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Cooperative relay broadcast channels with partial causal channel state information

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Abstract: The authors consider 'partially' and 'fully cooperative' state-dependent relay broadcast channels (RBCs), where partial channel state information (CSI) is available at the nodes causally. First, the authors derive an achievable rate region for general discrete memoryless partially cooperative RBC (PC-RBC) with partial causal CSI, by exploiting superposition coding at the source, decode-and-forward scheme at the relay and Shannon's strategy at the source and the relay. Then, they establish the capacity region of the discrete memoryless physically degraded PC-RBC with partial causal CSI. They also characterise the capacity region of discrete memoryless PC-RBC with feedback and partial causal CSI, and show that feedback does not affect the capacity region of the physically degraded channel. Moreover, for the fully cooperative RBC (FC-RBC) with partial causal CSI the authors obtain the same results as for the state-dependent PC-RBC. The authors' results subsume the previously known results for the degraded broadcast and relay channels with causal CSI. Finally, they extend their achievable rate regions to the Gaussian cases. Providing some numerical examples for the Gaussian cases, they compare the achievable rate regions derived for different situations.

1 Introduction

Cooperative relaying is a powerful technique which improves throughput and reliability of networks that include broadcast transmissions such as wireless networks [1]. In order to exploit relaying and user cooperation to achieve higher throughput in 'downlink' communication channels, relay broadcast channel (RBC) has been introduced in [2, 3], wherein partially cooperative RBC (PC-RBC) and fully cooperative RBC (FC-RBC) models have been investigated. The three-terminal RBC is a communication channel which consists of a source node and two destination nodes (e.g. user 1 and user 2). The source node transmits a common message to both destinations and also a private message to each of them. Destination nodes cooperate by relaying information to each other. Hence, the building blocks of the RBC are the relay channel (RC) [4] and the broadcast channel (BC) [5]. In the PC-RBC, only one of the two destinations (e.g. user 1) acts as a relay. So, this node is called the relay node or user 1. The FC-RBC is a more general model in which both destinations can relay information to each other.

On the other hand, owing to wide range of applications, channels whose probability transition function depends on a random state (state-dependent channels) have attracted considerable attention, recently [6]. In these systems, channel state information (CSI) can be available at the nodes causally or non-causally. For causal CSI at the transmitter (CSIT), at each time instant the transmitter knows only the past and the present channel-state sequence, whereas in the non-causal case the transmitter knows in advance the realisation of the entire state sequence from the beginning to the end of the block (for a review on channels with 'non-casual' CSI, see [6, 7] and the references therein). Note that, for the destination equipped with CSI, there is no difference between causal and non-causal cases, because it waits until the end of the block before decoding.

For the first time, Shannon characterised the capacity of a state-dependent single user channel where CSI is causally known to the transmitter [8]. Shannon showed that optimal codes for this channel are constructed over an extended input alphabet, which is the set of all functions from the state alphabet to the channel input alphabet, called 'Shannon's strategies'.

The Gaussian channel with additive interference causally known to the transmitter is called dirty tape channel (DTC) [9]. Although, a closed-form formula for the capacity of the DTC is still unknown [6], some techniques have been presented in [10, 11] to obtain achievable rates for this channel, which are extended to multi-user models in [12].

State-dependent multi-user channels have also been studied in several papers [7, 13–20]. In these systems, each node may have access to 'perfect' CSI (full knowledge of the channel state), or 'partial' CSI. Steinberg in [13] has derived the capacity region of a degraded BC when perfect CSI is causally known to the source. In [14], the capacity of a physically degraded RC where perfect CSI is causally available at the source and relay, has been established. Some classes of RC with partial causal CSI have been investigated in [19].

In this study, we consider state-dependent PC-RBC and FC-RBC where partial CSI is causally known to the nodes. In the sequel, we refer to these channels as PC-RBC (FC-RBC) with partial causal CSI, or as state-dependent PC-RBC (FC-RBC) in brief. First, we derive an achievable rate region for general discrete memoryless PC-RBC with partial causal CSI. Our achievability proof is based on using Shannon's strategy [8] at the source and the relay which are informed of CSI, superposition coding [5] and regular encoding/sliding-window decoding [1]. Moreover, we consider PC-RBC with feedback and partial casual CSI. In the state-dependent PC-RBC with feedback, we assume that the output at user 2 is provided to user 1 and the outputs at both users are provided to the source (as in [2]). Also, in our model for the state-dependent PC-RBC with feedback, we assume that the available CSI at user 2 is fed back to user 1, and the available CSI at both users are provided to the source, all through perfect feedback links. For the general discrete memoryless state-dependent PC-RBC with feedback as defined above, we establish the capacity region. This derived capacity region for the state-dependent PC-RBC with feedback is an outer bound on the capacity region of the general state-dependent PC-RBC, and coincides with our derived achievable rate region for the state-dependent PC-RBC in the physically degraded case. Hence, the capacity region of the discrete memoryless physically degraded PC-RBC with partial causal CSI is also established, and it is shown that feedback does not affect the capacity region of it.

Thereafter, we consider state-dependent FC-RBC with and without feedback. All results for the state-dependent PC-RBC with and without feedback are correspondingly generalised to the state-dependent FC-RBC with and without feedback. In particular, we derive an achievable rate region for general discrete memoryless FC-RBC with partial causal CSI, and establish the capacity region of the physically degraded channel. The capacity region of the discrete memoryless FC-RBC with feedback and partial causal CSI is also established, and it is shown that feedback does not increase the capacity region in the physically degraded case.

We also show that our results for PC-RBC and FC-RBC with partial causal CSI, include different situations in which perfect causal CSI is available at some or all of the nodes in these channels, as special cases. Our results also can readily be applied to the RC with causal CSI, as a special case of PC-RBC, when only a private message is sent to user 2. They can also be applied to the BC with causal CSI when user 1 only decodes its message but does not relay information to user 2.

We then extend our achievable rate regions to the Gaussian PC-RBC and FC-RBC with additive Gaussian interference modelled as channel state. Our achievable rate regions are based on the compensation strategy which has been proposed in [10] to obtain an achievable rate for the DTC. In the compensation scheme, the transmitter expends part of its power to clean the known interference from the channel, and uses the remaining power for information transmission. Providing some numerical examples for the Gaussian cases, we compare the achievable rate regions derived for different situations.

The rest of the study is organised as follows. In Section 2, channel models and definitions are presented. In Section 3, we investigate the discrete memoryless state-dependent PC-RBC. The discrete memoryless state-dependent FC-RBC is addressed in Section 4. The Gaussian cases are investigated in Section 5. The study is concluded in Section 6.

2 Definitions and channel models

Throughout this study, the following notations are used: upper case letters (e.g. X) are used to denote random variables (RVs), and their realisations are shown by lower case letters (e.g. x). X_i^j indicates a sequence of RVs $(X_i, X_{i+1}, \ldots, X_j)$. For brevity, X^j is used instead of X_1^j. $p_X(x)$ denotes the probability mass function (p.m.f.) of the RV X on a set $\mathcal{X}$, where subscript X is occasionally omitted. $A_\epsilon^n(X, Y)$ denotes the set of all jointly ϵ-typical sequences of length n based on $p(x, y)$ [21], which is abbreviated by A_ϵ^n in the sequel. $\mathcal{N}(0, \sigma^2)$ denotes the Gaussian distribution with zero mean and variance σ^2.

2.1 State-dependent PC-RBC

Definition 1: A discrete memoryless state-dependent PC-RBC (see Fig. 1) is denoted by $(\mathcal{X}, \mathcal{X}_1, \mathcal{S}_c, p(s_c), p(y_1, y_2|x, x_1, s_c), \mathcal{Y}_1, \mathcal{Y}_2)$, where $p(y_1, y_2|x, x_1, s_c)$ is the probability transition function of the channel, $X \in \mathcal{X}$ and $X_1 \in \mathcal{X}_1$ are the source and user 1 (the relay) inputs, respectively, $Y_1 \in \mathcal{Y}_1$ and $Y_2 \in \mathcal{Y}_2$ are the channel outputs at user 1 (the relay) and user 2, respectively. All the alphabet sets are of finite size. It should be noted that in this channel, user 1 acts as a relay too, so we refer to it as 'user 1' or as 'the relay'. The source sends a common message W_0, which is decoded by both users, and private messages W_1 and W_2 which are decoded by user 1 and user 2, respectively. Therefore $(\hat{W}_0^{(1)}, \hat{W}_1)$ and $(\hat{W}_0^{(2)}, \hat{W}_2)$ are the estimated messages at user 1 and user 2, respectively. Similar to the model considered in [22] for the state-dependent single user channel, here we assume that the underlying channel is

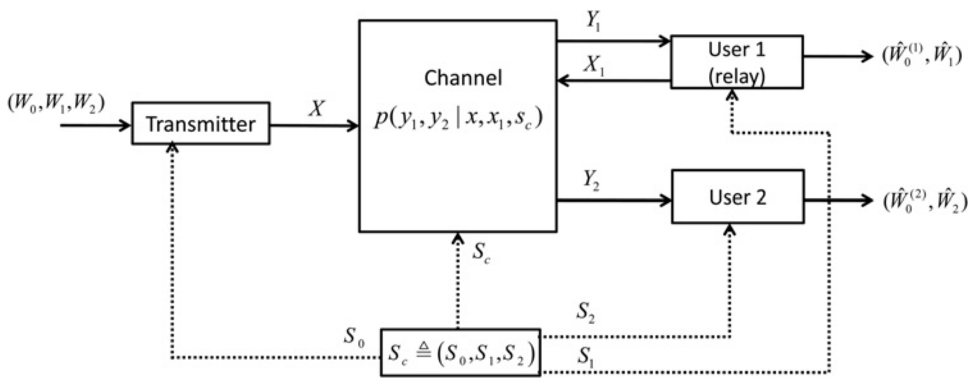

Fig. 1 *State-dependent PC-RBC*

Dotted lines show that the transmitter, user 1 and user 2 have access to S_0, S_1 and S_2, respectively, which are parts of the channel state S_c

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governed by a triple state, that is

$$S_c \triangleq (S_0, S_1, S_2)$$

where S_0 is available causally at the transmitter, and S_1 and S_2 are available causally at user 1 and user 2, respectively. By dotted lines in Fig. 1, we show that the transmitter, user 1 and user 2 have access to S_0, S_1 and S_2, respectively, which are parts of the channel state S_c. The CSI available at the source, user 1 and user 2 are perfect if $S_0 = S_1 = S_2 = S_c$.

The channel state process is assumed to be independent identically distributed (i.i.d.), and is drawn according to the known distribution $p(s_c)$ as follows

$$p(s_c^n) \triangleq p(s_0^n, s_1^n, s_2^n) = \prod_{t=1}^{n} p(s_{0,t}, s_{1,t}, s_{2,t}), \quad n \geq 1$$

Definition 2: A $((2^{nR_0}, 2^{nR_1}, 2^{nR_2}), n)$ code for a PC-RBC with partial causal CSI consists of the following:

- Three message sets $\mathcal{W}_0 = \{1, \ldots, 2^{nR_0}\}$, $\mathcal{W}_1 = \{1, \ldots, 2^{nR_1}\}$ and $\mathcal{W}_2 = \{1, \ldots, 2^{nR_2}\}$, and three independent messages W_0, W_1 and W_2 that are uniformly distributed over $\mathcal{W}_0$, $\mathcal{W}_1$ and $\mathcal{W}_2$, respectively.
- An encoder at the source

$$\phi_t: \mathcal{W}_0 \times \mathcal{W}_1 \times \mathcal{W}_2 \times \mathcal{S}_0^t \to \mathcal{X},$$
$$X_t = \phi_t(W_0, W_1, W_2, S_0^t) \text{ for } t = 1, \ldots, n$$

- A sequence of encoding functions at the relay (user 1)

$$\phi_{1,t}: \mathcal{Y}_1^{t-1} \times \mathcal{S}_1^t \to \mathcal{X}_1,$$
$$X_{1,t} = \phi_{1,t}(Y_1^{t-1}, S_1^t) \text{ for } t = 1, \ldots, n$$

- Two decoding functions, $\mathcal{D}_1$ and $\mathcal{D}_2$, where $\mathcal{D}_i$ at user i, $i = 1, 2$, is defined as

$$\mathcal{D}_i: \mathcal{Y}_i^n \times \mathcal{S}_i^n \to \mathcal{W}_0 \times \mathcal{W}_i, \text{ s.t. } (\hat{W}_0^{(i)}, \hat{W}_i) = \mathcal{D}_i(Y_i^n, S_i^n)$$

Thus, the joint p.m.f. of the PC-RBC with partial causal CSI is given by

$$\begin{aligned} &p(w_0, w_1, w_2, s_0^n, s_1^n, s_2^n, x^n, x_1^n, y_1^n, y_2^n) \\ &= p(w_0)p(w_1)p(w_2)\prod_{t=1}^{n}[p(s_{0,t}, s_{1,t}, s_{2,t})p(x_t|w_0, w_1, w_2, s_0^t) \\ &\quad \times p(x_{1,t}|y_1^{t-1}, s_1^t)p(y_{1,t}, y_{2,t}|x_t, x_{1,t}, s_{0,t}, s_{1,t}, s_{2,t})] \end{aligned} \quad (1)$$

The probability of error when the message triple (w_0, w_1, w_2) is sent, is defined as

$$\begin{aligned} P_e^{(n)}(w_0, w_1, w_2) = \Pr((&\hat{W}_0^{(1)}, \hat{W}_1) \neq (w_0, w_1) \text{ or} \\ &(\hat{W}_0^{(2)}, \hat{W}_2) \neq (w_0, w_2)|(w_0, w_1, w_2) \text{ is sent}) \end{aligned}$$

The average probability of error is given by

$$P_e^{(n)} = \frac{1}{2^{nR_0}2^{nR_1}2^{nR_2}} \sum_{w_0=1}^{2^{nR_0}} \sum_{w_1=1}^{2^{nR_1}} \sum_{w_2=1}^{2^{nR_2}} P_e^{(n)}(w_0, w_1, w_2) \quad (2)$$

Definition 3: The rate tuple (R_0, R_1, R_2) is said to be achievable for the PC-RBC with causal CSI, if there exists a sequence of $((2^{nR_0}, 2^{nR_1}, 2^{nR_2}), n)$ codes with average probability of error $P_e^{(n)} \to 0$ as $n \to \infty$. The capacity region is the closure of the set of all achievable rate tuples (R_0, R_1, R_2).

In the following, we define the physically degraded state-dependent PC-RBC. Note that the physically degraded BC [5] has been defined such that the better receiver can also decode the message intended for the weaker receiver in addition to its own message. Similarly, to define the physically degraded state-dependent PC-RBC we look for conditions under which the better receiver can also decode the message of the weaker receiver. Since the state-dependent PC-RBC is completely described by the conditional p.m.f. $p(y_1, y_2|x, x_1, s_0, s_1, s_2)$ and the p.m.f. of the channel state $p(s_0, s_1, s_2)$, the conditions should be determined by applying constraints on these p.m.s. Based on these issues, we define the physically degraded state-dependent PC-RBC as follows:

Definition 4: The discrete memoryless state-dependent PC-RBC with conditional p.m.f. $p(y_1, y_2|x, x_1, s_c)$, where $S_c \triangleq (S_0, S_1, S_2)$, is said to be physically degraded if the following conditions are satisfied simultaneously:

- $H(S_2|S_1) = 0$, or equivalently S_2 is a deterministic function of S_1. In fact, this constraint results in a situation where user 1 can perceive the CSI available at user 2.
- The conditional p.m.f. of the channel satisfies

$$p(y_1, y_2|x, x_1, s_0, s_1, s_2) = p(y_1|x, x_1, s_0, s_1, s_2)p(y_2|y_1, x_1, s_1) \quad (3)$$

2.2 State-dependent PC-RBC with feedback

We also consider the PC-RBC with feedback and partial causal CSI (see Fig. 2), where at each time instant t, the output sequence at user 2 up to the time instant $t-1$, that is, y_2^{t-1}, and its CSI, that is, s_2^{t-1}, are provided to user 1. Moreover, the outputs and also the available channel states at both users 1 and 2, that is, $(s_1^{t-1}, y_1^{t-1}, s_2^{t-1}, y_2^{t-1})$, are provided to the transmitter through perfect feedback links. In Fig. 2, dashed lines indicate the feedback from user 2 to user 1, and from both users to the transmitter.

So, the joint p.m.f. of the PC-RBC with feedback and partial causal CSI is given by

$$\begin{aligned} &p(w_0, w_1, w_2, s_0^n, s_1^n, s_2^n, x^n, x_1^n, y_1^n, y_2^n) \\ &= p(w_0)p(w_1)p(w_2)\prod_{t=1}^{n}[p(s_{0,t}, s_{1,t}, s_{2,t}) \\ &\quad \times p(x_t|w_0, w_1, w_2, s_0^t, s_1^{t-1}, y_1^{t-1}, s_2^{t-1}, y_2^{t-1}) \\ &\quad \times p(x_{1,t}|y_1^{t-1}, s_1^t, y_2^{t-1}, s_2^{t-1}) \\ &\quad \times p(y_{1,t}, y_{2,t}|x_t, x_{1,t}, s_{0,t}, s_{1,t}, s_{2,t})] \end{aligned} \quad (4)$$

The average probability of error for the state-dependent PC-RBC with feedback is defined the same as (2).

2.3 State-dependent FC-RBC

Definition 5: A discrete memoryless state-dependent FC-RBC (see Fig. 3) is denoted by $(\mathcal{X}, \mathcal{X}_1, \mathcal{X}_2, \mathcal{S}_c, p(s_c), p(y_1, y_2|x,$

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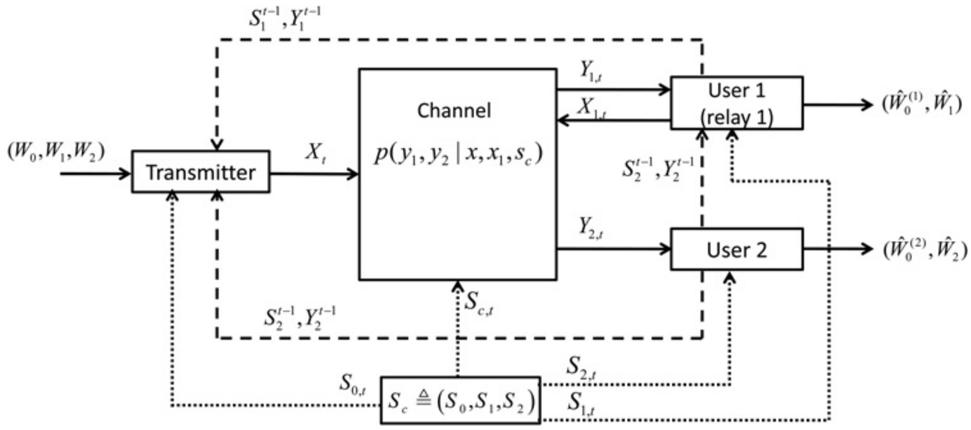

Fig. 2 *State-dependent PC-RBC with feedback*

Dotted lines show that the transmitter, user 1 and user 2 have access to S_0, S_1 and S_2, respectively, which are parts of the channel state S_c. Dashed lines indicate feedback

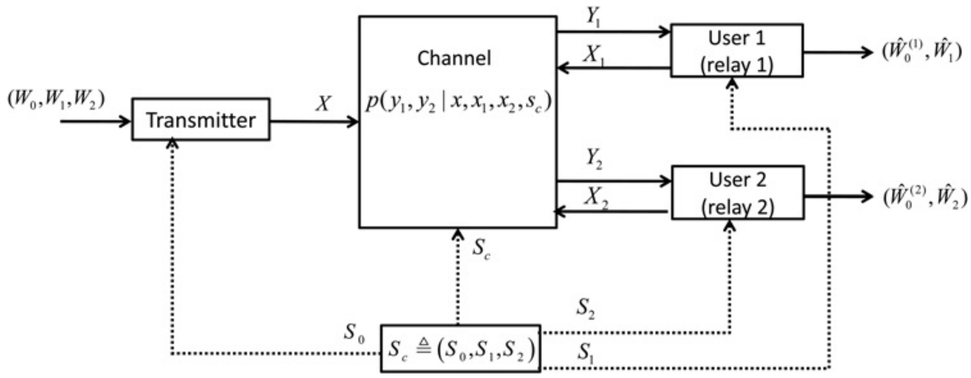

Fig. 3 *State-dependent FC-RBC*

Dotted lines show that the transmitter, user 1 and user 2 have access to S_0, S_1 and S_2, respectively, which are parts of the channel state S_c

$x_1, x_2, s_c), \mathcal{Y}_1, \mathcal{Y}_2)$, where $p(y_1, y_2|x, x_1, x_2, s_c)$ is the probability transition function of the channel, $X \in \mathcal{X}$ is the source input, $X_1 \in \mathcal{X}_1$ and $X_2 \in \mathcal{X}_2$ are the user 1 and user 2 inputs, respectively. $Y_1 \in \mathcal{Y}_1$ and $Y_2 \in \mathcal{Y}_2$ are the channel outputs at user 1 and user 2, respectively. All the alphabet sets are of finite size. In fact, the state-dependent FC-RBC has the same definition as the state-dependent PC-RBC except that in FC-RBC there are two relay sender alphabets $\mathcal{X}_1$, $\mathcal{X}_2$, each corresponds to the related user. Since user 2 acts as a relay too, we refer to it as 'user 2' or 'relay 2' (see Fig. 3). As in the state-dependent PC-RBC, we also assume that the state-dependent FC-RBC is governed by a triple state, that is, $S_c \triangleq (S_0, S_1, S_2)$, where S_0 is available at the transmitter, S_1 is available at user 1 and S_2 is available at user 2. The CSI available at the source, user 1 and user 2 are perfect if $S_0 = S_1 = S_2 = S_c$.

For the state-dependent FC-RBC, definition of a $((2^{nR_0}, 2^{nR_1}, 2^{nR_2}), n)$ code is the same as Definition 2 except that a sequence of encoding functions at relay 2 (user 2) is defined as well, where for causal CSI they are as follows

$$\phi_{2,t}: \mathcal{Y}_2^{t-1} \times \mathcal{S}_2^t \to \mathcal{X}_2, \quad X_{2,t} = \phi_{2,t}(Y_2^{t-1}, S_2^t) \quad \text{for } t = 1, \ldots, n$$

Therefore, the joint p.m.f. of the FC-RBC with partial causal CSI is as follows

$$\begin{aligned} p(w_0, w_1, w_2, s_0^n, s_1^n, s_2^n, x^n, x_1^n, x_2^n, y_1^n, y_2^n) \\ = p(w_0)p(w_1)p(w_2) \prod_{t=1}^{n} [p(s_{0,t}, s_{1,t}, s_{2,t})p(x_t|w_0, w_1, w_2, s_0^t) \\ \times p(x_{1,t}|y_1^{t-1}, s_1^t)p(x_{2,t}|y_2^{t-1}, s_2^t) \\ \times p(y_{1,t}, y_{2,t}|x_t, x_{1,t}, x_{2,t}, s_{0,t}, s_{1,t}, s_{2,t})] \end{aligned} \quad (5)$$

The average probability of error for the state-dependent FC-RBC is defined the same as (2). Now, similar to Definition 4 we define the physically degraded state-dependent FC-RBC as follows:

Definition 6: The discrete memoryless state-dependent FC-RBC with conditional p.m.f. $p(y_1, y_2|x, x_1, x_2, s_c)$ where $S_c \triangleq (S_0, S_1, S_2)$, is said to be physically degraded if the following conditions are satisfied simultaneously:

- $H(S_2|S_1) = 0$, or equivalently S_2 is a deterministic function of S_1. This constraint results in the situation where user 1 can perceive the CSI available at user 2.
- The conditional p.m.f. of the channel satisfies

$$\begin{aligned} p(y_1, y_2|x, x_1, x_2, s_0, s_1, s_2) = p(y_1|x, x_1, x_2, s_0, s_1, s_2) \\ \times p(y_2|y_1, x_1, x_2, s_1) \end{aligned} \quad (6)$$

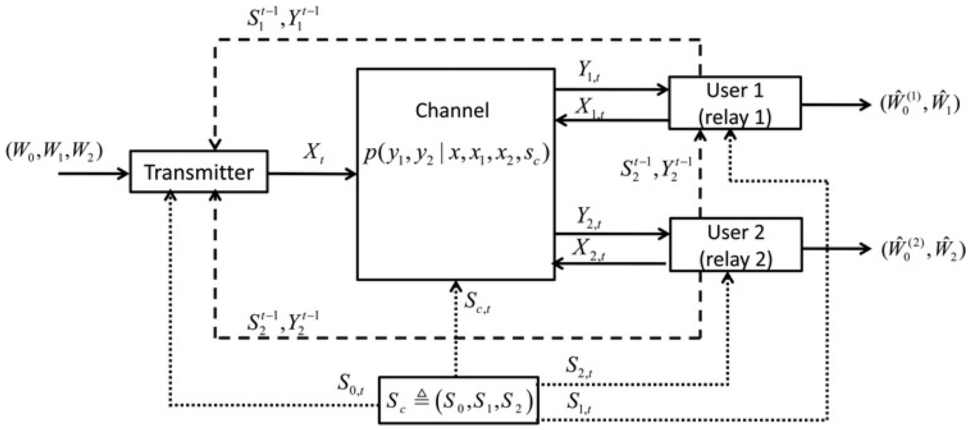

Fig. 4 *State-dependent FC-RBC with feedback*

Dotted lines show that the transmitter, user 1 and user 2 have access to S_0, S_1 and S_2, respectively, which are parts of the channel state S_c. Dashed lines indicate feedback

Note that in the above definition, the indices 1 and 2 can be exchanged.

2.4 State-dependent FC-RBC with feedback

We consider the FC-RBC with feedback and partial causal CSI (see Fig. 4), where at each time instant t, the output sequence at user 2 up to the time instant $t-1$, that is, y_2^{t-1}, and its CSI, that is, s_2^{t-1}, are provided to user 1. Moreover, the outputs and also the available channel states at both users 1 and 2, that is, $(s_1^{t-1}, y_1^{t-1}, s_2^{t-1}, y_2^{t-1})$, are provided to the transmitter through perfect feedback links.

So, the joint p.m.f. of the FC-RBC with feedback and partial causal CSI is given by

$$\begin{aligned} &p(w_0, w_1, w_2, s_0^n, s_1^n, s_2^n, x^n, x_1^n, x_2^n, y_1^n, y_2^n) \\ &= p(w_0)p(w_1)p(w_2)\prod_{t=1}^{n}[p(s_{0,t}, s_{1,t}, s_{2,t}) \\ &\quad \times p(x_t|w_0, w_1, w_2, s_0^t, s_1^{t-1}, y_1^{t-1}, s_2^{t-1}, y_2^{t-1}) \\ &\quad \times p(x_{1,t}|y_1^{t-1}, s_1^t, y_2^{t-1}, s_2^{t-1})p(x_{2,t}|y_2^{t-1}, s_2^t) \\ &\quad \times p(y_{1,t}, y_{2,t}|x_t, x_{1,t}, x_{2,t}, s_{0,t}, s_{1,t}, s_{2,t})] \end{aligned} \quad (7)$$

The average probability of error for state-dependent FC-RBC with feedback is defined the same as (2).

3 PC-RBC with partial causal CSI

In this section, we investigate the discrete memoryless PC-RBC with partial causal CSI. First, we establish an achievable rate region for this channel. Then, we derive the capacity region of the channel with feedback. Moreover, we establish the capacity region of the physically degraded channel, and show that feedback does not affect its capacity region.

Theorem 1: For the discrete memoryless PC-RBC with partial CSI, in which S_0, S_1 and S_2 are causally known to the transmitter, user 1 (relay 1) and user 2, respectively, the following rate region is achievable

$$\mathcal{R}_1 = \bigcup \left\{ \begin{array}{l} R_0, R_1, R_2 \geq 0 \\ R_1 \leq I(V; Y_1|U, V_1, S_1) \\ R_0 + R_2 \leq \min\{I(U, V_1; Y_2|S_2), I(U; Y_1|V_1, S_1)\} \end{array} \right\} \quad (8)$$

for all joint p.m.f. of the form

$$\begin{aligned} &p(s_0, s_1, s_2)p(u, v, v_1)p(y_1, y_2|x = f(u, v, s_0), \\ &\quad x_1 = f_1(v_1, s_1), s_0, s_1, s_2) \end{aligned} \quad (9)$$

where U, V, V_1 are auxiliary RVs, and $f(\cdot)$, $f_1(\cdot)$ are two arbitrary deterministic functions.

Proof: Our proof is based on a combination of Shannon's strategy [8], superposition coding [5], and regular encoding/sliding window decoding (similar to the coding scheme for PC-RBC in [2]). In this case, the random codebook is constructed using the auxiliary RVs V and V_1 which are used to apply Shannon's strategy at the source and the relay, respectively. Also, U denotes the auxiliary RV resulted from superposition coding. Since the superposition encoding is used to prove achievability, the message W_2 is also decoded at user 1. Therefore part of the rate R_2 can be considered as the common rate R_0 and it is enough to prove the achievability of the rate region in (8) for $R_0 = 0$. Now, assume a sequence of $B-1$ messages $(W_{1,b}, W_{2,b}) \in [1, 2^{nR_1}] \times [1, 2^{nR_2}]$ is transmitted in B blocks, each of length n. Note that as $B \to \infty$, the rate pair $(R_1, R_2) \times ((B-1)/B)$ approaches to (R_1, R_2).

Random coding: Fix a joint p.m.f. $p(u, v, v_1)$. Two statistically independent random codebooks 1, 2 are generated as follows:

(1) Generate 2^{nR_2} i.i.d. v_1^n sequences each with probability $p(v_1^n) = \prod_{t=1}^{n} p(v_{1,t})$. Label these $v_1^n(\overline{w}_2)$, where $\overline{w}_2 \in [1, 2^{nR_2}]$.
(2) For each $v_1^n(\overline{w}_2)$, generate 2^{nR_2} i.i.d. u^n sequences each with probability $p(u^n|v_1^n) = \prod_{t=1}^{n} p(u_t|v_{1,t})$. Index them as $u^n(\overline{w}_2, w_2)$, where $w_2 \in [1, 2^{nR_2}]$.
(3) For each $v_1^n(\overline{w}_2)$ and $u^n(\overline{w}_2, w_2)$, generate 2^{nR_1} i.i.d. v^n sequences each with probability $p(v^n|u^n, v_1^n) = \prod_{t=1}^{n} p(v_t|u_t, v_{1,t})$. Index them as $v^n(\overline{w}_2, w_2, w_1)$, where $w_1 \in [1, 2^{nR_1}]$.

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Encoding (at the beginning of block b): Messages are encoded using codebooks 1 and 2 for blocks with odd and even indices. Let $(w_{1,b}, w_{2,b})$ be the new message pair to be sent by the source in block b, and let $(w_{1,b-1}, w_{2,b-1})$ be the message pair which has been sent by the source in block $b-1$. Upon receiving $s_{0,t}(b)$, the source sends $x_t = f(u_t(w_{2,b-1}, w_{2,b}), v_t(w_{2,b-1}, w_{2,b}, w_{1,b}), s_{0,t}(b))$, $1 \le t \le n$. Moreover, we assume that at the end of block $b-1$, user 1 has correctly decoded the message $w_{2,b-1}$ sent from the source in block $b-1$. Therefore upon receiving $s_{1,t}(b)$, it sends $x_{1,t} = f_1(v_{1,t}(w_{2,b-1}), s_{1,t}(b))$, $1 \le t \le n$. Note that $s_{0,t}(b)$ and $s_{1,t}(b)$ are the values of the CSI at the transmitter and user 1, respectively, at time t in block b for $1 \le t \le n$.

Decoding (at the end of block b):

(1) User 1 (relay node), having known $w_{2,b-1}$ (from the previous block) and $s_1^n(b)$, finds a unique $\hat{w}_{2,b}$, such that

$$(v_1^n(w_{2,b-1}), u^n(w_{2,b-1}, \hat{w}_{2,b}), s_1^n(b), y_1^n(b)) \in A_\epsilon^n$$

For sufficiently large n, the decoding at this step is done with arbitrarily small probability of error, provided

$$R_2 \le I(U; Y_1, S_1 | V_1) \tag{10}$$

(2) Having known $w_{2,b-1}$ and $w_{2,b}$, user 1 then finds a unique $\hat{w}_{1,b}$ such that

$$(v^n(w_{2,b-1}, w_{2,b}, \hat{w}_{1,b}), v_1^n(w_{2,b-1}), u^n(w_{2,b-1}, w_{2,b}), s_1^n(b), y_1^n(b)) \in A_\epsilon^n$$

For sufficiently large n, this step can be done with arbitrarily small probability of error, provided

$$R_1 \le I(V; Y_1, S_1 | U, V_1) \tag{11}$$

(3) Finally, using sliding window decoding technique [1], user 2 declares that $\hat{\hat{w}}_{2,b-1}$ has been sent in block $b-1$, if there is a unique $\hat{\hat{w}}_{2,b-1}$ such that

$$(v_1^n(w_{2,b-2}), u^n(w_{2,b-2}, \hat{\hat{w}}_{2,b-1}), s_2^n(b-1), y_2^n(b-1)) \in A_\epsilon^n$$

and

$$(v_1^n(\hat{\hat{w}}_{2,b-1}), s_2^n(b), y_2^n(b)) \in A_\epsilon^n$$

It can be shown that the decoding error at this step is arbitrarily small for sufficiently large n, provided

$$\begin{aligned} R_2 &\le I(U; Y_2, S_2 | V_1) + I(V_1; Y_2, S_2) \\ &= I(U, V_1; Y_2, S_2) \end{aligned} \tag{12}$$

Combining (10)–(12), we obtain that the following rate region is achievable

$$\left\{ \begin{array}{l} R_1 \le I(V; Y_1, S_1 | U, V_1) \\ R_2 \le \min\{I(U, V_1; Y_2, S_2), I(U; Y_1, S_1 | V_1)\} \end{array} \right\} \tag{13}$$

Since by (9), U, V, V_1 are independent of $S_c \triangleq (S_0, S_1, S_2)$, the mutual information functions in (13) can be rewritten as in (8). This completes the proof. □

Now, we derive the capacity region of the discrete memoryless state-dependent PC-RBC with feedback.

Theorem 2: The capacity region of the discrete memoryless state-dependent PC-RBC with feedback defined in (4), is given by (see (14))
for all joint p.m.f of the form

$$\begin{aligned} &p(s_0, s_1, s_2)p(u, v, v_1)p(y_1, y_2 | x = f(u, v, s_0), \\ &\quad x_1 = f_1(v_1, s_1), s_0, s_1, s_2) \end{aligned} \tag{15}$$

where U, V, V_1 are auxiliary RVs, and $f(\cdot)$, $f_1(\cdot)$ are two arbitrary deterministic functions.

Proof:

Achievability: The achievability proof follows the same lines as in Theorem 1 in which Y_1 is replaced by the triple (Y_1, Y_2, S_2) as the output at user 1. Hence, we obtain the following to be achievable

$$\left\{ \begin{array}{l} R_1 \le I(V; Y_1, Y_2, S_2 | U, V_1, S_1) \\ R_0 + R_2 \le \min\{I(U, V_1; Y_2 | S_2), I(U; Y_1, Y_2, S_2 | V_1, S_1)\} \end{array} \right\} \tag{16}$$

Since U, V, V_1 are independent of $S_c \triangleq (S_0, S_1, S_2)$, we have

$$I(V; Y_1, Y_2, S_2 | U, V_1, S_1) = I(V; Y_1, Y_2 | U, V_1, S_1, S_2) \tag{17}$$

$$I(U; Y_1, Y_2, S_2 | V_1, S_1) = I(U; Y_1, Y_2 | V_1, S_1, S_2) \tag{18}$$

By substituting (17) and (18) in (16), the achievability of (14) is derived.

Converse: For a PC-RBC with feedback and causal CSI, consider a sequence of $((2^{nR_0}, 2^{nR_1}, 2^{nR_2}), n)$ codes with probability of error $P_e^{(n)}$, where $P_e^{(n)} \to 0$ as $n \to \infty$. Note that, the joint p.m.f. on the ensemble $\mathcal{W}_0 \times \mathcal{W}_1 \times \mathcal{W}_2 \times \mathcal{X}^n \times \mathcal{X}_1^n \times \mathcal{Y}_1^n \times \mathcal{Y}_2^n \times \mathcal{S}_0^n \times \mathcal{S}_1^n \times \mathcal{S}_2^n$ is given by (4).Define new RVs V_t, $V_{1,t}$, U_t for $t = 1, 2, \ldots, n$ as follows

$$\begin{aligned} V_t &\triangleq (W_1, S_0^{t-1}, S_1^{t-1}, S_2^{t-1}) \\ V_{1,t} &\triangleq (Y_1^{t-1}, Y_2^{t-1}, S_1^{t-1}, S_2^{t-1}) \\ U_t &\triangleq (W_0, W_2, Y_1^{t-1}, Y_2^{t-1}) \end{aligned} \tag{19}$$

$$\mathcal{C}_{\text{SD-FB}}^{\text{PC-RBC}} = \bigcup \left\{ \begin{array}{l} R_0, R_1, R_2 \ge 0, \\ R_1 \le I(V; Y_1, Y_2 | U, V_1, S_1, S_2) \\ R_0 + R_2 \le \min\{I(U, V_1; Y_2 | S_2), I(U; Y_1, Y_2 | V_1, S_1, S_2)\} \end{array} \right\} \tag{14}$$

By Fano's inequality [21, Section 2.10], we have

$$
\begin{aligned}
& n(R_0+R_2)-n\epsilon_{1,n} \\
& \quad \le I(W_0, W_2; Y_2^n, S_2^n) \\
& \quad = \sum_{t=1}^{n} I(W_0, W_2; Y_{2,t}, S_{2,t} | Y_2^{t-1}, S_2^{t-1}) && (20)\\
& \quad = \sum_{t=1}^{n} [I(W_0, W_2; S_{2,t} | Y_2^{t-1}, S_2^{t-1}) \\
& \qquad + I(W_0, W_2; Y_{2,t} | Y_2^{t-1}, S_2^{t-1}, S_{2,t})] && (21)\\
& \quad = \sum_{t=1}^{n} I(W_0, W_2; Y_{2,t} | Y_2^{t-1}, S_2^{t-1}, S_{2,t}) && (22)\\
& \quad \le \sum_{t=1}^{n} H(Y_{2,t}|S_{2,t}) - H(Y_{2,t}|Y_2^{t-1}, S_2^{t-1}, S_{2,t}, \\
& \qquad W_0, W_2, Y_1^{t-1}, S_1^{t-1}) && (23)\\
& \quad = \sum_{t=1}^{n} I(W_0, W_2, Y_2^{t-1}, S_2^{t-1}, Y_1^{t-1}, S_1^{t-1}; Y_{2,t} | S_{2,t}) \\
& \quad = \sum_{t=1}^{n} I(U_t, V_{1,t}; Y_{2,t} | S_{2,t}) && (24)
\end{aligned}
$$

where $\epsilon_{1,n} \to 0$ as $n \to \infty$. The equalities in (20) and (21) hold because of the chain rule. For causal encoding for memoryless channel with i.i.d. state process, $S_{2,t}$ is independent of W_0, W_2, Y_2^{t-1}, S_2^{t-1} [note that this independency can be easily verified by the p.m.f. in (4)], so the first term in (21) equals zero and (22) is resulted. The inequality in (23) holds because of the fact that conditioning does not increase the entropy. Furthermore, by Fano's inequality, we have

$$
\begin{aligned}
& n(R_0+R_2)-n\epsilon_{1,n} \\
& \quad \le I(W_0, W_2; Y_2^n, S_2^n) \\
& \quad \le I(W_0, W_2; Y_2^n, S_2^n, Y_1^n, S_1^n) && (25)\\
& \quad = \sum_{t=1}^{n} I(W_0, W_2; Y_{1,t}, Y_{2,t}, S_{1,t}, S_{2,t} | Y_1^{t-1}, \\
& \qquad Y_2^{t-1}, S_1^{t-1}, S_2^{t-1}) && (26)\\
& \quad = \sum_{t=1}^{n} [I(W_0, W_2; S_{1,t}, S_{2,t} | Y_1^{t-1}, Y_2^{t-1}, S_1^{t-1}, S_2^{t-1}) \\
& \qquad + I(W_0, W_2; Y_{1,t}, Y_{2,t} | Y_1^{t-1}, Y_2^{t-1}, S_1^{t-1}, S_2^{t-1}, S_{1,t}, S_{2,t})] && (27)\\
& \quad = \sum_{t=1}^{n} I(W_0, W_2; Y_{1,t}, Y_{2,t} | Y_1^{t-1}, Y_2^{t-1}, S_1^{t-1}, S_2^{t-1}, S_{1,t}, S_{2,t}) && (28)\\
& \quad = \sum_{t=1}^{n} I(W_0, W_2, Y_1^{t-1}, Y_2^{t-1}; Y_{1,t}, Y_{2,t} | Y_1^{t-1}, \\
& \qquad Y_2^{t-1}, S_1^{t-1}, S_2^{t-1}, S_{1,t}, S_{2,t}) && (29)\\
& \quad = \sum_{t=1}^{n} I(U_t; Y_{1,t}, Y_{2,t} | V_{1,t}, S_{1,t}, S_{2,t}) && (30)
\end{aligned}
$$

where the equalities in (26) and (27) hold because of the chain rule. Again, for causal encoding for memoryless channel with i.i.d. state process, $(S_{1,t}, S_{2,t})$ are independent of $(W_0, W_2, Y_1^{t-1}, Y_2^{t-1}, S_1^{t-1}, S_2^{t-1})$, so the first term in (27) equals zero and (28) is resulted. The equality in (29) holds due to the fact that conditioning does not increase the entropy. Finally, to derive the first bound in (14), by Fano's inequality we have

$$
\begin{aligned}
nR_1 - n\epsilon_{2,n} & \le I(W_1; Y_1^n, S_1^n, Y_2^n, S_2^n) \\
& \le I(W_1; Y_1^n, S_1^n, Y_2^n, S_2^n, W_0, W_2) && (31)\\
& = I(W_1; Y_1^n, S_1^n, Y_2^n, S_2^n | W_0, W_2) && (32)\\
& = \sum_{t=1}^{n} I(W_1; Y_{1,t}, S_{1,t}, Y_{2,t}, S_{2,t} | W_0, W_2, \\
& \qquad Y_1^{t-1}, S_1^{t-1}, Y_2^{t-1}, S_2^{t-1}) \\
& = \sum_{t=1}^{n} [I(W_1; S_{1,t}, S_{2,t} | W_0, W_2, Y_1^{t-1}, S_1^{t-1}, Y_2^{t-1}, S_2^{t-1}) \\
& \qquad + I(W_1; Y_{1,t}, Y_{2,t} | W_0, W_2, Y_1^{t-1}, S_1^{t-1}, Y_2^{t-1}, \\
& \qquad S_2^{t-1}, S_{1,t}, S_{2,t})] && (33)\\
& = \sum_{t=1}^{n} I(W_1; Y_{1,t}, Y_{2,t} | W_0, W_2, Y_1^{t-1}, S_1^{t-1}, \\
& \qquad Y_2^{t-1}, S_2^{t-1}, S_{1,t}, S_{2,t}) && (34)\\
& \le \sum_{t=1}^{n} I(W_1, S_0^{t-1}, S_1^{t-1}, S_2^{t-1}; Y_{1,t}, Y_{2,t} | W_0, W_2, \\
& \qquad Y_1^{t-1}, S_1^{t-1}, Y_2^{t-1}, S_2^{t-1}, S_{1,t}, S_{2,t}) && (35)\\
& = \sum_{t=1}^{n} I(V_t; Y_{1,t}, Y_{2,t} | U_t, V_{1,t}, S_{1,t}, S_{2,t}) && (36)
\end{aligned}
$$

where $\epsilon_{2,n} \to 0$ as $n \to \infty$. The equality in (32) holds because of the fact that W_0, W_1 and W_2 are independent (see Definition 2). Similar to the one mentioned before, for causal encoding for memoryless channel with i.i.d. state process, the first term in (33) equals zero and (34) is resulted.

Now, we define a time-sharing RV Q independent of everything and uniformly distributed over the set $\{1, \ldots, n\}$, and re-define: $V = (Q, V_Q)$, $V_1 = (Q, V_{1,Q})$, $U = (Q, U_Q)$, $X = X_Q, X_1 = X_{1,Q}$, $S_i = S_{i,Q}$ for $i = 0, 1, 2$ and $Y_j = Y_{j,Q}$ for $j = 1, 2$.

Then by (36), we obtain

$$
\begin{aligned}
nR_1 - \epsilon_{2,n} & \le \frac{1}{n}\sum_{t=1}^{n} I(V_Q; Y_{1,Q}, Y_{2,Q} | U_Q, V_{1,Q}, S_{1,Q}, S_{2,Q}, Q=t) \\
& = I(V_Q; Y_{1,Q}, Y_{2,Q} | U_Q, V_{1,Q}, S_{1,Q}, S_{2,Q}, Q) \\
& = I(V_Q, Q; Y_{1,Q}, Y_{2,Q} | U_Q, V_{1,Q}, S_{1,Q}, S_{2,Q}, Q) \\
& = I(V; Y_1, Y_2 | U, V_1, S_1, S_2) && (37)
\end{aligned}
$$

Proceeding likewise, we have

$$
R_0 + R_2 \le \min\{I(U, V_1; Y_2|S_2), I(U; Y_1, Y_2|V_1, S_1, S_2)\} + \epsilon_{1,n}
$$

Noticing that the state distribution is fixed and cannot be controlled by time-sharing scheme ($S_{i,Q}$ is independent

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of Q for $i = 0, 1, 2$), U, V, V_1 are independent of $S_c \triangleq (S_0, S_1, S_2)$. Moreover, X is a deterministic function of (U, V, S_0), X_1 is a deterministic function of (V_1, S_1) and the p.m.f. of (X, X_1, S_c, Y_1, Y_2) is consistent with the channel $p(y_1, y_2|x, x_1, s_c)$. So, by letting n tend to infinity in the latter bounds, the region in (4) is derived. This completes the proof of the converse part. □

Now, we characterise the capacity region of the degraded channel, and show that the feedback does not affect its capacity region.

Proposition 1: The capacity region of the discrete memoryless physically degraded PC-RBC with partial causal CSI, defined in Definition 4, is given by (see (38))
for all joint p.m.f. of the form

$$p(s_0, s_1, s_2)p(u, v, v_1)p(y_1, y_2|x = f(u, v, s_0), x_1 = f_1(v_1, s_1), s_0, s_1, s_2) \quad (39)$$

where U, V, V_1 are auxiliary RVs, and $f(\cdot), f_1(\cdot)$ are two arbitrary deterministic functions. Furthermore, feedback does not affect the capacity region of this degraded channel.

Remark 1:

- By setting $R_0 = R_1 = 0, U \triangleq V$ (since U denotes the auxiliary RV resulted from superposition coding, it can be set to V), $S_0 \triangleq S_1 \triangleq S$ and $S_2 \triangleq \emptyset$ in (38), the capacity region in (38) reduces to the capacity of the degraded RC with causal CSI at both the source and the relay [14, Theorem 2].
- By setting $R_0 = 0$, $S_0 \triangleq S$, $S_1 \triangleq S_2 \triangleq \emptyset$ and $V_1 \triangleq \emptyset$ in (38), the capacity region in (38) reduces to the capacity region of the degraded BC without common message and with causal CSI only at the source [13, Theorem 4].

Corollary 1:

- By setting $S_0 \triangleq S_1 \triangleq S$ and $S_2 \triangleq \emptyset$ in (38), the capacity region in (38) reduces to the capacity region of the degraded PC-RBC with perfect causal CSI at both the source and user 1 (relay 1) [23, Theorem 1].
- By setting $S_1 \triangleq S$, $S_0 \triangleq S_2 \triangleq \emptyset$ and $V \triangleq X$ in (38), the capacity region in (38) reduces to the capacity region of the degraded PC-RBC with perfect causal CSI only at user 1 (relay 1) [23, Theorem 2].
- By setting $S_0 \triangleq S$, $S_1 \triangleq S_2 \triangleq \emptyset$ and $V_1 \triangleq X_1$ in (38) , the capacity region in (38) reduces to the capacity region of the degraded PC-RBC with perfect causal CSI only at the source [23, Theorem 3].

Proof of Proposition 1: The proof of achievability can be obtained from Theorem 1. For the converse part, it is clear that the region in (14) which is the capacity region of the state-dependent PC-RBC with feedback, is also an outer bound on the capacity region of the state-dependent PC-RBC without feedback. In the following, we show that for degraded channel, the achievable rate region given in (8), and the outer bound given in (14) are equivalent, and therefore the capacity region of the degraded case is characterised.

For the degraded state-dependent PC-RBC defined in Definition 4, the first bound in (14) becomes as follows

$$\begin{aligned} I(V; Y_1, Y_2|U, V_1, S_1, S_2) &\overset{(a)}{=} I(V; Y_1, Y_2|U, V_1, S_1) \\ &= I(V; Y_1|U, V_1, S_1) \\ &\quad + I(V; Y_2|U, V_1, S_1, Y_1) \end{aligned} \quad (40)$$

where equality (a) is due to the fact that for the degraded channel as defined in Definition 4, S_2 is a deterministic function of S_1, that is, $H(S_2|S_1) = 0$.

Now, we show that for the joint p.m.f. in (39), the second term in (40) equals zero. For the degraded state-dependent PC-RBC, the joint p.m.f. in (39) becomes

$$\begin{aligned} &p(s_0, s_1, s_2, u, v, v_1, y_1, y_2) \\ &\quad = p(s_0, s_1, s_2)p(u, v, v_1)p(y_1|x = f(u, v, s_0), \\ &x_1 = f_1(v_1, s_1), s_0, s_1, s_2)p(y_2|y_1, x_1 = f_1(v_1, s_1), s_1) \end{aligned} \quad (41)$$

Now, based on (41), it is clear that the following Markov chain [21] holds

$$(U, V) \rightarrow (Y_1, V_1, S_1) \rightarrow Y_2 \quad (42)$$

that is, $p(y_2|y_1, v_1, s_1, u, v) = p(y_2|y_1, v_1, s_1)$. So, $I(U, V; Y_2|Y_1, V_1, S_1) = 0$, and owing to the chain rule it yields

$$\begin{aligned} I(U, V; Y_2|Y_1, V_1, S_1) &= I(U; Y_2|Y_1, V_1, S_1) \\ &\quad + I(V; Y_2|Y_1, V_1, S_1, U) = 0 \end{aligned}$$

Since mutual information is always non-negative, we obtain that

$$I(U; Y_2|Y_1, V_1, S_1) = I(V; Y_2|Y_1, V_1, S_1, U) = 0 \quad (43)$$

Hence, for the joint p.m.f. in (39), the second term in (40), that is, $I(V; Y_2|Y_1, V_1, S_1, U)$ equals zero. Furthermore, for the second term in 'min' operand of the second bound in (14), we have

$$\begin{aligned} &I(U; Y_1, Y_2|V_1, S_1, S_2) \\ &\overset{(a)}{=} I(U; Y_1, Y_2|V_1, S_1) \\ &= I(U; Y_1|V_1, S_1) + I(U; Y_2|V_1, S_1, Y_1) \quad (44) \\ &= I(U; Y_1|V_1, S_1) \quad (45) \end{aligned}$$

where equality (a) is obtained using $H(S_2|S_1) = 0$. Moreover, based on (43) the second term in (44) equals zero and (45) is resulted.

By substituting (40), (43) and (45) in (14), (38) is obtained. Therefore for the degraded state-dependent PC-RBC the achievable rate region in Theorem 1 coincides with the capacity region of the state-dependent PC-RBC with feedback in Theorem 2. This yields the capacity region of the degraded channel and shows that in this case feedback does not increase the capacity region. This completes the proof. □

$$C_{\text{SD-Deg}}^{\text{PC-RBC}} = \bigcup \left\{ \begin{array}{l} R_0, R_1, R_2 \geq 0, \\ R_1 \leq I(V; Y_1|U, V_1, S_1) \\ R_0 + R_2 \leq \min\{I(U, V_1; Y_2|S_2), I(U; Y_1|V_1, S_1)\} \end{array} \right\} \quad (38)$$

4 FC-RBC with partial causal CSI

In this section, we investigate discrete memoryless FC-RBC (both users 1 and 2 can act as relay) with partial causal CSI, and similar to state-dependent PC-RBC, at first, we establish an achievable rate region for this channel. Then, we derive the capacity region of FC-RBC with feedback and partial causal CSI. We also derive the capacity region of physically degraded channel, and show that feedback does not affect its capacity region.

Theorem 3: For the discrete memoryless FC-RBC with partial CSI, in which S_0, S_1 and S_2 are causally known to the transmitter, user 1 and user 2, respectively, the following rate region is achievable (see 46))
for all joint p.m.f. of the form

$$p(s_0, s_1, s_2)p(u, v, v_1, v_2)p(y_1, y_2|x = f(u, v, s_0)),\\ x_1 = f_1(v_1, s_1), x_2 = f_2(v_2, s_2), s_0, s_1, s_2) \quad (47)$$

where U, V, V_1, V_2 are auxiliary RVs, and $f(\cdot)$, $f_1(\cdot)$ and $f_2(\cdot)$ are arbitrary deterministic functions.

Proof: This theorem can be proved similar to that one for the state-dependent PC-RBC in Theorem 1. In this case, the random codebook is constructed using the auxiliary RVs V, V_1 and V_2 which are used to apply Shannon's strategy at the source, user 1 and user 2, respectively. Also, U denotes the auxiliary RV resulted from superposition coding. At each time instant t, the source node sends $x_t = f(u_t, v_t, s_{0,t})$, the first user employs the decode-and-forward (DF) scheme [4] and sends $x_{1,t} = f_1(v_{1,t}, s_{1,t})$ over the channel, whereas the second user always selects a single codeword constructed from $\mathcal{V}_2$ (the alphabet of the auxiliary RV V_2) and then sends $x_{2,t} = f_2(v_{2,t}, s_{2,t})$ for $1 \le t \le n$, over the channel. Note that although the codeword v_2^n is fixed, $x_{2,t}$ depends on $s_{2,t}$. Moreover, decoding at each user is performed using (Y_i, S_i) for $i = 1, 2$. □

Note that due to symmetry in the structure of the channel, another achievable rate region can be obtained for this channel by exchanging indices 1 and 2 in $\mathcal{R}_3$ (i.e. switching two relaying schemes). The union of these two latter achievable rate regions is also an achievable rate region for the state-dependent FC-RBC.

Theorem 4: The capacity region of the discrete memoryless state-dependent FC-RBC with feedback defined in (7), is given by (see 48))

for all joint p.m.f. of the form

$$p(s_0, s_1, s_2)p(u, v, v_1, v_2)p(y_1, y_2|x = f(u, v, s_0)),\\ x_1 = f_1(v_1, s_1), x_2 = f_2(v_2, s_2), s_0, s_1, s_2) \quad (49)$$

where U, V, V_1, V_2 are auxiliary RVs, and $f(\cdot)$, $f_1(\cdot)$ and $f_2(\cdot)$ are arbitrary deterministic functions.

Proof:
Achievability: The achievability proof follows the same lines as in Theorem 3 in which Y_1 is replaced by (Y_1, Y_2, S_2) as the output at user 1. Furthermore, note that (U, V, V_1, V_2) are independent of $S_c \triangleq (S_0, S_1, S_2)$. So, we have

$$I(V; Y_1, Y_2, S_2|U, V_1, V_2, S_1)\\ = I(V; Y_1, Y_2|U, V_1, V_2, S_1, S_2) \quad (50)$$

$$I(U; Y_1, Y_2, S_2|V_1, V_2, S_1) = I(U; Y_1, Y_2|V_1, V_2, S_1, S_2) \quad (51)$$

This yields the achievability of the rate region in (48).
Converse: For a state-dependent FC-RBC with feedback and causal CSI, consider a sequence of $((2^{nR_0}, 2^{nR_1}, 2^{nR_2}), n)$ codes with probability of error $P_e^{(n)}$, where $P_e^{(n)} \to 0$ as $n \to \infty$. Note that, the joint p.m.f. on the ensemble $\mathcal{W}_0 \times \mathcal{W}_1 \times \mathcal{W}_2 \times \mathcal{X}^n \times \mathcal{X}_1^n \times \mathcal{X}_2^n \times \mathcal{Y}_1^n \times \mathcal{Y}_2^n \times \mathcal{S}_0^n \times \mathcal{S}_1^n \times \mathcal{S}_2^n$ is given by (7).

Define new RVs V_t, $V_{1,t}$, $V_{2,t}$, U_t for $t = 1, 2, \ldots, n$ as follows

$$\begin{aligned} V_t &\triangleq (W_1, S_0^{t-1}, S_1^{t-1}, S_2^{t-1})\\ V_{1,t} &\triangleq (Y_1^{t-1}, Y_2^{t-1}, S_1^{t-1}, S_2^{t-1})\\ V_{2,t} &\triangleq (Y_2^{t-1}, S_2^{t-1})\\ U_t &\triangleq (W_0, W_2, Y_1^{t-1}, Y_2^{t-1}) \end{aligned} \quad (52)$$

By Fano's inequality, we have

$$\begin{aligned} n(R_0 + R_2) - n\epsilon_{1,n} &\le I(W_0, W_2; Y_2^n, S_2^n)\\ &= \sum_{t=1}^{n} I(W_0, W_2; Y_{2,t}, S_{2,t}|Y_2^{t-1}, S_2^{t-1})\\ &= \sum_{t=1}^{n} [I(W_0, W_2; S_{2,t}|Y_2^{t-1}, S_2^{t-1})\\ &\quad + I(W_0, W_2; Y_{2,t}|Y_2^{t-1}, S_2^{t-1}, S_{2,t})] \end{aligned} \quad (53)$$

$$\mathcal{R}_3 = \bigcup \left\{ \begin{array}{l} R_0, R_1, R_2 \ge 0,\\ R_1 \le I(V; Y_1|U, V_1, V_2, S_1)\\ R_0 + R_2 \le \min\{I(U, V_1; Y_2|V_2, S_2), I(U; Y_1|V_1, V_2, S_1)\} \end{array} \right\} \quad (46)$$

$$\mathcal{C}_{\text{SD-FB}}^{\text{FC-RBC}} = \left\{ \begin{array}{l} R_0, R_1, R_2 \ge 0,\\ R_1 \le I(V; Y_1, Y_2|U, V_1, V_2, S_1, S_2)\\ R_0 + R_2 \le \min\{I(U, V_1; Y_2|V_2, S_2), I(U; Y_1, Y_2|V_1, V_2, S_1, S_2)\} \end{array} \right\} \quad (48)$$

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$$= \sum_{t=1}^{n} I(W_0, W_2; Y_{2,t}|Y_2^{t-1}, S_2^{t-1}, S_{2,t}) \quad (54)$$

$$\leq \sum_{t=1}^{n} I(W_0, W_2, Y_1^{t-1}, S_1^{t-1}, Y_2^{t-1}, S_2^{t-1}; Y_{2,t}|Y_2^{t-1}, S_2^{t-1}, S_{2,t})$$

$$= \sum_{t=1}^{n} I(U_t, V_{1,t}; Y_{2,t}|V_{2,t}, S_{2,t}) \quad (55)$$

where $\epsilon_{1,n} \to 0$ as $n \to \infty$. For causal encoding for memoryless channel with i.i.d. state process, $S_{2,t}$ is independent of $(W_0, W_2, Y_2^{t-1}, S_2^{t-1})$, so the first term in (53) equals zero and (54) is resulted.

To derive the second bound on $R_0 + R_2$ in (48), proceeding like (25)–(29) we obtain

$$n(R_0 + R_2) - n\epsilon_{1,n} \leq \sum_{t=1}^{n} I(W_0, W_2, Y_1^{t-1}, Y_2^{t-1}; Y_{1,t}, Y_{2,t}|\ Y_1^{t-1}, Y_2^{t-1}, S_1^{t-1}, S_2^{t-1}, S_{1,t}, S_{2,t})$$

$$= \sum_{t=1}^{n} I(U_t; Y_{1,t}, Y_{2,t}|V_{1,t}, V_{2,t}, S_{1,t}, S_{2,t}) \quad (56)$$

Finally, to derive the bound on R_1 in (48), proceeding like (31)–(35), we obtain

$$nR_1 - n\epsilon_{2,n} \leq \sum_{t=1}^{n} I(W_1, S_0^{t-1}, S_1^{t-1}, S_2^{t-1}; Y_{1,t}, Y_{2,t}|W_0, W_2, Y_1^{t-1}, S_1^{t-1}, Y_2^{t-1}, S_2^{t-1}, S_{1,t}, S_{2,t})$$

$$= \sum_{t=1}^{n} I(V_t; Y_{1,t}, Y_{2,t}|U_t, V_{1,t}, V_{2,t}, S_{1,t}, S_{2,t}) \quad (57)$$

where $\epsilon_{2,n} \to 0$ as $n \to \infty$.

The remaining part of the converse proof is resulted by applying a standard time-sharing argument similar to the one that we mentioned for the converse part of Theorem 2, and by considering that $X_{2,t}$ is a deterministic function of $(V_{2,t}, S_{2,t})$, and $U_t, V_t, V_{1,t}, V_{2,t}$ are independent of $S_{c,t} \triangleq (S_{0,t}, S_{1,t}, S_{2,t})$. This completes the proof of the converse part. □

Now, we characterise the capacity region of the degraded channel, and show that feedback does not increase its capacity region.

Proposition 2: The capacity region of the discrete memoryless physically degraded state-dependent FC-RBC defined in Definition 6, is given by (see 58))

$$C_{\text{SD-Deg}}^{\text{FC-RBC}} = \bigcup \left\{ \begin{array}{l} R_0, R_1, R_2 \geq 0, \\ R_1 \leq I(V; Y_1|U, V_1, V_2, S_1) \\ R_0 + R_2 \leq \min\{I(U, V_1; Y_2|V_2, S_2), I(U; Y_1|V_1, V_2, S_1)\} \end{array} \right\} \quad (58)$$

for all joint p.m.f. of the form

$$p(s_0, s_1, s_2)p(u, v, v_1, v_2)p(y_1, y_2|x = f(u, v, s_0), x_1 = f_1(v_1, s_1), x_2 = f_2(v_2, s_2), s_0, s_1, s_2) \quad (59)$$

where U, V, V_1, V_2 are auxiliary RVs, and $f(\cdot), f_1(\cdot)$ and $f_2(\cdot)$ are arbitrary deterministic functions. Furthermore, feedback does not affect the capacity region of this degraded channel.

Corollary 2:

- By setting $S_0 \triangleq S_1 \triangleq S_2 \triangleq S$ in (58), the capacity region in (58) reduces to the capacity region of the degraded FC-RBC with perfect causal CSI at the source and both relay nodes [23, Theorem 4].
- By setting $S_1 \triangleq S_2 \triangleq S$, $S_0 \triangleq \emptyset$ and $V \triangleq X$ in (58), the capacity region in (58) reduces to the capacity region of the degraded FC-RBC with perfect causal CSI only at the relay nodes (users 1 and 2) [23, Theorem 5].
- By setting $S_0 \triangleq S$, $S_1 \triangleq S_2 \triangleq \emptyset$, $V_1 \triangleq X_1$ and $V_2 \triangleq X_2$ in (58), the capacity region in (58) reduces to the capacity region of the degraded FC-RBC with perfect causal CSI only at the source [23, Theorem 6].

Proof of Proposition 2: The proof of achievability can be obtained from Theorem 3. Moreover, it is clear that the region in (48) which is the capacity region of the state-dependent FC-RBC with feedback, is also an outer bound on the capacity region of the state-dependent FC-RBC without feedback. Therefore it is sufficient to show that for degraded channel, the achievable rate region derived in (46) is equal to the capacity region of the state-dependent FC-RBC with feedback which is given by (48).

For the degraded state-dependent FC-RBC defined in Definition 6, the p.m.f. in (59) becomes

$$p(s_0, s_1, s_2)p(u, v, v_1, v_2)p(y_1|x = f(u, v, s_0), x_1 = f_1(v_1, s_1), x_2 = f_2(v_2, s_2), s_0, s_1, s_2) \times p(y_2|y_1, x_1 = f_1(v_1, s_1), x_2 = f_2(v_2, s_2), s_1)$$

So, the following Markov chain holds

$$(U, V) \to (Y_1, V_1, V_2, S_1, S_2) \to Y_2$$

Moreover, since S_2 is a deterministic function of S_1 we obtain

$$(U, V) \to (Y_1, V_1, V_2, S_1) \to Y_2 \quad (60)$$

Therefore for the first bound in (48), we have

$$I(V; Y_1, Y_2|U, V_1, V_2, S_1, S_2) \overset{(a)}{=} I(V; Y_1, Y_2|U, V_1, V_2, S_1) = I(V; Y_1|U, V_1, V_2, S_1) + I(V; Y_2|U, V_1, V_2, S_1, Y_1) \quad (61)$$

$$= I(V; Y_1|U, V_1, V_2, S_1) \quad (62)$$

where equality (a) is obtained using $H(S_2|S_1) = 0$. Moreover, based on the Markov chain in (60), the second term in (61) equals zero and (62) is resulted.

Furthermore, for the second term in the 'min' operand of the second bound in (48), we have

$$\begin{aligned} I(U; Y_1, Y_2|V_1, V_2, S_1, S_2) &\overset{(a)}{=} I(U; Y_1, Y_2|V_1, V_2, S_1) \\ &= I(U; Y_1|V_1, V_2, S_1) \\ &\quad + I(U; Y_2|V_1, V_2, S_1, Y_1) \end{aligned} \tag{63}$$

$$= I(U; Y_1|V_1, V_2, S_1) \tag{64}$$

where equality (a) is obtained using $H(S_2|S_1) = 0$. Based on the Markov chain in (60), the second term in (63) equals zero and (64) is resulted.

Now, by substituting (62) and (64) in (48), (58) is obtained. Therefore for the degraded state-dependent FC-RBC, the rate region given in (58) coincides with the capacity region of the state-dependent FC-RBC with feedback derived in Theorem 4. This yields the capacity region of the degraded channel and shows that feedback does not affect the capacity region of it. This completes the proof. □

5 Gaussian state-dependent RBCs

In this section, we consider the general Gaussian PC-RBC and FC-RBC with additive Gaussian interference and noise signals. We assume that the Gaussian interference which is modelled as channel state, is known partially and causally to the nodes. We use the results in Sections 3 and 4 to derive achievable rate regions for Gaussian models.

5.1 Channel models for the Gaussian cases

We consider the Gaussian PC-RBC with additive interference, in which the channel outputs at user 1 (relay) and user 2 at time instant t $(1 \le t \le n)$ are formulated by

$$\begin{aligned} Y_{1,t} &= X_t + Z_{1,t} + S_{0,t} + S_{1,t} + S_{2,t} \\ Y_{2,t} &= X_t + X_{1,t} + Z_{2,t} + S_{0,t} + S_{1,t} + S_{2,t} \end{aligned} \tag{65}$$

where the noises $Z_{1,t}$ and $Z_{2,t}$ are independent zero mean Gaussian RVs with variances N_1 and N_2, respectively, X_t and $X_{1,t}$ are the signals transmitted by the transmitter and user 1 with individual average power constraints P and P_1, respectively, that is, $(1/n)\sum_{t=1}^{n} E[X_t^2] \le P$ and $(1/n)\sum_{t=1}^{n} E[X_{1,t}^2] \le P_1$. Also, $\{S_{0,t}, S_{1,t}, S_{2,t}\}_{t=1}^{\infty}$ is an i.i.d. random process, where $S_{0,t}$, $S_{1,t}$, $S_{2,t}$ are independent Gaussian distributed RVs with zero mean and variances P_{s0}, P_{s1}, P_{s2}, respectively. Similar to the model considered in [24] for state-dependent multiple-access channel, here we assume that $S_{0,t} + S_{1,t} + S_{2,t}$ is the total interference which is added to the channel at time t, and can be modelled as the channel state. In fact, we assume that $S_{0,t}$, $S_{1,t}$ and $S_{2,t}$ are parts of the channel state, which are causally known to the transmitter, user 1 and user 2, respectively.

Also, we consider the Gaussian FC-RBC with additive interference, in which the channel outputs at user 1 and user 2 at time instant t $(1 \le t \le n)$ are formulated by

$$\begin{aligned} Y_{1,t} &= X_t + X_{2,t} + Z_{1,t} + S_{0,t} + S_{1,t} + S_{2,t} \\ Y_{2,t} &= X_t + X_{1,t} + Z_{2,t} + S_{0,t} + S_{1,t} + S_{2,t} \end{aligned} \tag{66}$$

where $Z_{1,t}$, $Z_{2,t}$, $S_{0,t}$, $S_{1,t}$, $S_{2,t}$, X_t, $X_{1,t}$ are defined as for the Gaussian PC-RBC in (65). $X_{2,t}$ is the signal transmitted by user 2 with average power constraint P_2, that is, $(1/n)\sum_{t=1}^{n} E[X_{2,t}^2] \le P_2$.

Channel models in the presence of feedback for Gaussian channels in (65) and (66) are defined the same as discrete channels, that is, at time instant t, the output sequence at user 2 up to the time $t-1$, that is, y_2^{t-1}, and its CSI, that is, s_2^{t-1}, are provided to user 1, and the outputs and available channel states at both users 1 and 2, that is, $(s_1^{t-1}, y_1^{t-1}, s_2^{t-1}, y_2^{t-1})$, are provided to the transmitter, all through perfect feedback links.

5.2 Achievable rate regions for the Gaussian cases

In the following, denote $\bar{x} \triangleq 1 - x$, where $x \in [0, 1]$, $\mathbb{C}(x) \triangleq 1/2 \log(1 + x)$.

Theorem 5: For the Gaussian PC-RBC with additive interference in (65), in which S_0, S_1, and S_2 are causally known to the transmitter, user 1 and user 2, respectively, the following rate region is achievable (see 67)) where $\alpha, \beta \in [0, 1]$, $\beta_0 \in [0, \sqrt{P/P_{s0}}]$, $\beta_1 \in [0, \sqrt{P_1/P_{s1}}]$, $P' \triangleq P - \beta_0^2 P_{s0}$ and $P_1' \triangleq P_1 - \beta_1^2 P_{s1}$.

Proof: The achievable rate region $\mathcal{R}_1$ in Theorem 1 can be extended to the discrete-time Gaussian memoryless case with continuous alphabets by standard arguments [21]. Hence, we compute (8) with an appropriate choice of distributions to obtain (67). Assume $\beta_0 \in [0, \sqrt{P/P_{s0}}]$, $\beta_1 \in [0, \sqrt{P_1/P_{s1}}]$, $\alpha, \beta \in [0, 1]$ be arbitrary real numbers. We consider the following mapping for the generated codebooks in Theorem 1 with respect to the p.m.f. (9).

Let $V_1 \sim \mathcal{N}(0, P_1')$, $U' \sim \mathcal{N}(0, \beta\bar{\alpha}P')$, $V' \sim \mathcal{N}(0, \alpha P')$ where V_1, U', V' are independent. Let $U = \sqrt{(\bar{\beta}\bar{\alpha}P'/P_1')}V_1 + U'$ and $V = U + V'$. Define $X = V - \beta_0 S_0$ and $X_1 = V_1 - \beta_1 S_1$. Note that V_1, U', V' are independent of S_0, S_1, S_2. In this mapping, based on the compensation strategy the source expends a part $\beta_0^2 P_{s0}$ of its power to clean S_0 from the channel, and uses the rest of it, that is, $P' = P - \beta_0^2 P_{s0}$, for information transmission. Also, user 1 expends a part $\beta_1^2 P_{s1}$ of its power to clean S_1 from the channel, and uses the rest of it, that is, $P_1' = P_1 - \beta_1^2 P_{s1}$, for information transmission. Now, based on the channel model (65) and by using RVs V, V_1, U, X, X_1 as defined above, it is easy to compute the mutual information terms in (8). For example, to compute the

$$\mathcal{R}_1^* = \bigcup_{\substack{\alpha,\beta \\ \beta_0,\beta_1}} \left\{ \begin{aligned} & R_1 \le \mathbb{C}\left(\frac{\alpha P'}{(1-\beta_0)^2 P_{s0} + P_{s2} + N_1}\right) \\ & R_0 + R_2 \le \min\left\{\mathbb{C}\left(\frac{\bar{\alpha}P' + P_1' + 2\sqrt{\bar{\beta}\bar{\alpha}P'P_1'}}{\alpha P' + (1-\beta_0)^2 P_{s0} + (1-\beta_1)^2 P_{s1} + N_2}\right), \mathbb{C}\left(\frac{\beta\bar{\alpha}P'}{\alpha P' + (1-\beta_0)^2 P_{s0} + P_{s2} + N_1}\right)\right\} \end{aligned} \right\} \tag{67}$$

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first mutual information in (8) we have

$$R_1 \le I(V; Y_1|U, V_1, S_1)$$
$$= h(Y_1|U, V_1, S_1) - h(Y_1|U, V_1, S_1, V)$$

where $h(\cdot)$ denotes differential entropy [21]. Due to the channel model in (65) and by using the RVs V, V_1, U, X, X_1 as defined above, we obtain $Y_1 = U + V' + (1-\beta_0)S_0 + S_1 + S_2 + Z_1$.

Now, for the Gaussian distribution and the above mapping

$$h(Y_1|U, V_1, S_1) = \frac{1}{2}\log[(2\pi e)(\alpha P' + (1-\beta_0)^2 \times P_{s0} + P_{s2} + N_1)]$$
$$h(Y_1|U, V_1, S_1, V) = h(Y_1|U, V_1, S_1, V, V') = \frac{1}{2}\log[(2\pi e)((1-\beta_0)^2 P_{s0} + P_{s2} + N_1)]$$

Therefore, $I(V; Y_1|U, V_1, S_1)$ can be derived. Following the same procedure, other mutual information terms in (8) can be computed, and the achievable rate region in (67) is obtained. This completes the proof. □

Theorem 6: For the Gaussian PC-RBC with additive interference and with feedback, in which S_0, S_1 and S_2 are causally known to the transmitter, user 1 and user 2, respectively, the following rate region is achievable (see 68))
where $\alpha, \beta \in [0, 1]$, $\beta_0 \in [0, \sqrt{P/P_{s0}}]$, $\beta_1 \in [0, \sqrt{P_1/P_{s1}}]$, $P' \triangleq P - \beta_0^2 P_{s0}$ and $P'_1 \triangleq P_1 - \beta_1^2 P_{s1}$.

Proof: To prove this theorem, we compute (14) with the same mapping as in Theorem 5. Hence, the achievable rate region in (68) is established. □

Comparing (67) with (68), we can see that in (68) P_{s2} is omitted. Further, N_1 is replaced with $(N_1N_2)/(N_1+N_2)$. This relation is similar to the result obtained for the Gaussian PC-RBC (without channel state) in [2, Theorem 8].

Theorem 7: For the Gaussian FC-RBC with additive interference in (66), in which S_0, S_1 and S_2 are causally known to the transmitter, user 1 and user 2, respectively, the following rate region is achievable (see 69))
where $\alpha, \beta \in [0, 1]$, $\beta_0 \in [0, \sqrt{P/P_{s0}}]$, $\beta_1 \in [0, \sqrt{P_1/P_{s1}}]$, $\beta_2 \in [0, \sqrt{P_2/P_{s2}}]$, $P' \triangleq P - \beta_0^2 P_{s0}$ and $P'_1 \triangleq P_1 - \beta_1^2 P_{s1}$.

Proof: To prove this theorem, it is sufficient to compute (46) with an appropriate choice of distributions. As mentioned in Theorem 3, to find the achievable rate region (46) for the state-dependent FC-RBC, we assume that user 2 always selects a single codeword constructed from the alphabet $\mathcal{V}_2$, and then transmits a function of V_2 and S_2. In the Gaussian case, we assume that $V_2 = \emptyset$ and user 2 transmits a scaled version of the channel state which is available for it, that is, $X_2 = -\beta_2 S_2$. The power constraint P_2 at user 2 yields $\beta_2 \in [0, \sqrt{P_2/P_{s2}}]$. The RVs V, V_1, U, X, X_1 are defined the same as for Theorem 5. Now, based on the channel model in (66) and by substituting the RVs $V, V_1, V_2, U, X, X_1, X_2$ in (46), the achievable rate region in (69) is established. This completes the proof. □

Similar to the relation between $\mathcal{R}_1^*$ and $\mathcal{R}_2^*$, to obtain an achievable rate region for the Gaussian FC-RBC with additive interference and feedback, it is enough to replace N_1 with $(N_1N_2)/(N_1+N_2)$ and also omit P_{s2} in (69). Applying these modification, (69) becomes the same as (68). This result is analogous to [2, Theorem 18].

5.3 Numerical examples for the Gaussian state-dependent RBCs

Here, we provide some numerical examples for the rate regions $\mathcal{R}_1^*$ in Theorem 5, $\mathcal{R}_2^*$ in Theorem 6 and $\mathcal{R}_3^*$ in Theorem 7. Considering these derived achievable rate regions for the Gaussian channels, we investigate the effects of knowing the additive interference, user cooperation, feedback and the usage of compensation strategy in these channels.

Fig. 5 compares $\mathcal{R}_1^*$ and $\mathcal{R}_3^*$ (i.e. achievable rate regions for the Gaussian PC-RBC and FC-RBC with additive interference, respectively), for the same channel parameters and for different values of P_{s2}. As mentioned before, in the FC-RBC both users can assist the transmitter, whereas in the PC-RBC only one of the users (i.e. user 1) acts as a relay. Also, as mentioned in Section 5-B, to derive $\mathcal{R}_3^*$, we have assumed that user 2 (relay 2) assists the transmitter by sending a scaled version of S_2, that is, $X_2 = -\beta_2 S_2$, to cancel part of the interference S_2 from Y_1, where β_2 depends on its power constraint (i.e. P_2) and P_{s2}. Therefore it can be seen that when $P_{s2} = 0$ (i.e. $S_2 \triangleq \emptyset$), $\mathcal{R}_3^*$ coincides with $\mathcal{R}_1^*$. As P_{s2} increases, $\mathcal{R}_1^*$ decreases, while as

$$\mathcal{R}_2^* = \bigcup_{\substack{\alpha,\beta\\ \beta_0,\beta_1}} \left\{ \begin{array}{l} R_1 \le \mathbb{C}\left(\dfrac{\alpha P'}{(1-\beta_0)^2 P_{s0} + (N_1N_2)/(N_1+N_2)}\right) \\ R_0 + R_2 \le \min\left\{\mathbb{C}\left(\dfrac{\bar{\alpha}P' + P'_1 + 2\sqrt{\bar{\beta}\bar{\alpha}P'P'_1}}{\alpha P' + (1-\beta_0)^2 P_{s0} + (1-\beta_1)^2 P_{s1} + N_2}\right), \mathbb{C}\left(\dfrac{\beta\bar{\alpha}P'}{\alpha P' + (1-\beta_0)^2 P_{s0} + (N_1N_2)/(N_1+N_2)}\right)\right\} \end{array} \right\} \quad (68)$$

$$\mathcal{R}_3^* = \bigcup_{\substack{\alpha,\beta\\ \beta_0,\beta_1,\beta_2}} \left\{ \begin{array}{l} R_1 \le \mathbb{C}\left(\dfrac{\alpha P'}{(1-\beta_0)^2 P_{s0} + (1-\beta_2)^2 P_{s2} + N_1}\right) \\ R_0 + R_2 \le \min\left\{\mathbb{C}\left(\dfrac{\bar{\alpha}P' + P'_1 + 2\sqrt{\bar{\beta}\bar{\alpha}P'P'_1}}{\alpha P' + (1-\beta_0)^2 P_{s0} + (1-\beta_1)^2 P_{s1} + N_2}\right), \mathbb{C}\left(\dfrac{\beta\bar{\alpha}P'}{\alpha P' + (1-\beta_0)^2 P_{s0} + (1-\beta_2)^2 P_{s2} + N_1}\right)\right\} \end{array} \right\} \quad (69)$$

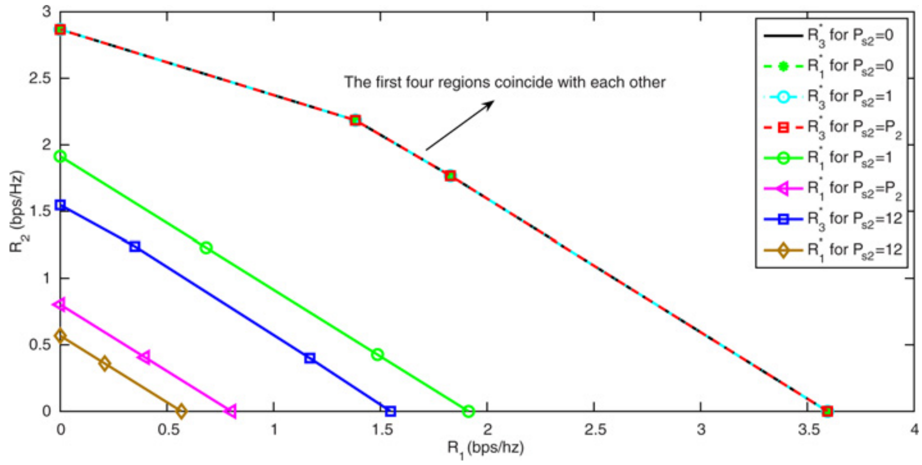

Fig. 5 *Achievable rate regions $\mathcal{R}_1^*$ and $\mathcal{R}_3^*$, for $P = P_1 = 15$, $P_2 = 7$, $N_1 = 0.1$, $N_2 = 1$, $P_{s0} = P_{s1} = 0.5$ and different values of P_{s2}*

long as $P_{s2} \le P_2$, the resulted achievable rate regions $\mathcal{R}_3^*$ coincide with that of the case where $P_{s2} = 0$. This happens because for these cases in the FC-RBC, the decoder of user 2 cancels S_2 from Y_2, and the encoder of user 2 cancels the interference S_2 from Y_1 by sending $X_2 = -S_2$ over the channel. For $P_{s2} > P_2$, $\mathcal{R}_3^*$ also decreases, although it still outperforms $\mathcal{R}_1^*$. In fact, this happens because the encoder of user 2 in the FC-RBC can cancel part of the interference S_2 from Y_1 up to its maximum power P_2, however, it cannot completely cancel S_2 from Y_1 for $P_{s2} > P_2$.

In Fig. 6 the achievable rate region for the Gaussian PC-RBC with additive interference and feedback, that is, $\mathcal{R}_2^*$ given in (68), is plotted for different values of P_{s0}, P_{s1} and P_{s2}. Note that since user 2 always knows the interference S_2, it cancels S_2 from Y_2. Also, S_2 can be cancelled from Y_1, because S_2 is available at user 1 via the feedback link. Therefore the interference S_2 with power P_{s2} has no effect on the achievable rate region R_2^* derived for the Gaussian PC-RBC with feedback, as it can be seen from (68). In Fig. 6, we investigate the effect of using the compensation strategy at the transmitter and the encoder of user 1. Setting $\beta_0 = 0$ (or $\beta_1 = 0$) denotes the case where the compensation strategy at the transmitter (or user 1) is ignored. It can be seen that without the compensation strategy, the achievable rate region decreases significantly with respect to the case where $\beta_0 \neq 0$ (or $\beta_1 \neq 0$).

In Fig. 7, considering $\mathcal{R}_1^*$, we assume that the transmitter and user 1 share the total amount of power $P_{\rm T}$, that is, $P_{\rm T} = P + P_1$, where $P = \gamma P_{\rm T}$ and $P_1 = (1-\gamma)P_{\rm T}$ for $\gamma \in [0,\ 1]$. In Fig. 7, the achievable rate region $\mathcal{R}_1^*$ is plotted under the conditions $P_{s1} = P_{s2} = 0$ and $N_1 < N_2$. For each $\gamma \in [0,\ 1]$, a different achievable rate region is obtained. The value of $\gamma = 1$ denotes the case where the relay encoder is off and the channel reduces to a Gaussian BC. As we see from Fig. 7, when γ decreases the maximum achievable rate of R_1 (the cut-off point on R_1-axis) decreases. This is because the link from the transmitter to user 1 becomes weak. The variations of the maximum achievable rate of R_2 (the cut-off point on R_2-axis) with respect to γ is different. As we see from Fig. 7, by decreasing γ from 1 down to a critical value denoted by γ^* (for the channel parameters in Fig. 7, $\gamma^* \simeq 0.6$), the maximum achievable rate of R_2 (the cut-off point on R_2-axis) increases. This is because the dedicated power to the relay node [i.e. $(1-\gamma)P_{\rm T}$] increases, and it can assist the transmitter for relaying the message W_2 to user 2. For $\gamma \le \gamma^*$, however, as γ decreases the maximum achievable rate of R_2 (the cut-off point on R_2-axis) also decreases, because for these values of γ the assigned power to the transmitter, that is, $P = \gamma P_{\rm T}$, is substantially reduced and the link from the transmitter to the relay becomes weak. The weaker the link from the transmitter to the relay is, the smaller gain can be achieved by using the relay, although a larger P_1 is assigned to the relay. Note that since for each γ an achievable rate region can be obtained, the

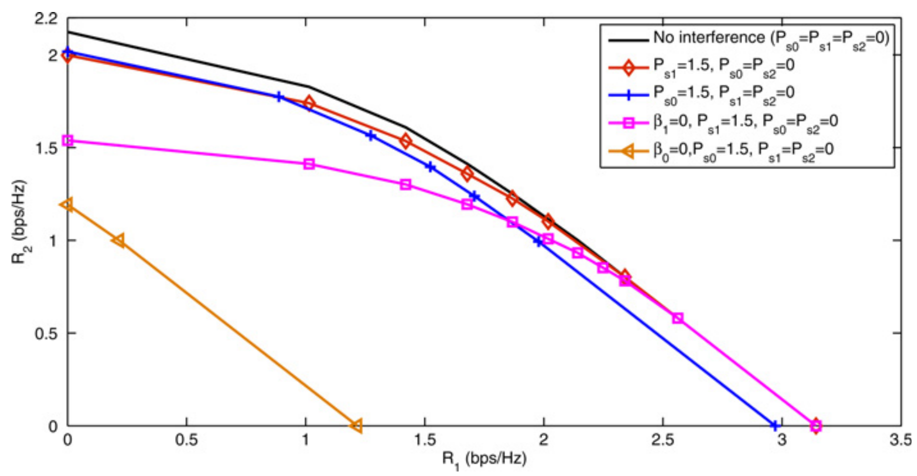

Fig. 6 *Achievable rate region $\mathcal{R}_2^*$ for $P = 7$, $P_1 = 3$, $N_1 = 0.1$, $N_2 = 1$, and different values of P_{s0}, P_{s1} and P_{s2}*

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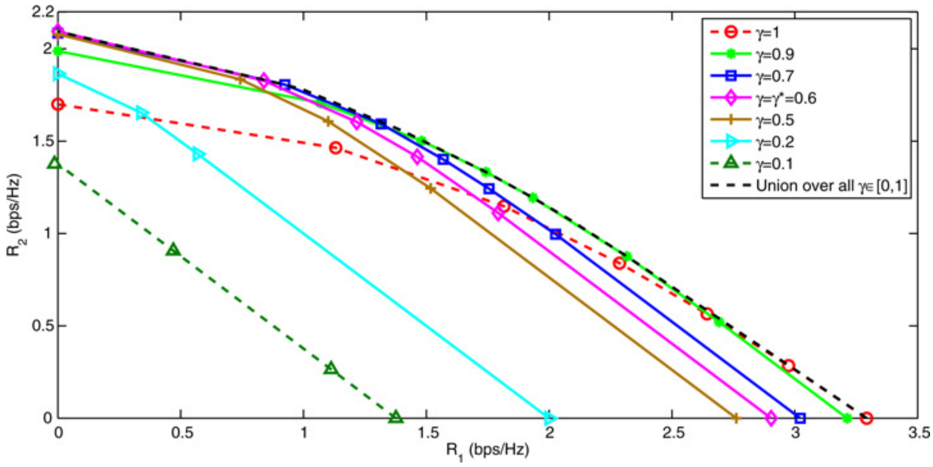

Fig. 7 *Achievable rate region $\mathcal{R}_1^*$ under total power constraint, for $P_T = 10$, $N_1 = 0.1$, $N_2 = 1$, $P_{s0} = 0.5$ and $P_{s1} = P_{s2} = 0$*

union of these derived regions is also achievable. So, we let γ get all values of the interval [0, 1]. Then, taking the union of all $\mathcal{R}_1^*$ derived by varying $\gamma \in [0, 1]$, an achievable rate region is established for the Gaussian PC-RBC with additive interference which is denoted as 'Union over all $\gamma \in [0, 1]$' in Fig. 7.

6 Conclusion

In this study, we investigated PC-RBC and FC-RBC with partial causal CSI. We derived an achievable rate region for the general discrete memoryless PC-RBC with causal CSI, and established the capacity region of the degraded case. We also derived the capacity region of discrete memoryless PC-RBC with feedback and causal CSI, and showed that feedback does not increase the capacity region of the degraded channel. Similarly, we derived an achievable rate region for the general FC-RBC with causal CSI and established the capacity region for the physically degraded case. The capacity region of the discrete memoryless FC-RBC with feedback and causal CSI was also established, and it was shown that feedback does not increase the capacity region of the degraded channel. Also, we investigated the Gaussian PC-RBC and FC-RBC with additive interference which is modelled as the channel state, and we established achievable rate regions for these channels. Providing numerical examples, we investigated the effects of knowing the additive interference, user cooperation, feedback and the usage of compensation strategy in these channels.

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