$ is the shape parameter, and $\Gamma(t) = \int_0^\infty e^{-u} u^{t-1} \, du$ is the Gamma function.

In our watermarking approach, as will be discussed in the next section, the contourlet coefficients $x$ is multiplied by a monotonous strength function $f(x)$. Thus, if we show the watermarked coefficients by $w$, we have $w = xf(x)$. As discussed in the next section, the $f(x)$ is selected in such a way that this $xf(x)$ function is still monotonous in the practical range of $x$. Therefore, the distribution of $y$ can be defined as:

$$P(w) = \frac{P(x)}{xf'(x) + f(x)} \tag{2}$$

At the receiver, we receive the coefficients attacked by noise or other kinds of attacks. We assume that the attacked noise is zero mean AWGN with the distribution of $N(0, \sigma_n^2)$. Thus, the received coefficients are $y = w + n$. Since the contourlet coefficients are considered to be independent of the noise term, we have $P(y) = P(w) * P(n)$, where $*$ is the convolution operator. Thus, considering the GG distribution of contourlet coefficients $x$, we have:

$$P(y) = \int_{-\infty}^{\infty} \frac{Ce^{-[\alpha|g(z)|]^\beta}}{g(z)f'(g(z)) + f(g(z))} \cdot \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-\frac{(y-z)^2}{2\sigma_n^2}} \, dz \tag{3}$$

where $g(x)$ is the inverse function of the $w = xf(x)$ in the practical range of $x$; that is, $g(w) = x$.

To find a closed form answer for $P(y)$, we estimate the Gaussian function with a triangular function, as follows:

$$\Lambda_G(x) = \begin{cases} \frac{-x+3\sigma_n}{9\sigma_n^2} & 0 < x \leq 3\sigma_n \\ \frac{x+3\sigma_n}{9\sigma_n^2} & -3\sigma_n \leq x < 0 \\ 0 & |x| > 3\sigma_n \end{cases} \tag{4}$$

Then, substituting this function in (3), and using the trapezoidal rule, we can compute the integral as:

$$P(y) \simeq \frac{3\sigma_n}{2} \cdot [\dots$$
$$\dots \frac{K(y + \frac{3\sigma_n}{2})\Lambda_G(-\frac{3\sigma_n}{2}) + K(y + 3\sigma_n)\Lambda_G(-3\sigma_n)}{2}$$
$$+ \frac{K(y)\Lambda_G(0) + K(y + \frac{3\sigma_n}{2})\Lambda_G(-\frac{3\sigma_n}{2})}{2}$$
$$+ \frac{K(y)\Lambda_G(0) + K(y - \frac{3\sigma_n}{2})\Lambda_G(\frac{3\sigma_n}{2})}{2}$$
$$+ \frac{K(y - \frac{3\sigma_n}{2})\Lambda_G(\frac{3\sigma_n}{2}) + K(y - 3\sigma_n)\Lambda_G(3\sigma_n)}{2}] \tag{5}$$

where,

$$K(y) = \frac{Ce^{-[\alpha|g(y)|]^\beta}}{g(y)f'(g(y)) + f(g(y))} \tag{6}$$

Then, after some simplifications, (5) is converted to:

$$P(y) \simeq \frac{1}{2}[\frac{K(y - \frac{3\sigma_n}{2})}{2} + K(y) + \frac{K(y + \frac{3\sigma_n}{2})}{2}] \tag{7}$$
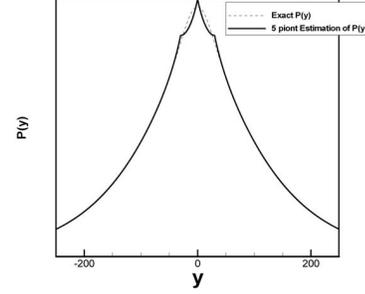


**Fig. 1**. The distribution function of the noisy watermarked contourlet signal, P(y) (3), compared with the five point (5) estimation.

To verify the accuracy of the estimation used in calculation of $P(y)$ in (7), we have simulated this estimation along with (3) in Fig. 1 for an example case of $\beta = 1.2$, $\sigma_x = 50$, and $\sigma_n = 20$. As we can see, the estimation is well matched with the exact density function.

## 3. PROPOSED METHOD

### 3.1. Watermark Embedding

Imperceptibility of the watermarking algorithm is commonly achieved by exploiting the weaknesses of the HVS. For this purpose, we utilize $N$ blocks with the highest entropy of the image to embed the watermark code.

We then apply the contourlet transform to each selected block. Then calculating the energy of the coefficients in each directional subband of the finest scale, we choose the directional subband with the highest energy for embedding purpose. This way, we hide the code in the most significant direction of each block. We embed a single bit of '0' or '1' to each block by manipulating the contourlet coefficients $x_i$ in this significant directional subband based on the following strategy:

$$w_i = \begin{cases} x_i \cdot f_1(x_i) & \text{For embedding 1} \\ x_i \cdot f_0(x_i) & \text{For embedding 0} \end{cases} \tag{8}$$

where $f_1(x)$ and $f_0(x)$ are strength functions which are chosen to be monotonous exponentially functions. To achieve the best performance, we define them as follows:

$$f_1(x) = -0.3e^{-0.2|x|} + 1.65; \tag{9}$$

$$f_0(x) = 0.15e^{-0.2|x|} + 0.65; \tag{10}$$

$f_1(x)$ is the exponentially ascending function for $x > 0$ which is larger than one and $f_0(x)$ is the exponentially descending function for $x > 0$ which is smaller than one. These functions are chosen exponentially in order that larger coefficients changes more than smaller ones during the watermarking process, as the larger coefficients are related to the strong edges in the supposed directional subband. However, as we mentioned in Section 2, these functions must be

defined in a way that the monotony of $xf(x)$ is satisfied for the practical range of $x$. The proposed functions in (10), (9) satisfy these condition and $xf_0(x)$ and $xf_1(x)$ are ascending monotonous function.

By applying the inverse contourlet transform, we reconstruct the watermarked block. Repositioning each block in its position in the image, we create the watermark embedded image. The block positions and the GGD parameters ($\sigma_x$ and $\beta$) should be sent along with the watermarked image.

### 3.2. Watermark Detection

For detecting the watermark data in each block, we suggest a detection scheme based on an optimum detector.

Suppose that $x_i$ represents the contourlet coefficients of the most energetic directional subband of a specific block. As discussed in Section 2, we considered these coefficients to have an iid GG distribution. Besides, we approximated the distribution of the watermarked coefficients attacked by AWGN by (7).

In order to have ML decision we must have:

$$P(y_1, y_2, ..., y_N|1) \gtrless_0^1 P(y_1, y_2, ..., y_N|0) \quad (11)$$

where the left term is the distribution of the coefficients in a specific block with $N$ coefficients for '1' embedding and the right term is the same distribution for '0' embedding. Considering the iid distribution of the contourlet coefficients, these distributions are defined as:

$$P(y_1, ..., y_N|1) =$$
$$\prod_{i=1}^{N} \frac{K_1(y_i - \frac{3\sigma_n}{2}) + 2K_1(y)_i + K_1(y_i + \frac{3\sigma_n}{2})}{4}$$
$$P(y_1, ..., y_N|0) =$$
$$\prod_{i=1}^{N} \frac{K_0(y_i - \frac{3\sigma_n}{2}) + 2K_0(y_i) + K_0(y_i + \frac{3\sigma_n}{2})}{4} \quad (12)$$

where, $K_1(y)$ and $K_0(y)$ are computed using (6) by the strength functions $f_1(x)$ and $f_0(x)$, respectively.

By inserting (12) in (11) we can find the watermarked bit using the optimum detector.

As we can see, the best decision depends on the noise standard deviation in the supposed directional subband, $\sigma_n$. To estimate this parameter, we can use a Monte-Carlo method as suggested in [7].

For the noise free environment, (11) can be simplified more as:

$$\sum_{i=1}^{N} [(\alpha|g_0(y_i)|)^\beta - (\alpha|g_1(y_i)|)^\beta] \gtrless_0^1 T \quad (13)$$

where,

$$T = \sum_{i=1}^{N} \ln \frac{g_1(y_i)f_1'(g_1(y_i)) + f_1(g_1(y_i))}{g_0(y_i)f_0'(g_0(y_i)) + f_0(g_0(y_i))}$$



(a) Original Image



(b) Watermarked Image

**Fig. 2**. Original and watermarked test images; Left to right: *Baboon*, *Barbara*, *Bridge*, and *Couple*

where, $g_1(y)$ and $g_0(y)$ are the inverse functions of $xf_1(x)$ and $xf_0(x)$, respectively.

Considering (1), we need to estimate the standard deviation $\sigma_x$ and the shape parameter $\beta$ of the GGD function for the supposed directional subband in each block. The $\beta$ parameter can be found using the kurtosis of the GGD. To find the $\sigma_x$ parameter we suggest an estimator which is fitted for our ML detector.

Suppose we have $N$ GGD coefficients in the current subband. Thus, the distribution of these coefficients can be defined as:

$$P(\beta, \sigma_x; x) = \prod_{i=1}^{N} GG(x_i) \quad (14)$$

By applying a logarithm function to both sides, we have:

$$L(\beta, \sigma_x; x) = N \ln C(\sigma_x, \beta) - \alpha(\sigma_x, \beta)^\beta \sum_{i=1}^{N} |x_i|^\beta \quad (15)$$

Computing the root of $\frac{\partial L(\beta, \sigma_x; x)}{\partial \sigma_x} = 0$, we have:

$$\hat{\sigma}_x = [\frac{\Gamma(\frac{3}{\beta})}{\Gamma(\frac{1}{\beta})}]^{\frac{1}{2}} (\frac{\beta}{N} \sum_{i=1}^{N} |x_i|^\beta)^{\frac{1}{\beta}} \quad (16)$$

### 4. SIMULATION RESULTS

We have performed several experiments to test the proposed algorithms and evaluate its performance against AWGN and JPEG attacks which are common in our application. For the contourlet transform, we use the 9-7 biorthogonal filters with three levels of pyramidal decomposition for the multiscale decomposition stage and the PKVA filters used in [6] for the multidirectional decomposition stage. We partition the finest scale to eight directional subbands. All the results are obtained by averaging over 20 runs with 20 different pseudorandom binary sequences as the watermarking signal.

For this study, we use various natural images of size $512 \times 512$. These images consist of *Baboon*, *Barbara*, *Bridge*,
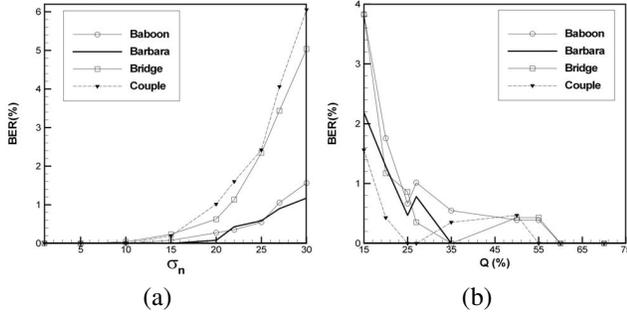
**Fig. 3**. (a) AWGN attack for various noise variances. (b) JPEG compression attack for various quality factors.
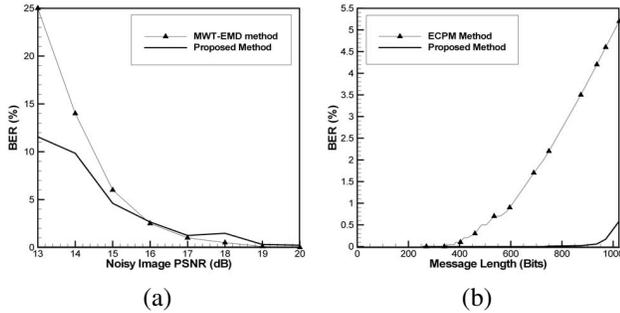


**Fig. 4**. Comparison between our watermarking method and (a) MWT-EMD method [8]: BER (%) under AWGN attack. (b) ECPM method [9] : BER (%) under AWGN attack for $\sigma_n^2 = 5$ with different message lengths.

and *Couple*. The original test images and their watermarked version using the proposed method are shown in Fig. 2. The mean Peak-Signal-to-Noise-Ratio (PSNR) of the watermarked images are 39.53dB, 36.63dB, 42.40db, and 42.48dB, respectively. We segment the image to $16 \times 16$ blocks; thus, we have a 128 bit date rate.

In the first experiment, we investigate the effect of AWGN to the proposed watermarking scheme. Fig. 3(a) shows the Bit Error Rate of the proposed method for various images versus different noise power. As we expect, the method has a great resistance against noise attack. This is because the receiver is optimized for noisy environment.

In the second experiment, the proposed technique is tested against JPEG compression with different quality factor. As demonstrated in Fig 3(b), the proposed method is highly robust against JPEG with different quality factor up to 10%.

To compare our watermarking algorithm with other watermarking schemes, we use the same bit rate and PSNR as the bit rate and PSNR used in other techniques. The simulation results are shown in Fig. 4. We see that the robustness of our method against AWGN attack are considerably better than Multiband Wavelets and Empirical Mode Decomposition (MWT-EMD) [8] and Ergodic Chaotic Parameter Modulation (ECPM) [9] methods as our detector is optimized for the noisy environment.

## 5. CONCLUSION

In this paper, we have presented a new robust multiplicative image watermarking technique in the contourlet transform domain. Since the contourlet transform concentrates the image's energy in the limited number of edge coefficients, using multiplicative approach in this domain yields high robustness accompanied by great transparency. We model the distribution of contourlet coefficients by GGD. Then, the distribution of watermarked noisy coefficients are calculated analytically. Using ML decision rule, the optimum detector has been proposed. The optimal detector guarantees the suggested method is well suited for high noisy environment. Experimental results over several images confirm the imperceptibility and the excellent robustness of the proposed method in comparison with other reported techniques.

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