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Power allocation scheme in time division multiple access distributed multiple-input multiple-output interference channels

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Abstract: In this study, the authors propose a novel power allocation scheme in time division multiple access (TDMA)-based distributed multiple-input multiple-output (MIMO) channels. By modelling the problem with a TDMA MIMO interference channel, the authors have derived closed-form expressions for power and time slot assigned to each MIMO transmission. Although our scheme is sub-optimal, with the aid of Jensen's inequality, it is shown to be superior to solutions based on channel inversion. In addition, the proposed scheme is compared with the optimal solution, and also with the scheme proposed by Dohler. By applying our proposed power allocation to the hierarchical cooperation strategy, the authors have shown that this scheme significantly improves the overall network throughput compared with the channel inversion solution. However, the improvement gain (which is at high-signal-to-noise ratio values) reduces when the method is compared with Dohler's solution.

1 Introduction

The idea of forming distributed (cooperative) multiple-input multiple-output (MIMO) transmissions by clustering multiple single-antenna nodes in wireless networks, has been a very popular strategy in recent years. In such strategy, neighbouring nodes cooperate to form distributed MIMO transmitters and receivers and benefit from the multiplexing/diversity gain in MIMO transmissions [1–3]. This will result in great energy saving in *ad hoc* and sensor networks [4–6]. In addition, this idea has been the core of the most successful achievable schemes in large wireless networks [7–10]. By clustering the network into smaller sub-networks and communicating between the sub-networks in a distributed MIMO manner, one can achieve linear throughput scaling in wireless networks [7, 10]. This strategy will help us benefit from interference in a MIMO framework, instead of being degraded without such strategy.

In this paper, we focus on performance of MIMO transmissions originating from clusters of nodes forming virtual MIMO transmitters, to the clusters of nodes forming virtual MIMO receivers. Such process can be modelled by a MIMO interference channel in which MIMO transmitters are in fact distributed MIMO transmitters formed by neighbouring nodes. The same is true for MIMO receivers. Accordingly, we consider a MIMO interference channel which is orthogonalised by the time division multiple access (TDMA) scheme (Fig. 1) [In this paper, the original MIMO interference channel is reduced to an interference-free scenario through the TDMA strategy.]. The channel between each transmitter–receiver pair is modelled by a random phase change with a path-loss factor depending on their distance. It is required that equal average rate [Average rate is the number of bits transmitted by a session divided by the TDMA round time.] is allocated to each transmitter–receiver pair at each TDMA round. Taking into account the above assumptions, and assuming total power constraints for each transmitter, we propose a novel power allocation between transmitting nodes with the objective of achieving high transmission rates.

Although, using the TDMA distributed MIMO technique, as described in the preceding paragraph, is very popular in operating *ad hoc* wireless networks (see e.g. [7–10]), no analytical result is currently available in the literature addressing proper power allocation among clusters. In this regard, this paper is the first work considering such issue in TDMA-based distributed MIMO schemes.

It should be noted that the scheme proposed in this paper is based on solving a convex optimisation problem. A similar optimisation problem also arises in resource allocation in multi-hop distributed MIMO systems. Whereas most of the papers dealing with resource allocation in the aforementioned systems address the problem numerically, [1] is the only reference presenting a closed-form sub-optimal solution for the optimisation problem. In other words, [1] and our paper are the only papers proposing closed-form sub-optimal solutions to this optimisation problem and since our goal is also finding an analytical solution, we have provided a comparison of our results with those of [1]. In addition, we compare our solution with the optimal solution obtained numerically.

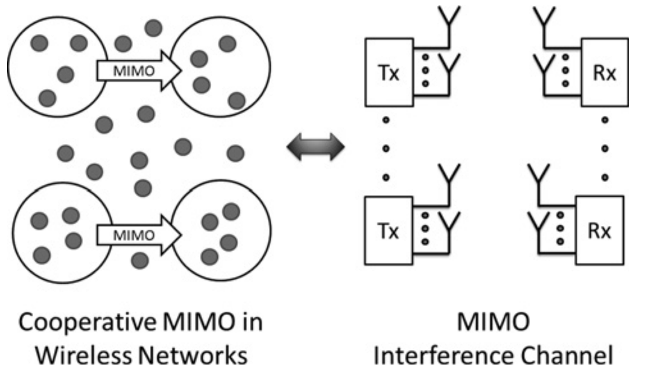

Fig. 1 *Modelling a distributed (cooperative) MIMO scheme with a MIMO interference channel*

It should also be mentioned that in terms of solving the optimisation problem, an idea related to our proposal has been introduced in [11] in the context of regenerative single-antenna multi-hop links. In [11] the ratio of the power of each hop to the time fraction of that hop is constant, like in our proposal. Based on this assumption, they derive a numerical approximate sub-optimal result for the optimisation problem. However, with the same assumption, we have derived closed-form relations for the resource allocation strategy and the resulting transmission rates. Such closed-form relations have provided a proper platform for us to rigorously compare our proposal with other schemes without any approximations, in contrast with [11].

The rest of the paper is organised as follows. In Section 2, we describe the model of system. In Section 3, the proposed power allocation scheme is presented and its performance is evaluated. In Section 4, application of the power allocation scheme to the hierarchical cooperation strategy in wireless networks is discussed. Finally, Section 5 concludes the paper.

2 System model

As indicated in the earlier section, we model the distributed MIMO transmissions by a MIMO interference channel. Consider n transmitters each having M transmit antennas, and n receivers each having M receive antennas. These transmitters and receivers establish n MIMO transmission sessions by one-to-one pairing. We use TDMA to eliminate the interference between different communication sessions [It should be noted that the synchronisation mechanism which is required for implementation of the TDMA strategy is not addressed in this paper.]. Each transmitter has the same number of bits to transmit to the corresponding receiver at each TDMA round. The channel matrix for the ith transmission session (from transmitter i to receiver i) is denoted by the $M \times M$ matrix $\boldsymbol{H}_i$. We separate the path-loss and the phase change effects as follows [Matrices are indicated in boldface, and $\boldsymbol{I}$ is the identity matrix of appropriate size.]

$$\boldsymbol{H}_i = \frac{\boldsymbol{G}_i}{\gamma_i} \tag{1}$$

where $\gamma_i = r_{SD_i}^{\alpha/2}$, r_{SD_i} is the distance for the ith transmission session, α is the path-loss exponent, and $\boldsymbol{G}_i$ is an $M \times M$ matrix with random elements $(\boldsymbol{G}_i)_{jk} = \exp\left(\sqrt{-1}\theta_{ijk}\right)$. In the mentioned model, θ_{ijk} is a random variable uniformly distributed in $[0, 2\pi]$, and all θ_{ijk}s are independent. Thus, the matrices $\boldsymbol{G}_i$ are statistically equivalent, and whenever it is applicable, we call them $\boldsymbol{G}$. This model is a broadly accepted model in the literature, and also used in [7] for the channel between nodes in an *ad hoc* network. It should be noted that due to the lack of channel state information at the transmitter (CSIT) and the distributed nature of the MIMO transmissions, we have assumed no MIMO pre-coding.

As mentioned above, we eliminate the interference between different MIMO sessions by a TDMA strategy. In this strategy, without loss of generality, the total time is assumed to be unity, and the fraction α_i of the total time is allocated to the session i. Thus, the average rate (the number of transmitted bits in a TDMA round, divided by the total time of a TDMA round which we assume to be unity) of the ith session is

$$R_i = \alpha_i \mathbb{E}\left[\log\det\left(\boldsymbol{I} + \frac{P_i \boldsymbol{H}_i \boldsymbol{H}_i^{\mathrm{H}}}{\alpha_i N_0}\right)\right], \quad i = 1, \ldots, n \tag{2}$$

where $\boldsymbol{I}$ is the $M \times M$ identity matrix, P_i is the average power of each antenna in the ith transmitter, and N_0 is the variance of noise at each receive antenna. Also, the expectation is with respect to the variables θ_{ijk}. It should be noted that since P_i is the average power, and we are only using the fraction α_i of the total time for this transmitter, the power is boosted by $1/\alpha_i$ during this time fraction.

3 Power allocation scheme

We consider the total power constraint $\sum_{i=1}^{n} P_i \leq P$. Since the distance between different transmitters and receivers (i.e. r_{SD_i}s) are not the same, if we allocate equal power to each of the sessions (i.e. $P_1 = P_2 = \cdots = P_n$), and equal time fractions (i.e. $\alpha_1 = \alpha_2 = \cdots = \alpha_n$), average rates (i.e. R_is) will not necessarily be the same. However, because of our assumptions, we assume that the number of transmitted bits in the total TDMA round time, for each transmission session is the same. Therefore average rates should be the same.

One possible power allocation approach is based on some form of channel inversion, where by proper power allocation, channel inversion scheme compensates for the path-loss effect as follows

$$P_i = P\frac{\gamma_i^2}{\sum_k \gamma_k^2} = P\frac{r_{SD_i}^{\alpha}}{\sum_k r_{SD_k}^{\alpha}} \tag{3}$$

Based on this compensation strategy, we will have symmetric induced channels, and, thus we will have

$$\alpha_i = \frac{1}{n} \tag{4}$$

which means the time fractions for different sessions are the same. This solution results in the following common average rate for the sessions, which we call R_{C1}

$$R_{C1} = \frac{1}{n}\mathbb{E}\left[\log\det\left(\boldsymbol{I} + \frac{nP\boldsymbol{G}\boldsymbol{G}^{\mathrm{H}}}{N_0 \sum_k \gamma_k^2}\right)\right] \tag{5}$$

The above solution is the original solution proposed in the

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MIMO phase of [7]. However, as will be proved shortly, it will not constitute the optimum solution for this problem.

In fact, we are faced with the following optimisation problem

$$\max_{P_1,\ldots,P_n\alpha_1,\ldots,\alpha_n} R \tag{6}$$

subject to

$$R_i = \alpha_i \mathbb{E}\left[\log\det\left(\boldsymbol{I} + \frac{P_i \boldsymbol{G}\boldsymbol{G}^{\mathrm{H}}}{\alpha_i \gamma_i^2 N_0}\right)\right], \quad i = 1, \ldots, n \tag{7}$$

$$R_1 = R_2 = \cdots = R_n \triangleq R$$

$$\sum_{i=1}^{n} \alpha_i = 1, \ \alpha_i \geq 0$$

$$\sum_{i=1}^{n} P_i \leq P$$

Although the above problem is convex and can be solved numerically through standard techniques, finding the optimal solution of the above convex optimisation problem in closed-form is not trivial. Thus, we look for a closed-form sub-optimal solution.

In order to find a sub-optimal solution for the above optimisation problem, we assume that the power allocated to the ith transmission session is proportional to its corresponding time fraction. Accordingly we have [In fact, we have considered this assumption in analogy with the similar broadcast problem where a base station wants to transmit to different mobile stations in a TDMA manner with a constant power. In this broadcast problem (8) is inherent in the problem formulation. This analogy has just provided an intuition for us to assume (8).]

$$P_i = \alpha_i P \tag{8}$$

Thus, we will have

$$R_i = \alpha_i \mathbb{E}\left[\log\det\left(\boldsymbol{I} + \frac{P \boldsymbol{G}\boldsymbol{G}^{\mathrm{H}}}{\gamma_i^2 N_0}\right)\right], \quad i = 1, \ldots, n \tag{9}$$

$$R_1 = R_2 = \cdots = R_n \triangleq R$$

$$\sum_{i} \alpha_i = 1, \ \alpha_i \geq 0$$

where no degrees of freedom is left for the optimisation. This will result in a sub-optimal solution to (6). Based on (9), we will have [It should be noted that in order to evaluate α_i and β_i values, we only need the p.d.f. of elements of $\boldsymbol{G}$, as presented in the system model and no CSIT is available.] (see (10) and (11))

which results in the following average rate

$$R_{C2} = \frac{1}{\sum_k \left(\mathbb{E}\left[\log\det\left(\boldsymbol{I} + \left(P\boldsymbol{G}\boldsymbol{G}^{\mathrm{H}}/N_0\gamma_k^2\right)\right)\right]\right)^{-1}} \tag{12}$$

We are now in a position to investigate the results, and clarify the improvement that our proposed method provides. We define

$$f(x) \triangleq \frac{1}{\mathbb{E}\left[\log\det\left(\boldsymbol{I} + \left(P\boldsymbol{G}\boldsymbol{G}^{\mathrm{H}}/N_0 x\right)\right)\right]} \tag{13}$$

$$x_k \triangleq \gamma_k^2$$

$$p_k \triangleq \frac{1}{n}$$

Thus, we have

$$R_{C2} = \frac{1}{n}\frac{1}{\sum_k p_k f(x_k)} \tag{14}$$

and

$$R_{C1} = \frac{1}{n}\frac{1}{f\left(\sum_k p_k x_k\right)} \tag{15}$$

According to the following lemma, the function $f(x)$ is a strictly concave function, which is rigorously proved in the appendix.

Lemma 1: If $\boldsymbol{I}$ is the identity matrix, $\boldsymbol{B}$ is a positive-definite matrix, and x is a positive scalar, then the function

$$f(x) = \frac{1}{\mathbb{E}[\log\det(\boldsymbol{I} + \boldsymbol{B}/x)]} \tag{16}$$

is a strictly concave function.

Proof: See appendix.

Consequently, according to Jensen's inequality, we will have

$$R_{C_2} > R_{C_1} \tag{17}$$

Therefore although our proposed power allocation method is sub-optimal, it improves the common rate of transmitters, comparing with the channel inversion solution used in [7]. Also, as mentioned earlier, [1] proposes a sub-optimal analytical solution for the same optimisation problem we have considered here, and we call it the Dohler's solution. Therefore in Fig. 2 we have compared the performance of our proposed scheme with the channel inversion solution, Dohler's solution, and the numerically evaluated optimal solution of the convex problem. In this figure, we have

$$\alpha_i = \frac{1}{\mathbb{E}\left[\log\det\left(\boldsymbol{I} + \left(P\boldsymbol{G}\boldsymbol{G}^{\mathrm{H}}/N_0\gamma_i^2\right)\right)\right]\sum_{k=1}^{n}\left(\mathbb{E}\left[\log\det\left(\boldsymbol{I} + \left(P\boldsymbol{G}\boldsymbol{G}^{\mathrm{H}}/N_0\gamma_k^2\right)\right)\right]\right)^{-1}} \tag{10}$$

$$P_i = \frac{P}{\mathbb{E}\left[\log\det\left(\boldsymbol{I} + \left(P\boldsymbol{G}\boldsymbol{G}^{\mathrm{H}}/N_0\gamma_i^2\right)\right)\right]\sum_{k=1}^{n}\left(\mathbb{E}\left[\log\det\left(\boldsymbol{I} + \left(P\boldsymbol{G}\boldsymbol{G}^{\mathrm{H}}/N_0\gamma_k^2\right)\right)\right]\right)^{-1}} \tag{11}$$

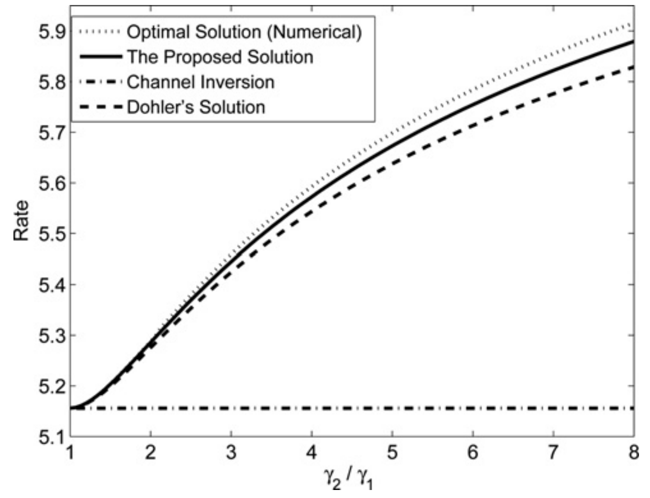

Fig. 2 *Performance comparison of different solutions for a two-user 2 × 2 channel*

The results are evaluated at ($P/N_0 = 30$ dB)

focused on the role of asymmetry on performance improvement in a two user interference channel. The *x*-axis is the ratio γ_2/γ_1 which is a measure of asymmetry between the users, and the *y*-axis is the rate. We have assumed that $\gamma_1^2 + \gamma_2^2$ is constant whereas γ_2/γ_1 is varying. As observed in the figure, the channel inversion solution completely ignores the asymmetry in the channels. However, the other solutions benefit from the asymmetry. Also, we see that our solution ranks below the optimal solution and above Dohler's solution.

Moreover, Figs. 3–5 show the performance of the proposed solution along with Dohler's solution normalised by the optimum solution for different signal-to-noise ratio (SNR) values. The γ_2/γ_1 ratio in these figures are 3, 5, and 8, respectively. These figures show that at higher SNRs, the performance of the proposed solution outperforms Dohler's solution, whereas in low-SNRs Dohler's solution has a better performance. An interesting observation is that, the SNR value at which our method loses its superiority is higher in the more asymmetric scenarios; This critical SNR is about 15, 18 and 19 dB for the cases $\gamma_2/\gamma_1 = 3$, 5, and 8, respectively.

It is observed that our method has its best performance at high-SNR values, whereas Dohler's solution has better performance at lower SNR values. This effect may be due to the fact that the logarithm function involved in the optimisation problem behaves more like a linear function at lower SNR values and mainly exhibits its non-linear nature at higher SNR values.

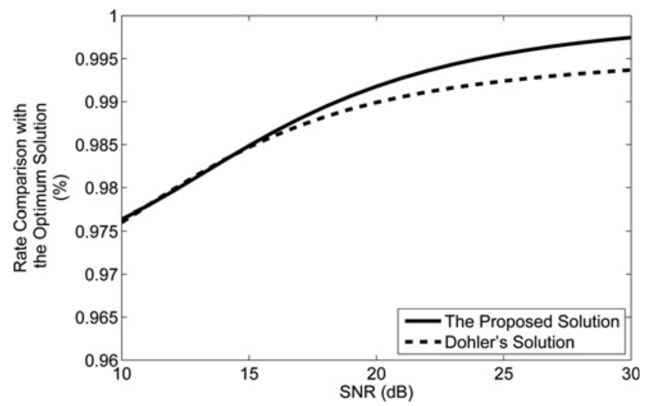

Fig. 3 *Performance comparison of the proposed solution with Dohler's solution for a two-user 2 × 2 channel*

The *y*-axis is normalised to the optimal solution rate and is specified in percentage. The results are evaluated at ($\gamma_2/\gamma_1 = 30$)

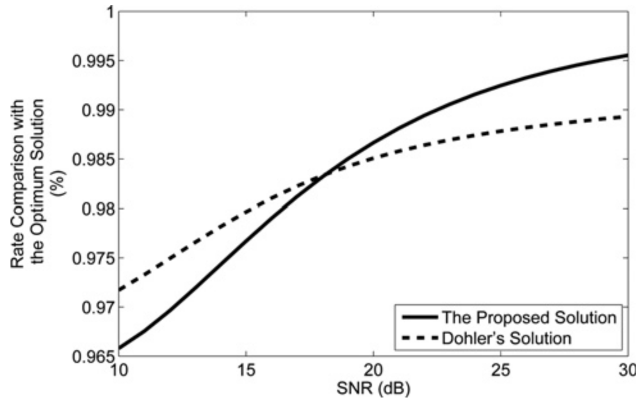

Fig. 4 *Performance comparison of the proposed solution with Dohler's solution for a two-user 2 × 2 channel*

The *y*-axis is normalised to the optimal solution rate and is specified in percentage. The results are evaluated at ($\gamma_2/\gamma_1 = 5$)

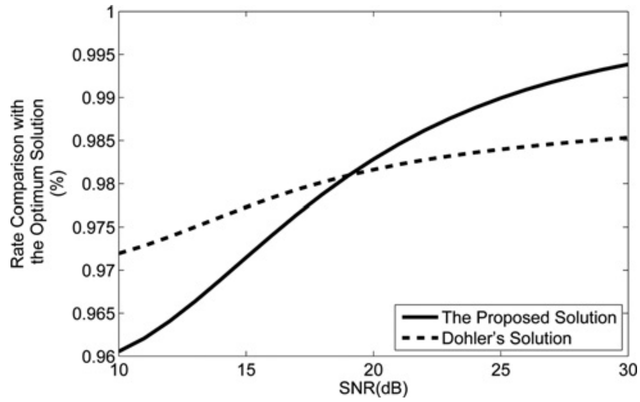

Fig. 5 *Performance comparison of the proposed solution with Dohler's solution for a two-user 2 × 2 channel*

The *y*-axis is normalised to the optimal solution rate and is specified in percentage. The results are evaluated at ($\gamma_2/\gamma_1 = 8$)

4 Throughput enhancement in the hierarchical cooperation scheme

Hierarchical cooperation scheme proposed in [7] for *ad hoc* networks is proven to achieve linear aggregate throughput scaling. Consider *n* nodes distributed randomly and uniformly on a square area. In the scheme proposed in [7], the network is partitioned into clusters of *M* nodes. The scheme consists of three phases. In the first phase, the nodes inside a cluster distribute their own data between other nodes inside the same cluster. In the second phase, each source cluster will perform a MIMO transmission shot towards its corresponding destination cluster. Such MIMO transmissions are planned by a TDMA strategy. Since we assume to have *n* source nodes, we will have a total of *n* MIMO shots. Finally, in the third phase, nodes in the receive cluster cooperate to decode their own data. By handling the first and the third phases in a hierarchical manner and repeating the recursion, one can show that the total aggregate throughput will be linear in the number of nodes. In other words, the first and third phase can be seen as the original problem in a smaller scale. We can continue this recursion h times which we call a 'h-level hierarchical cooperation'. We call the above scheme the 'original hierarchical cooperation' scheme.

In the scheme proposed in [7], there are individual power constraints on all nodes. By relaxing these individual power constraints to a total power constraint, we apply our proposed power allocation scheme to the second phase, whereas maintaining the other phases unchanged. In this section, we analyse how such power allocation scheme for the MIMO phase will enhance the network aggregate throughput.

By following the exact method stated in [8], one can prove that the aggregate throughput of the hierarchical cooperation scheme is as follows [The only difference between (18) and the result derived in [8] is that in this relation we have not considered $C_0 = C_1$, but in (17) of [8] authors have put $C_0 = C_1 = R$. Thus, (18) is more general than (17) of [8]. For details of the derivation refer to [8].]

$$T_{\text{HC}} = \left(\frac{C_0}{C_1}\right)^{(1/h)} \frac{C_1}{h[4(1+Q/C)]^{(h-1)/2}} \left(\frac{n}{2}\right)^{1-(1/h)} \quad (18)$$

where C_0 and Q are constants, C_1 is proportional to the second (MIMO) phase capacity [If the MIMO rate between each source and destination cluster in the second phase is R, we have $C_1 = (R/M)$.], and h is the number of hierarchy levels.

Our proposed power allocation scheme enhances the term C_1 in (18), which will translate into network aggregate throughput enhancement. As it does not affect other phases, in order to consider its effect, it suffices to replace the term C_1 in (18) by the enhanced value. In other words, by the power allocation scheme used in [7], we should put $C_1 = R_{C1}$, and by our proposed method we should put $C_1 = R_{C_2}$. If we define $\xi \triangleq R_{C_2}/R_{C_1}$ to be the second phase throughput enhancement, for the enhanced aggregate throughput (with our proposed power allocation in the second phase) of the network we have

$$T_{\text{HC}_2} = \left(\frac{C_0}{\xi R_{C_1}}\right)^{(1/h)} \frac{\xi R_{C_1}}{h\left[4(1+\left(Q/\xi R_{C_1}\right)\right]^{(h-1)/2}} \left(\frac{n}{2}\right)^{1-(1/h)} \quad (19)$$

where the aggregate throughput with the original power allocation in the second phase, is

$$T_{\text{HC}_1} = \left(\frac{C_0}{R_{C_1}}\right)^{(1/h)} \frac{R_{C_1}}{h\left[4\left(1+Q/R_{C_1}\right)\right]^{(h-1)/2}} \left(\frac{n}{2}\right)^{1-(1/h)} \quad (20)$$

We define $\psi_{\text{HC}} \triangleq (T_{\text{HC}_2}/T_{\text{HC}_1})$ to be the network aggregate throughput enhancement because of the proposed power allocation in the second phase.

Also, in [9], a modified version of the hierarchical cooperation scheme is introduced which has a better delay performance. We call this scheme the 'modified hierarchical cooperation' scheme. By following the exact method stated in [10], one can prove that the aggregate throughput of the modified hierarchical cooperation scheme is as follows [The only difference between (21) and the result derived in [10] is that in this relation we have not considered $C_0 = C_1$, but in [10], author has put $C_0 = C_1 = R$. For details of the derivation refer to [10].]

$$T_{\text{MHC}} = \frac{C_0}{h\left(1+C_1/Q\right)^{(h-1)/h}\left(4Q/C_1\right)^{(h-1)/2}} \left(\frac{n}{2}\right)^{(h-1/h)} \quad (21)$$

As before, increasing the term C_1 will result in aggregate network throughput enhancement in (21), whereas maintaining the other phases unchanged. In other words, we have

$$T_{\text{MHC}\,2} = \frac{C_0}{h\left(1+\xi R_{C1}/Q\right)^{(h-1)/h}\left(4Q/\xi R_{C1}\right)^{(h-1)/2}} \left(\frac{n}{2}\right)^{(h-1/h)} \quad (22)$$

and

$$T_{\text{MHC}1} = \frac{C_0}{h(1+R_{C1}/Q)^{(h-1)/h}(4Q/R_{C1})^{(h-1)/2}} \left(\frac{n}{2}\right)^{(h-1)/h}. \quad (23)$$

Also, we define $\psi_{\text{MHC}} \triangleq \left(T_{\text{MHC}\,2}/T_{\text{MHC}\,1}\right)$.

Before the network starts to operate, each node should be informed of its power and time allocation. Therefore one important issue regarding the network operation is the calculation of α_is and P_is. As it is clear from (10) and (11), in order to calculate these parameters, we need to know all the γ_is. Thus, this task is a centralised calculation, where the distance information of nodes is available altogether. This calculation can be performed in the network initialisation phase. Since, we do not consider node mobility in our network model, α_is and P_is remain constant during the network operation. After concluding the initialisation phase, the data transmission phase can be performed in a distributed manner. In this paper we do not focus on the initialisation phase, however, it is an important phase which is critical for network operation.

In addition, from (10) and (11) it can be noticed that, in order to evaluate α_i and P_i values, we should calculate expectation values of some functions of $\boldsymbol{G}$ (the functions stated in (10) and (11)). Thus, we only need to know the probability distribution function of elements of $\boldsymbol{G}$ – not the instantaneous values of elements of $\boldsymbol{G}$ – which is presented in the model of the system; Each element of $\boldsymbol{G}$ is a complex exponential with random phase uniformly distributed on $[0, 2\pi]$. Therefore there is no CSIT available to benefit from the classical MIMO pre-coding techniques.

Fig. 6 shows a numerical example for the network aggregate throughput enhancement (ψ_{HC} and ψ_{MHC}) as a function of path-loss exponent, for a network consisting of 120 nodes in both low- and high-SNR regimes. This figure shows the improvement ratio of the proposed method with respect to the channel inversion solution. It is clear from this figure that this method leads to more performance enhancement at larger path-loss exponents. Also, Fig. 7 shows the same numerical example when the comparison is with Dohler's solution. As shown in this figure, at SNR = 30 dB our scheme has a better performance than Dohler's solution, whereas at SNR = 10 dB, Dohler's solution outperforms our solution. Also, by comparing Figs. 6 and

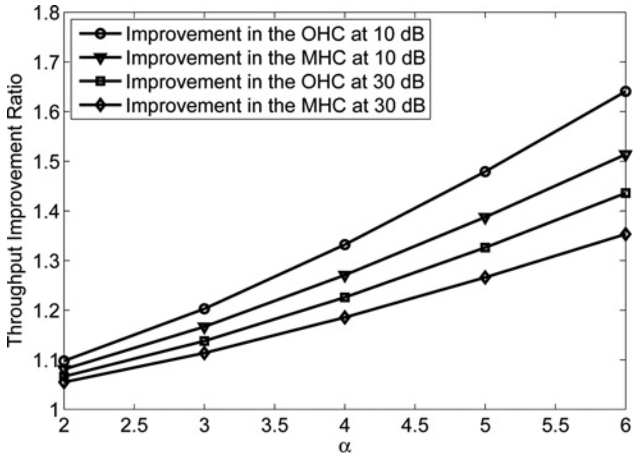

Fig. 6 *Network aggregate throughput improvement ratio (ψ_{HC} and ψ_{MHC}) as a function of path-loss exponent (α)*

The improvement ratio is calculated with respect to the channel inversion method. (OHC: original hierarchical cooperation, MHC: modified hierarchical cooperation)

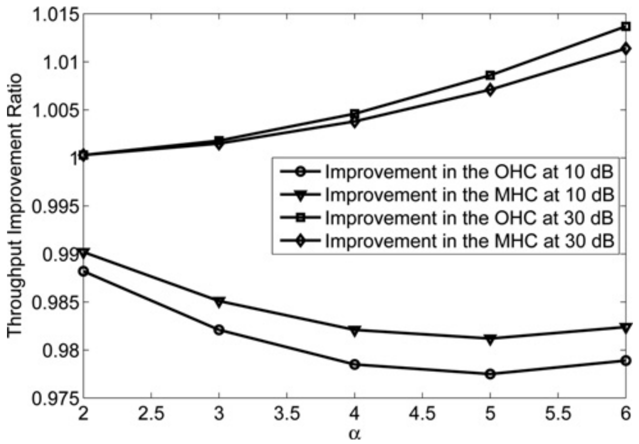

Fig. 7 *Network aggregate throughput improvement ratio (ψ_{HC} and ψ_{MHC}) as a function of path-loss exponent (α)*

The improvement ratio is calculated with respect to the Dohler's solution. (OHC: original hierarchical cooperation, MHC: modified hierarchical cooperation)

7, we conclude that the improvement gain of the proposed method, at high-SNR values, with respect to the Dohler's solution is much less than the improvement gain with respect to the channel inversion method.

5 Conclusions

In this paper, we proposed a novel power allocation scheme for distributed-MIMO TDMA interference channels. As our work is the only paper addressing this issue for distributed-MIMO TDMA networks, we have compared our work with the solution of a similarly formulated problem in the context of distributed-MIMO multi-hop networks [1]. The proposed sub-optimal solution is in closed-form, and is shown to outperform the solution proposed in [1] in the high-SNR regime. As an application of our sub-optimal solution, we have evaluated the improvement in network throughput that our scheme provides in *ad hoc* networks operating under the hierarchical cooperation strategy. Our results clearly demonstrates the critical role of power allocation in ad-hoc networks based on distributed-MIMO strategy.

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8 Appendix

Proof of Lemma 1: We use similar guidelines used in [12]. We have

$$\frac{1}{\mathbb{E}[\log\det(\boldsymbol{I}+\boldsymbol{B}/x)]}=\frac{1}{\mathbb{E}\left[\log\det\left(\boldsymbol{I}+\boldsymbol{B}^{1/2}(x\boldsymbol{I})^{-1}\boldsymbol{B}^{1/2}\right)\right]} \tag{24}$$

and since $x\boldsymbol{I}\mapsto\boldsymbol{B}^{1/2}x\boldsymbol{I}\boldsymbol{B}^{1/2}$ is a strict positive linear map, we have to prove that the following is strictly concave:

$$\frac{1}{\log(1+1/x)} \tag{25}$$

which is trivial by taking its derivatives. □

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