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An Improved Image Denoising Technique Using Cycle Spinning

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***Abstract*—Denoising of corrupted images has been a classical problem in image processing. In this paper we propose a new approach for image noise reduction using wavelet transform. In this method an improved version of thresholding neural networks (TNN) is used to find the optimum threshold values in the sense of minimum mean square error (MMSE). Based on these optimum thresholds a novel cycle-spinning based method is used to reduce image artifacts. In this method, we utilize two thresholding schemes as the thresholding operator of cycle-spinning. A neighbor dependent thresholding scheme is employed as its first shrinkage step and a simple wavelet thresholding with the optimum derived threshold values is used as the second thresholding step. Using this approach we will achieve a smooth, artifact free denoised image. Experimental results indicate that the proposed method outperforms several other established wavelet denoising techniques, in terms of peak-signal-to-noise-ratio (PSNR) and visual quality.**

***Index Terms*— Image enhancement, wavelet transforms, neural networks.**

I. Introduction

Digital images are often degraded by noise due to various factors during its acquisition and transmission phases. The presence of noise is visually annoying and makes it more difficult to perform processing tasks such as segmentation, recognition and interpretation. Hence, denoising is a crucial step before image analysis. The aim of image denoising is to effectively reduce the noise while preserving significant image details such as edges.

Traditionally, linear methods such as Wiener filtering are used for this purpose. But these methods tend to reduce the important image features such as edges while removing the noise. Thus, over the last decade, a wide range of nonlinear and especially wavelet-based methods and ideas have been proposed for image noise reduction [1-26]. Donoho and Johnstone pioneered a wavelet denoising scheme by proposing their wavelet shrinkage method [1-3]. This method consists of wavelet transforming of the noisy image, soft or hard thresholding of the resulting wavelet coefficients and performing inverse 2D discrete wavelet transform to obtain the denoised image. This approach can significantly reduce noise, due to the excellent localization property of wavelet transforms which concentrates signal energy on a few number of large wavelet coefficients while uniformly spreading the noise out across all coefficients. The choice of the threshold parameters is an important factor in the success of these methods. Thus, several wavelet thresholding methods with different approaches for determining the threshold parameters have been proposed. Among those methods are VisuShrink [1], SureShrink [3] and BayersShrink [4]. Another methods to find the optimum threshold values is *Thresholding Neural Network* (TNN), introduced by Zhang [5-7]. In this method, wavelet coefficients of the corrupted signal are applied to TNN to perform thresholding by using a class of smooth nonlinear functions. In this method, threshold values are adaptively adjusted for a given nonlinear function.

Although the method of thresholding offers the advantages of smoothness and adaptation, but as Coifman and Donoho [8] suggested, it exhibits visual artifacts known as Gibbs phenomena in the vicinity of discontinuities. Therefore, they proposed a translation invariant (TI) denoising method, also known as cycle spinning, to suppress such artifacts by averaging over the denoised signals of all circular shifts. As the wavelet transform is not translation invariant, this approach will result in different estimates of the original image with statistically different noises, which will be reduced by averaging. Also, several methods have been proposed by considering the effect of neighboring wavelet coefficients on the current coefficient. This approach is based on this fact that a large wavelet coefficient will probably have large wavelet coefficients at its neighbour locations [9]. Chen et al. proposed several methods which consider neighboring dependencies [9-11]. They claimed that their neighboring wavelet thresholding method outperforms traditional simple thresholding methods.

In this paper we propose a new wavelet image denoising method. In this method an improved version of thresholding neural networks (TNN) [26] is used to find the optimum threshold values in the sense of minimum mean square error (MMSE). Then a novel cycle spinning technique is employed which utilizes a two step thresholding technique. As the first step, it uses the neighbor wavelet coefficient dependencies in the thresholding process. A simple wavelet thresholding scheme is implemented for the second thresholding step using a smooth thresholding function which employs the optimum derived threshold values. This approach will lead to a smooth, artifact free denoised image. Experimental results indicate that the proposed method outperforms several other established denoising methods, in terms of PSNR and visual quality.

II. BACKGROUND

A. Wavelet Thresholding

Symbolically, let x be the unknown clean image, n the additive independent Gaussian noise and y the observed noisy image, i.e. $y=x+n$. Then denoising is defined as retrieving a reconstructed image $\hat{x}$, which minimizes the mean squared error. Donoho and Johnstone suggested a wavelet denoising scheme known as wavelet thresholding [1-3]. This method consists of three steps: Wavelet transforming of the noisy image, soft or hard thresholding of the resulting wavelet coefficients and performing inverse 2D discrete wavelet transform to obtain the denoised image.

If we denote the 2-D wavelet transform by W and the thresholding function by $\eta(.)$, the shrinkage is performed as $\hat{x} = W^{-1}(\eta(W(y)))$. The most common choices for $\eta(.)$, are hard and soft-thresholding functions, which respectively are:

$$\eta_h(x,\lambda) = x.\max(0, abs(x)-\lambda) \quad (1)$$

$$\eta_s(x,\lambda) = (x - \operatorname{sgn}(x).\lambda).\max(0, abs(x)-\lambda) \quad (2)$$

The hard-thresholding could excellently preserve the detailed characteristics of the image edges; however it produces more artifacts due to its discontinuity at the threshold value. On the other hand, soft-thresholding yields a smoother image, but it can create distortion along the image edges [26]. To define the optimal threshold value, Donoho suggested the universal threshold of $\lambda = \sigma\sqrt{2\log(M^2)}$, for an $M\times M$ image which has corrupted by a white Gaussian noise of variance σ.

B. Cycle spinning

The basic thresholding method of Donoho [1-2] exhibits visual artifacts and oscillations in the vicinity of signal discontinuities, called pseudo-Gibbs phenomena. Therefore, Coifman and Donoho [8] proposed a translation invariant denoising scheme called cycle spinning. In this algorithm we can compute different estimates of the unknown signal by using different shifts of the noisy image and then linearly averaging these estimates. As the wavelet transform is not translation invariant, this approach will result in different estimates of the original image with statistically different noises, which is reduced by averaging.

If we denote the 2-D circular shift by $S_{i,j}$, the wavelet transform by W, and the threshold operator by $\eta(.)$, the cycle spinning will be performed as :

$$\hat{y} = \frac{1}{k_1 k_2}\sum_{i=1,j=1}^{k_1,k_2} S_{-i,-j}(W^{-1}(\eta(W(S_{i,j}(y))))) \quad (3)$$

where k_1 and k_2 are the maximum number of shifts which would cause an improvement in denoising, that is equal to the number of decomposition levels used for wavelet transform, which is maximally equal to log_2M for an $M\times M$ image [26]. This approach reduces the image artifacts.

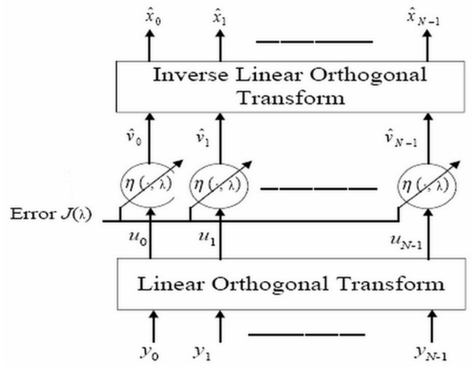


Fig. 1. Zhang's thresholding neural networks (TNN) [5].

C. NeighShrink

As we know a large wavelet coefficient will probably have large coefficients at its neighbors [9]. Therefore, Chen et al. [9] proposed a wavelet denoising scheme, known as *NeighShrink*, by incorporating neighboring coefficients in the thresholding process. In this method for every wavelet coefficient $d_{j,k}$, a neighborhood window $B_{j,k}$ is considered around it. Then, using the summation $S_{j,k}^2 = \sum_{(i,l)\in B_{j,k}} d_{i,l}^2$ and the threshold λ we can compute the thresholded coefficient as:

$$\hat{d}_{j,k} = d_{j,k}\cdot\max(1-\lambda^2 / S_{j,k}^2, 0) \quad (4)$$

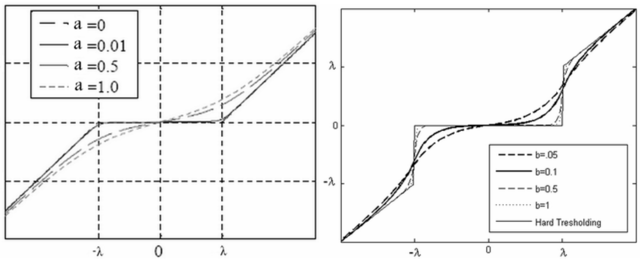
In this method each subband should be thresholded independently, because the boundaries of different subbands are not correlated. Chen has shown in his paper [9] that this method of thresholding outperforms traditional *VisuShrink*, the universal soft thresholding of Donoho.

D. Thresholding Neural Network (TNN) and Improved TNN

Zhang introduced TNN to find the optimum threshold values in the transform domain to achieve noise reduction [5-7]. The neural network structure of the TNN is shown in Fig. 1. The input of the TNN is noisy samples, $y_i = x_i + n_i$, where x is the true signal and n is additive noise. The transform shown in Fig.1 is an orthogonal wavelet transform. Here the thresholding function $\eta(x,\lambda)$ is employed as the nonlinear activation function of the neural network. Zhang suggested a class of activation functions as follows.

$$\eta_a(x,\lambda) = x + \tfrac{1}{2}\left(\sqrt{(x-\lambda)^2 + a} - \sqrt{(x+\lambda)^2 + a}\right) \quad (5)$$

This function is the smooth version of the soft-thresholding function which is an entire function for $a>0$. Fig. 2(a) shows this function for various values of a. In TNN algorithm a neural network based scheme is used to obtain the estimate $\hat{v}_i$ of the true image DWT coefficients v_i, which minimize the MSE risk:



(a) (b)

Fig. 2. a) Zhang's thresholding function $\eta_a(x,\lambda)$ [5], b) The smooth version of hard thresholding function $\eta_b(x,\lambda)$ [26].

$$J(\lambda) = E\{(\hat{v}_i - v_i)^2\} = \frac{1}{N}\sum_{i=1}^{N}[\eta_a(u_i,\lambda_i) - v_i]^2 \tag{6}$$

where u_i and v_i denote the data stream of the 2-D DWT coefficients of the input noisy image y and the true image x, respectively. Moreover, λ_i is the threshold value used for the i^{th} wavelet coefficient, which will be adjusted by TNN to minimize the risk $J(\lambda)$. Since we do not have information about the original image x and cannot utilize its DWT coefficients v_i as reference to estimate the risk $J(\lambda)$, Zhang has suggested a practical approach to this problem. His suggestion is to use another noisy image y' as the reference. This image is obtained from the same true image x plus the noise term n' that is uncorrelated to n. This is a reasonable assumption, since in some applications we may have an array of sensors and obtain more than one corrupted version of the signal [5]. Zhang proved that using such noisy reference signal leads to the same optimum threshold as using the true signal [6]. Also if we have not any available reference signal, it is possible to use TNN. Zhang suggested using *Stein's Unbiased Risk Estimate* (SURE) as an estimator of the MSE [7].

In TNN, gradient-based LMS stochastic adaptive learning algorithm is used to obtain the optimum thresholds. To do so, in the j^{th} iteration, the threshold parameter λ at position i is adjusted by $\lambda_i^{j+1} = \lambda_i^j - \Delta\lambda_i^j$ where

$$\Delta\lambda_i^j = \alpha_i^j . \frac{\partial \hat{v}_i^j}{\partial t} . \varepsilon_i^j \tag{7}$$

where α_i^j is a learning parameter, $\varepsilon_i^j = \hat{v}_i^j - v_i'$ is the instantaneous error for j^{th} wavelet coefficient and v'_i denotes the data stream of the 2-D DWT coefficients of the reference image y'. Thus the optimum thresholds which minimize the risk $J(\lambda)$ is obtained and can be used to denoise the image.

Considering the fact that hard-thresholding could better preserve the detailed characteristics of the image edges than soft thresholding, an improved TNN method was introduced in [26] using a new class of smooth nonlinear thresholding functions as the activation function. In this method a new smooth differentiable version of hard-thresholding function was used instead of Zhang's smooth semi-soft thresholding function which can keep the good properties of the standard hard-thresholding. This function is as follows:

$$\eta_b(x,\lambda) = \begin{cases} a(e^{b|x|} - 1).\mathrm{sgn}(x) & |x| \le \lambda \\ (|x| + ce^{-b|x|}).\mathrm{sgn}(x) & |x| > \lambda \end{cases} \tag{8}$$

Fig. 2(b) shows this function for various values of b. As we see this function is the smooth version of the basic hard-thresholding and estimates two segments of hard-thresholding with two exponential functions. The parameter b determines the degree of the thresholding effect. Also parameters a and c is determined such that the continuity of the thresholding function and its derivative is preserved at the threshold value λ. Since this new thresholding function is the smooth version of the basic hard-thresholding, it can produce some artifacts in the image. To compensate these artifacts a cycle spinning is used in [26] to improve the resulted image from the TNN step.

III. Proposed Method

Now we will describe the proposed method. In this method, first we use an improved thresholding neural network with smooth nonlinear thresholding function (8), introduced in [26] as the activation function. Then we use the optimum derived thresholds for over new cycle spinning based scheme.

Thus we first train four separate TNNs for HH, HL, LH and LL subbands of the noisy image using (7). This approach will lead us to the optimum threshold values. After that we use a new cycle-spinning based method whose thresholding operator consists of two thresholding steps. As the first step, it utilizes the idea of *NeighShrink* which use neighbor wavelet coefficient dependency in the thresholding process. It means that the following average is calculated for a neighborhood window $B_{j,k}$ of size $N \times N$ around each wavelet coefficient.

$$S_{j,k}^2 = \frac{1}{N^2}\sum_{(i,l)\in B_{j,k}} d_{i,l}^2 \tag{9}$$

Then using (4) with threshold value $\lambda_1 = \alpha(\sigma\sqrt{2\log(M^2)})$, we perform the first stage of thresholding. Here, α will be calculated practically to achieve the best performance.

In the second step we perform a wavelet thresholding on the resultant wavelet coefficients of the previous step, $\hat{d}_{j,k}$, using the smooth thresholding function (8). For this function we use the optimum threshold values derived from the TNN step. This approach will reduce the artifacts appeared by thresholding.

Hence we can summarize our method as follows:

1) Train four separate TNNs for HH, HL, LH and LL subbands of the noisy image using (7) to obtain optimum threshold values.

2) Perform the cycle spinning using (3) and in each circular shift do following tasks:

a) Apply the neighboring shrinkage using (4) with λ_1. b) Apply smooth thresholding function (8), with the optimum threshold values resulted in step 1.

(a) Original image (b) Zhang's TNN [5] (PSNR=26.6db)

(c) Improved TNN method [26] (PSNR=27.18db) (d) Our method (PSNR=27.43db)

Fig. 3. Comparison of the proposed method with Zhang's TNN method [5] and improved TNN method [26]. (PSNR of noisy image is 20db).

IV. EXPERIMENTAL RESULTS

In this section we perform several experiments to test the proposed algorithm and compare it with other image denoising techniques. Throughout our experiments we use the smooth thresholding function (8) with *b=0.1* and implement the neighbor coefficient dependencies using (4) for a 5×5 window size. Also we use the Daubechies length-8 symlet filters with five levels of decomposition to compute the 2-D DWT. All the results are obtained by averaging on five runs.

In the first experiment we compare our method with the TNN method proposed by Zhang [5] and improved TNN method [26]. To do so, the 256×256 *Cameraman* image is used as a test image. The original image is shown in Fig. 3(a). The PSNR results, for different noise levels, are shown in Table 1. This table confirms that our proposed algorithm outperforms traditional and improved TNN methods [5, 26]. Fig. 3 compares the visual quality of these methods for a noisy image with PSNR=20db. As we see, our method gives better visual result besides the PSNR improvement.

In the second experiment we compare the proposed method with several well-known denoising methods. These methods include: *VisuShrink* [2], MATLAB®'s spatially adaptive image filtering algorithm *Wiener2*, Donoho's *SureShrink* of soft-thresholding [3], *BayesShrink* [4], *NeighShrink* [9], *improved TNN* [26], simple cycle-spinning with wavelet hard thresholding (CS-WHT), *Mihcak's LAWMAP* method [12], *FOE* [13], *variance adaptive* [14], *Steerable Complex* [15], *multifractal* [16], *tight frame* [17], *moving-window based* [18] and several recent directional transform methods such as *curvelet*, *contourlet*, *Non-Subsampled contourlet* (NSCT) [23], *Directional multiscale modeling of contourlet* (DMMC) [24] and *oriented wavelet* [25]. We tested our algorithm on a number of images, but only report results for *Lena* of size 512×512. The PSNR results of these methods are shown in Table 2. We can see that for low and moderate noise levels our method outperforms all these established denoising approaches but for higher power of noise this approach slightly degrades due to the amount of introduced artifacts but still has high PSNR values. Fig. 4 displays part of the reconstructed noisy *Lena* image of $\sigma_n=20$ using cycle-spinning with wavelet hard thresholding (CS-WHT), *improved TNN* [26], *Curvelet* [21], NSCT [23], and the proposed method. As we see, our method gives better visual result with less artifacts and better edge preserving besides the PSNR improvement.

TABLE I
Comparison between the PSNRs (dB) resulted from Zhang's TNN method [5], improved TNN method [23] and our method for denoising *Cameraman*.

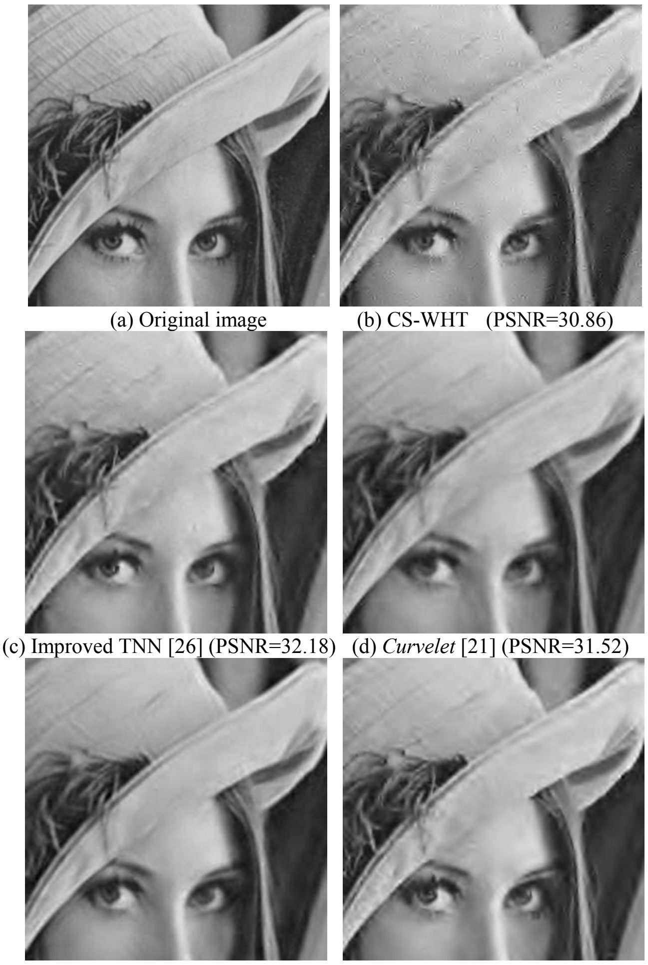
Noisy	Zhang's TNN	Improved TNN	Proposed Method
20	26.64	27.18	**27.43**
25	29.97	30.67	**30.80**
30	33.73	33.74	**34.10**

TABLE II
Comparison of PSNRs (dB) for different denoising methods using *Lena* image with different noise levels.
('-' is placed for cases that have not reported in the literatures.)

Denoising Method	σ_n=10	σ_n=15	σ_n=20	σ_n=25	σ_n=30
VisuShrink [2]	30.40	28.65	27.39	26.46	25.74
Wiener2	33.06	30.90	29.40	28.28	27.40
SureShrink [3]	33.50	31.50	30.17	29.18	28.41
BayesShrink [4]	33.40	31.41	30.23	29.22	28.49
NeighShrink [6]	34.36	32.42	31.03	29.98	29.12
Improved TNN [26]	34.81	33.50	32.18	31.13	30.21
CS-WHT	34.42	32.34	30.86	29.69	28.70
LAWMAP [12]	34.31	32.36	31.01	29.98	-
FOE [13]	35.04	33.27	31.92	30.82	-
Var.Adapt. [14]	34.75	32.73	31.35	30.32	-
Comp.steer. [15]	32.81	-	31.07	-	29.69
Multifractal [16]	-	32.72	31.36	30.17	-
Tight frame [17]	34.92	33.24	31.99	31.00	30.14
Mov. Wind. [18]	33.83	-	30.04	-	27.62
NSCT [23]	34.69	-	32.03	-	**30.35**
Curvelet [23]	34.17	-	31.52	-	30.01
Contourlet [23]	-	32.3	30.9	29.8	-
DMMC [24]	-	-	-	-	28.35
Orient. WT [25]	-	32.9	31.6	30.4	-
Proposed Method	**35.06**	**33.58**	**32.30**	**31.20**	30.31

V. CONCLUSION

In this paper, we proposed a new efficient wavelet image denoising approach. In this method we used improved TNN method to find optimal threshold values and employed these thresholds in a novel cycle-spinning denoising scheme to reduce image artifacts. As the thresholding operator of the cycle-spinning, we implemented a two step thresholding



(a) Original image (b) CS-WHT (PSNR=30.86)

(c) Improved TNN [26] (PSNR=32.18) (d) *Curvelet* [21] (PSNR=31.52)

(e) NSCT [23] (PSNR=32.03) (f) Proposed Method (PSNR=32.30)

Fig. 4. Comparison of the proposed method with CS-WHT, Improved TNN [26], *Curvele* [21] and NSCT [23] methods.

approach consists of a neighbor dependent thresholding technique and a simple wavelet thresholding with a smooth thresholding function witch uses optimum derived threshold values. This approach has several outstanding properties such as employing improved TNN to find optimal threshold values, utilizing the smoothness property of cycle spinning and edge preserving of hard-thresholding while producing less artifact with the smooth thresholding function and incorporating neighbor wavelet coefficient dependencies in thresholding step to prevent eliminating of too many wavelet coefficients. Experimental results clearly showed the denoising ability of the proposed method and its superiority to several other established wavelet denoising techniques, in terms of PSNR and visual quality. Future work may be done by considering new thresholding functions and also using other transforms especially directional transforms such as *contourlet transform*.

VI. ACKNOWLEDGEMENT

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