Asymptotically optimum detection of primary user in cognitive radio networks

A. Taherpour, Y. Norouzi, M. Nasiri-Kenari, A. Jamshidi and Z. Zeinalpour-Yazdi

Abstract: Traditionally, the frequency spectrum is licensed to users by government agencies in a fixed manner where the licensee has exclusive right to access the allocated band. However, with increasing demand for the spectrum and scarcity of vacant bands, a spectrum policy reform seems inevitable. Meanwhile, recent measurements suggest the possibility of sharing spectrum among different parties subject to interference-protection constraints. In order to enable access to an unused licensed spectrum, a secondary user has to monitor licensed bands and opportunistically transmit whenever no primary signal is detected. Spectrum-sharing between a primary licensee and a group of secondary users has been studied. The structure of an asymptotically optimum detector based on the measurements of all secondary users is derived and the effect of the quantisation error in such a system is evaluated. Also, it is shown that by using the proposed detector in a sequential detection structure, it is possible to shorten the decision time needed by the detector. The results show the superiority of the proposed detector to other schemes.

1 Introduction

Demand for ubiquitous wireless services has been ever-increasing, and this is expected to continue in the future. Today's wireless networks are characterised by a fixed spectrum assignment policy and according to frequency allocation bodies around the world, few frequency resources are currently available for future wireless applications [1].

Recent measurements by the spectrum policy task force (SPTF) within FCC indicate that many portions of the licensed spectrum are not used for significant periods of time [1]. This observation suggests that it is not an actual spectrum shortage that is worrisome, but rather the inefficient usage of the licensed spectrum. As the number of users and their data rates steadily increase, inefficient spectrum usage is no longer feasible. One proposal for alleviating the scarcity of spectrum is using cognitive or agile radios.

First introduced by Mitola [2], a cognitive radio is a low cost, highly flexible, single protocol wireless device which uses spectrum 'holes' for its signal transmission. A spectrum hole is a band of frequencies assigned to a primary user, but unused at a particular time and geographic location [3–5]. Eliminating these holes dramatically increases spectral efficiency.

Detection of the presence or absence of primary users in a particular frequency band is vital to the operation of cognitive radios, and must be performed quickly and accurately. This is one of the major challenges of implementing this technology. Several efficient methods have been proposed for detection of the primary user presence [6–10]. Recently, collaborative spectrum sensing to detect the primary user has been considered in various papers [7–9].

In [6], the local oscillator (LO) leakage power of RF receivers in TV broadcasting bands is used to detect the primary user receiver's location. In [7, 8], a collaborative sensing is proposed to detect the primary user, which is achieved via a logical OR operation over all secondary user decisions. In other words, first the decision on the presence of the primary user is made internally and individually by each secondary user by comparing sufficient statistics with a specific threshold. Then, the decision result is transmitted to the other users by one bit (0 or 1), and whenever at least one user decides the presence of the primary user, the presence of the user is accepted by all secondary users and none of the secondary users will transmit its signal in that frequency band. In [9, 10], the authors proposed using cooperative protocols such as amplify and forward between two secondary users to detect the primary user while transmitting the users' signals via cooperation. In that work, the authors have analysed their cooperation protocol in a Rayleigh fading channel and showed that by the cooperation, the required time for the spectrum sensing would decrease.

In an optimum detector, which has not been considered previously, the observations of all the secondary users must be taken under consideration in a signal detection function.

We consider the problem of primary user sensing in general and attempt to find an optimum solution. Our proposed detector considers all the observations and then makes its decision on the presence or absence of the primary user based on these observations. The optimum detector requires knowledge of some parameters which are actually unknown. To alleviate this problem, the proposed detector is modified into a generalised likelihood ratio (GLR) detector, which uses estimations of the unknown parameters in the detector structure. The performance of the proposed GLR detector is investigated. The effect of quantisation errors on the performance is evaluated using simulation. Also, it is shown that by using the

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The authors are with the Wireless Research Lab, Sharif University of Technology, Azadi Ave., PO Box 11365-9363, Iran

E-mail: mnasiri@sharif.edu

proposed detector in a sequential detection structure, it is possible to shorten the required spectrum sensing time.

The paper is organised as follows. In Section 2, we describe the basic assumptions and system model. In Section 3, the problem of the primary user detection using likelihood ratio method is formulated, and then by using some reasonable approximations, the GLR detector will be introduced to simplify the optimal detector to a practical one. In Section 4, the performance of the proposed detector is evaluated in fading channels. In Section 5, the effect of the quantisation error on the performance of the detector is evaluated by simulation. In Section 6, a sequential implementation of the detector is introduced to shorten the time required for each decision. Finally, Section 7 concludes the paper.

2 Basic assumptions and system model

An important requirement of a cognitive radio network is to sense the spectrum and decide about the presence or absence of the primary user in a special frequency band. In practice, it is difficult for a cognitive radio to have a direct measurement of the channel between a primary receiver and a transmitter. Thus, the most recent works focus on primary transmitter detection based on local observations of cognitive radios [11].

The goal of spectrum sensing is to decide between the following two hypotheses

$$y_j(t) = \begin{cases} n_j(t), & H_0 \\ h_j s(t) + n_j(t), & H_1 \end{cases}; j = 1, 2, \ldots, n \qquad (1)$$

where n is the number of available secondary users that collaborate with each other in order to sense the presence of the primary user, $y_j(t)$ is the signal received by jth secondary user, $s(t)$ is the primary user's transmitted signal, $n(t)$ is the additive white Gaussian noise (AWGN) and finally h_j is the amplitude gain of the channel between the primary user and jth secondary user.

H_0 is a null hypothesis, which states that there is no licensed user signal in a certain spectrum band. H_1 is the alternative hypothesis, which indicates that there exists a licensed user signal.

When the primary user signal is known to the secondary user, the optimal detector in an AWGN channel is the matched filter, as it maximises the received signal-to-noise ratio (SNR). The main advantage of the matched filter is that it requires the least time to achieve a reliable decision because of its coherency. However, it requires a priori knowledge of the primary user signal, such as the modulation type, the pulse shape and the packet format. Hence, if this information is not accurate, the matched filter performs poorly.

In general, the primary user signal is unknown. A common method for detection of an unknown signal in noise is using energy detector (radiometry). In the energy detector, an input band-pass filter selects the centre frequency, f_c, and the bandwidth of interest, W. This filter is followed by a squaring device to measure the received energy and an integrator which determines the observation interval, that is, T. Now, if the secondary user wants to make decision individually on the signal presence or absence, the output of the integrator will be compared with a threshold. In this case, each secondary user decides about the primary user presence individually. This non-cooperative method requires a long integration time to achieve a reliable decision, as will be discussed below.

Let us denote the output of the integrator for the secondary user jth by Y_j which serves as the decision statistic. Then, Y_j can be shown to have the following distribution [11, 12]

$$Y_j = \begin{cases} \chi^2_{2TW}, & H_0 \\ \chi^2_{2TW}(2\gamma_j), & H_1 \end{cases}; \quad j = 1, 2, \ldots, n \qquad (2)$$

In the above expression, the parameter γ_j is the instantaneous SNR of jth secondary user, and χ^2_{2TW} and $\chi^2_{2TW}\ (2\gamma_j)$ denote the central and non-central chi-square distributions, respectively, each with $2TW$ degrees of freedom and a non-centrality parameter of $2\gamma_j$ for the latter distribution.

The secondary users are interested in a low false alarm probability (P_{fa}) and a high detection probability (P_d). To this end, the product WT must be large enough. Since the bandwidth is constant, the large value of WT means a large observation time for each secondary user. In practice, each secondary user is confronted with many frequency bands and must choose one of them for transmission. These bands have to be sensed for the presence or absence of the active primary use. Only after detection of a free frequency band can the secondary user use the band for the transmission of its signal. If the secondary user waits for T seconds to decide on the presence or absence of the primary user in each of these frequency bands, the process can take a very long time.

If we assume that there is more than one secondary user in a special region and all the secondary users are interested in using free frequency bands concurrently, they have opportunity to collaborate in order to reduce the required spectrum sensing time.

Using collaborative protocols has another advantage. Consider a case in which one secondary user is in shadowing and will not sense the primary user's presence. Consequently, the user may decide incorrectly that a frequency band is free, and start transmitting its signal within the band, disturbing the regular operation of the primary user. However if other secondary users which are sensing the spectrum are considered here, the probability of missed detection will be far less. The greater the number of collaborative secondary users, the less the probability of mis-detection will be.

For these reasons, many authors have proposed the cooperative or collaborative detection of the primary user [7–10]. Fig. 1 shows the cognitive radio physical and link layers' functionalities as proposed by most previous works [13–17]. As can be realised from this figure, a reliable channel with low capacity (common control channel) is usually considered to exchange sensing information among secondary users.

In all previous works, it has been assumed that each secondary user has an internal detector to decide about the presence of the primary user, and then transmit its decision by one bit (0 or 1) to the other secondary users. Such detection algorithms are known as 'decentralised detection' and were first proposed for radar detection applications in the 80s. Recently, these methods have been proposed for use in sensor networks. Important advantages of these algorithms include their simplicity and low transmission rates among secondary users (0 or 1). However, as stated in [18], the decentralised detection is not an optimum detection and for an optimum decision, the signal sensed by each secondary user (or equivalently the sufficient statistics of the observed signal) must be transmitted to other secondary users. Since the spectrum sensing phase is one of the most important phases of a cognitive radio's operation,

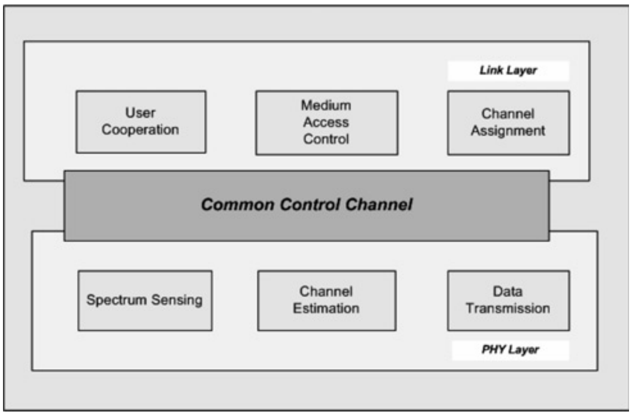

Fig. 1 *Physical and link layer functionalities of a typical cognitive radio*

using several bits to transmit the observed energy to the other secondary users is quite reasonable. (Note that, even in the simple decentralised method, each secondary user must consider some bits to inform the other users about the searched frequency.) In this work, we consider a cooperative method, in which the energy sensed by each secondary user is transmitted to the other secondary users. Then, by taking into account its own observed energy and the other secondary users' observed energies, each secondary user attempts to decide about the spectrum hole. We describe our proposed detector in the following section.

3 Proposed detector

As stated previously, each secondary user has an energy detector to measure the received signal energy. From (2), the output of the detector has a central chi-square distribution under the H_0 hypothesis, although it has a non-central chi-square distribution under the H_1 hypothesis, each with $2TW$ degrees of freedom and a non-centrality parameter of λ for the latter distribution. The parameter λ is proportional to SNR, that is, $\lambda = 2\gamma$. For the sake of simplicity, it is assumed that the time-bandwidth product TW is an integer number denoted by m. If y_j denotes the output of the jth secondary user's detector, we then have

$$f_{Y_j}(y_j|H_0) = \frac{1}{2^m\Gamma(m)} y_j^{m-1} \mathrm{e}^{-y_j/2} \tag{3}$$

$$f_{Y_j}(y_j|H_1) = \frac{\mathrm{e}^{-(y_j+\lambda_j)/2} y_j^{m-(1/2)} \sqrt{\lambda_j}}{2(\lambda_j y_j)^{m/2}} I_{m-1}\left(\sqrt{\lambda_j y_j}\right) \tag{4}$$

where $I_\alpha(x)$ denotes the first kind modified Bessel function of degree α, λ_j is the instantaneous SNR of jth secondary user, and $\Gamma(.)$ is the gamma function.

If we assume that the detection will be based only on y_j, s, and if we choose the ML, maximum a posterior probability or Neyman-Pearson detector, the likelihood ratio must be compared with a threshold. If we denote the observed energies of all secondary users by a vector $\bar{\boldsymbol{Y}} = (y_1, y_2, \ldots, y_n)$, then the likelihood ratio of this vector is obtained as

$$L(\bar{\boldsymbol{Y}}) = \frac{f_{\bar{Y}}(y_1, y_2, \ldots, y_n|H_1)}{f_{\bar{Y}}(y_1, y_2, \ldots, y_n|H_0)} \tag{5}$$

As y_j's can be assumed to be independent and identical distributed random variables (i.i.d.), the likelihood ratio $L(\bar{\boldsymbol{Y}})$ can be rewritten as

$$L(\bar{\boldsymbol{Y}}) = \prod_{j=1}^{n} L(y_j) \tag{6}$$

So, first we consider $L(y_j)$. From (3) and (4), the likelihood ratio for y_j can be computed as follows

$$L(y_j) = \frac{f_{Y_j}(y_j|H_1)}{f_{Y_j}(y_j|H_0)} = 2^{m-1}\Gamma(m)\cdot \mathrm{e}^{-\lambda_j/2} \cdot \frac{I_{m-1}\left(\sqrt{\lambda_j y_j}\right)}{\left(\sqrt{\lambda_j y_j}\right)^{m-1}} \tag{7}$$

Referring to (7), $L(y_j)$ is a complicated function of y_j. To properly approximate this function with a simple one, we consider the behaviour of the function at very small and large values of y_j.

For small values of x ($x \ll 1$), the modified Bessel function can be approximated by [19]

$$I_\alpha(x) \simeq \frac{1}{\Gamma(\alpha+1)}\left(\frac{x}{2}\right)^\alpha \quad 0 < x \ll \sqrt{\alpha+1} \tag{8}$$

Similarly for large values of x ($x \gg 1$), the approximation takes the form [19]

$$I_\alpha(x) \simeq \frac{1}{\sqrt{2\pi x}}\mathrm{e}^x \quad x \gg \left|\alpha^2 - \frac{1}{4}\right| \tag{9}$$

Using the above approximations, from (7) we obtain

$$\sqrt{\lambda_j y_j} \ll \sqrt{m} \Longrightarrow L(y_j) \simeq 2^{m-1}\Gamma(m)\cdot \mathrm{e}^{-(\lambda_j/2)} \times \frac{1}{\Gamma(m)} \cdot \frac{\left(\sqrt{\lambda_j y_j}/2\right)^{m-1}}{\left(\sqrt{\lambda_j y_j}\right)^{m-1}} = \mathrm{e}^{(-\lambda_j/2)} = \text{cte} \tag{10}$$

According to (10), a very small value of $\lambda_j y_j$ has no information about the presence or absence of the primary user. This is not the case for a large value of $\lambda_j y_j$

$$\sqrt{\lambda_j y_j} \gg \left|(m-1)^2 - \frac{1}{4}\right| \Longrightarrow L(y_j) \simeq 2^{m-1}\Gamma(m)\cdot \mathrm{e}^{(-\lambda_j/2)} \cdot \frac{\mathrm{e}^{\sqrt{\lambda_j y_j}}}{\sqrt{2\pi}\left(\sqrt{\lambda_j y_j}\right)^{1/2}\left(\sqrt{\lambda_j y_j}\right)^{m-1}} = \frac{2^{m-1}\Gamma(m)}{\sqrt{2\pi}} \cdot \frac{\mathrm{e}^{\sqrt{\lambda_j y_j}-(\lambda_j/2)}}{\left(\sqrt{\lambda_j y_j}\right)^{m-(1/2)}} \tag{11}$$

As the function ln(.) is a monotonic function, we can use ln $L(y_j)$ instead of $L(y_j)$. Now by taking the logarithm from both sides of (11), we will obtain

$$\ln L(y_j) = -\frac{1}{2}\ln(2\pi) + (m-1)\ln 2 + \ln\Gamma(m) - \frac{2m-1}{4}\ln(\lambda_j y_j) + \left(\sqrt{\lambda_j y_j} - \frac{\lambda_j}{2}\right) \tag{12a}$$

Then, by substituting (12a) in (6), we have

$$L(\bar{Y}) = \prod_{j=1}^{n} L(y_j) \Longrightarrow \ln L(\bar{Y}) = \sum_{j=1}^{n} \ln L(y_j)$$
$$= n\left(-\frac{1}{2}\ln(2\pi) + (m-1)\ln 2 + \ln\Gamma(m)\right)$$
$$-\frac{2m-1}{4}\sum_{j=1}^{n}\ln(\lambda_j y_j) + \sum_{j=1}^{n}\left(\sqrt{\lambda_j y_j} - \frac{\lambda_j}{2}\right) \quad (12b)$$

Note that for a large value of $\lambda_j y_j$, the term $\ln(\lambda_j y_j)$ in (12b) can be neglected compared to $\sqrt{\lambda_j y_j}$ and also the constant terms have no effect on the decision. So (12b) can be simplified as

$$\ln L(\bar{Y}) \cong \text{cte} + \sum_{j=1}^{n}\left(\sqrt{\lambda_j y_j} - \frac{\lambda_j}{2}\right) \quad (13)$$

So, considering the above approximation, the ML detection of the primary user based on the observations of all the secondary users will be

$$\sum_{j=1}^{n}\left(\sqrt{\lambda_j y_j} - \frac{\lambda_j}{2}\right) \underset{H_0}{\overset{H_1}{\gtrless}} \tau \quad (14)$$

where τ denotes the threshold of the detector. The problem with the above detector is that the values of λ_j, the SNR of the secondary users, are not available, and they depend on the channel coefficients, defined in (1), and the received energy by each secondary user. These values thus have to be estimated by a proper method.

We can use ML estimation of the unknown parameter in a GLR detector. Even though this detector is not necessarily optimal, its efficiency is shown in many works (e.g. see [20–22]) so we also use this method for our problem. The ML estimation of λ_j is a value that maximises the function $f(y_j|H_1)$, defined in (4)

$$\lambda_{\text{ML}} = \arg\max_{\lambda} f(y_j|H_1) \quad (15)$$

We must find the value of λ_j for each secondary user based on (15). Analytical solution of this equation is not possible and the equation must be solved numerically. Fig. 2 shows the plots of the solution λ_{ML}, against y_j, for various values of m. As shown, λ_{ML} can be approximately expressed as a linear function of y_j

$$\lambda_{\text{ML}} = f(m) + y_j \quad (16a)$$

where at small values of m, $f(m)$ is nearly zero, and therefore we have

$$\lambda_{\text{ML}} \simeq y_j \quad (16b)$$

Substituting this approximation in (14) results to

$$\lambda_{\text{ML}} \simeq y_j \Longrightarrow \sum_{j=1}^{n}\left(\sqrt{y_j y_j} - \frac{y_j}{2}\right) \underset{H_0}{\overset{H_1}{\gtrless}} \tau$$
$$= \sum_{j=1}^{n} |y_j| - \frac{y_j}{2} = \frac{1}{2}\cdot\sum_{j=1}^{n} y_j \underset{H_0}{\overset{H_1}{\gtrless}} \tau \quad (17)$$

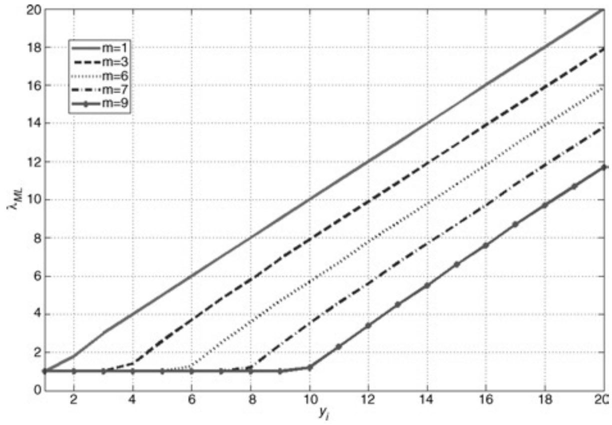

Fig. 2 *Plots of λ_{ML} against y_j*

The final structure of the detector thus reduces to

$$U = \sum_{i=1}^{n} y_j \underset{H_0}{\overset{H_1}{\gtrless}} \eta \quad (18)$$

As can be realised from (18), the sum of the secondary users' observed energies is computed and then compared with a threshold for the primary user's detection.

The proposed detector introduced in (18) is asymptotically optimum. That is, when the SNRs of the secondary users are large enough, the detector approaches the optimum detector. Our numerical results indicate that even for small values of SNR, the detector performs quite well.

In Fig. 3, we compare the performance of the proposed simplified detector (18), the optimum detector (7), and the OR detector. For comparison, we use the complementary receiver operating characteristics (ROC) curves (the plots of P_m against P_{fa}) (note that $P_m = 1 - P_d$ is the probability of a missed detection), which is commonly used to compare the various detectors with each other. If y_j's under H_0 have distribution χ^2_{2m}, then the decision variable U defined in (18), which is the summation of n i.i.d. random variables, will have distribution χ^2_{2mn}. Also if y_j's under H_1 have distribution $\chi^2_{2m}(\lambda_j)$, then U will have distribution $\chi^2_{2mn}(\mu)$ which μ, the non-centrality of the chi-square distribution, from (18) is equal to $\mu = \sum_{j=1}^{n} \lambda_j = \sum_{j=1}^{n} 2\gamma_j$.

For computing the ROC of our proposed detector, we first compute the probabilities, P_d and P_{fa}, in a non-fading environment where h_j; $j = 1, 2, \ldots, n$ in (1) are

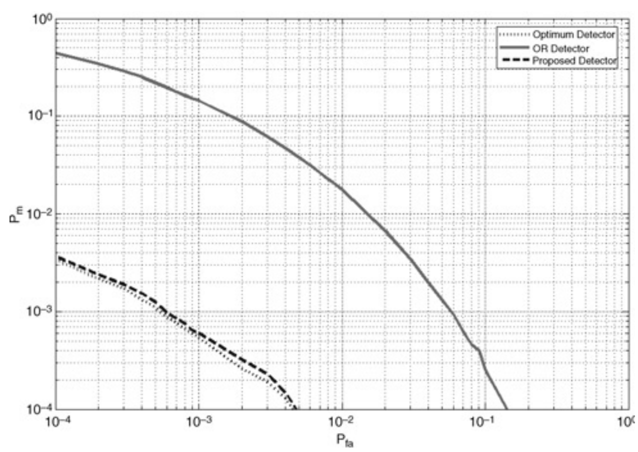

Fig. 3 *Complementary ROC curves for different detectors in AWGN channel, $\bar{\gamma} = 10$ dB, $m = 5$, $n = 5$*

deterministic. Probabilities are easily computed as

$$P_d = P\{U > \eta | H_1\} = Q_{mn}(\sqrt{\mu}, \sqrt{\eta}) \qquad (19)$$

$$P_{\mathrm{fa}} = P\{U > \eta | H_0\} = \frac{\Gamma(mn, \eta/2)}{\Gamma(mn)} \qquad (20)$$

where Γ (.,.) and Q(.,.) are the regularised incomplete lower gamma function and regularised Marcum Q function, respectively.

In an AWGN channel, it is possible to compute the complementary ROC of the proposed detector analytically. Let α be the false alarm probability, that is, P_{fa}, then by the definition, we have

$$\begin{aligned} P_{\mathrm{fa}} &= P(U \geq \eta | H_0) = \alpha \Longrightarrow 1 - F_{\chi^2_{2mn}}(\eta) \\ &= \alpha \Longrightarrow \eta = F^{-1}_{\chi^2_{2mn}}(1-\alpha) \end{aligned} \qquad (21)$$

where the function $F^{-1}_{\chi^2_{2mN}}$(.) is the inverse cumulative function of the central chi-square distribution with $2mn$ degree of freedom. Similarly, the probability of missed detection, that is, P_m, for the proposed detector will be

$$P_m = P(U \leq \eta | H_1) \Rightarrow P_m = F_{\chi^2_{2mn}(\mu)}(\eta) \qquad (22)$$

Then, from (18) and (19), we have

$$\eta = F^{-1}_{\chi^2_{2mn}}(1-\alpha) \Rightarrow P_m = F_{\chi^2_{2mn}(\mu)}(F^{-1}_{\chi^2_{2mn}}(1-\alpha)) \qquad (23)$$

where the function $F^{-1}_{\chi^2_{2mn}(\mu)}$(.) is the inverse cumulative function of the non-central chi-square distribution.

Fig. 3 shows the plots of the complementary ROC for different detectors in AWGN channel. The secondary users' average SNR, that is, $\bar{\gamma}$, and m have been assumed to be 10 dB and 5, respectively. As can be realised, the proposed detector performs substantially better than the OR detector, and almost like the optimum detector. For instance, at $P_{\mathrm{fa}} = 10^{-3}$, P_m equals 10^{-1}, 6×10^{-4} and 5×10^{-4} for the OR detector, the proposed detector and the optimum detector, respectively.

4 Performance evaluation in fading channels

In this section, we first derive the average detection probability for the proposed detector in fading channels. In fading channels, the unconditional detection probability is derived by taking the average of the conditional detection probability of (19) as

$$\bar{P}_d = \int_x P_{d/\mu} \cdot f_\mu(x)\,\mathrm{d}x \qquad (24)$$

Note that the probability of false alarm, that is, P_{fa}, in (20) will remain the same under any fading channels, as it is independent of the SNRs received by the secondary users. In the following sections, we evaluate the performance of the proposed detector under Rayleigh fading and shadowing scenarios.

4.1 Rayleigh fading

For a Rayleigh fading channel, the received SNR of each secondary user, that is, γ_j, has an exponential probability density function (PDF). For computing (24), the PDF of $\mu = \sum_{j=1}^{n} \lambda_j$ must be computed. λ_j's are independent and identically exponentially distributed random variables with parameter $\bar{\lambda}_j = 2\bar{\gamma}_j; j = 1, 2, \ldots, n$. If we assume that all the secondary users have the same average SNR, $\bar{\gamma}_j = \bar{\gamma}; j = 1, 2, \ldots, n$, then the parameter μ will have a gamma distribution with parameters n and $\bar{\lambda} = 2\bar{\gamma}$

$$f(\mu) = \frac{1}{\Gamma(n)}\left(\frac{1}{\bar{\lambda}}\right)\mu^{n-1}\exp\left(\frac{\mu}{\bar{\lambda}}\right) \quad \mu > 0 \qquad (25)$$

By substituting (25) in (24), after some simplifications, a closed form for $\bar{P}_d$ can easily be obtained as [12]

$$\bar{P}_d = a\left[G_1 + b\sum_{k=1}^{m-1}\frac{(h/2)^k}{2(k!)^1}F_1\left(n; k+1; \frac{h}{2}\frac{\bar{l}}{1+\bar{l}}\right)\right] \qquad (26)$$

where $\alpha = 1/\Gamma(n)2^{n-1}(1/\bar{\lambda})^n$ and $\beta = \Gamma(n)(2\bar{\lambda}/(1+\bar{\lambda}))^n \mathrm{e}_1^{-\eta/2}F_1(.;.;.)$ is the confluent hypergeometric function, and

$$G_1 = \int_0^\infty x^{2n-1}\exp\left(-\frac{x^2}{2\bar{\lambda}}\right)Q(x, \sqrt{\eta})\,\mathrm{d}x \qquad (27)$$

In Fig. 4, the performance of the proposed detector, the OR detector and the optimum detector has been compared in Rayleigh fading channels. The secondary users' average SNR, that is, $\bar{\gamma}$, and m have been assumed to be 10 dB and 5, respectively. As can be realised, the proposed detector has better performance than the OR detector in Rayleigh fading channels. For instance, at $P_{\mathrm{fa}} = 10^{-2}$, P_m equals 8×10^{-3}, 9×10^{-4} and 6×10^{-4} for the OR detector, the proposed detector and the optimum detector, respectively.

4.2 Log-normal shadowing

Shadowing is caused by the cumulative effects of different objects in the propagation path which result in large-scale variations of the received power. Empirical measurements suggest that these variations when represented in dB units follow a normal distribution (see e.g. [23]). Log-normal shadowing is usually characterised in terms of its dB-spread, σ_{dB}. A typical range for σ_{dB} is between 2 and 12 dB [24].

Again as λ_j's are independent, $\mu = \sum_{j=1}^{n} \lambda_j = \sum_{j=1}^{n} 2\gamma_j$ is the sum of the n log-normal independent random variables. In this case, as there is no closed form for f_μ (.), the performance of the proposed detector is evaluated by simulation.

Fig. 5 shows the complementary ROC curves for the proposed detector for different values of the shadowing variance when the number of secondary users, n, is 5. The various shadowing variances considered are $\sigma_{\mathrm{dB}} = 2$, 6 and 10 which reflect small, medium and sever shadowing scenarios respectively. Again, the parameter $\bar{\gamma}$ and m are

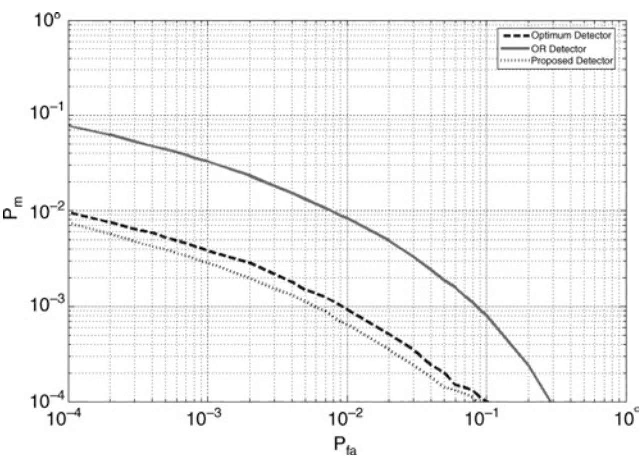

Fig. 4 *Complementary ROC (P_m against P_{fa}) of different detectors in Rayleigh fading $\bar{\gamma}$ = 10 dB, m = 5, n = 10*

set to 10 dB and 5, respectively. It can be realised that decrease of the shadowing variance improves performance. For instance, at $P_{fa} = 10^{-3}$, P_m equals 1.5×10^{-3}, 6×10^{-3}, 2×10^{-2} and 6×10^{-4} for the shadowing channel with $\sigma_{dB} = 2$, 6, 10 and the AWGN channel, respectively.

Fig. 6 shows the complementary ROC curves of the proposed and OR detectors under log-normal shadowing and ideal AWGN channels. As shown, the proposed detector outperforms the OR detector in shadowing environment. For instance, in the shadowing channel with $\sigma_{dB} = 6$ at $P_{fa} = 10^{-2}$, P_m equals 2×10^{-2}, and 1.5×10^{-1} for the proposed detector and the OR detector, respectively. For the AWGN channel, the values are 5×10^{-3}, and 5×10^{-2}, respectively. Furthermore, from Figs. 5 and 6 it is apparent that the performance of both the proposed and OR detectors improves with increase of n. By cooperative protocol, the deleterious effect of the fading channel is substantially reduced.

5 Quantisation effect

In the previous sections, we have not considered the effect of the quantisation errors on the performance of the proposed detector and have assumed that the secondary users' energies could be presented with infinite precision. In practice, the analogue energy observed by each secondary user is first quantised by a finite number of bits, and then is transmitted.

In this section, we assume that the observed energies, y_j's, are presented by few bits. Also, like [8–10], we assume ideal channels among secondary users without error in transmission. Note that, as mentioned before, many proposed spectrum sharing methods assume a common control channel which facilitates several spectrum sharing functionalities such as sensing information exchange.

Computer systems are commonly 16, 32 or 64 bits. However, smaller hardwares and embedded computers can have word lengths of 4 or 8 bits. Some typical ADC converter resolution lengths are 4, 8, 12 or 16 bits. In order to evaluate the performance of the proposed detector, it is assumed that each secondary user has a standard ADC with above precisions.

Fig. 7 shows the performance of the proposed detector under Rayleigh fading and shadowing when the observed energy is quantised. Again the parameters $\bar{\gamma}$ and m have been set equal to 10 dB and 5, respectively. As can be seen, for the number of quantisation bits equal to 6, for all

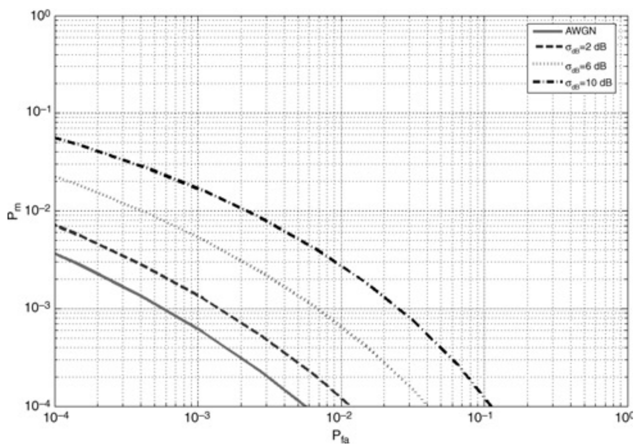

Fig. 5 *Complementary ROC (P_m against P_{fa}) of proposed in shadowing and AWGN channel $\bar{\gamma}$ = 10 dB, m = 5, n = 5*

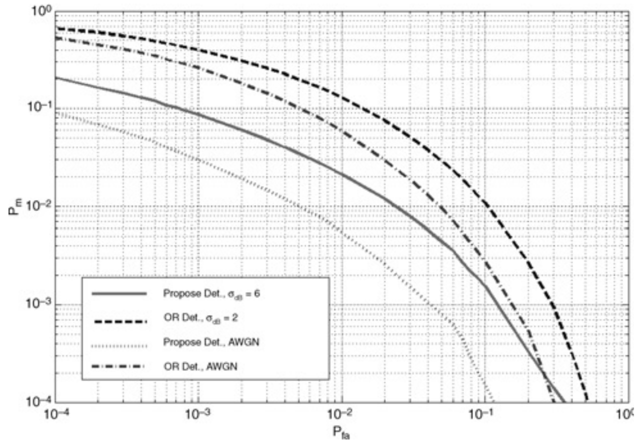

Fig. 6 *Complementary ROC (P_m against P_{fa}) of different detectors in shadowing and AWGN channel, $\bar{\gamma}$ = 10 dB, m = 5, n = 3*

the cases considered, the performance of the detector is close to that of the detector which uses unquantised energies.

6 Sequential detection

In the previous sections, it has been assumed that the number of secondary users which collaborate with each other to sense a particular frequency band is fixed. But the exchange of all spectrum sensing observations may take a large amount of time that in turn results in a decrease spectrum utilisation. It is possible to reduce the observation time using sequential detection. The sequential probability

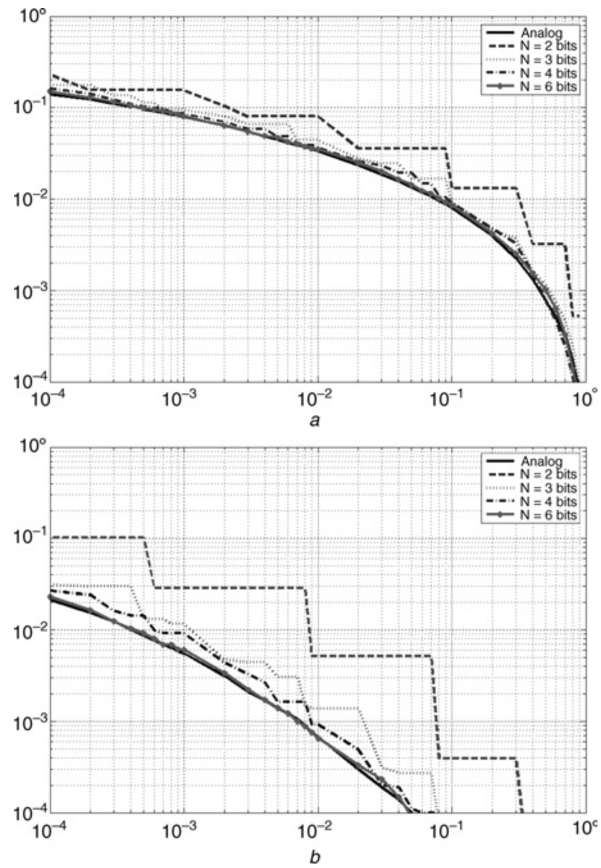

Fig. 7 *Complementary ROC (P_m against P_{fa}) of proposed detector with quantisation*

a Rayleigh fading channel
b Shadowing channel with $\bar{\gamma}$ = 10 dB, m = 5, σ_{dB} = 6 dB, n = 5

ratio test has been suggested by Wald [25] for sequential binary hypothesis testing problems. Sequential tests outperform fixed sample size tests by requiring fewer samples on average to achieve the same level of error performance.

In this section, we present a sequential algorithm for the implementation of the proposed detector. Considering the approximate structure in (18), the detector can be sequentially implemented as follows

$$U_k = \sum_{j=1}^{k} y_j$$

$$\begin{cases} \text{if } U_k \geq B, & \text{Then decide on } H_1 \\ \text{if } U_k \leq A, & \text{Then decide on } H_0 \\ \text{if } A < U_k < B, & \text{Then take another sample} \end{cases} \quad (28)$$

In the other words, in kth stage, each secondary user compares the total observed energies with lower and upper predefined thresholds to decide on the presence of the primary user. In the case that the decision cannot be made, another secondary user observation is considered for the next step.

The values A and B are related to the acceptable values of P_{fa} and P_d. In this case, the definition of P_{fa} is as before, and indicates the probability of deciding the primary user is ON when they are in fact OFF. However, as there is no significant difference between an OFF primary user and a primary user with a very low level of received signal, the probability of correct detection of an ON primary user, that is, P_d, must be determined for a minimum level of the received signal. Therefore we define the following constraints for the sequential detector:

- If primary user is OFF, then the probability of being detected as ON, that is, P_{fa} must have the maximum value of α.
- If primary user is ON and the minimum received energy by the secondary users is μ_{min}, the primary user will be detected as ON with a minimum probability P_{fa}.

Considering the above constraints, the parameters A and B are obtained accordingly.

In Fig. 8, the performance of the sequential and non-sequential detectors is compared. This figure shows the plots of the number of required samples against the SNR at $P_d \geq 0.95$ and $P_{\text{fa}} \leq 10^{-4}$. Note that although the number of samples is fixed for the non-sequential detector, it is a random variable for the sequential one. Besides, the average number of samples required by the sequential detector is different under H_0 and H_1 assumptions. However, for both hypotheses, the average number of samples required for the sequential detector is much less than that of the non-sequential one. That means by using the sequential detection, it is possible to make decisions about the presence of the primary user with fewer observations, and hence to improve the spectrum utilisation in a particular frequency band.

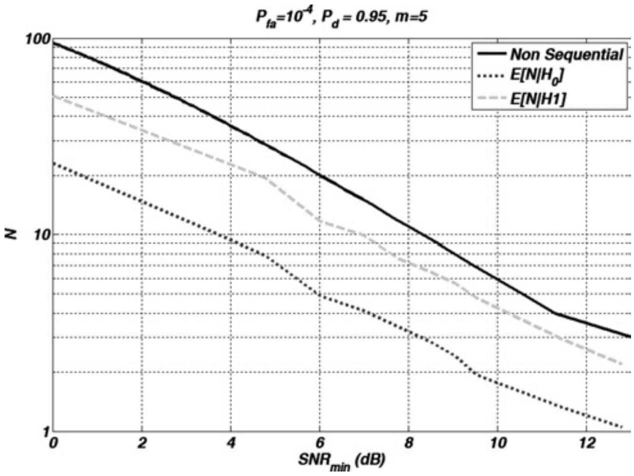

Fig. 8 *Number of samples needed for a decision using sequential and non-sequential method $\bar{\gamma} = 10$ dB, $m = 5$, $P_{fa} = 10^{-4}$, $P_d = 0.95$*

7 Conclusion

We have considered spectrum-sharing between a primary licensee and a group of secondary users and proposed an efficient detector which directly uses the energies observed by all the secondary users. The proposed detector is asymptotically optimum, and has a simple structure to implement. Our numerical results have indicated that the proposed detector has better performance than the OR detector in all the cases considered, including Rayleigh fading and shadowing environments, and also it performs almost as well as the optimum detector. However, it must be noted that the information exchange rate among the secondary users in our method is higher than the OR detector. Since this rate is generally very low, it does not introduce an excess load on the common control channel. Also, the effect of quantisation errors on the performance of the proposed detector has been considered. It has been shown that at about 6 bits quantisation of the observed energies, the performance approaches that of the detector which uses the unquantised energies. Sequential implementation of the proposed detector has also been considered in order to decrease the time required to detect the primary user. The results show that, by using the sequential detection, the average number of the required samples substantially decreases.

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