The Capacity Region of Fading Multiple Access Channels with Cooperative Encoders and Partial CSIT

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Abstract—**In this paper, we study the two-user Gaussian fading Multiple Access Channel (MAC) with cooperative encoders. Two different scenarios are studied: the Gaussian fading MAC with a common message, and the Gaussian fading MAC with conferencing encoders. The throughput capacity region of these channels with partial Channel State Information (CSI) at the transmitters (CSIT) and perfect CSI at the receiver (CSIR) is established. For the Gaussian fading systems with only CSIR (transmitters have no access to CSIT), some numerical examples and simulation results are provided for Rayleigh fading models.**

***Keywords*-Multiple access channel; cooperative encoders; conferencing; fading channels; channel state information.**

I. INTRODUCTION

The Multiple access channel (MAC) with a common message was first investigated in [1], (see also [2]), where the capacity region of a discrete two-user MAC with common information was established. The capacity region of the Gaussian channel model has been recently characterized in [3]. A common message known to the encoders is also useful to model communication channels in which partial cooperation is allowed between the encoders via conferencing. The notion of conferencing between transmitters of MAC was first introduced by Willems [4], where the capacity region of the two-user discrete memoryless MAC with conferencing encoders was established; in the latter scenario, transmitters are connected by communication links with finite capacities, which permit both encoders to communicate over noise-free bit-pipes of given capacity. The capacity region of the two-user Gaussian MAC with conferencing encoders was characterized in [5].

Cooperation between different nodes of a communication system via common message and conferencing was also studied for other channel models. Specifically, in [6] a three-transmitter one-receiver Gaussian MAC with encoders partially cooperating over a ring of finite-capacity unidirectional links was studied. In [7] the capacity region of the compound MAC (a two-transmitter/two-receiver MAC) with a common message, also with conferencing encoders was derived. The authors in [7] also studied the interference channel with common information and also with conferencing encoders and the capacity region was established for some special cases. A two-user broadcast channel where the decoders cooperate via conferencing was investigated in [8]. The compound MAC with a common message and conferencing decoders and also the compound MAC when both encoders and decoders cooperate via conferencing links, was considered in [9] and the capacity region was established for some cases, specially, the physically degraded channel.

In this paper, we first investigate the Gaussian fading MAC with a common message and stationary ergodic state process. The Gaussian fading MAC without common message was studied in [10] (see also [11]). The authors in [10] considered a Gaussian fading $\wp$ -transmitter/one-receiver MAC (without common message) and obtained throughput capacities, delay limited capacities and the associated optimal power allocation for this channel model. Now, we establish the throughput capacity region for the Gaussian fading MAC with a common message, when the Channel State Information (CSI) is perfectly known at the receiver, while partial CSI is available at the transmitters. Precisely speaking, partial CSI at the transmitters (CSIT) in our model refers to the case where a version of CSI is available at each encoder which is a deterministic function of the channel state process, while perfect CSI at the receivers (CSIR) refers to the case for which the state process is available at the receiver completely. This description of partial CSIT and perfect CSIR for the Gaussian fading MAC was previously considered in [12].

Similar to the MAC with a common message, we also study a two-user Gaussian fading MAC with conferencing encoders. We extend Willems' approach [4], (who derived the capacity region of the MAC with conferencing encoders using the MAC with a common message,) to the case of Gaussian fading MAC, and determine the capacity region of this channel with partial CSIT and perfect CSIR. In our model for the Gaussian fading MAC with conferencing encoders, the transmitters know partial CSIT after the conference.

The rest of the paper is organized as follows: In Section II, preliminaries and channel models are introduced. The main results are presented in Section III, where in Subsection III-A the Gaussian fading MAC with a common message and in Subsection III-B the Gaussian fading MAC with conferencing encoders are investigated. In Section IV, numerical examples and simulation results are provided. Finally, the paper is concluded in Section V.

II. CHANNEL MODELS AND DEFINITIONS

In this paper, we use the following notations: random variables (r.v.) are denoted by upper case letters (e.g. X) and lower case letters are used to show their realizations (e.g. x). The probability distribution function (p.d.f.) of a r.v. X with alphabet set $\mathcal{X}$, is denoted by $P_X(x)$ where $x \in \mathcal{X}$ and $P_{X|Y}(x|y)$ denotes the conditional p.d.f. of X given Y. $\mathbb{E}[.]$ indicates the expectation operator, where sometimes to be more precise, we use $\mathbb{E}_X[.]$ to denote expectation with respect to the distribution of the r.v. X. We also use $\boldsymbol{h}(.)$ to represent differential entropy.

Gaussian Fading MAC: A two-user Gaussian fading MAC is a channel with formulation as:

$$Y_t = S_{1,t}X_{1,t} + S_{2,t}X_{2,t} + Z_t, \quad t \geq 1 \tag{1}$$

where $\{Y_t\}_{t=1}^{\infty}, \{X_{i,t}\}_{t=1}^{\infty}, \{S_{i,t}\}_{t=1}^{\infty}, i = 1,2$, are the $\mathbb{R}$ -valued received signals at the receiver, the transmitted signal of the i^{th} transmitter and the fading coefficient experienced by the signal of the i^{th} transmitter at the receiver , respectively; $\{Z_t\}_{t=1}^{\infty}$ is independent identically distributed (i.i.d.) Additive White Gaussian Noise (AWGN) with zero mean and variance P_z, at the receiver. The fading processes $\{S_{i,t}\}_{t=1}^{\infty}, i = 1,2$ are assumed to be jointly stationary and ergodic but not necessarily independent of each other. For the system in (1), we define the state process of

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the channel, S_t, to be $S_t \triangleq (S_{1,t}, S_{2,t}),\ t \geq 1$. The state process of the channel $\{S_t\}_{t=1}^{\infty}$ is independent of the AWGN process $\{Z_t\}_{t=1}^{\infty}$.

Partial CSIT and perfect CSIR: In this paper, we consider the channel model in (1) with partial CSIT: the i^{th} transmitter at each time instant knows a version of the current CSI determined by a deterministic function of the state process of the channel. In other words, the CSI available at the i^{th} transmitter is given by $\xi_i\big(S = (S_1, S_2)\big)$, where $\xi_i(.)\colon \mathbb{R}^2 \to \mathcal{E}_i, i = 1,2$, and $\mathcal{E}_i$ is an arbitrary set (maybe finite) associated to the i^{th} transmitter. We also assume that the decoder has access to the perfect CSIR, i.e. the state of the channel $S = (S_1, S_2)$ is completely known at the receiver.

MAC with a common message: For the Gaussian fading MAC described in (1) with a common message, a length-n block code $\mathfrak{C}_n$ with a common message set $\mathcal{W}_0 = \{1, \dots, 2^{nR_0}\}$ and private message sets $\mathcal{W}_i = \{1, \dots, 2^{nR_i}\}, i = 1,2$, is defined by two set of encoder functions $\{\mathfrak{E}_{i,t}\}_{t=1}^{n}$ such that:

$$\mathfrak{E}_{i,t}\colon \mathcal{W}_0 \times \mathcal{W}_i \times \mathcal{E}_i \to \mathbb{R}, \qquad t = 1, \dots, n, \qquad i = 1,2$$

where the input symbol of the channel from the i^{th} transmitter at each time instant t, is given by $X_{i,t} = \mathfrak{E}_{i,t}\big(W_0, W_i, E_{i,t}\big)$, $E_{i,t} = \xi_i(S_t) \in \mathcal{E}_i$; a decoder function which is given by:

$$\mathfrak{D}\colon \mathcal{Y}^n \times (\mathbb{R}^2)^n \to \mathcal{W}_0 \times \mathcal{W}_1 \times \mathcal{W}_2$$

which estimates the messages as: $\big(\widehat{W}_0, \widehat{W}_1, \widehat{W}_2\big) = \mathfrak{D}(\mathcal{Y}^n, S^n)$, where $S_t = \big(S_{1,t}, S_{2,t}\big), t = 1, \dots, n$, is the state process of the channel at time instant t.

MAC with Conferencing Encoders: A length-n block code $\mathfrak{C}(n, K, C_{12}, C_{21})$, for the Gaussian MAC described in (1), with conferencing encoders, has two private message sets $\mathcal{W}_i = \{1, \dots, 2^{nR_i}\}, i = 1,2$: before transmission of the messages over the channel, two encoders hold a conference, i.e., the code $\mathfrak{C}(n, K, C_{12}, C_{21})$ consists of two sets of K communicating functions $\{\mathscr{k}_{i,1}, \dots, \mathscr{k}_{i,K}\}, i = 1,2$, and two sets of (finite) conferencing alphabets $\{\mathcal{V}_{i,1}, \dots, \mathcal{V}_{i,K}\}, i = 1,2$. Each communicating function $\mathscr{k}_{i,k}, i = 1,2$, for $k = 1, \dots, K$, maps the message W_i and the sequence of the previously received symbols from the other transmitter into the k^{th} symbol $V_{i,k} \in \mathcal{V}_{i,k}$. In notations we have:

$$\mathscr{k}_{1,k}\colon \mathcal{W}_1 \times \mathcal{V}_2^{k-1} \to \mathcal{V}_{1,k}, \qquad V_{1,k} = \mathscr{k}_{1,k}(W_1, V_2^{k-1})$$
$$\mathscr{k}_{2,k}\colon \mathcal{W}_2 \times \mathcal{V}_1^{k-1} \to \mathcal{V}_{2,k}, \qquad V_{2,k} = \mathscr{k}_{2,k}(W_2, V_1^{k-1})$$

A conference is said to be (C_{12}, C_{21})-permissible [4], if the set of communicating functions are such that:

$$\sum_{k=1}^{K} \log\|\mathcal{V}_{1,k}\| \leq nC_{12},\ \sum_{k=1}^{K} \log\|\mathcal{V}_{2,k}\| \leq nC_{21}$$

where $\|\mathcal{V}_{i,k}\|$ denotes cardinality of $\mathcal{V}_{i,k}$.

As we mentioned before, the CSIT is known to the encoders after the conference. Assume that the information that the first transmitter obtains after the conference to be $V_2^K = \big(V_{2,1}, \dots, V_{2,K}\big)$ and for the second transmitter to be $V_1^K = \big(V_{1,1}, \dots, V_{1,K}\big)$. By these assumptions the i^{th} encoder consists of a set of functions $\{\mathfrak{E}_{i,t}\}_{t=1}^{n}$, $i = 1,2$, where for $t = 1, \dots, n$:

$$\begin{Bmatrix} \mathfrak{E}_{1,t}\colon \mathcal{W}_1 \times V_2^K \times \mathcal{E}_1 \to \mathbb{R} \\ \mathfrak{E}_{2,t}\colon \mathcal{W}_2 \times V_1^K \times \mathcal{E}_2 \to \mathbb{R} \end{Bmatrix}$$

Thus, the input symbol of the channel from the first transmitter at each time instant t, is now given by $X_{1,t} = \mathfrak{E}_{1,t}\big(W_1, V_2^n, E_{1,t}\big)$, where $E_{1,t} = \xi_1(S_t) \in \mathcal{E}_1$, (respectively, $X_{2,t} = \mathfrak{E}_{2,t}\big(W_2, V_1^n, E_{2,t}\big)$, where $E_{2,t} = \xi_2(S_t) \in \mathcal{E}_2$). The decoder function is given by:

$$\mathfrak{D}\colon \mathcal{Y}^n \times (\mathbb{R}^2)^n \to \mathcal{W}_1 \times \mathcal{W}_2$$

which estimates the messages as $\big(\widehat{W}_1, \widehat{W}_2\big) = \mathfrak{D}(Y^n, S^n)$, where $S_t = \big(S_{1,t}, S_{2,t}\big), t = 1, \dots, n$, is the state process of the channel at time instant t.

For both channel with a common message and also channel with conferencing encoders, the i^{th} transmitter is subject to an average power constraint $P_i \in \mathbb{R}^+$, $i = 1,2$. Precisely speaking, for the codewords of the i^{th} transmitter we have:

$$\frac{1}{n}\mathbb{E}\big[\textstyle\sum_{t=1}^{n} X_{i,t}^2\big] \leq P_i \qquad i = 1,2 \tag{2}$$

In the next section, we state our main results for channel models presented here.

III. Main results

III-A) Gaussian fading MAC with a common message:

In this section, we study the Gaussian fading MAC defined in (1), with a common message. In the following theorem, we present our main result and characterize the throughput capacity region of the underlying channel:

Theorem 1: Consider the Gaussian fading MAC in (1) with a common message, where CSI is partially known to encoders and the receiver has access to perfect CSIR. The capacity region denoted by $\mathcal{C}_{cm}^{Gf-MAC}$ is given as:

$$\mathcal{C}_{cm}^{Gf-MAC} = \bigcup_{\substack{\varphi_i(.)\colon \mathcal{E}_i \to \mathbb{R}^+ \\ \varrho_i(.)\colon \mathcal{E}_i \to [0,1] \\ i=1,2}} \left\{ \begin{array}{l} 0 \leq R_0, R_1, R_2: \\ R_1 \leq \mathbb{E}_S\left[\mathbb{C}\left(\frac{|S_k|^2 \varphi_1(E_1)\left(1-\varrho_1^2(E_1)\right)}{P_z}\right)\right] \\ R_2 \leq \mathbb{E}_S\left[\mathbb{C}\left(\frac{|S_2|^2 \varphi_2(E_2)\left(1-\varrho_2^2(E_2)\right)}{P_z}\right)\right] \\ R_1 + R_2 \leq \mathbb{E}_S\left[\mathbb{C}\left(\frac{\sum_{k=1}^{2}\left(|S_k|^2 \varphi_k(E_k)\left(1-\varrho_k^2(E_k)\right)\right)}{P_z}\right)\right] \\ R_0 + R_1 + R_2 \leq \mathbb{E}_S\left[\mathbb{C}\left(\frac{|S_1|^2\varphi_1(E_1) + |S_2|^2\varphi_2(E_2) + 2S_1S_2\varrho_1(E_1)\varrho_2(E_2)\sqrt{\varphi_1(E_1)\varphi_2(E_2)}}{P_z}\right)\right] \end{array} \right\} \tag{3}$$

where $\mathbb{C}(x) \equiv \frac{1}{2}\log(1 + x)$, $|.|$ stands for the absolute value, $S = (S_1, S_2) \in \mathbb{R}^2$ is a r.v. representing the state process of the channel, $E_i \in \mathcal{E}_i, i = 1,2$ is a r.v. representing the CSIT available at the i^{th} transmitter, which is a deterministic function of S: $E_i = \xi_i(S)$. In addition, $\varphi_i(.)\colon \mathcal{E}_i \to \mathbb{R}^+$ is the power allocation policy for the i^{th} transmitter, satisfying the following constraint:

$$\mathbb{E}[\varphi_i(E_i)] \leq P_i,\ i = 1,2 \tag{4}$$

and $\varrho_i(.)\colon \mathcal{E}_i \to [0,1]$ is an arbitrary deterministic function, taking its values in the interval [0,1].

Remark: By setting $R_0 = 0$, $\varrho_i(.) \equiv 0, i = 1,2$, and also $E_i \equiv S, i = 1,2$, (perfect CSIT), the rate region in (3), reduces to the capacity region of the two-user Gaussian fading MAC without common message with perfect CSIT and also perfect CSIR, obtained previously in [10, Theorem 2.1].

Proof of theorem 1: <u>*Achievability part*</u>:

Lemma 1: Let (U, V_1, V_2) be a triple of auxiliary r.v.s with alphabets $U \in \mathcal{U}$ and $V_i \in \mathcal{V}_i, i = 1,2$. Then for the Gaussian fading MAC in (1) with a common message, with partially CSIT and perfect CSIR, the following rate region is achievable:

$$\bigcup_{\substack{P_U(u)\prod_{i=1}^{2} P_{V_i|U}(v_i|u), \\ f_i(.)\colon \mathcal{V}_i \times \mathcal{U} \times \mathcal{E}_i \to \mathbb{R}: \\ X_i = f_i(V_i, U, E_i) \\ i=1,2}} \left\{ \begin{array}{l} 0 \leq R_0, R_1, R_2: \\ R_1 \leq I(V_1; Y|V_2, U, S) \\ R_2 \leq I(V_2; Y|V_1, U, S) \\ R_1 + R_2 \leq I(V_1, V_2; Y|U, S) \\ R_0 + R_1 + R_2 \leq I(U, V_1, V_2; Y|S) \end{array} \right\} \tag{5}$$

where $Y = S_1X_1 + S_2X_2 + Z$, and the joint p.d.f. of the r.v.s $(S, E_1, E_2, U, V_1, V_2)$ is given by:

$$P_{SE_1E_2UV_1V_2}(s, e_1, e_2, u, v_1, v_2) = P_{SE_1E_2}(s, e_1, e_2)P_U(u)P_{V_1|U}(v_1|u)P_{V_2|U}(v_2|u). \quad (6)$$

Furthermore, $f_i(.): \mathcal{V}_i \times \mathcal{U} \times \mathcal{E}_i \to \mathbb{R}$ are two deterministic functions such that $X_i = f_i(V_i, U, E_i)$ satisfies the power constraint policy of the i^{th} transmitter: $\mathbb{E}[X_i^2] \le P_i, i = 1,2$.

Proof: The proof could be seen in [13, proof of Theorem 1] by assuming two-user MAC. We do not provide it here because of the lack of space.

Now, to prove the achievability of (3), let (U, V_1, V_2) be a triple of Gaussian distributed r.v.s with zero mean and unit variance, and independent of each other and also independent of the state S. Furthermore, assume $\{\varphi_i: \mathcal{E}_i \to \mathbb{R}^+\}_{i=1}^2$ to be two power allocation policy functions which satisfy the power constraints in (4) and $\varrho_i(.): \mathcal{E}_i \to [0,1], i = 1,2$, to be two arbitrary deterministic functions with range [0,1]. Define the r.v.s X_1, X_2 as:

$$X_i \triangleq \sqrt{\varphi_i(E_i)}\left(\varrho_i(E_i)U + \sqrt{1-\varrho_i^2(E_i)}V_i\right),\ i = 1,2 \quad (7)$$

Note that in definition (7), E_1, E_2 indicates CSITs with joint distribution determined by $P_{SE_1E_2}(s, e_1, e_2)$. One can easily check that the r.v.s X_1, X_2 defined in (7) satisfy:

$$\mathbb{E}_S[X_i^2] = \mathbb{E}_S[\varphi_i(E_i)] \le P_i,\ i = 1,2$$

Then by substituting $\langle U, V_1, V_2, X_1, X_2\rangle$, into the rate region given by (5), we obtain the achievability of (3). The details of computations are relegated to [13, Theorem 1] by assuming two-user MAC.

<u>Converse part</u>: To prove the converse, we first derive an upper bound on $\mathcal{C}_{cm}^{Gf-MAC}$, in terms of mutual information functions, in the following lemma.

Lemma 2: $\mathcal{C}_{cm}^{Gf-MAC}$ is upper bounded by:

$$\mathcal{C}_{cm}^{Gf-MAC} \subseteq \bigcup_{P_U P_{X_1|UE_1} P_{X_2|UE_2}} \left\{\begin{array}{l} 0 \le R_0, R_1, R_2: \\ R_1 \le I(X_1; Y|X_2, U, S) \\ R_2 \le I(X_2; Y|X_1, U, S) \\ R_1 + R_2 \le I(X_1, X_2; Y|U, S) \\ R_0 + R_1 + R_2 \le I(X_1, X_2; Y|S) \end{array}\right\} \quad (8)$$

over all joint p.d.f.s of the form:

$$P_{SE_1E_2UX_1X_2}(s, e_1, e_2, u, x_1, x_2) = P_{SE_1E_2}(s, e_1, e_2)P_U(u)P_{X_1|UE_1}(x_1|u, e_1)P_{X_2|UE_2}(x_2|u, e_2) \quad (9)$$

for which the power constraints: $\mathbb{E}[X_i^2] \le P_i,\ i = 1,2$ hold. Note that in (8), $S = (S_1, S_2) \in \mathbb{R}^2$ is a r.v. representing the state process of the channel and $E_i \in \mathcal{E}_i$ is the r.v. representing the CSIT available at the i^{th} transmitter, which is a deterministic function of S : $E_i = \xi_i(S), i = 1,2$. We also note that $P_{SE_1E_2}(s, e_1, e_2)$ is obtained from the (first order) distribution of the state process of channel $P_S(s)$, which describes the behavior of the fading coefficients.

Proof: See [13, Appendix II] by assuming two-user MAC.

Now, to see that the rate region in (3), is also an upper bound on the capacity region of the underlying channel, i.e., $\mathcal{C}_{cm}^{Gf-MAC}$, it is enough to show that the rate region (8), is a subset of that one in (3). However, we prove a stronger result, which is the fact that, the rate region described in (3) is equivalent to that one given by (8). First notice that one can define the pair of r.v.s (X_1, X_2) as in (7), to satisfy the joint p.d.f. of (9) and also the power constraints $\mathbb{E}[X_i^2] \le P_i,\ i = 1,2$. So by substituting $\langle U, X_1, X_2\rangle$ as defined through (7) in the rate region described in (8), one can see that (3) is resulted. Conversely, we show that the rate region in (8) does not exceed any points of (3). First, we state some useful technical lemmas from probability theory in the following.

For arbitrary r.v.s X and Y, $VAR(X|Y = y)$ is defined as:

$$VAR(X|Y = y) \triangleq \mathbb{E}[X^2|Y = y] - (\mathbb{E}[X|Y = y])^2 \quad (10)$$

that is in fact the variance of X with respect to the distribution $P_{X|Y}(x|y)$, and so it is a positive quantity.

Lemma 3: Consider three r.v.s X, Y and Z:
I) Assume Y and Z are independent. Then the following holds:

$$\mathbb{E}_Y\big[\mathbb{E}[X^2|Y, Z = z]\big] = \mathbb{E}[X^2|Z = z] \quad (11)$$

II) Assume X and Z are real valued and $X \to Y \to Z$ form a Markov chain. Then:

$$VAR(X + Z|Y = y) = VAR(X|Y = y) + VAR(Z|Y = y) \quad (12)$$

Proof: By direct computation.

Lemma 4: Consider r.v.s $(S, E_1, E_2, U, X_1, X_2)$, with the joint p.d.f. given as (9), where X_1 and X_2 are real-valued and there exist deterministic functions $\xi_i(.)$, such that: $E_i = \xi_i(S), i = 1,2$. Then, the following inequality holds:

$$\mathbb{E}[X_1X_2|S = s] \le \sqrt{\mathbb{E}_U[(\mathbb{E}[X_1|U, E_1 = \xi_1(s)])^2]} \times \sqrt{\mathbb{E}_U[(\mathbb{E}[X_2|U, E_2 = \xi_2(s)])^2]} \quad (13)$$

Proof: See [13, Lemma 3].

Now, consider the rate region described in (8). Define deterministic functions $\varphi_i(.): \mathcal{E}_i \to \mathbb{R}^+, i = 1,2$, such that:

$$\forall\ e_i \in \mathcal{E}_i\ :\quad \varphi_i(e_i) \triangleq \mathbb{E}[X_i^2|E_i = e_i] \quad (14)$$

and also deterministic functions $\varrho_i(.): \mathcal{E}_i \to [0,1], i = 1,2$, as follows:

$$\forall\ e_i \in \mathcal{E}_i\ :\quad \varrho_i(e_i) \triangleq \sqrt{\frac{\mathbb{E}_U[(\mathbb{E}[X_i|U, E_i = e_i])^2]}{\varphi_i(e_i)}} \quad (15)$$

Notice that:

$$\mathbb{E}[X_i^2|U = u, E_i = e_i] - (\mathbb{E}[X_i|U = u, E_i = e_i])^2 = VAR(X_i|U = u, E_i = e_i) \ge 0 \quad (16)$$

which yields:

$$\mathbb{E}[X_i^2|E_i = e_i] - \mathbb{E}_U[(\mathbb{E}[X_i|U, E_i = e_i])^2] \overset{(a)}{=} \mathbb{E}_U[\mathbb{E}[X_i^2|U, E_i = e_i] - (\mathbb{E}[X_i|U, E_i = e_i])^2] \ge 0 \quad (17)$$

where (a) is because of part I of Lemma 3. Therefore, for each $e_i \in \mathcal{E}_i$, $\varrho_i(e_i)$ belongs to the interval [0,1] and the notation $\varrho_i(.): \mathcal{E}_i \to [0,1]$ is well-defined. Furthermore from (14), we have:

$$\mathbb{E}[\varphi_i(E_i)] = \mathbb{E}\big[\mathbb{E}[X_i^2|E_i]\big] = \mathbb{E}[X_i^2] \le P_i,\ i = 1,2$$

Now for the mutual information function terms in the rate region (8), we can proceed as:

$$\begin{aligned} I(X_1; Y|X_2, U, S) &= \boldsymbol{h}(S_1X_1 + S_2X_2 + Z|X_2, U, S) \\ &\quad - \boldsymbol{h}(S_1X_1 + S_2X_2 + Z|X_1, X_2, U, S) \\ &\overset{(a)}{=} \boldsymbol{h}(S_1X_1 + Z|U, S) - \tfrac{1}{2}\log(2\pi eP_z) \\ &\overset{(b)}{=} \mathbb{E}_S\big[\mathbb{E}_U[\boldsymbol{h}(S_1X_1 + Z|U = u, S = s)]\big] \\ &\quad - \tfrac{1}{2}\log(2\pi eP_z) \\ &\le \mathbb{E}_S\left[\mathbb{E}_U\left[\tfrac{1}{2}\log\big(2\pi eVAR(S_1X_1 + Z|U = u, S = s)\big)\right]\right] \\ &\quad - \tfrac{1}{2}\log(2\pi eP_z) \end{aligned}$$

$$= \mathbb{E}_S\left[\mathbb{E}_U\left[\frac{1}{2}\log\left(2\pi e\left(|s_1|^2 VAR(X_1|U=u,S=s)+P_z\right)\right)\right]\right] - \frac{1}{2}\log(2\pi e P_z)$$
$$\overset{(c)}{=} \mathbb{E}_S\left[\mathbb{E}_U\left[\mathbb{C}\left(\frac{|s_1|^2 VAR(X_1|U=u,E_1=e_1)}{P_z}\right)\right]\right]$$
$$= \mathbb{E}_S\left[\mathbb{E}_U\left[\mathbb{C}\left(\frac{|s_1|^2\left(\mathbb{E}\left[X_1^2|U=u,E_1=e_1\right]-(\mathbb{E}[X_1|U=u,E_1=e_1])^2\right)}{P_z}\right)\right]\right]$$
$$\overset{(d)}{\le} \mathbb{E}_S\left[\mathbb{C}\left(\frac{|s_1|^2\mathbb{E}_U\left[\mathbb{E}\left[X_1^2|U,E_1=e_1\right]-(\mathbb{E}[X_1|U,E_1=e_1])^2\right]}{P_z}\right)\right]$$
$$\overset{(f)}{=} \mathbb{E}_S\left[\mathbb{C}\left(\frac{|S_1|^2\varphi_1(E_1)\left(1-\varrho_1^2(E_1)\right)}{P_z}\right)\right] \quad (18)$$

where (a) is true since $X_2 \to (U,S) \to X_1$ forms a Markov chain, (b) is true since U and S are independent of each other, (c) is due to the fact that from (9) $S \to (U,E_i) \to X_i, i=1,2$, form Markov chains, (d) is due to Jensen's inequality and (f) is obtained from definitions (14) and (15). In a similar way one can show:

$$I(X_2;Y|X_1,U,S) \le \mathbb{E}_S\left[\mathbb{C}\left(\frac{|S_2|^2\varphi_2(E_2)\left(1-\varrho_2^2(E_2)\right)}{P_z}\right)\right]$$
$$I(X_1,X_2;Y|U,S) \le \mathbb{E}_S\left[\mathbb{C}\left(\frac{\sum_{i=1}^2\left(|S_i|^2\varphi_i(E_i)\left(1-\varrho_i^2(E_i)\right)\right)}{P_z}\right)\right] \quad (19)$$

Moreover for the bound on the sum rate in (8), we have:

$$I(X_1,X_2;Y|S) = \boldsymbol{h}(S_1X_1+S_2X_2+Z|S) - \boldsymbol{h}(S_1X_1+S_2X_2+Z|X_1,X_2,S)$$
$$= \boldsymbol{h}(S_1X_1+S_2X_2+Z|S) - \frac{1}{2}\log(2\pi e P_z)$$
$$= \mathbb{E}_S[\boldsymbol{h}(S_1X_1+S_2X_2+Z|S=s)] - \frac{1}{2}\log(2\pi e P_z)$$
$$\le \mathbb{E}_S\left[\tfrac{1}{2}\log(2\pi e\mathbb{E}[(S_1X_1+S_2X_2+Z)^2|S=s])\right] - \frac{1}{2}\log(2\pi e P_z)$$
$$= \mathbb{E}_S\left[\mathbb{C}\left(\frac{|s_1|^2\mathbb{E}\left[X_1^2|S=s\right]+|s_2|^2\mathbb{E}\left[X_2^2|S=s\right]+2s_1s_2\mathbb{E}[X_1X_2|S=s]}{P_z}\right)\right]$$
$$\overset{(a)}{\le} \mathbb{E}_S\left[\mathbb{C}\left(\frac{|s_1|^2\mathbb{E}\left[X_1^2|S=s\right]+|s_2|^2\mathbb{E}\left[X_2^2|S=s\right]+2s_1s_2\sqrt{\mathbb{E}_U[(\mathbb{E}[X_1|U,E_1=\xi_1(s)])^2]}\sqrt{\mathbb{E}_U[(\mathbb{E}[X_2|U,E_2=\xi_2(s)])^2]}}{P_z}\right)\right]$$
$$\overset{(b)}{=} \mathbb{E}_S\left[\mathbb{C}\left(\frac{|S_1|^2\varphi_1(E_1)+|S_2|^2\varphi_2(E_2)+2S_1S_2\varrho_1(E_1)\varrho_2(E_2)\sqrt{\varphi_1(E_1)\varphi_2(E_2)}}{P_z}\right)\right] \quad (20)$$

where (a) is because $S \to E_i \to X_i, i=1,2$ form Markov chains and also Lemma 4 has been applied. (b) is obtained from definitions (14) and (15).

Now substituting (18)-(19) in (8), we obtain the fact that no point outside of the rate region given by (3) is achievable. This completes the proof of Theorem 1. ■

III-B) Gaussian fading MAC with conferencing encoders:

In this section, we investigate the Gaussian fading MAC described in (1) with conferencing encoders. We recall that the capacity region of a discrete two-user MAC with conferencing encoders was obtained in [4]. To prove his result, Willems [4] used the capacity region of discrete MAC with a common message, which has already been obtained in [1] by Slepian and Wolf. In the following theorem, we establish the capacity region of the two-user Gaussian fading MAC in (1), with conferencing encoders:

Theorem 2: The capacity region of the two-user Gaussian fading MAC in (1), with conferencing encoders connected to each other by links of capacity C_{12} and C_{21}, denoted by $\mathcal{C}_{conf}^{Gf-MAC}(C_{12},C_{21})$, when the transmitters have access to partial CSIT and the decoder has access to perfect CSIR, is given by:

$$\mathcal{C}_{conf}^{Gf-MAC}(C_{12},C_{21}) = \bigcup_{\substack{\varphi_1(.),\varphi_2(.)\\ \varrho_1(.),\varrho_2(.)}} \left\{\begin{array}{l} 0 \le R_1, R_2: \\ R_1 \le \mathbb{E}_S\left[\mathbb{C}\left(\frac{|S_1|^2\varphi_1(E_1)\left(1-\varrho_1^2(E_1)\right)}{P_z}\right)\right] + C_{12} \\ R_2 \le \mathbb{E}_S\left[\mathbb{C}\left(\frac{|S_2|^2\varphi_2(E_2)\left(1-\varrho_2^2(E_2)\right)}{P_z}\right)\right] + C_{21} \\ R_1+R_2 \le \mathbb{E}_S\left[\mathbb{C}\left(\frac{\sum_{i=1}^2|S_i|^2\varphi_i(E_i)\left(1-\varrho_i^2(E_i)\right)}{P_z}\right)\right] + C_{12}+C_{21} \\ R_1+R_2 \le \mathbb{E}_S\left[\mathbb{C}\left(\frac{|S_1|^2\varphi_1(E_1)+|S_2|^2\varphi_2(E_2)+2S_1S_2\varrho_1(E_1)\varrho_2(E_2)\sqrt{\varphi_1(E_1)\varphi_2(E_2)}}{P_z}\right)\right] \end{array}\right\} \quad (21)$$

where $S=(S_1,S_2) \in \mathbb{R}^2$ is a r.v. representing the state process of the channel, $E_i \in \mathcal{E}_i$, is a r.v. for the CSIT available at the i^{th} transmitter, $i=1,2$, which is a deterministic function of S, i.e. $E_i = \xi_i(S)$. Furthermore, $\varphi_i(.): \mathcal{E}_i \to \mathbb{R}^+$, is the power allocation policy for i^{th} transmitter, satisfying the following constraint:

$$\mathbb{E}[\varphi_i(E_i)] \le P_i, \; i=1,2 \quad (22)$$

and $\varrho_i(.): \mathcal{E}_i \to [0,1], i=1,2$, are two arbitrary deterministic functions, taking their values in the interval [0,1].

Proof of theorem 2: The proof will be relegated to [13, Theorem 3]. Here, we only notice that the achievability of (21) is obtained using the capacity region of the Gaussian fading MAC with a common message given by (3), and by following the same approach used in [4].

IV. Numerical Examples and Simulations

Now, we examine a few implications of our results for the channel model in (1), with a Rayleigh fading environment. Here, we only consider the Gaussian fading channels with no CSIT (the transmitters have no knowledge about the CSI) and perfect CSIR. At such situation, we denote the capacity region of the Gaussian fading MAC in (1) with a common message by $\mathcal{C}_{cm}^{no-CSIT}$, which is obtained from (3) by setting $E_i \equiv \emptyset, i=1,2$, and also the capacity region of the channel with conferencing encoders by $\mathcal{C}_{conf}^{no-CSIT}(C_{12},C_{21})$, which is obtained from (21) by setting $E_i \equiv \emptyset, i=1,2$. Assume that the fading processes $\{S_{1,t}\}_{t=1}^{\infty}$ and $\{S_{2,t}\}_{t=1}^{\infty}$ in (1) are independent of each other and $S_{i,t}, i=1,2$, is a Rayleigh-distributed r.v. with a p.d.f. given by:

$$P_{S_i}(s) = 2se^{-s^2}, s \ge 0. \quad (23)$$

We plot the capacity region of the two-user Gaussian MAC with common message, $\mathcal{C}_{cm}^{no-CSIT}$, in Fig. 1, under different values of the power ratio of transmitters $\frac{P_1}{P_2}$ (boundaries in each planes with solid lines). The sum power of two transmitters, i.e., P_1+P_2 is fixed for all plots. As we see from figure, when two transmitters have the same power, i.e., $\frac{P_1}{P_2}=1$, the maximum rate of R_0, is attained. This can be justified as follow. It can be easily verified from (3), that maximum achievable rate of the common message R_0, is given by:

$$\max_{R_0 \in \mathcal{C}_{cm}^{no-CSIT}} R_0 = \mathbb{E}_S\left[\mathbb{C}\left(\frac{|S_1|^2P_1+|S_2|^2P_2+2S_1S_2\sqrt{P_1P_2}}{P_z}\right)\right] \quad (24)$$

On the other hand, for fixed value of P_1+P_2, the geometric average $\sqrt{P_1P_2}$ is maximum when $P_1=P_2$, (note that S_1 and S_2 are positive-valued i.i.d. r.v.s). So, when the powers P_1 and P_2, become far from each other, the maximum achievable value of the rate R_0, i.e. the cut-off point on the R_0 axis decreases. We note that for the values of $\frac{P_1}{P_2}$ that are greater than 10, the variation of the power P_1 is less than 10 percent, and so increment in the maximum achievable rate of R_1, i.e. the cut-off point on the R_1-axis, is not significant, as can be seen from Fig. 1.

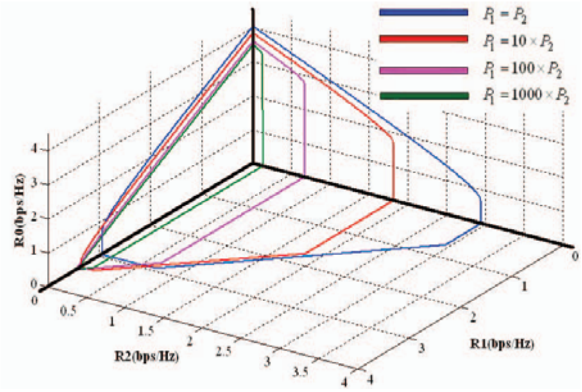

Figure 1. The capacity region of the two-user Gaussian fading MAC with a common message, $\mathcal{C}_{cm}^{no-CSIT}$. The sum power for all plots is fixed and is equal to $P_1 + P_2 = 23.01$ dB, and also $P_z = 0$ dB.

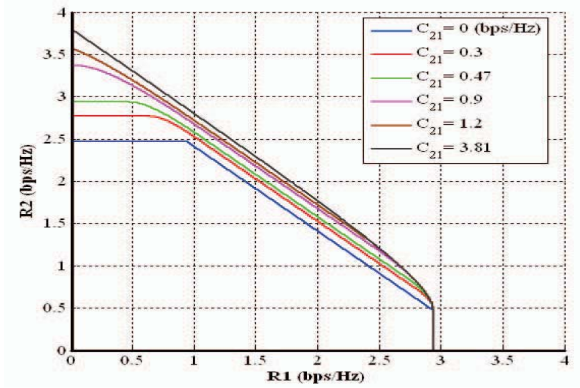

Figure 2. The capacity region of the two-user Gaussian fading MAC with one encoder communicating to the other, $\mathcal{C}_{conf}^{no-CSIT}(0, C_{21})$, for $P_1 = 23.01$ dB, $P_2 = 20$ dB, and also $P_z = 0$ dB.

In Fig. 2, the capacity region of the two-user Gaussian fading MAC with one encoder communicating to the other, i.e., $\mathcal{C}_{conf}^{no-CSIT}(0, C_{21})$, has been plotted. So, in this situation the second encoder can communicate to the first encoder via a link with capacity C_{21}. It is clear from the figure that as the rate of conferencing increases, the capacity region enlarges, however, since only the second encoder can cooperate its message with the first one via conferencing, the maximum achievable rate of R_1 (the cut-off point on R_1-axis), does not change. In Fig. 2 it has assumed that the associated power of the second encoder is half of that one for the first encoder, i.e. $P_2 = \frac{1}{2}P_1$. This causes the capacity region with no conferencing ($C_{21} = 0$), to be an asymmetric region (the region with blue boundary in Fig. 2), such that the value of the cut-off point on the R_2-axis is less than that one on the R_1-axis. As we see from Fig. 2, by holding a conferencing with the rate of about $C_{21} = 0.47$ (bps/Hz), (the region with green boundary), the second encoder can communicate some part of its message to the first one, which makes the maximum achievable rate of R_2 to be the same as that one for R_1, i.e. the cut-off points on both axes are the same. This means that conferencing can compensate for the lack of power. In general, one can see that when the second transmitter is restricted by $P_2 = \alpha P_1$ where $0 < \alpha < 1$, a conferencing link with the capacity $C_{21}(\alpha) = \mathbb{E}_S\left[\mathbb{C}\left(\frac{|S_1|^2(1-\alpha)P_1}{P_z+|S_2|^2\alpha P_2}\right)\right]$ could compensate for the lack of power, depicted by $C_{21}(0.5) = 0.47$ (bps/Hz), in Fig. 2. Moreover, for values of C_{21} greater than $\mathbb{E}_S\left[\mathbb{C}\left(\frac{|S_1|^2P_1+|S_2|^2P_2+2S_1S_2\sqrt{P_1P_2}}{P_z}\right)\right]$, (in Fig. 2 for $C_{21} \geq 3.81$ (bps/Hz)), no further improvement in the capacity region is possible. In fact, for values of C_{21} equal or greater than $\mathbb{E}_S\left[\mathbb{C}\left(\frac{|S_1|^2P_1+|S_2|^2P_2+2S_1S_2\sqrt{P_1P_2}}{P_z}\right)\right]$, by holding a conference, the second transmitter can cooperate its message perfectly with the first one, and the channel acts as a MAC with *degraded messages*: a common message is sent by both transmitters and a private message by just the first one. Therefore, the form of the capacity region is the same as the capacity region of MAC with a common message in (R_0, R_1)-plane, depicted in Fig. 1.

V. CONCLUSION

The two-user Gaussian fading MAC with cooperative encoders was studied. Two different scenarios were investigated: the Gaussian fading MAC with a common message and the Gaussian fading MAC with conferencing encoders. The capacity region of these channels with partial CSIT and perfect CSIR was established.

For the Gaussian fading systems with only CSIR (transmitters have no access to CSIT), some numerical examples and simulation results are provided for Rayleigh fading models and the effect of conferencing on the improvement of the capacity region was investigated. In future work, we will study the problem of optimal power allocation for the Gaussian fading channels with partial CSIT and perfect CSIR, investigated here.

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