Connectivity Analysis of One-Dimensional Ad Hoc Networks with Arbitrary Spatial Distribution for Variable and Fixed Number of Nodes

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Abstract—In this paper, we propose an analytical approach to compute the probability of connectivity for one-dimensional ad hoc networks. The proposed analysis gives the exact probability of connectivity for an arbitrary distribution of nodes, provided that nodes are independently and identically distributed. We conduct separate analyses for two cases; in the first case, the number of nodes varies by time under a stationary distribution and in the second case, there is a fixed (known) number of nodes in the network. Using the approaches presented in this work, we are able to derive closed-form formulas for the probability of connectivity for some spatial distributions, while for more complicated distributions, our approach leads to tractable numerical algorithms. As an example, we apply our method to a special case (uniform distribution) and derive a closed-form formula for its probability of connectivity. Finally, we confirm the validity of our analytical approach by simulation for several distributions and show higher accuracy and applicability of the proposed approach compared with existing methods.

Index Terms—Ad hoc network, connectivity, spatial density, spatial distribution.

1 Introduction

Mobile ad hoc networks (MANETs) have attracted the attention of many researchers in recent years due to their numerous applications in different communication scenarios. One of the desirable properties of a practical ad hoc network is having a high probability of connectivity, because connectivity affects the main performance measures of networks such as throughput and delay [1]. Besides, there are some problems (related to design of multihop wireless networks) that depend upon computation of this probability, e.g., optimal power allocation to satisfy a certain minimum probability of connectivity [2], [3], [4], finding the minimum number of nodes needed to ensure network connectedness with high probability (e.g., in sensor networks) [2], and design of efficient packet routing algorithms in vehicular ad hoc networks (VANETs), especially at intersections [5], [6], [7].

Connectivity of ad hoc networks was first studied in [8], [9]. Up to now, there have been several efforts to compute exact or approximate expressions for this probability. Authors in [10], [11], [12], [13], [14] provided exact formulas for probability of connectivity in some specific one-dimensional (1D) networks. Authors in [10] derived exact formulas for the case of n uniformly distributed nodes in a 1D path. A correction on [10] along with another similar model was verified in [11]. An exact expression for the probability that these networks (with uniform distribution) are composed of at most c clusters was given in [12], where in the case of $c = 1$ it converts to the probability of network connectivity. Misra et al. [13] concerned the connectivity of a 1D uniform circular network, i.e., when n nodes are uniformly distributed on a circle. A simple closed-form formula as an approximation for the probability of connectivity for a network with n uniformly distributed nodes was derived in [14]. Also there has been a lot of interest in connectivity of 1D and 2D networks in asymptotic cases [15], [16], [17], i.e., when the number of nodes goes to infinity. Generally, in most networks studied so far, the assumptions of uniform distribution and a fixed (known) number of nodes have been applied.

The aforementioned scenarios are far from reality in dynamic environments, i.e., when the number of nodes is varying continually. In this case, in order to acquire the probability of connectivity, first, one should find the steady-state spatial distribution of nodes for the network scenario and then compute the probability of connectivity corresponding to each state of the spatial distribution. On the other hand, uniform distribution considered in the literature [2], [3], [10], [11], [12], [13], [14], is not applicable somewhere. In this regard, several mobility models have been presented in the literature, such as Random Waypoint [18], Gauss-Markov model [19], and Reference Point Group [20] that lead to nonuniform spatial distributions. As an example of where the uniform distribution assumption fails, in VANETs, intuitively we expect to have more traffic load in the last parts of streets (in the vicinity of crosses), and middle parts are supposed to be less crowded since vehicles move faster in these parts. In this case, uniform

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distribution along the street fails to model the practical situation and more complex distributions should be concerned. Some of the research works in the literature are able to give the spatial distribution of vehicles in a VANET. In this respect, in a new research work [21] we have focused on VANETs in dense scenarios that the vehicles affect each other. In that paper we have proposed a new analytical mobility model capable of considering interdependence among vehicles. The output of such a mobility model is a spatial traffic distribution. Furthermore, VANETs are examples of dynamic environments, in which the number of nodes is varying with time.

Nevertheless, there have been some research works that considered nonuniform distribution or variable number of nodes. In [22], the authors studied the probability of connectivity for a special VANET, and proposed an analysis to compute a good approximation for the probability of connectivity at that network. In their model, they assumed nodes arrive in a highway and depart from it through certain traffic entry points according to a Poisson process and move with constant speed along their path, hence, the number of nodes is variable and distribution is not uniform. Authors in [23] proposed an analytical mobility model for VANETs (with variable number of nodes) and derived loose upper and lower bounds for the probability of connectivity. The research work in [24] provided an analysis to compute approximate probability of connectivity, where a fixed number of nodes are arbitrarily distributed in a 1D network. Although this approximation is of relatively high precision for dense networks (i.e., when the number of nodes is large, hence, the probability of connectivity is close to one), it doesn't yield good approximation for sparse networks in which the probability of connectivity is small. To the best of our knowledge there is still no analytical method to compute the exact probability of connectivity for an arbitrary distribution of nodes in either case of variable or the case of fixed number of nodes.

In this paper, we propose an analytical method to find the exact probability of connectivity for any arbitrary spatial distribution of nodes, provided that nodes are independently and identically distributed (iid). Our proposal includes two cases, i.e., variable and fixed number of nodes. In the first case, we assume that the number of nodes changes with time under a stationary distribution. As an example, in a VANET scenario it is assumed that the arrival rates of the vehicles, the driving habits of the drivers, restrictive road conditions, etc., remain nearly constant for a sufficiently long period of time, such that spatial distribution of the vehicles reaches steady state. Moreover, since at a typical street in the VANET scenario the effective factors on the mobility pattern (e.g., traffic signs, crosswalks, etc.) are the same for all nodes, the iid assumption is justifiable. Our approach gives integral form solutions that can be simplified to closed-form formulas for some distributions, whereas for more complicated distributions it provides tractable numerical algorithms to find the probability of connectivity. For the second case, i.e., fixed (known) number of nodes, our proposed method is able to give exact probability of connectivity for any arbitrary spatial distribution. It is worth saying that our proposed approach gives exact results for both dense and sparse situations.

In our approach, we present separate analyses for two cases. For the first case where the number of nodes is variable, we first analyze discrete 1D networks in which the path is sectorized and each sector can be empty or occupied by nodes according to a given probability. By increasing the number of sectors to infinity (or equivalently shrinking the size of the sectors to zero), an approach to compute the exact probability of connectivity for continuous networks is derived, which is then applied to a special case, namely uniform distribution, giving a closed-form formula for its probability of connectivity. For the second case where the number of nodes is fixed, we conduct a separate analysis directly for continuous networks. It is then shown through numerical examples that our results match with simulation and are compatible with existing methods. Furthermore, we present some examples in order to show the importance of exact computation of probability of connectivity in an environment with variable number of nodes and a known spatial density function. Moreover, it is shown that our approach is much more accurate than the approximate method presented in [24].

The paper is organized as follows: In Section 2, the connectivity analysis of discrete networks with variable number of nodes is proposed. Section 3 is devoted to the connectivity analysis of continuous networks where the number of nodes is variable. Closed-form formula for the probability of connectivity in the case of uniform distribution is derived in Section 4. Section 5 provides connectivity analysis of continuous networks with a fixed number of nodes. In Section 6, we confirm our analysis by simulation and present several numerical results to show the applicability of our approach for different cases. Finally, the paper is concluded in Section 7.

2 Connectivity of Discrete Networks in the Case of a Variable Number of Nodes

In this section, we propose an analytical approach to compute the probability of connectivity for discrete 1D networks. A discrete 1D network is composed of M subregions each of length L/M, constructing a path of length L. We assume two more subregions; first, a fixed wireless transmitter, called the source, placed in subregion 0 and second, a fixed wireless receiver in subregion $M+1$, called the destination. A subregion is occupied if there is at least one node in that subregion. Let r_d denote the transmission range normalized to the length of a subregion, such that there is a radio link between any two consecutive nodes if their corresponding subregions have a difference less than r_d. For example, nodes in the subregions $1, 2, \ldots, r_d - 1$ are always connected to the source, but if these subregions are empty and there is a node in subregion r_d, then this node will be disconnected. A node is said to be connected if there is a connected path between this node and the source. Here, we are interested in connectivity of the whole network, that is a connected path between the source and the destination.

The discrete network is like centering the nodes on their corresponding subregions. In our analysis, we use the concept of connectivity of positions instead of connectivity of nodes. A position (here, a subregion) is said to be connected if its center is within the transmission range of a centeralized connected node. A connected position may be either occupied or empty. In other words, a connected subregion is a subregion that if we pose a new node in it, this node becomes connected. Thus, connectivity of the whole path is equivalent to the connectivity of subregion $M+1$ (i.e., position of the destination).

Definition 1 (Spatial density function of a discrete network). *The probability that the* jth *subregion of a 1D discrete network is occupied is denoted by* $f_d(j)$. f_d *is called spatial density function of the discrete network.*

Note that the source is in subregion 0, thus we always have: $f_d(0)=1$. Define

$$g(m) \triangleq f_d(m) \prod_{k=m+1}^{m+r_d-1} (1-f_d(k)), \quad 0 \leq m \leq M-r_d+1, \quad (1)$$

which denotes the probability that the subregion m is occupied but its $r_d - 1$ following subregions are empty.

In the equations throughout the paper, we indicate the phrases "Disconnected," "Connected," "Occupied," and "Empty" by their abbreviations $'DC'$, $'C'$, $'O'$, and $'E'$, respectively. The following lemma is the basis of our analysis:

Lemma 1. *Let* $P_C(m)$ *denote the probability that subregion* m *is connected to the source, then*

$$P_C(m) = 1, \quad m < r_d,$$

$$P_C(m) = P_C(m-1) - P_C(m-r_d)g(m-r_d), \quad m \geq r_d. \quad (2)$$

Proof. The first equation is clear from the definition of transmission range. For the second equation

$$\begin{aligned}
P_{DC}(m) &\triangleq P(m \text{ is } 'DC') \\
&= P(m \text{ is } 'DC',\ m-1 \text{ is } 'DC') \\
&\quad + P(m \text{ is } 'DC',\ m-1 \text{ is } 'C') \\
&= P(m-1 \text{ is } 'DC') + P(m-r_d \text{ is } 'C', m-r_d \\
&\quad \text{ is } 'O',\ m-r_d+1, \ldots, m-1 \text{ are } 'E') \\
&= P_{DC}(m-1) + P_C(m-r_d)g(m-r_d) \\
&\Rightarrow P_C(m) = P_C(m-1) - P_C(m-r_d)g(m-r_d),
\end{aligned}$$

where the last equality is due to $P_C(m) = 1 - P_{DC}(m)$. □

Thus, for finding the probability of connectivity in a discrete network, one can utilize the following two-step approach:

Approach 1.

1. Compute $g(m)$ for $m = 0, 1, \ldots, M-r_d+1$, from (1).
2. Set $P_C(m) = 1$ for $m < r_d$ and find $P_C(m)$ for $m = r_d \ldots, M+1$ using (2).

Clearly, the number of iterations is $O(M)$. Hence, the method is well tractable.

3 Connectivity of Continuous Networks in the Case of a Variable Number of Nodes

The 1D continuous network is a path of length L with a fixed transmitter and a fixed receiver located in positions 0 and L, called the source and the destination. Denote by r the transmission range due to which if $|x-y| < r$, then the nodes in positions x and y are connected to each other; but, if $|x-y| \geq r$ and the interval (x, y) is empty, there would be no radio link between these nodes. A node is said to be connected if there is a connected path between this node and the source. Similar to the discrete case, a position is connected if it is within the transmission range of a connected node and the connectivity of the whole path is equivalent to the connectivity of position L.

Definition 2 (Filling a network according to a spatial density function *dynamically*). *A continuous network is said to be dynamically filled according to a spatial density function,* f, *if each differential interval,* $[x, x+dx)$, *of the network is occupied with probability* $f(x)dx$, *independent of the state of the other parts. In this case, the number of nodes is variable and can be any nonnegative integer.*

In the following, we construct a function, g, similar to the previous section and state a similar lemma for the continuous case, which is then used to derive integral form formula for the probability of connectivity.

By taking the assumption into account that occupancy state of a differential interval $[x, x+dx)$, is independent of the state of the other intervals, the probability of emptiness for a typical interval, $[x, x+r)$, is equal to the product of probabilities that each differential interval in this region is empty. In order to compute such a product, by taking logarithm of both sides of the equality, the infinite product is converted to integration of logarithms of probabilities that the differential intervals are empty. Hence, if we define $g(x)dx$ as the probability that $[x, x+dx)$ is occupied but $[x+dx, x+r)$ is empty, then we have

$$\begin{aligned}
&\ln(g(x)dx) \\
&\quad = \ln(P([x, x+dx) \text{ is } 'O')P([x+dx, x+r) \text{ is } 'E')) \\
&\quad = \ln(P([x, x+dx) \text{ is } 'O')) \\
&\qquad + \ln(P([x+dx, x+r) \text{ is } 'E')) \\
&\quad = \ln(f(x)dx) + \int_{y=x^+}^{x+r} \ln(1-f(y)dy) \\
&\quad = \ln(f(x)dx) - \int_{y=x^+}^{x+r} f(y)dy,
\end{aligned}$$

where in the last equation, we used the equality $\ln(1+z) = z$, for asymptotically small $|z|$. A fixed wireless node in the middle of the path (e.g., in position x_0) can be modeled by a unit impulse, $\delta(x-x_0)$, in the spatial density function, f. We have assumed that there is no fixed node in the middle of the path (except for the source) and consequently there is no singularity in the density function, f (except for position 0). Hence, the above expression for $g(x)$ can be simplified in the form of the following equation:

$$g(x) = \begin{cases} f(x)e^{-\int_{y=x^+}^{x+r} f(y)dy}, & x > 0, \\ e^{-\int_{y=0^+}^{r} f(y)dy}\delta(x), & x = 0. \end{cases} \quad (3)$$

The following lemma is the continuous version of Lemma 1 in the previous section.

Lemma 2. *If $P_C(x)$ denote the probability that position x is connected, then we have*

$$\begin{aligned} &P_C(x) = 1, \quad x < r, \\ &\frac{dP_C(x)}{dx} = P_C(x-r)g^-(x-r), \quad x \geq r; \end{aligned} \tag{4}$$

where $g^-(x) = -g(x)$, $\forall x$.

Proof. Continuous networks are limits of discrete networks when the length of subregions tends to zero. The proof of this lemma is similar to Lemma 1 (note the difference between definition of spatial density functions and g functions in the two cases). □

The differential equation in (4) can be solved numerically. The complexity of numerically solving (4) is proportional to the complexity of integrating g. It can be simply carried out by sectorizing the x-axis into M subregions to form a discrete network with $f_d(n) = \min(1, \int_{z=(n-1)L/M}^{nL/M} f(z)dz)$, $1 \leq n \leq M$ (which is equal to $\frac{L}{M} f((n-\frac{1}{2})\frac{L}{M})$ for large M), and then finding the probability of connectivity using Approach 1. It is clear that the larger the number of subregions, the more accurate is the answer. In brief:

Approach 2. In order to find the probability of connectivity of a continuous network, divide the continuous path into small subregions and use Approach 1.

However, finding a closed-form formula for this kind of equations is not generally an easy job. In the following, we derive an integral form solution for this system.

From elementary calculus we know that

$$P_C(x) = P_C(r^-) + \int_{y=r^-}^{x} \frac{dP_C(y)}{dy} dy, \quad x \geq r. \tag{5}$$

From (4) and (5) we have

$$\begin{aligned} P_C(x) &= 1 + \int_{y=r^-}^{x} g^-(y-r)P_C(y-r)dy \\ &= 1 + \int_{y=0^-}^{x-r} g^-(y)P_C(y)dy, \quad x \geq r. \end{aligned} \tag{6}$$

Let $k = \lfloor \frac{x}{r} \rfloor$. By using (6) again and again, $P_C(x)$ can be explicitly described in terms of g-function, as in the following:

$$\begin{aligned} P_C(x) &= 1 + \int_{y_1=0^-}^{x-r} g^-(y_1)P_C(y_1)dy_1 \\ &= 1 + \int_{y_1=0^-}^{x-r} g^-(y_1)\left[1 + \int_{y_2=0^-}^{y_1-r} g^-(y_2)P_C(y_2)dy_2\right]dy_1 \\ &= 1 + \int_{y_1=0^-}^{x-r} g^-(y_1)\left[1 + \int_{y_2=0^-}^{y_1-r} g^-(y_2)\left[1 + \cdots \right.\right. \\ &\quad \left.\left.\left[1 + \int_{y_k=0^-}^{y_{k-1}-r} g^-(y_k)P_C(y_k)dy_k\right] \cdots \right]dy_2\right]dy_1 \\ &= 1 + \int_{y_1=0^-}^{x-r} g^-(y_1)\left[1 + \int_{y_2=0^-}^{y_1-r} g^-(y_2)\left[1 + \cdots \right.\right. \\ &\quad \left.\left.\left[1 + \int_{y_k=0^-}^{y_{k-1}-r} g^-(y_k)dy_k\right] \cdots \right]dy_2\right]dy_1. \end{aligned}$$

Note that in the last equality $0 \leq y_k \leq x - kr = x - \lfloor \frac{x}{r} \rfloor r < r$. Hence, according to (4), $P_C(y_k) = 1$. By reshaping this equation, we obtain

$$\begin{aligned} P_C(x) = 1 &+ \int_{y_1=0^-}^{x-r} g^-(y_1)dy_1 \\ &+ \int_{y_1=0^-}^{x-r} g^-(y_1) \int_{y_2=0^-}^{y_1-r} g^-(y_2)dy_2dy_1 \\ &\vdots \\ &+ \int_{y_1=0^-}^{x-r} g^-(y_1) \cdots \int_{y_k=0^-}^{y_{k-1}-r} g^-(y_k)dy_k \cdots dy_1. \end{aligned} \tag{7}$$

For the sake of simplicity, define functions $h_i(x)$, $0 \leq i \leq k$, as follows:

$$\begin{aligned} &h_0(x) \triangleq u(x), \\ &h_i(x) \triangleq \int_{y=0^-}^{x-r} h_{i-1}(y)g^-(y)dy, \quad 1 \leq i \leq \left\lfloor \frac{x}{r} \right\rfloor, \end{aligned} \tag{8}$$

where $u(x)$ is the unit step function with $u(0) = 1$. Now, (7) can be rewritten as

$$P_C(x) = \sum_{i=0}^{\lfloor \frac{x}{r} \rfloor} h_i(x). \tag{9}$$

Hence, the probability of connectivity for continuous networks can be found using the following approach. It also leads to closed-form formulas for some special distributions.

Approach 3.

1. Compute $g^-(x)$ from (3) for $x \in [0, L-r]$.
2. Find functions $h_i(x)$ from (8).
3. Calculate $P_C(L)$ (the probability of network connectivity), using (9).

In this approach, $O(L/r)$ integrations should be computed. For some special spatial density functions (such as uniform distribution) these integrations can be solved analytically to give closed-form formulas. However, in general, there is no closed-form solution and the integrations must be calculated numerically. In this case, Approach 2 would be more computationally efficient.

4 A Special Case, Uniform Distribution

As stated in the previous section, with the aid of Approach 3, closed-form formulas for probability of connectivity can be found only for some special distributions. By a special spatial distribution we mean density functions whose corresponding g-functions have closed-form solutions when integrated for many times. This is due to the fact that $P_C(x)$ is computed by integrating the product of g with its integrals (see (8) and (9)). Spatial distributions with corresponding g-functions in the form of polynomials (especially uniform g-function whose f-function is also uniform), exponential functions, and sinusoids are some examples for which the probability of connectivity has closed-form solution. However, it is clear that f-functions of these distributions may have complicated expressions.

In this section, we find a closed-form formula for probability of connectivity in networks with uniform spatial density, i.e., $f(x) = f,\ 0 < x < L$. Here again, the number of nodes is variable. In order to find a closed-form formula for the probability of connectivity using Approach 3, we should compute the functions $g^-(x)$ and $h_i(x)$. From (3), we have

$$g^-(x) = -fe^{-rf}u(x) - e^{-rf}\delta(x),$$

where $\delta(x)$ is the unit impulse function. For simplicity, let

$$\begin{aligned} g^-(x) &= g_0u(x) - e^{-rf}\delta(x), \\ g_0 &\triangleq -fe^{-rf}. \end{aligned} \tag{10}$$

The functions h_i can be computed from (8)

$$\begin{aligned} h_0(x) &= u(x), \\ h_1(x) &= \int_{y=0^-}^{x-r} h_0(y)g^-(y)dy \\ &= \int_{y=0^-}^{x-r} u(y)\big[g_0u(y) - e^{-rf}\delta(y)\big]dy \\ &= \big[g_0(x-r) - e^{-rf}\big]u(x-r), \\ h_2(x) &= \int_{y=0^-}^{x-r} h_1(y)g^-(y)dy = \int_{y=0^-}^{x-r} \big[(y-r)g_0 \\ &\quad - e^{-rf}\big]u(y-r)\big[g_0u(y) - e^{-rf}\delta(y)\big]dy \\ &= \int_{y=r}^{x-r} \big[(y-r)g_0 - e^{-rf}\big]u(y-r)g_0u(y)dy \\ &= \int_{y'=0}^{x-2r} \big[g_0^2y' - g_0e^{-rf}\big]u(y')dy' \\ &= \left[\frac{g_0^2}{2}(x-2r)^2 - g_0e^{-rf}(x-2r)\right]u(x-2r). \end{aligned}$$

In the same way for $1 \le j \le \lfloor \frac{x}{r} \rfloor$:

$$h_j(x) = \left[\frac{g_0^j}{j!}(x-jr)^j - \frac{g_0^{j-1}e^{-rf}}{(j-1)!}(x-jr)^{j-1}\right]u(x-jr), \tag{11}$$

where g_0 is given by (10). Finally, exact expression for the probability of network connectivity can be obtained from (9)

$$P_C(L) = 1 + \sum_{j=1}^{\lfloor \frac{L}{r} \rfloor} \frac{(-fe^{-rf})^j}{j!}(L-jr)^{j-1}\left[(L-jr) + \frac{j}{f}\right]. \tag{12}$$

It is easy to check that $P_C(x)$ given by (12) satisfies the equations in (4).

5 Connectivity in the Case of a Fixed Number of Nodes

In this section, we propose expressions and algorithms for computing the probability of connectivity for 1D networks with a fixed number of nodes. The idea is similar to the previous case where the number of nodes was variable. Here, we only study continuous networks.

Definition 3 (Distributing n nodes in the network directly). *n nodes are said to be directly distributed in a 1D network according to a probability density function, f_p, if each node is posed on the path according to f_p independent of the positions of other nodes, that is each node sits in the interval $[x, x+dx)$ with probability $f_p(x)dx$.*

For direct distribution, one should first place the first node on the path according to the given probability density function, then do the same for the second, third, etc., regardless of the positions of the previous nodes. This definition may look trivial at the first glance, but a meticulous reader will point out its contrast with the following definition (note the difference between probability density function in Definition 3 and spatial density function in Definition 4).

Definition 4 (Distributing n nodes in the network indirectly). *Indirect distribution of n nodes in a network according to a spatial density function, f, is filling the network according to f dynamically (see Definition 2), subject to having exactly n nodes in the network.*

Given a spatial density function, f, when we want to distribute n nodes in our 1D network according to f indirectly, we can first assume an empty network and dynamically fill it according to f (see Definition 2). At this point there would be some nodes in the network (we may have zero or infinitely many nodes). If the network has exactly n nodes, then the network is filled with n nodes indirectly. But if the number of nodes is more or less than n, we should empty the network and fill it again. We continue refilling the network according to f again and again until reaching a network with exactly n nodes. This network is said to be indirectly filled with n nodes according to the spatial density function, f. Let $P_{nodes}(x, n)$ denote the probability that there are exactly n nodes in $(0, x)$ when the network is filled according to a spatial density function dynamically.

Most of the papers on network connectivity assume that nodes are directly distributed in the network. However, in order to use the ideas of the previous sections in our future analysis, we distribute nodes in the network indirectly. The following lemma expresses the relation between these two ways of filling a network:

Lemma 3. *If f is a spatial density function and f_p is a probability density function that is constructed from f by normalization, i.e.:*

$$f_p(x) = \frac{f(x)}{F(L)}, \tag{13}$$

where $F(x) \triangleq \int_{y=0^+}^{x} f(y)dy$; then, the direct distribution of n nodes according to f_p is equivalent to the indirect distribution of n nodes according to f.

Proof. Denote by $p_{direct_n}(x_1, \ldots, x_n)$ and $p_{indirect_n}(x_1, \ldots, x_n)$ the pdfs of direct and indirect distributions of n nodes according to f_p and f, respectively. Then

$$\begin{aligned} &p_{direct_n}(x_1, \ldots, x_n)dx_1 \ldots dx_n \\ &\triangleq P\begin{pmatrix} [x_1, x_1+dx), \ldots, [x_n, x_n+dx) \text{ are} \\ \text{'}O\text{' in direct distrib. of } n \text{ nodes} \end{pmatrix} \\ &= n!f_p(x_1)\cdots f_p(x_n)dx_1\cdots dx_n \\ &= \frac{n!}{F(L)^n}f(x_1)\cdots f(x_n)dx_1\cdots dx_n, \end{aligned} \tag{14}$$

where $n!$ is due to permutation (note that the nodes are indistinguishable). On the other hand

$$\begin{aligned}
&p_{indirect_n}(x_1,\ldots,x_n)dx_1\cdots dx_n \\
&\triangleq P\begin{pmatrix}[x_1,x_1+dx),\ldots,[x_n,x_n+dx) \text{ are} \\ 'O' \text{ in indirect distrib. of } n \text{ nodes}\end{pmatrix} \\
&= P\left(\begin{array}{l}[x_1,x_1+dx),\ldots,[x_n,x_n+dx) \\ \text{are } 'O' \text{ in dynamically filled networks}\end{array}\middle| n \text{ nodes}\right) \\
&= \frac{P\begin{pmatrix}[x_1,x_1+dx),\ldots,[x_n,x_n+dx) \text{ are } 'O' \text{ when the} \\ \text{network is filled dynamically, and exactly } n \text{ nodes}\end{pmatrix}}{P(\text{exactly } n \text{ nodes in the network})} \\
&= \frac{f(x_1)\cdots f(x_n)P(\text{other parts are empty})dx_1\cdots dx_n}{P_{nodes(L,n)}} \\
&= \frac{P(\text{entire network is } 'E')}{P_{nodes}(L,n)} f(x_1)\cdots f(x_n)dx_1\cdots dx_n,
\end{aligned} \tag{15}$$

where the last equality is due to

$$\frac{P(\text{entire network is } 'E')}{P(\text{other parts are } 'E')} = \prod_{i=1}^{n}(1-f(x_i)dx_i) = 1-O(x).$$

From (14) and (15), we have

$$\begin{aligned}
&\frac{p_{indirect_n}(x_1,\ldots,x_n)dx_1\cdots dx_n}{p_{direct_n}(x_1,\ldots,x_n)dx_1\cdots dx_n} \\
&\quad = \frac{F(L)^n P(\textit{entire network is } 'E')}{n! P_{nodes}(L,n)} \\
&\quad = const. \quad (\text{independent of } x_i{}'s).
\end{aligned} \tag{16}$$

Since both P_{direct_n} and $P_{indirect_n}$ are probability density functions and their integrals are equal to unity, this constant ratio is one and the two pdfs are equal. □

Corollary 1. *Since the constant in (16) is equal to one,* $P_{nodes}(L,n)$ *is*

$$\begin{aligned}
P_{nodes}(L,n) &= \frac{F(L)^n P(\text{entire network is } 'E')}{n!} \\
&= \frac{F(L)^n e^{-\int_{y=0^+}^{L} f(y)dy}}{n!} = \frac{F(L)^n e^{-F(L)}}{n!}.
\end{aligned}$$

In a similar way,

$$P_{nodes}(x,n) = \frac{F(x)^n e^{-F(x)}}{n!}, \quad 0 < x \le L. \tag{17}$$

In the following, we study the probability of connectivity for continuous networks where n nodes are distributed in the network indirectly. Lemma 3 assures us that for any direct distribution, there is an equivalent indirect distribution whose density function is obtained by scaling $f_p(x)$ with an arbitrary scaling factor. We use the following notation in the remaining of this section:

$$\begin{aligned}
P_C(x|n) &= P\big(x \text{ is } 'C' \text{ s.t. exactly } n \text{ nodes in } (0,x)\big), \\
P_C(x,n) &= P(x \text{ is } 'C' \text{ and exactly } n \text{ nodes in } (0,x)), \\
P_{DC}(x,n) &= P(x \text{ is } 'DC' \text{ and exactly } n \text{ nodes in } (0,x)).
\end{aligned}$$

In order to find $P_C(x|n)$, we first compute $P_{DC}(x,n)$ and then calculate $P_C(x|n)$ from

$$\begin{aligned}
P_C(x|n) &= \frac{P_C(x,n)}{P_{nodes}(x,n)} = \frac{P_{nodes}(x,n) - P_{DC}(x,n)}{P_{nodes}(x,n)} \\
&= 1 - \frac{P_{DC}(x,n)}{P_{nodes}(x,n)}.
\end{aligned} \tag{18}$$

The following lemma enables us to compute $P_{DC}(x,n)$:

Lemma 4. $P_{DC}(x,n)$ *satisfies the following equations:*

$$\begin{aligned}
&P_{DC}(x,n) = 0, \quad x < r,\ n \ge 0, \\
&P_{DC}(x,0) = e^{-F(x)}u(x-r), \quad \forall x, \\
&\frac{dP_{DC}(x,n)}{dx} = -P_{DC}(x,n)f(x) \\
&\qquad + P_{DC}(x,n-1)f(x) \\
&\qquad + P_{nodes}(x-r,n-1)g(x-r) \\
&\qquad - P_{DC}(x-r,n-1)g(x-r), \\
&\qquad\qquad \forall x \ge r,\ n \ge 1.
\end{aligned} \tag{19}$$

Proof. The first equation is clear from the definition of transmission range. For the second equation note that $e^{-F(x)}$ is equal to the probability of emptiness of $(0,x)$. The proof of the third equality is similar to the proof of Lemma 2.

$$\begin{aligned}
P_{DC}(x,n) &\triangleq P\big(x \text{ is } 'DC', n \text{ nodes in } (0,x)\big) \\
&= P\big(x \text{ is } 'DC', x-dx \text{ is } 'DC', n \text{ nodes in } (0,x)\big) \\
&\quad + P\big(x \text{ is } 'DC', x-dx \text{ is } 'C', n \text{ nodes in } (0,x)\big) \\
&= P\big(x-dx \text{ is } 'DC', n \text{ nodes in } (0,x-dx), \\
&\quad [x-dx,x) \text{ is } 'E'\big) + P\big(x-dx \text{ is } 'DC', \\
&\quad n-1 \text{ nodes in } (0,x-dx), [x-dx,x) \text{ is } 'O'\big) \\
&\quad + P\big(x-r \text{ is } 'C', n-1 \text{ nodes in } (0,x-r), \\
&\quad [x-r,x-r+dx) \text{ is } 'O', [x-r+dx,x) \text{ is } 'E'\big) \\
&= P_{DC}(x-dx,n)(1-f(x-dx)dx) \\
&\quad + P_{DC}(x-dx,n-1)f(x-dx)dx \\
&\quad + P_C(x-r,n-1)g(x-r)dx \\
\Rightarrow \frac{dP_{DC}(x,n)}{dx} &= \frac{P_{DC}(x,n) - P_{DC}(x-dx,n)}{dx} \\
&= [P_{DC}(x-dx,n-1) - P_{DC}(x-dx,n)]f(x-dx) \\
&\quad + P_C(x-r,n-1)g(x-r).
\end{aligned}$$

Similar to Section 3, we assume that the spatial density function, f, doesn't have any singularity in $(0,L)$. Hence, according to the last equality, $\frac{dP_{DC}(x,n)}{dx}$ has no singularity (note that $P_C(0,n) \le P_{nodes}(0,n) = P(n \text{ nodes in } (0,0)) = 0,\ \forall n$). Hence, $P_{DC}(x,n)$ is a continuous function for $n \ge 1$, and $x-dx$ can be replaced with x in the right hand of the last equation to obtain (19). □

We start from $n=0$ and find $P_{DC}(x,n)$ for $n = 1,2,\ldots,$ using the recursive equation (with respect to n) in (19). Having $P_{DC}(x,n-1)$, the third equation in (19) is a first order Ordinary Differential Equation (ODE) with respect to x, and can be solved numerically with the aid of numerous algorithms for solving ODEs [25].

Given a pdf f_p (i.e., direct distribution), the probability of connectivity can be computed numerically, using the following approach:

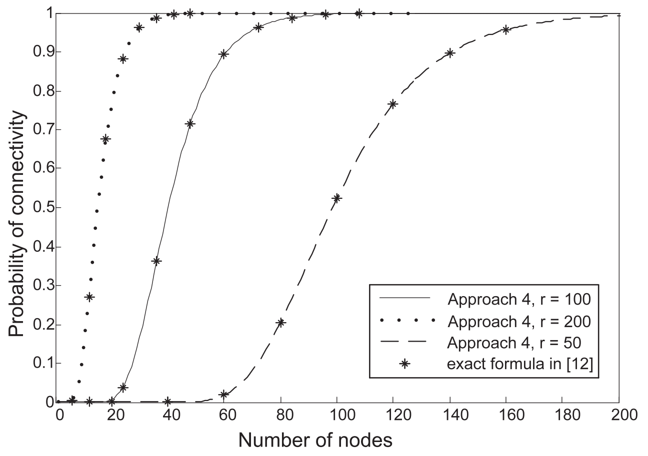

Fig. 1. Probability of connectivity versus the number of nodes for different transmission ranges, r; where n nodes are uniformly distributed in a 1D network.

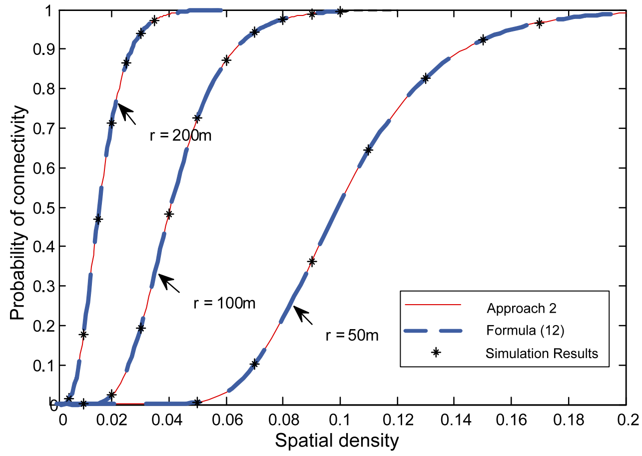

Fig. 2. Probability of connectivity of 1D networks with uniform spatial density function, where the number of nodes is variable.

Approach 4.

1. Form a spatial density function f by scaling f_p with an arbitrary scaling factor (e.g., let $f(x) = f_p(x)$, $\forall x > 0$) and compute $P_{nodes}(x, n)$ using (17).
2. Use (19) to compute $P_{DC}(x, k)$ for $k = 0, \dots, n$, $x \in [0, L]$.
3. Calculate the probability of network connectivity, $P_C(L|n)$, using (18).

In this approach, $O(n)$ ODEs should be solved, which may sound very costly. However, note that $P_{DC}(x, k)$ is also being computed for $0 < x < L$ and $0 \leq k \leq n$, when Approach 4 aims to find $P_C(L|n)$; from which $P_C(x|k)$, $0 < x < L, 0 \leq k \leq n$ can be calculated directly, using (18). In other words, when there are n nodes in the network, this approach finds the probability of connectivity not only for the destination, but also for every position in the path and every number of nodes in the network less than n, when run once. To the best of our knowledge, among the papers focused on connectivity, the approach proposed in [24] to compute an approximate probability of connectivity with an arbitrary distribution is of relatively high accuracy. It is worth mentioning that the approach presented in [24] needs to compute the same order ($O(n)$) of definite integrations. In the next section we will show that our proposed approach is much more accurate than the approach proposed in [24], especially for small number of nodes corresponding to sparse situation.

6 Numerical Results

In this section, we present numerical results for probability of connectivity obtained from our approaches and compare them with simulation and existing methods. We also obtain exact values for probability of connectivity for some distributions, that could not be handled by the existing methods in the literature.

Example 1. Uniform distribution of n nodes.

In this example, we compute the probability of connectivity for networks where n nodes are distributed independently and uniformly, using Approach 4 and compare the obtained probability with the probability of connectivity computed from the exact formula of [12]. These probabilities are plotted in Fig. 1 versus the number of nodes, for different transmission ranges. Here, we consider a path of length 1 km and transmission ranges equal to 50, 100, and 200 meters. It can be seen that the results obtained from our approach are in perfect match with the exact formula of [12], validating the accuracy of the proposed approach (note that the formula of [12] works only for uniform distribution). For each transmission range the formula in [12] is applied for some points in the rising regions of the corresponding curves, because these regions have more information content. Hence, the "*" marks in Fig. 1 are drawn in different number of nodes for different curves.

Example 2. Uniform spatial density in the case of variable number of nodes.

Here, we are going to validate (12) by comparing it with the probability of connectivity computed from Approach 2 and simulation results. The resulted probabilities are plotted versus density of nodes in Fig. 2. Here, we consider a 1 km-long path and compute the probability of connectivity for different transmission ranges. In Approach 2, we divide the path into subregions of length 1 cm which is a very good approximation of being continuous. The accuracy of Approach 2 can be even improved by shortening the subregions. The probabilities obtained from simulation are averages over connectivity of 10,000 randomly distributed sets of nodes.

Example 3. Different distributions of n nodes.

In this example, we obtain the probability of connectivity in the case that n nodes are distributed in the network according to three different pdfs, namely Random Waypoint [18], Triangle, and uniform distributions.

Random Waypoint was introduced in [18] as a generic mobility model. The pdf corresponding to the distance of each node from the source in steady-state was first derived in [26]

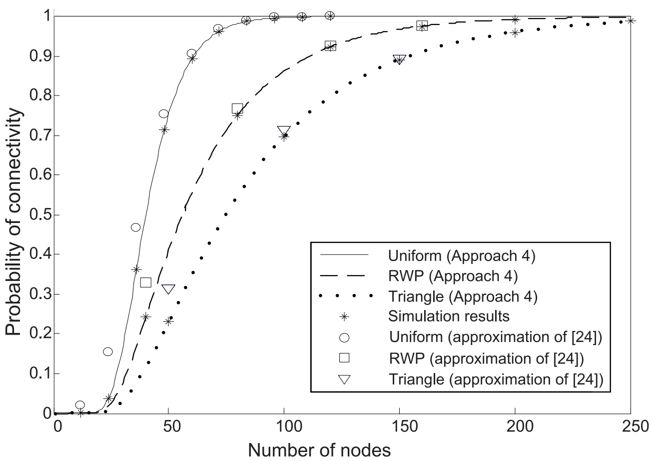

Fig. 3. Probability of connectivity of 1D networks with different distributions of fixed number of nodes, computed from Approach 4, the approximate analysis in [24], and simulation.

$$f(x) = \frac{6}{L^2}x - \frac{6}{L^3}x^2, \quad 0 \leq x \leq L.$$

The pdf for the distance of each node from the source in the Triangle distribution is

$$f(x) = \begin{cases} \frac{4x}{L^2}, & 0 \leq x \leq \frac{L}{2}, \\ \frac{4(L-x)}{L^2}, & \frac{L}{2} < x \leq L. \end{cases}$$

We assume a path of length 1 km and transmission range equal to 100 meters, use Approach 4 to find the exact probability of connectivity for the distributions and compare the results with the approximate probability computed from [24, (5)]. The results have been illustrated versus the number of nodes in Fig. 3. The simulation results shown in this figure are averages over connectivity of 5,000 randomly distributed sets of nodes. It can be seen from the figure that although [24] gives a good approximation for probabilities close to one, it has large relative errors where the probability of connectivity is smaller (e.g., in the case of uniform distribution, where the exact probabilities of connectivity are 0.5 and 0.2, the relative errors of the approximation of the approach in [24] are 15 and 62 percent, respectively). Sparse ad hoc networks [2] and delay tolerant networks (DTNs) are examples when the connectedness occurs with low probability. In these cases it is important to have an exact evaluation of this probability.

TABLE 1
Simulation and Analytical (Approach 2) Results for Connectivity of the Network in Fig. 4

Transmission range	P_C from Approach 2	P_C from Simulation	Relative difference
60m	0.2254	0.2254	.03%
100m	0.8656	0.8656	.01%

Example 4. Applying our approach to a real-world scenario.

In this example we aim to find the probability of network connectivity in a real-world scenario. Imagine a 1D VANET with network topology shown in Fig. 4a, in which the drivers are supposed to reduce speed near junctions, shopping centers, gas stations, and pedestrian crossings. Hence, we expect a higher density of vehicles in these parts. Let Fig. 4b present the density function related to the steady state of this network. Here, we just generated this distribution; however, in real-world applications, this density function should be computed either through analytical approaches or by simulation tools after considering a certain mobility model which governs the movement of vehicles [27].

Here, the number of nodes is variable and Approach 2 can be used to compute the probability of connectivity. We use a density function whose values numerically match with

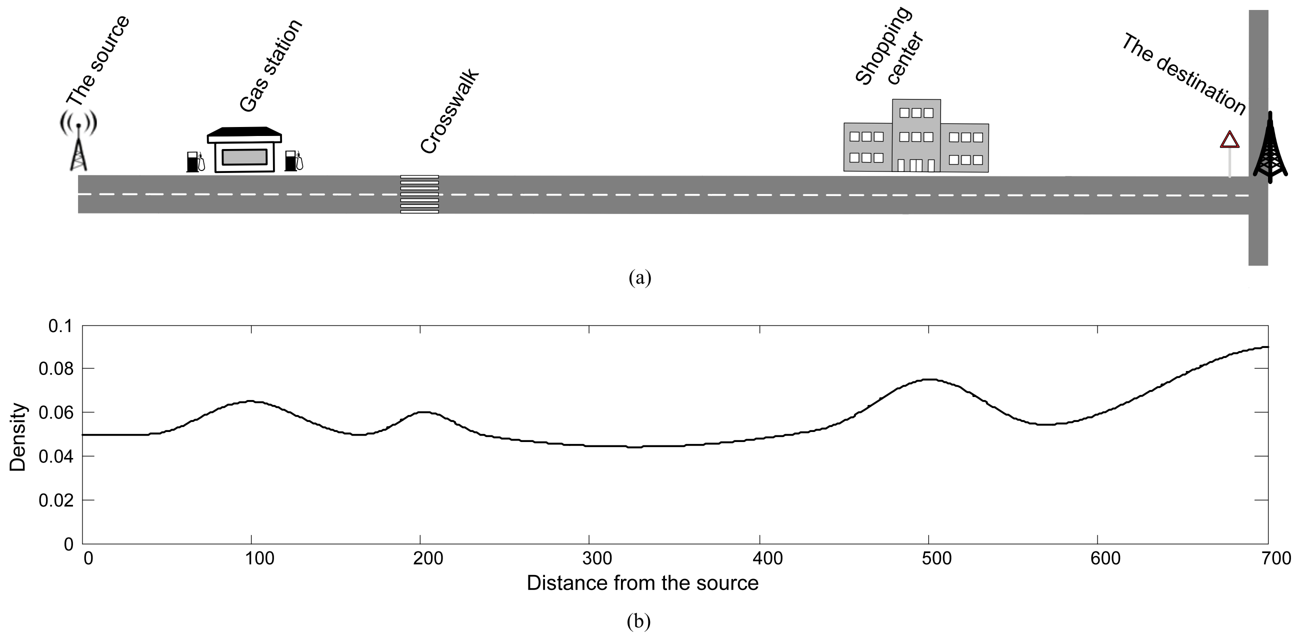

Fig. 4. A sample road topology and corresponding spatial density function in a typical VANET. (a) Road topology, (b) spatial density function.

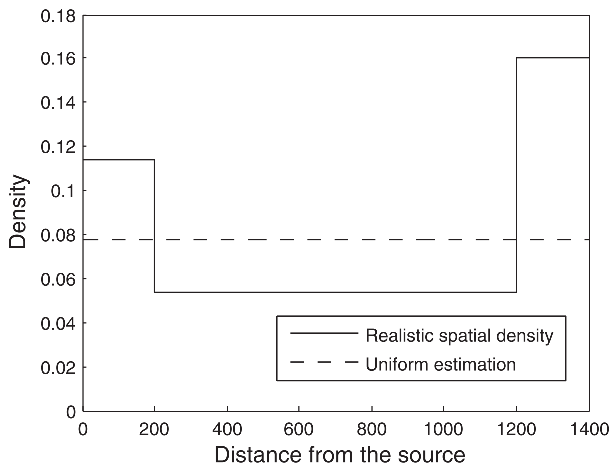

Fig. 5. Density function of simplified version of the network presented in [23] and its uniform estimation for arrival rate of vehicles to the street being 0.8.

the curve in Fig. 4b. The probability of network connectivity calculated by Approach 2, as well as simulation results for two different transmission ranges of nodes, have been shown in Table 1. The simulation results are acquired by averaging over connectivity of 1e6 random distribution of nodes according to density function of Fig. 4b.

Example 5. Probability of connectivity and network protocol design.

Probability of connectivity computed by the approaches presented in this work can be utilized for network protocol design purposes. Several routing algorithms proposed in VANETs have been founded on the connectivity status of different streets [5], [6], [7]. Adaptive Connectivity Aware Routing (ACAR) protocol proposed in [6] is an example where routing decisions are made based on probability of connectivity at different routes. In [6] it is assumed that vehicles are provided with information about the traffic statistics of different parts of the network such as traffic density and average velocity of vehicles, via GPS and navigation systems. Having the density of vehicles, a uniform distribution is assumed at each road segment (i.e., portion of a street between two adjacent junctions) with a corresponding density. The ACAR protocol selects an optimal route with the best transmission quality, which is defined as the multiplication of data delivery ratio (i.e., $1 -$ packet error rate (PER)) and the probability of connectivity of the route. However, the assumption of uniform distribution is debatable and more realistic distributions can be considered at the road segments to reflect the impact of many factors such as traffic lights on the distribution of vehicles, as stated in [6].

TABLE 2
Connectivity of the Street with Spatial Density Shown in Fig. 5

Arriaval rate	P_C of realistic spatial density	P_C of uniform estimation	Relative difference
0.64	0.5626	0.8620	53%
0.7	0.6659	0.9163	38%
0.8	0.7975	0.9652	21%

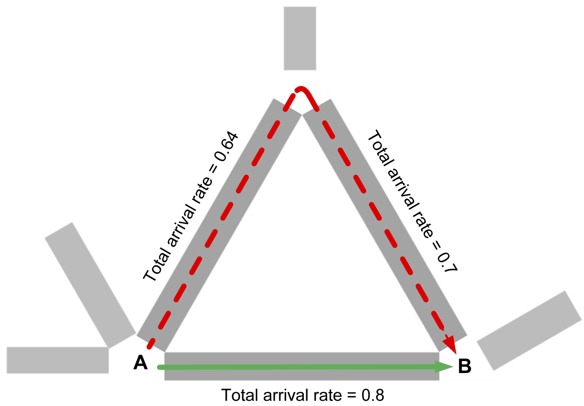

Fig. 6. A sample network topology, where ACAR route selection differs when the probability of connectivity is computed based on realistic spatial density and uniform estimation.

In this example, we are going to verify the adverse effect of simplifying assumptions, such as uniform distribution, on probability of connectivity and performance of ACAR. We employ a simplified version of the network model presented in [23] (as our realistic spatial density). We show the ACAR peformance for two cases. In the first case, we utilize the probability of connectivity regarding realistic spatial density, and in the second case, a uniform estimation at each street (that is a street having the same total density but with uniform distribution (see Fig. 5)) is utilized in computing the probability of connectivity. The streets of the network studied in [23] consist of three parts, namely front part, middle part, and end part, with lengths of 200, 1,000, and 200 m, respectively. It is assumed that the vehicles take the average speed of 7, 15, and 5 m/s in these parts, respectively. The spatial density of vehicles is then obtained from the arrival rates of the vehicles to the street. Fig. 5 shows the density function of a street in this network with arrival rate of 0.8. The vehicles arrival rate at each street is determined based on the departure rates from other streets as well as vehicle mobility patterns at intersections. The probabilities of connectivity of the realistic spatial density and the uniform estimation at three streets are computed for three typical total arrival rates (i.e., at both directions of the street) with transmission range of 100 m, whose results have been shown in Table 2. It can be observed that using the uniform estimation instead of realistic spatial density may lead to large errors in probability of connectivity.

Now, consider the network topology shown in Fig. 6, where node **A** has a packet to send for node **B** via two available routes (solid line (green) and dashed line (red) routes). If the effect of PER on transmission quality of the street segments is neglected (i.e., the data delivery ratio equals one), the transmission quality of the two routes would be equal to the multiplication of the probabilities of connectivity of the road segments at each route. Table 3

TABLE 3
Transmission Qualities of Different Routes in Fig. 6

	Green route	Red route
Accurate spatial density	0.5626	0.5311
Uniform estimation	0.8620	0.8843

shows the transmission qualities of the solid line and the dashed line routes when the probabilities of connectivity are computed based on the realistic spatial density and the uniform estimation (given in Table 2). It is observed that the solid line route would be selected by ACAR if the probability of connectivity is computed based on the realistic spatial density, while the dashed line route is selected when the uniform estimation is utilized.

It is worth mentioning that for estimating the probability of connectivity at each street in the above example, we need an approach to be able to work based on vehicle density, because the information on the number of vehicles at each street is not easily available at each time instant.

7 Conclusions

Connectivity is one of the important aspects of ad hoc networks. In this work, we treated the concept of connectivity in a new way and considered connectivity of positions whose contribution led us to simple expressions and tractable algorithms for calculating the exact probability of connectivity of nodes and connectivity of the path. We considered two cases and conducted separate analyses for them. In the first case where the number of nodes was variable, we proposed an analytical approach to compute the exact probability of connectivity of discrete and continuous networks for any spatial density function, provided that nodes are independently and identically distributed. For continuous networks, we derived an integral formula for the probability of connectivity using which we could obtain closed-form formulas for some special density functions. As an example, we derived exact expression for the probability of connectivity corresponding to uniform spatial density. In the second case in which the number of nodes was assumed to be fixed, we proposed an analytical approach to compute the probability of connectivity in the networks where we were given an arbitrary probability distribution function, according to which n nodes are placed in the network independently. Finally, we compared the results of analysis to some previously reported methods for the second case and derived the probability of connectivity for some spatial densities. We also validated our approach by simulation. Considering more realistic transmission ranges due to fading and other effects, and nodes with different transmission ranges are of our future works.

Acknowledgments

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References

[1] A. Javanmard and F. Ashtiani, "Analytical Evaluation of Average Delay and Maximum Stable Throughput along a Typical Two-Way Street for Vehicular Ad Hoc Networks in Sparse Situations," *Elsevier Trans. Computer Comm.*, vol. 32, pp. 1768-1780, 2009.

[2] P. Santi and D.M. Blough, "The Critical Transmitting Range for Connectivity in Sparse Wireless Ad Hoc Networks," *IEEE Trans. Mobile Computing*, vol. 2, no. 1, pp. 25-39, Jan.-Mar. 2003.

[3] C. Bettstetter, "On the Minimum Node Degree and Connectivity of a Wireless Multihop Network," *Proc. Third ACM Int'l Symp. Mobile Ad Hoc Networking & Computing*, pp. 80-91, 2002.

[4] P. Gupta and P.R. Kumar, "Critical Power for Asymptotic Connectivity in Wireless Networks," *Stochastic Analysis, Control, Optimization and Applications: A Volume in Honor of W. H. Fleming*, Springer, 1998.

[5] J. Zhao and G. Cao, "VADD: Vehicle-Assisted Data Delivery in Vehicular Ad Hoc Networks," *IEEE Trans. Vehicular Technology*, vol. 57, no. 3, pp. 1910-1922, May 2008.

[6] Q. Yang, A. Lim, S. Li, J. Fang, and P. Agrawal, "ACAR: Adaptive Connectivity Aware Routing Protocol for Vehicular Ad Hoc Networks," *Proc. 17th Int'l Conf. Computer Comm. and Networks (ICCCN '08)*, pp. 1-6, 2008.

[7] M. Ghaffari and F. Ashtiani, "A New Routing Algorithm for Sparse Vehicular Ad-Hoc Networks with Moving Destinations," *Proc. IEEE Conf. Wireless Comm. & Networking (WCNC)*, 2009.

[8] T.K. Philips, S.S. Pandwar, and A.N. Tantawi, "Connectivity Properties of a Packet Radio Network Model," *IEEE Trans. Information Theory*, vol. 35, no. 5, pp. 1044-1047, Sept. 1989.

[9] Y.C. Cheng and T.G. Robertazzi, "Critical Connectivity Phenomena in Multihop Radio Models," *IEEE Trans. Comm.*, vol. 37, no. 7, pp. 770-777, July 1989.

[10] M. Desai and D. Manjunath, "On the Connectivity in Finite Ad Hoc Networks," *IEEE Comm. Letters*, vol. 6, no. 10, pp. 437-439, Oct. 2002.

[11] A.D. Gore, "Comments on 'On the Connectivity in Finite Ad Hoc Networks'," *IEEE Comm. Letters*, vol. 10, no. 2, pp. 88-90, Feb. 2006.

[12] A. Ghasemi and S. Nader-Esfahani, "Exact Probability of Connectivity in One-Dimensional Ad Hoc Wireless Networks," *IEEE Comm. Letters*, vol. 10, no. 4, pp. 251-253, Apr. 2006.

[13] A. Misra, G. Teltia, and A. Chaturvedi, "On the Connectivity of Circularly Distributed Nodes in Ad Hoc Wireless Networks," *IEEE Comm. Letters*, vol. 12, no. 10, pp. 717-719, Oct. 2008.

[14] C.H. Foh and B.S. Lee, "A Closed Form Network Connectivity Formula for One-Dimensional Manets," *Proc. IEEE Int'l Conf. Comm. (ICC '04)*, vol. 6, pp. 3739-3742, June 2004.

[15] D. Goeckel, B. Liu, D. Towsley, L. Wang, and C. Westphal, "Asymptotic Connectivity Properties of Cooperative Wireless Ad Hoc Networks," *IEEE J. Selected Areas in Comm.*, vol. 27, no. 7, pp. 3739-3742, Sept. 2009.

[16] P.J. Wan and C.W. Yi, "Asymptotic Critical Transmission Radius and Critical Neighbor Number for K-Connectivity in Wireless Ad Hoc Networks," *Proc. ACM MobiHoc*, pp. 1-8, 2004.

[17] P.J. Wan, C.W. Yi, and L. Wang, "Asymptotic Critical Transmission Radius for K-Connectivity in Wireless Ad Hoc Networks," *IEEE Trans. Information Theory*, vol. 56, no. 6, pp. 2867-2874, June 2010.

[18] D.B. Johnson and D.A. Maltz, "Dynamic Source Routing in Ad Hoc Wireless Networks," *Mobile Computing*, T. Imielinski and H. Korth, eds., pp. 153-181, Springer, 1996.

[19] Z.J. Haas and B. Liang, "Predictive Distance Based Mobility Management for PCS Networks," *Proc. IEEE INFOCOM*, 1999.

[20] X. Hong, M. Gerla, G. Pei, and C.C. Chiang, "Group Mobility Model for Ad Hoc Wireless Networks," *Proc. ACM/IEEE Second ACM Int'l Workshop Modeling, Analysis and Simulation of Wireless and Mobile Systems (MSWiM '99)*, pp. 53-60, 1999.

[21] A. Sharif-Nassab and F. Ashtiani, "Micro-Scale Mobility Modeling and Analysis of Spatial Distribution in Vanets, A Queueing Theoretical Approach," *unpublished manuscript.*

[22] M. Khabazian and M.K.M. Ali, "A Performance Modeling of Connectivity in Vehicular Ad-Hoc Networks," *IEEE Trans. Vehicular Technology*, vol. 57, no. 4, pp. 2440-2450, July 2008.

[23] G.H. Mohimani, F. Ashtiani, A. Javanmard, and M. Hamdi, "Mobility Modeling, Spatial Traffic Distribution, and Probability of Connectivity for Sparse and Dense Vehicular Ad Hoc Networks," *IEEE Trans. Vehicular Technology*, vol. 58, no. 4, pp. 1998-2007, May 2009.

[24] C. Foh, G. Liu, B. Lee, B. Seet, K. Wong, and C. Fu, "Network Connectivity of One-Dimensional Manets with Random Waypoint Movement," *IEEE Comm. Letters*, vol. 9, no. 1, pp. 31-33, Jan. 2005.

[25] J.C. Butcher, *Numerical Methods for Ordinary Differential Equations.* John Wiley and Sons, 2003.

[26] C. Bettstetter, G. Resta, and P. Santi, "The Node Distribution of the Random Waypoint Mobility Model for Wireless Ad Hoc Networks," *IEEE Trans. Mobile Computing*, vol. 2, no. 3, pp. 257-269, July-Sept. 2003.

[27] J. Harri, F. Filali, and C. Bonet, "Mobility Models for Vehicular Ad Hoc Networks: A Survey and Taxonomy," *IEEE Comm. Surveys & Tutorials*, vol. 11, no. 4, pp. 19-41, Oct.-Dec. 2009.

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