Block Adaptive Compressive Sensing for Distributed MIMO Radar in Clutter Environment

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Abstract— For the target parameter estimation in MIMO radars in the presence of strong clutter, non-adaptive compressive sensing methods have a poor performance. On the other hand, the adaptive ones usually need higher data acquisition time. In this paper, we propose an adaptive compressive sensing method called LDBGTM that has tolerable data acquisition time. Furthermore, we have presented the essential changes in a distributed MIMO radar to exploit an adaptive group testing compressive sensing method.

Index Terms—block compressive sensing, adaptive compressive sensing, clutter, MIMO radar.

I. INTRODUCTION

Distributed Multi-Input Multi-Output (MIMO) radars are radars with multiple transmitters and multiple receivers that are widely separated. In these systems, the received samples of all the receivers should be sent to a common processing center (fusion center). Hence, decreasing the sampling rate at the receivers leads to a great decrease in the cost. Exploiting Compressive Sensing (CS), this reduction can be possible.

Compressive sensing is a signal processing method that lets us reduce the sampling rate of a sparse signal such as \( x_{N 	imes 1} \) to a sub-Nyquist rate. In CS, we achieve \( y_{M 	imes 1} = \Phi_{M 	imes N} x_{N 	imes 1} \) instead of \( x \) (when \( M < N \)) and then try to recover \( x \) from \( y \) whenever it is necessary [1]. Matrix \( \Phi \) is called measurement matrix. Compressive sensing methods are divided into two groups: adaptive CS methods and non-adaptive ones. In adaptive CS methods, each row of the measurement matrix is determined according to the previous measurements. However, in non-adaptive ones, the measurement matrix can be completely determined before achieving the measurements. It has been shown that in the presence of noise, adaptive compressive sensing methods outperform the non-adaptive ones [2]–[4]. Among the existing adaptive methods, the adaptive group testing method proposed in [4] has a very simple measurement procedure and can be used in a wider variety of received signal modeling. However, like the other adaptive CS methods, this method suffers from the large data acquisition time.

In distributed MIMO radar systems like other radar systems, considering the unwanted echoes that are typically returned from ground, sea, rain, insects and atmospheric turbulences called clutter is necessary for most of the practical cases. However, up to now usually this problem is neglected in the CS-based distributed MIMO radar systems [5]–[9]. In this paper, clutter is considered in the received signal modeling of a distributed MIMO radar. It seems that in this case, an adaptive CS method with tolerable data acquisition time is a good choice for the multiple targets parameters estimation. Hence, a block adaptive CS method called LDBGTM is proposed in this paper that has an acceptable data acquisition time and a good performance. Then, the essential changes in MIMO radars for exploiting adaptive group testing CS methods are discussed. Simulation results show the superiority of the proposed block adaptive method over group lasso and the traditional group testing compressive sensing method in distributed MIMO radars.

This paper is organized as follows: In section II, the received signal of a block CS-based distributed MIMO radar is modeled in the presence of clutter. Section III is allocated to the proposed LDBGTM method. Essential conditions for the adaptive group testing compressive sensing methods and structural changes in MIMO radars for fulfilling them are presented in section IV. In section V, simulation results are discussed, and finally, we make some concluding remarks in section VI.

II. RECEIVED SIGNAL MODELING IN CLUTTER ENVIRONMENT

In this section, we model the received signal at the fusion center of a distributed MIMO radar with \( M_t \) transmitters and \( N_r \) receivers in the presence of homogeneous clutter. The transmitters and the receivers are located at \( t_i = [t_{ix}, t_{iy}] \) for \( i = 1, \ldots, M_t \), and \( r_l = [r_{lx}, r_{ly}] \) for \( l = 1, \ldots, N_r \) on a Cartesian coordinate system, respectively. The transmit signal from the \( i^{th} \) transmitter has the baseband form of \( x_i(t) \) with duration \( T_p \). The carrier frequency and the Pulse Repetition Interval (PRI) are respectively denoted by \( f_c \) and \( T \), and for the estimation process, \( N_p \) pulses are used. Let us consider the position space as the estimation space. In this paper, we assume that the targets and the clutter sources are all stationary and all the antennas and targets are located in some plane. However, the results can easily be extended to the moving targets and clutter sources case in 3 dimensional space. If there is no target in the search area \( S \), under a narrow band assumption on the transmit waveforms, the \( l^{th} \) receiver receives the following baseband signal from the \( i^{th} \) transmitter:

\[
z_l(t) = \int_{(x,y) \in S} c^{il}(x,y)x_i(t - t^{il}([x,y]))e^{-j2\pi f_c t^{il}([x,y])}dxdy + \Pi_l(t)
\]

(1)
where \( \tilde{n}_d(t) \) and \( e^H(\ldots) \) are respectively the thermal noise and the signal attenuation coefficient due to the clutter sources corresponding to the \( k \)th transmitter and \( l \)th receiver. Furthermore, \( \tau^d(\mathbf{p}) \) is the corresponding delay of point \( \mathbf{p} \) that can be written as

\[
\tau^d(\mathbf{p}) = \frac{1}{c} || \mathbf{p} - \mathbf{t}_i || + || \mathbf{p} - \mathbf{r}_i ||
\]

(2)

where \( c \) is the speed of light.

Let us discretize the estimation space to \( L \) points that are denoted by \( (x_h, y_h) \) for \( h = 1, \ldots, L \) where \( x_h \) and \( y_h \) are the positions of the points in directions \( x \) and \( y \), respectively. By mapping the points of the search area to the nearest points of this discretized estimation space (dividing the \( S \) into \( L \) areas that are denoted by \( S_h \) for \( h = 1, \ldots, L \), (1) can be written as

\[
z_d(t) = \sum_{h=1}^{L} g^d([x_h, y_h])x_h(t - \tau^d([x_h, y_h]))e^{-j2\pi f_r \tau^d([x_h, y_h])} + \tilde{n}_d(t)
\]

(3)

where

\[
g^d(x_h, y_h) = \int_{(x,y)\in S_h} c^d(x,y)dx dy
\]

(4)

Now, we assume that there are \( K \) point targets that are located at \( \mathbf{p}_k = [x_k, y_k]^T \) for \( k = 1, \ldots, K \). If \( x_h \) and \( y_h \) are constant within the estimation process duration and the received signal of each receiver pass through a bank of \( M_i \) matched filters corresponding to the transmit waveforms. Putting together the output of the matched filters at each receiver and then, putting together the output of all receivers at the fusion center, the received signal in the \( m \)th pulse can be expressed as

\[
z^m = \Psi^m(\mathbf{s} + \mathbf{g}) + \mathbf{e}^m
\]

(6)

where \( \text{diag} \{ \cdot \} \) denotes a diagonal matrix with diagonal entries of \( \{ \cdot \} \), we have

\[
\Psi^m = \text{diag} \{ \Psi_{1,1}, \ldots, \Psi_{m,N_1} \}, \ldots, \text{diag} \{ \Psi_{1,L}, \ldots, \Psi_{m,N_L} \},
\]

(7)

\[
\Psi_{k,h} = \text{diag} \{ e^{-j2\pi f_r \tau^i([x_k, y_k])}, \ldots, e^{-j2\pi f_r \tau^i([x_k, y_k])} \},
\]

(8)

and \( \mathbf{s} \) is defined as

\[
\mathbf{s} = [\beta_{1,1}]^T, \ldots, [\beta_{N_1,1}]^T, \ldots, [\beta_{1,L}]^T, \ldots, [\beta_{N_L,L}]^T.
\]

(9)

\[
\beta_{k,h} = \frac{\gamma_{k,h} M_i}{|| \mathbf{p}_k - \mathbf{p}_h ||^2},
\]

(10)

\[
\gamma_{k,h} = \begin{cases} 
\frac{\alpha_{k,k}}{L^{D}, & \text{if the } k \text{th target is at } (x_h, y_h), v_h, v_h^1 \\
0, & \text{otherwise}
\end{cases}
\]

(11)

In (6), \( \mathbf{g} \) is defined as

\[
\mathbf{g} = [\mathbf{g}_{1,1}]^T, \ldots, [\mathbf{g}_{N_1,1}]^T, \ldots, [\mathbf{g}_{1,L}]^T, \ldots, [\mathbf{g}_{N_L,L}]^T.
\]

(12)

and \( \mathbf{e}^m \) is

\[
\mathbf{e}^m = \left[ \left( \mathbf{e}^{m}_{1} \right)^T, \ldots, \left( \mathbf{e}^{m}_{N_r} \right)^T \right]^T,
\]

(13)

where \( \mathbf{e}^{m} = [\tilde{\mathbf{e}}^{m}_1, \ldots, \tilde{\mathbf{e}}^{m}_{N_r}]^T \) and \( \mathbf{e}^{m}_{n} = \left[ \tilde{\mathbf{e}}^{m}_{n,1}, \ldots, \tilde{\mathbf{e}}^{m}_{n,L} \right]^T \).

(14)

To improve the targets parameters estimation in the presence of noise, we can use \( N_p \) pulses in the estimation process. Stacking all the received signals together, we have

\[
\mathbf{z}_{(N_p \times M_i \times N_r \times 1)} = [(\mathbf{z}_1)^T, \ldots, (\mathbf{z}_{N_p})^T]^T = \Psi(\mathbf{s} + \mathbf{g}) + \mathbf{e}
\]

(15)

where

\[
\Psi(\mathbf{s} + \mathbf{g}) = \left[ (\mathbf{y}_1)^T, \ldots, (\mathbf{y}_{N_r})^T \right]^T
\]

(16)

\[
\mathbf{e}^{m}_{n} = [\mathbf{e}^{m}_{1} + \tilde{\mathbf{e}}^{m}_{n} , \ldots, \mathbf{e}^{m}_{N_r} + \tilde{\mathbf{e}}^{m}_{n}]^T
\]

(17)

It is clear that \( \mathbf{s} \) has only \( K \) non-zero blocks that are corresponding to the positions of the targets. Thus, \( \mathbf{s} \) is a block \( K \)-sparse signal with block-length of \( d = M_i \times N_r \) and we can reduce the sampling rate by multiplying \( \Phi_{(M_i \times N_r \times M_i \times N_r)} \) by the received signal at each receiver. Hence, the received signal at the fusion center can be written as

\[
y = \Phi \mathbf{z}
\]

(19)

As the received signal in a distributed MIMO radar is block sparse, block compressive sensing methods are superior over non-block ones in this application.

III. LENGTH-DEPENDENT BLOCK GROUP TESTING METHOD (LDBGTM)

In this section, we propose a block adaptive compressive sensing method called Length-Dependent Block Group Testing Method (LDBGTM). Compared to the traditional group testing method of [4], this adaptive method have more tolerable data acquisition time and better performance especially for small size block-sparse vectors.

In this section like [4], we consider \( \mathbf{z}_{dL \times 1} = \mathbf{s}_{dL \times 1} + \mathbf{g}_{dL \times 1} \) as the input of the proposed adaptive CS method where \( \mathbf{s} \) is a \( K \)-block-sparse vector with positive real entries and \( \mathbf{g} \) entries have identical distribution.

Let us define the sensing vector for \( I \), \( \mathcal{I}_I : \{1 \}, \mathcal{I}_I : \{1 \}, \ldots, \mathcal{I}_I : \{dL \} \}, \) as below:

\[
\mathcal{I}_I[i] = \begin{cases} 
1 & \text{if } i \in I \\
0 & \text{otherwise}
\end{cases}
\]

(20)

First, we consider \( \mathbf{s} \) as a 1-block-sparse vector with positive real entries and propose Algorithm 1 to recover it from \( \mathbf{z} \). In Algorithm 1, \( \mathcal{e}^{m} \) denotes the pointwise product and for the 1-block-sparse case we consider \( I = \{1, \ldots, L \} \). As it is clear, the sum of the entries of the measurement array \( \mathbf{m}_I \) is equal to the measurement number. Furthermore, we should have \( \sum_{i=1}^{L} M_i = L \). Hence, there is a tradeoff between the measurement number and the data acquisition time, i.e., the smaller \( m \) leads to
the smaller data acquisition time and the larger measurement number.

Now, we consider the $K$-block-sparse case for $s$ and propose LDBGTM in Algorithm 2. In this algorithm, for the recovery of small size vectors, first, we search for the block with largest norm. Then, we put this block aside and search for the second block with largest norm. This procedure until all the non-zero blocks are found. This searching can not be used for large vectors due to its large data acquisition time. Hence, in this cases, we divide the search area to $q$ sets that each have one non-zero norm block with almost high probabilities. Then, Algorithm 1 is used in each set to find the non-zero norm blocks. We choose $q$ as one of the first $2K[\log_2 N]$ prime numbers larger than $K$ [4].

IV. ADAPTIVE GROUP TESTING COMPRESSIVE SENSING METHODS FOR DISTRIBUTED MIMO RADARS

Let us consider (16) as the input signal of an adaptive group testing method such as the traditional group testing method of [4] and the proposed LDBGTM. For exploiting the adaptive group testing algorithms, the rows of the measurement matrix, $\Phi = \sum_{i=1}^{u} M_i - M_i$, for $i = 1, \ldots, m$ and $u = 1, \ldots, M_i$, should fulfill $I_{ai} = \Phi_{ai} = \frac{1}{\sum_{i=1}^{u} M_i - M_i} \Psi$. Hence, we can calculate them as the following

$$\Phi_{ai} = \frac{1}{\sum_{i=1}^{u} M_i - M_i} \Psi$$

(21)

where $\Psi$ denotes the pseudoinverse of matrix $\Psi$, and in this case, $T_{I_{ai}}$ is calculated as

$$T_{I_{ai}} = \Phi_{ai} = \frac{1}{\sum_{i=1}^{u} M_i - M_i} z$$

(22)

Other parts of the algorithms are the same as the identical matrix as the basis matrix case.

Algorithm 1 Finding the non-zero norm block in a 1-block-sparse vector with positive real entries

1: Input: the index vector $I$ and the measurement array $m_a = [M_1, M_2, \ldots, M_m]$
2: Output: the positions of the non-zero norm block entries $c_i$ and the values of the non-zero norm block entries $v_i$.
3: for $i = 1 : m$ do
4:     $n = \text{size}(I)$
5:     for $u = 1 : M_i$ do
6:         $S = ((u - 1) \frac{n}{M_i} + 1, u \frac{n}{M_i})$
7:         $I_{ai} = I | S$
8:         $T_{I_{ai}} = I_{ai} \Psi$
9:     end for
10:     $I = \arg \max \{ T_{I_{ai}} \}$
11: end for
12: return $c_i \leftarrow I$
13: return $v_i \leftarrow I \odot z$

Algorithm 2 Length Dependent Block Group Testing Method (LDBGTM)

1: Input: set number $q$, maximum block-sparsity number $K$, smallest non-zero norm of interest $v_{\text{min}}$, block number $L$, and the measurement array $m_a = [M_1, M_2, \ldots, M_m]$
2: Output: the values of the non-zero norm blocks entries $v = [v_1, v_2, \ldots, v_K]$ and the positions of the non-zero norm blocks entries $c = [c_1, c_2, \ldots, c_K]$
3: $\hat{c} \leftarrow \{\}$
4: $\hat{v} \leftarrow \{\}$
5: $I = \{1, \ldots, L\}$
6: if $L > L_1$ then
7:     for $j = 0 : q - 1$ do
8:         $I_{aq} = \bigcup_{i=1}^{q} [I_{ai} | I_{ai+1} | I_{ai+2} | \ldots | I_{ai+(i+1)q}]$
9:         $[\hat{c}, \hat{v}] = \text{Algorithm 1}(m_a, I_{aq})$
10: if $\|v\| > v_{\text{min}}$ then
11:     $\hat{v} \leftarrow \hat{v} \cup \hat{v}$ and $\hat{c} \leftarrow \hat{c} \cup \hat{c}$
12: end if
13: end for
14: Select $K$ max values of $\hat{v}$ as $v$ and its corresponding positions as $c$.
15: else
16:     for $j = 1 : K$ do
17:         $[\hat{c}, \hat{v}] = \text{Algorithm 1}(m_a, I)$
18: if $\|v\| > v_{\text{min}}$ then
19:     $v \leftarrow v \cup \hat{v}$ and $c \leftarrow c \cup \hat{c}$
20: end if
21:     $I = I - \hat{c}$
22: Deducting 1 from the entry of $m_a$ with largest index
23: end for
24: end if

There are two essential conditions for adaptive group testing compressive sensing methods:

1. Vector $g$ entries should be identically distributed.
2. Vectors $s$ and $g$ should be real.

The first condition is fulfilled in a homogenous environment. For fulfilling the second one, we need almost the same structure as [10] for the measuring process with one difference. We should put a push detector before the analog integrator. The appropriate measuring Process for an adaptive group testing compressive sensing method in discrete case is shown in Fig. 1 that can easily be extended to the continuous case.

![Fig. 1: Measuring Process](image-url)
V. SIMULATION RESULTS

In this section, we evaluate the performance of the proposed block adaptive method compared to the traditional group testing method of [4] and group lasso that is a non-adaptive block CS method. The simulations have been conducted on a core i7-2600K @ 3.4GHz processor with 8GHz RAM. Let us consider a distributed MIMO radar with 2 transmitters located at \( \mathbf{t}_1 = [100, 0] \) and \( \mathbf{t}_2 = [200, 0] \) and 2 receivers located at \( \mathbf{r}_1 = [0, 200] \) and \( \mathbf{r}_2 = [0, 100] \) (the distance unit is meter). The parameters of the MIMO radar system are as the following: \( f_c = 1GHz \), \( T = 1ms \), and \( N_r = 40 \). We assume that there are 2 targets that are randomly located on the estimation grid points. Two different Scenarios are considered for the estimation grid entries as mentioned in table I. Clutter in each grid has i.

### Table I: Estimation Grid Scenarios

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Estimation Grid</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( x_y \in (80, 90, \ldots, 170) ) ( g = 1, \ldots, 100 ), ( y_s \in (260, 270, \ldots, 150) ) ( g = 1, \ldots, 100 )</td>
</tr>
<tr>
<td>2</td>
<td>( x_y \in (80, 90, \ldots, 120) ) ( g = 1, \ldots, 200 ), ( y_s \in (60, 70, 80, 90) ) ( g = 1, \ldots, 20 )</td>
</tr>
</tbody>
</table>

i. d. complex Gaussian distribution and thermal noise entries are Gaussian with the variance of 0.02 and the mean of 0. Furthermore, the distribution of target attenuation coefficients is complex Gaussian with the mean of 0.407 and variance of 0.0907 for both real and imaginary parts.

The receiver operating characteristic (ROC) curves of LDBGTM, the group testing method of [4], and group lasso versus the clutter variance for Scenarios 1 and 2 are shown in Fig. 2 and Fig. 3, respectively. The measurement array for LDBGTM in Fig. 2 and Fig. 3 are respectively \( \mathbf{m}_r = [5, 4] \) and \( \mathbf{m}_d = [5, 4] \). The measurement number in the group lasso method is same as the average measurement number of LDBGTM. As it can be seen that in large estimation grids, adaptive methods outperform the non-adaptive group lasso method and the propose method outperform the group testing method of [4]. In small estimation grids, the group testing method of [4] has a poor performance when the performance of LDBGTM is perfect. Furthermore, table II shows the data acquisition time (at the receivers) and the processing time (at the fusion center) of the mentioned methods in the clutter variance of 0.24. According to this table, the data acquisition time of the proposed adaptive method is tolerable and for the large estimation grids, it is significantly less than the one of the group testing method of [4].

### Table II: Data Acquisition Time and Processing Time

<table>
<thead>
<tr>
<th>CS Method</th>
<th>Data Acquisition Time (at the receivers)</th>
<th>Processing Time (at the fusion center)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LDBGTM</td>
<td>Scenario 1: 11s</td>
<td>Scenario 1: 14s</td>
</tr>
<tr>
<td>Group Lasso</td>
<td>Scenario 1: 7s</td>
<td>Scenario 1: 13s</td>
</tr>
</tbody>
</table>

VI. CONCLUSION

In this paper, a block adaptive compressive sensing method called LDBGTM is proposed that has a tolerable data acquisition time. Furthermore, we have shown that under some conditions, we can exploit the proposed adaptive method in a distributed MIMO radar and according to the simulation results, improve the target parameter estimation in presence of strong clutter compared to exploiting its competitors.

### References