IEEE 7th Int. Symp. on Spread-Spectrum Tech. & Appl., Prague, Czech Republic, Sept. 2-5, 2002

EM-Based Multiuser Detections for Optical CDMA Networks

Abolfazl S. Motahary and Masoumeh Nasiri-Kenari, Member
Department of Electrical Engineering
Sharif University of Technology
P.O. Box 11365-9363 Tehran, Iran

Abstract– **In this paper, we introduce several EM-based multiuser detectors for an optical CDMA system. The detectors have two soft and hard stages. In soft stage, a soft estimation of the interference is obtained by solving an unconstrained maximum-likelihood (ML) problem via iterative Expectation-Maximization (EM) algorithm. Then, the hard stage detects the user information bit by solving a one-dimensional Boolean constrained problem conditioned on knowing the interference. Our results reveal that the proposed detectors have very low complexity and robust against changes in parameters. Moreover, the numerical results illustrate that despite of their simplicities, our detectors substantially outperform other well-known suboptimum detectors, such as multi-stage and decorrelating detectors.**

Index Terms--**Optical CDMA networks, multiuser and blind detection, interference cancellation.**

I. INTRODUCTION

IN any multiple-access system, the available resources are shared in some ways among all active users. In code division multiple access (CDMA) systems all resources in principle are available to all users, simultaneously. The users are distinguished from each other by user specific signature sequence (PN sequence). In a fiber optic network, CDMA is considered as a viable multiple-access technique due to its ability to establish an asynchronous and robust multiple access system for a number of users.

In [1], Salehi proposed optical orthogonal codes (OOCs) as spreading codes for intensity modulation/direct detection (IM/DD) optical CDMA networks, which yield low crosscorrelation and out of phase autocorrelation, and therefore suitable for high speed asynchronous networks using conventional correlation detectors [2]. The correlation detector, which has a very low complexity, is optimal for a single user system. However, as the number of simultaneous users increases, the simple correlation detector performance seriously degrades. This is due to the fact that the correlation detector does not take into account the existence of multiple-access interference (MAI) and treats them as a noise.

In order to overcome this deficiency, Verdù [3] proposed optimal multiuser detector, which is a maximum-likelihood (ML) detector for an equiprobable channel input data. The optimum detector, which is the solution of a Boolean constrained ML problem, provides a performance comparable to that of a single user system, but at the expense of a computational complexity that is known to be NP-hard. Therefore, the optimum detector is in general too complex for practical optical CDMA systems, even with a moderate number of users. To circumvent the complexity problem, much efforts have been devoted to developing suboptimum receivers.

In this paper, we propose several suboptimum multiuser detectors for an optical CDMA system. In order to reduce the complexity of the optimum receiver, we first solve an unconstrained ML problem on R_+[1] in which the symbols can take any positive real value (soft decision stage). We then use these soft decisions as an estimate of the interference and solve a one dimensional Boolean constraint ML problem, conditioned on knowing the interference. This stage called hard decision stage. By applying this approach, several new multiuser detectors are introduced.

For solving the unconstrained ML problem, we use the expectation maximization (EM) algorithm. This algorithm provides an iterative approach to likelihood based parameter estimation where direct maximization of the likelihood function may not be feasible [7]. Previous studies on the applications of EM algorithm for multiuser detection in CDMA systems, in the best knowledge of the authors, have been in radio frequency domain. In this paper, we use EM algorithm for the iterative soft decision stage as described above. Numerical results show that only a few iterations are required for practical convergence of the EM algorithm in our applications.

The rest of this paper is organized as follows. Section II describes the system model for an optical CDMA network. Section III introduces several multiuser detectors with two soft and hard stages, in which the soft stage utilizes the EM algorithm. Finally, numerical results are presented in Section IV, and a conclusion is then given in Section V.

Throughout this paper, scalars are lowercase, vectors are boldfaced lowercase, and matrices are boldfaced uppercase. The notation $(\tilde{o})$ and $(\hat{o})$ are used for soft and hard estimation operators, respectively.

---

[1] $R_+=\{[x]_+|\ x\in R\}$. Throughout this paper the notation $[x]_+$ implies that if $x>0$ then $[x]_+=x$ else $[x]_+=\varepsilon$, where ε is a small positive number near zero.

II. SYSTEM MODEL

We consider an intensity modulation/direct detection (IM/DD) optical CDMA network with M users. Each user uses an on/off keying (OOK) modulation for transmitting independent and equiprobable binary data over a common optical channel. The signature sequence is OOC with property $(F, w, \lambda_a, \lambda_c)$[2] , where F is the signature code length (or processing gain,) and w is the weight of codes.

Let $b_i[l]\in\{0,1\}$ be the transmitted bit of the ith user at lth bit interval and $c_i=[c_{1i},\ldots, c_{ji},\ldots, c_{Fi}]^T$ be its signature code. Then the received intensity signal can be written as

$$x(t)=\sum_{j=1}^{F}\sum_{i=1}^{M}\sum_{l}\lambda_i b_i[l]c_{ji}\Pi(t-lT-\tau_i)+\lambda_d\ , \qquad (1)$$

where λ_i, and $\tau_i\in[0,T)$ are the signal strength and the relative transmission delay of user i, respectively. The term λ_d represents the dark current effect. $\Pi(t)$ is a rectangular waveform with amplitude one and duration T.

For simplicity of presentation, the system is assumed synchronous. The generalization to the asynchronous case is straightforward and will not be considered here. For a synchronous system, a more convenient form of (1) is in discrete vector notation as follows

$$\mathbf{x}=[x_1,x_2,\ldots,x_F]^T=\mathbf{\Lambda}\mathbf{C}\mathbf{b}+\lambda_d\mathbf{u}\ , \qquad (2)$$

where $\mathbf{C}=[\mathbf{c}_1\ \mathbf{c}_2\ \ldots\mathbf{c}_M]$, $\mathbf{b}=[b_1\ b_2\ \ldots b_M]^T$, and $\Lambda=diag(\lambda_1, \lambda_2,\ldots, \lambda_M)$ represent the signature sequences matrix, users bits vector, and users power matrix, respectively, and $\mathbf{u}$ denotes a vector of all one. Also x_j is the received intensity at chip interval j, for $j=1,2,\ldots,F$. Note that in a fiber optic system, the background noise is weak, and thus it is neglected in (2). Fig.1 shows an equivalent discrete model of this network. Received signal is passed through photo detector. We usually expect a Poisson process at the output of a photo detector. That is if the photo counts collected from a chip position j is denoted by y_j, then y_j s can be modeled as Poisson random variables, i.e.,

$$\mathbf{y}=[y_1,y_2,\ldots,y_F]^T\approx Poisson(\mathbf{x})\ , \qquad (3)$$

where $\mathbf{x}$ is given by (2).

The correlation detector for user i detects the user's bit based on the following decision rule

$$\mathbf{c}_i\mathbf{y}^T \gtrless \eta\ , \qquad (4)$$

where η is a threshold which depends on the user power and dark current intensity. This simple detector is optimal only in a single user system. The ML detector decides based on the following rule [3]

$$\hat{\mathbf{b}}=\arg\max_{\mathbf{b}\in\{0,1\}^M} L_y(\mathbf{b})\ , \qquad (5)$$

[2] Even though we use the OOC as signature code for performance evaluation of our proposed detector, any other signature code can be used as well.

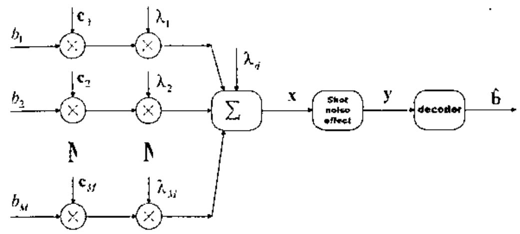

Fig. 1. The discrete model of an optical CDMA system.

where

$$L_y(\mathbf{b})=\log f(\mathbf{y}|\mathbf{b})\ , \qquad (6)$$

is the log-likelihood function and $f(\mathbf{y}|\mathbf{b})$ denotes the density function of $\mathbf{y}$ conditioned on $\mathbf{b}$. This detector incidentally is identical to the MAP detector for an equiprobable channel input data. The computational complexity of this detector is known to be NP-hard. Several suboptimum multiuser detectors for mitigating the complexity of the optimum receiver have been proposed. These detectors mostly have the following structure

$$\sum_{j=1}^{F}s_j y_j \lessgtr \eta\ , \qquad (7)$$

where s_j s are the weighting coefficients. In this structure, those chips observing stronger interference are weighted smaller compared to the other chips.

III. MULTIUSER DETECTION

In this section, we propose several EM-based multiuser detectors, which contain two soft and hard stages. The soft stage is implemented iteratively using EM algorithm. We describe the soft and hard decision stages in part A and B, respectively. In part C we summarize some advantages of the new detectors.

A. Soft decision stage

In this stage, we relax Boolean constrained in (5) and assume that the transmitted bits vector $\tilde{\mathbf{b}}$ belong to R_+^M. With this assumption, the log-likelihood function, i.e., $L_y(\tilde{\mathbf{b}})$, is concave [5] and bounded below, therefore it has only global maximum that can be found via EM algorithm, and in sequel the convergence to the global maximum is guaranteed [7].

Fig.2 shows a new equivalent mathematical model for the network. Note that the models of Figs.1 and 2 are equivalent if we set

$$\sum_{i=1}^{M}\lambda_d^{(i)}+\widetilde{\lambda}_d=\lambda_d\ , \qquad (8)$$

where $\lambda_d^{(i)}$ is the dark current assigned to the ith user. The photo counts of each user, i.e., $\mathbf{n}_i=[N_{1i}, N_{2i},\ldots,N_{Fi}]^T$, plays the role of the complete data, where N_{ji} represents the photo counts due to ith user collected at jth chip interval. The whole complete data is represented by $\mathbf{z}=\{\mathbf{N},\mathbf{r}\}$, where $\mathbf{N}=[\mathbf{n}_1\ \mathbf{n}_2\ \ \mathbf{n}_M]$ is photo counts matrix with the columns as described above,

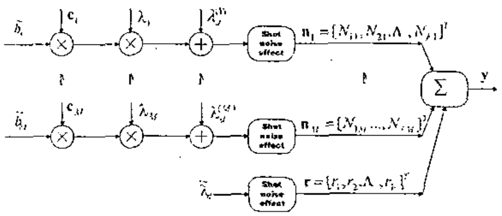

Fig. 2. The complete data for the EM based multiuser detector.

and $\mathbf{r}=[r_1, r_2,\ldots, r_F]^T$ is the photo counts vector due to $\tilde{\lambda}_d$. The observable data (incomplete data) is related to the complete data with the following many to one mapping

$$y_j = \sum_{i=1}^{M} N_{ji} + r_j \qquad for \quad j=1,2,\ldots,F. \tag{9}$$

the EM algorithm is implemented in two steps, expectation and maximization, as follow.

The E-step is defined by Q function

$$Q(\tilde{\mathbf{b}},\tilde{\lambda}_d \mid \tilde{\mathbf{b}}^{[k]},\tilde{\lambda}_d^{[k]}) = E[L_z(\tilde{\mathbf{b}},\tilde{\lambda}_d) \mid \mathbf{y},\tilde{\mathbf{b}}^{[k]},\tilde{\lambda}_d^{[k]}], \tag{10}$$

where $L_z(\tilde{\mathbf{b}}, \tilde{\lambda}_d) = \log f(\mathbf{z} \mid \tilde{\mathbf{b}}, \tilde{\lambda}_d)$.

By taking the expectation, we simply obtain

$$Q(\tilde{\mathbf{b}},\tilde{\lambda}_d \mid \tilde{\mathbf{b}}^{[k]},\tilde{\lambda}_d^{[k]}) \cong \sum_{j=1}^{F}\sum_{i=1}^{M} c_{ji}[\alpha_{ji}\log(\tilde{b}_i\lambda_i + \lambda_d^{(i)}) - \tilde{b}_i\lambda_i] + \sum_{j=1}^{F}[\beta_j \log\tilde{\lambda}_d - \tilde{\lambda}_d] \tag{11}$$

(the symbol "≅" denotes the equivalent up to a constant term which is independent from $\tilde{\mathbf{b}}$), where in appendix A we show

$$\alpha_{ji} = E[N_{ji} \mid \mathbf{y},\tilde{\mathbf{b}}^{[k]}] = \frac{c_{ji}(\tilde{b}_i^{[k]}\lambda_i + \lambda_d^{(i)})}{\sum_{i=1}^{M} c_{ji}\tilde{b}_i^{[k]}\lambda_i + \lambda_d} y_j, \tag{12}$$

$$\beta_j = E[r_j \mid \mathbf{y},\tilde{\mathbf{b}}^{[k]}] = \frac{\tilde{\lambda}_d^{[k]}}{\sum_{i=1}^{M} c_{ji}\tilde{b}_i^{[k]}\lambda_i + \lambda_d} y_j. \tag{13}$$

Since the Q function in (11) is a separable function of $\tilde{b}_1, \tilde{b}_2,\ldots, \tilde{b}_M, \tilde{\lambda}_d$, the following problem in the M-step:

$$[\tilde{\mathbf{b}}^{[k+1]},\tilde{\lambda}_d^{[k+1]}] = \arg\max_{\substack{\tilde{\mathbf{b}}\in R_+^M \\ \tilde{\lambda}_d\in R_+}} Q(\tilde{\mathbf{b}},\tilde{\lambda}_d \mid \tilde{\mathbf{b}}^{[k]},\tilde{\lambda}_d^{[k]}), \tag{14}$$

has a closed form solution for $\tilde{b}_i$ s and $\tilde{\lambda}_d$. After some algebra, we obtain

$$\tilde{b}_i^{[k+1]} = \left[\frac{\sum_{j=1}^{F}\alpha_{ji}c_{ji}}{w\lambda_i} - \lambda_d^{(i)}\right]_+, \tag{15}$$

$$\tilde{\lambda}_d^{[k+1]} = \frac{\sum_{j=1}^{F}\beta_j}{F}. \tag{16}$$

The equations (12), (13), (15), and (16) can be used iteratively to obtain the unconstraint ML solution.

We can divide the dark current among the users, i.e., $\lambda_d^{(i)}$s in (8), in two ways as follows:

1) Case I: known dark current

When the dark-current intensity is known, we do the following division

$$\lambda_d^{(i)} = \frac{\lambda_d}{M}\ ;\ \tilde{\lambda}_d = 0\,. \tag{17}$$

In this case, the detector is implemented iteratively using equations (12) and (15), and equations (13) and (16) are not used, also (11) is modified as

$$Q(\tilde{\mathbf{b}} \mid \tilde{\mathbf{b}}^{[k]}) \cong \sum_{j=1}^{F}\sum_{i=1}^{M} c_{ji}[\alpha_{ji}\log\tilde{b}_i\lambda_i - \tilde{b}_i\lambda_i]. \tag{18}$$

2) Case II: unknown dark current

In this case, information symbols and dark current must be estimated simultaneously, and thus we set

$$\lambda_d^{(i)} = 0\ ;\ \tilde{\lambda}_d = \lambda_d\,. \tag{19}$$

Now the detector is implemented iteratively using equations (12), (13), (15), and (16).

B. Hard decision stage

In this stage, the solution of the soft decision stage, i.e. $\tilde{\mathbf{b}}$, is converted onto a valid data point through a suboptimum mapping, as follows. Interference at the jth chip of user i is estimated using the soft decisions as:

$$\tilde{I}_{ji} = \sum_{\substack{k=1 \\ k\neq i}}^{M} c_{jk}\tilde{b}_k\lambda_k + \lambda_d, \tag{20}$$

where λ_d can be replaced with $\tilde{\lambda}_d$ if the dark current is unknown and must be estimated by (16). Then, the user i's bit is detected by solving the following one-dimensional Boolean constrained ML problem conditioned on having interference as (20)

$$\hat{b}_i = \arg\max_{b_i\in\{0,1\}} \log f(\mathbf{y} \mid \tilde{\mathbf{I}}_i, b_i), \tag{21}$$

where $\tilde{\mathbf{I}}_i=[\tilde{I}_{1i}, \tilde{I}_{2i},\ldots, \tilde{I}_{Fi}]$. Solving the above problem leads to

$$\hat{b}_i = \arg\max_{b_i\in\{0,1\}} \sum_{j=1}^{F} c_{ji}[y_j\log(b_i\lambda_i + \tilde{I}_{ji}) - b_i\lambda_i]. \tag{22}$$

C. Advantages of proposed detector

The proposed detector has some crucial advantages as follows:

- *Low complexity:* there is not any comparison required at soft decision stage. Only at hard decision stage, M comparisons are required, one for each user. Moreover, the numerical results show that two or three iterations are sufficient for the EM algorithm convergence.

- *Unique convergence:* in contrast to the multi-stage receiver, the performance of this detector does not depend on the initial estimation at first iteration, and global convergence can be achieved for any initial values.

- *Robustness:* in this detector, the knowledge of the dark current intensity is not required and in fact the dark current can be estimated simultaneously.

- *Blindness:* there is a very interesting result from (12), (13), and (15), as follows. If we want to decode information bit of only one user, we do not need to know the powers and bits of the interfering users, separately. We actually need to estimate $\tilde{\delta}_i = \tilde{b}_i \lambda_i$. By substituting $\tilde{\delta}_i$ in (12), (13), and (15) we obtain

$$\alpha_{ji} = \frac{c_{ji}(\tilde{\delta}_i^{[k]} + \lambda_d^{(i)})}{\sum_{i=1}^{M} c_{ji}\tilde{\delta}_i^{[k]} + \lambda_d} y_j , \tag{23}$$

$$\beta_j = \frac{\tilde{\lambda}_d^{[k]}}{\sum_{i=1}^{M} c_{ji}\tilde{\delta}_i^{[k]} + \lambda_d} y_j , \tag{24}$$

$$\tilde{\delta}_i^{[k+1]} = \frac{\sum_{j=1}^{F} \alpha_{ji}}{w} . \tag{25}$$

Now, the interference can be estimated as

$$\tilde{I}_{ji} = \sum_{\substack{k=1 \\ k \neq i}}^{M} c_{jk}\tilde{\delta}_k + \lambda_d . \tag{26}$$

The final hard stage can be implemented by Eq. (22). This detector requires the same knowledge as required by the decorrelating detector, even though, as our simulation results indicate, it substantially outperforms the decorrelating detector.

IV. Numerical Results

In this section, some numerical results are presented to demonstrate the performance of our proposed detectors. For simulation, we consider a chip synchronous OOK-CDMA with OOC signature sequence (200,3,1,1). The maximum number of user is 33 [1]. The bit error rates of the proposed detectors are obtained by Monte Carlo simulation and then compared with those of various previously proposed detectors, namely the decorrelating, multi-stage, and known interference detectors. Note that the performance of the known interference detector is a lower bound for the performance of the optimum detector [4]. Intensity of all users is assumed to be identical and the dark current intensity is chosen to be 0.1. As mentioned before, in our proposed detectors the initial value of the transmitted bits at soft stage can be chosen arbitrary. In simulation, we have, however, set the soft initial values of the other user bits equal to half value of the desired user (user one) power.

It must be noted that although the average performance of the system depends on the number of simultaneous users, the performance at a particular instance is dictated by the interference pattern. So, we consider three interference patterns. First, we consider weak interference pattern in which only one of the three marked chips of the desired user signature code collides with interfering pulses. In the second pattern, we assume only two marked chips collide with interfering pulses, and in last pattern, we consider the case in which all three marked chips will collide with interference pulses.

Figs. 3-4, present the plots of bit error rate versus the user's powers for cases 2-3, respectively. As can be observed, for all cases, EM multiuser detector substantially outperforms the multi-stage and decorrelating detectors.

In the above simulations, we have assumed that the dark current intensity is known. To investigate the robustness of the above receivers, we have evaluated

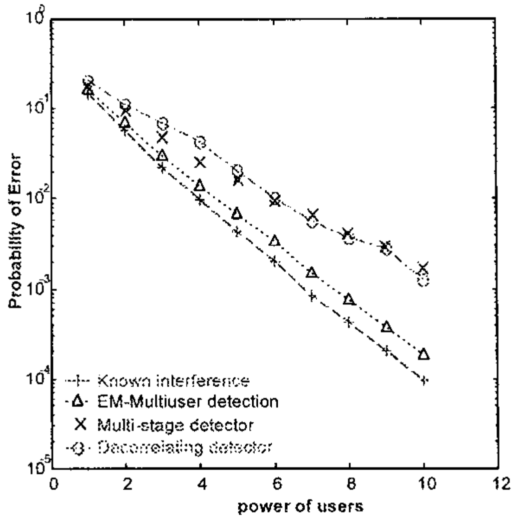

Fig. 3. bit error probability of user one versus intensity for case 2, where *F=200*, *w=3*, and *M=8*.

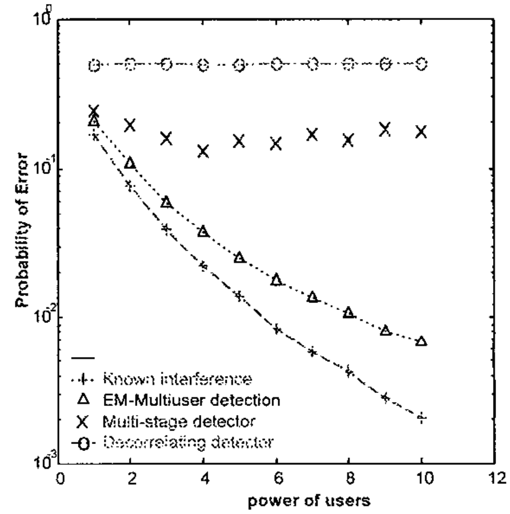

Fig. 4. bit error probability of user one versus intensity for case 3, where *F=200*, *w=3*, and *M=8*.

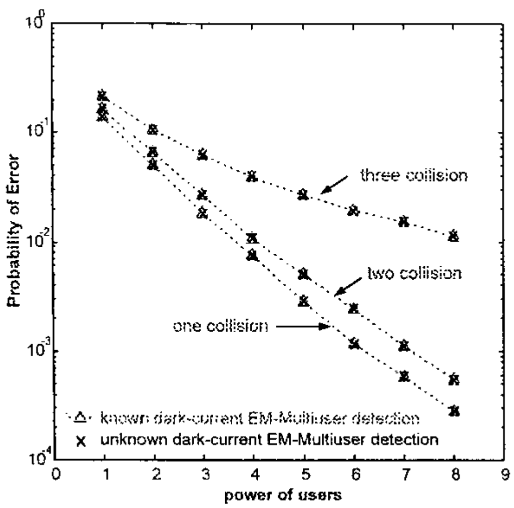

Fig. 5. bit error probability of user one versus intensity for EM-Multiuser detector in two cases: known dark-current and unkown dark-current, where *F=200*, *w=3*, and *M=8*.

the performance of the detectors when the dark current intensity is not known and must be estimated. Fig. 5 illustrates the performance of the proposed detectors for two cases, namely known and unknown dark current intensity, respectively. As expected, the performance for the above two cases are almost the same.

V. Conclusion

In this paper, we have introduced several multiuser detectors for an optical CDMA system based on EM-algorithm. These detectors have two stages in which at soft stage a soft estimation of the interference is obtained by solving an unconstrained likelihood function through EM algorithm. Then, at hard stage by solving a one-dimensional constrained Boolean likelihood function conditioned on knowing the interference, the input bit is detected.

The derived detectors have a very low computational complexity and are robust against the channel parameters variations. Our simulation results have shown that the proposed detectors substantially outperform the conventional, multistage, and decorrelating detectors in the cases considered. Moreover, the numerical results have indicated that at the soft stage, only a few iterations are required for the EM algorithm convergence.

Appendix A

Expectation computation

In this appendix, we derive the equation (12). First, the probability density function (pdf) of N_{ji} conditioned on y_j and $\mathbf{b}^{[k]}$ is computed. From Baye's formula, we have

$$f(N_{ji}|y_j,\mathbf{b}^{[k]}) = \frac{f(y_j|N_{ji},\mathbf{b}^{[k]})\,f(N_{ji}|\mathbf{b}^{[k]})}{f(y_j|\mathbf{b}^{[k]})}. \quad (A.1)$$

By denoting the intensity at the jth chip position as

$$I_j = \sum_{l=1}^{M} c_{jl} b_l^{[k]} \lambda_l + \lambda_d, \quad (A.2)$$

and the intensity of desired user, user i, as

$$I_{ji} = c_{ji} b_i^{[k]} \lambda_i + \lambda_d^{(i)}, \quad (A.3)$$

it can be easily shown that

$$f(N_{ji}|\mathbf{b}^{[k]}) = \frac{e^{-I_{ji}} I_{ji}^{N_{ji}}}{N_{ji}!}, \quad (A.4)$$

$$f(y_j|\mathbf{b}^{[k]}) = \frac{e^{-I_j} I_j^{y_j}}{y_j!}, \quad (A.5)$$

and

$$f(y_j|N_{ji},\mathbf{b}^{[k]}) = \frac{e^{-(I_j-I_{ji})}(I_j-I_{ji})^{y_j-N_{ji}}}{(y_j-N_{ji})!}. \quad (A.6)$$

By substituting these equations in (A.1), we obtain

$$f(N_{ji}|y_j,\mathbf{b}^{[k]}) = \binom{y_j}{N_{ji}} \rho^{N_{ji}} (1-\rho)^{y_j-N_{ji}}, \quad (A.7)$$

where $\rho = I_{ji}/I_j$. Thus, N_{ji} conditioned on y_j and $b^{[k]}$ has a binomial distribution with parameter ρ. As a result, the expectation of N_{ji} conditioned on y_j and $b^{[k]}$ is as

$$E[N_{ji}|y_j,\mathbf{b}^{[k]}] = \rho y_j, \quad (A.8)$$

and then (12) is derived.

Acknowledgment

The authors would like to thank Dr. J.A. Salehi, M. Razavi, and A. Keshavarzian for their helpful comments.

References

[1] J. A. Salehi, "Code division multiple-access techniques in optical fiber network–part I: fundamental principles," *IEEE Trans. Commun*, vol. com-37, no.8, pp. 824-833, Aug 1989.
[2] J. A. Salehi and C. A. Brackett, "Code division multiple-access techniques in optical fiber network–part II: system performance analysis," *IEEE Trans. Commun.*, vol. com-37 , no.8, pp.834-842 ,Aug 1989.
[3] S. Verdú, " Multiple-access channels with point process observation: optimum demodulation," *IEEE Trans. info. Theory*, vol. IT-32, no. 5 , pp. 642-651 , September 1986.
[4] M.B. Pearce and B. Aazhang, " Multiuser detection for optical code division multiple access," *IEEE Trans. Commun.* , Vol. 42, no. 2/3/4 , pp. 1801-1810, Feb/Mar/Apr 1994.
[5] ...., " Performance analysis of single user and multiuser detectors for optical code division multiple access communication systems," *IEEE Trans. Commun*. vol. 43, pp. 435-444, Feb/Mar/Apr 1995.
[6] L. B. Nelson and H. V. Poor, " Performance of multiuser detection for optical CDMA –part I: Error probabilities," *IEEE Trans. Commun.* , vol. 43 , no.11,pp.2803-2811.Nov 1995.
[7] A.P. Dempster, N.M. Laired, and D.B. Rubin, " Maximum likelihood from incomplete data via the EM algorithm," *J. Royal statistical society*, ser. B, vol. 39, no. 1, pp. 1-38, 1977.