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Decorrelating closely spaced antennas by pattern design in uniform scattering environments

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Abstract: The authors propose a novel solution for decorrelating closely spaced antennas with identical radiation patterns. By proper steering of the directional elements' patterns, the decorrelation distance is reduced to 0.25λ in comparison with the well-established 0.4λ result based on the Bessel-shaped correlation expression. In addition, by proposing the definition of Decorrelation Factor of an antenna pattern, a framework for comparison of correlation behaviour of different antennas is provided. Subsequently, a performance metric characterised by the Jensen upper bound for the capacity of closely spaced antennas is proposed. Finally, we apply our results to two practical directional antennas, namely, the half-wavelength dipole and the vertical electric dipole antennas and compare their correlation and capacity performance.

1 Introduction

MIMO communication techniques have proven to be promising tools for providing spectrally efficient communication systems [1]. However, a serious limitation for these systems is the correlation between individual subchannels [2], which can cause serious deficiencies in system performance [3]. This fact has motivated many researchers to mitigate the effect of correlation on MIMO system performance.

In this regard, a large part of the literature has been focused on the role of antenna array physics on the elements' correlation properties and MIMO system performance. Some works such as [4] and [5] have considered the effects of different polarisations and patterns of MIMO elements on the capacity. In addition, the role of antenna patterns on the correlation coefficients has been investigated in [6], whose results are supported by measured data. Furthermore, it has been shown that the interaction of the antennas' pattern and the environment's scattering characteristics has significant effect on the cross correlation function (CCF) of the MIMO channels in two-dimensional [7] and three-dimensional MIMO channels [8]. In [9], it is shown that by using directional antennas the close proximity of the array elements can be compensated in order to maintain low correlation between antenna elements. Also, the effect of using directional antennas on the MIMO capacity in low SNR regime is discussed in [10].

One approach to decrease the correlation of antenna elements by pattern design is to use orthogonal patterns to achieve pattern diversity in MIMO systems [11, 12]. The concept of pattern diversity in comparison with spatial and polarisation diversity is thoroughly investigated in [11]. In [13], different modes of circular patch antennas are excited to provide pattern diversity in indoor clustered MIMO channels, which is shown to be advantageous in terms of system capacity.

In this paper, we investigate the problem of decorrelating closely spaced directional antennas for isotropic scattering environments where antennas with identical patterns and the same steering angles are employed. Firstly, we review the MIMO system model and introduce the ergodic capacity as a measure of system performance. The statistical behaviour of capacity depends on the correlation matrices determined by the physical characteristics of the environment as well as the antenna radiation patterns. Next, we observe that in isotropic scattering environments, the correlation coefficients of the antenna elements are the main factors affecting maximum system capacity. Consequently, by assuming identical patterns for MIMO elements, we propose a steering scheme for the patterns which results in reduced correlation behaviour, and in turn increases the ergodic capacity. In fact, our proposal provides considerable performance enhancement in the spatial diversity context, apart from the orthogonal solutions introduced in the pattern diversity literature. Then, a metric for measuring the decorrelation capability of directional patterns in the regime of low antenna distance is defined, based on which we derive an approximate expression for an upper bound on capacity. Finally, the analysis is supported by numerical examples.

2 MIMO system model and ergodic capacity

Consider a MIMO communication system consisting of m transmit and n receive antennas. The MIMO channel is assumed to be linear and is modelled by the following equation

$$y = Hx + z \tag{1}$$

where x is the $m \times 1$ transmitted symbols vector, y is the $n \times 1$ received symbols vector, H is the $n \times m$ channel matrix and z is the $n \times 1$ additive white Gaussian noise vector.

As a well-established simplification for the channel model, we will consider the correlation of the channels to be composed of the transmitter and receiver correlations which are physically independent [14]. Consequently, we can write [3]

$$H = R_r^{1/2} H_w R_t^{1/2} \tag{2}$$

where H_w is a matrix of complex Gaussian fading correlation, and R_r and R_t are the receive and transmit correlation matrices, respectively. The element in the ith row and the jth column of R_r and R_t corresponds to the correlation coefficient of ith and jth receive and transmit antennas, respectively. Also, the average received power of ith antenna is the ith diagonal element of R_r. As shown in the next section, the receive and transmit correlation coefficients depend on the receiver antennas' distances, their patterns, and the distribution of scatterers in space.

A widely used MIMO system performance metric is the system's ergodic capacity introduced by Telatar [15]. It is well-established that the information theoretic capacity of a randomly changing MIMO channel (also called the ergodic capacity) when the channel state information (CSI) is only present at the receiver is formulated as follows [16]

$$C_e = E_H\left\{\log_2 \det\left[I + \left(\frac{\rho}{n}\right) HH^H\right]\right\} \tag{3}$$

where H is the channel matrix, ρ is the signal to noise ratio, n is the number of receive antennas and the expectation is with respect to the channel distribution. Equation (3) is applicable when we assume that the communication is carried out over the burst duration, which is short enough such that the channel remains constant during a burst transmission, but is long enough such that we can assume employing a long code to achieve the capacity [2]. By substituting (2) into (3) the ergodic capacity expression can be written as

$$C_e = E_H\left\{\log_2 \det\left[I + \left(\frac{\rho}{n}\right) R_r H_w^H R_t H_w\right]\right\} \tag{4}$$

3 Decorrelating directional antennas in isotropic scattering environments

We consider a 2×2 MIMO system and will focus on the receiver side antenna design. Thus, we will assume the transmitter side to be correlation-free with the correlation matrix equal to the identity matrix. For notation simplicity we will subsequently denote R_r by R. At the receiver side, correlation of two antennas placed apart at the distance Δ and with the same power gain pattern, is given by

$$r = 2\pi \int_0^{2\pi} e^{j2\pi\Delta/\lambda \cos(\theta)} G(\theta) f_\Theta(\theta)\, d\theta \tag{5}$$

where $f_\Theta(\theta)$ is the probability distribution function (p.d.f.) of the arrived signal from the scatterers around the antennas [7], $G(\theta)$ represents the antennae's pattern, and λ is the transmission wavelength. In isotropic scattering environment, by using omnidirectional antennas, (5) reduces to $J_0(2\pi\Delta/\lambda)$. (To arrive at this result we need to have $\theta = 1/2\pi$ which when taken outside the integral it results in the classic bessel integral.) By assuming an isotropic scattering environment and using directional antenna elements, the absolute value of the correlation is

$$\begin{aligned} |r| &= \left| \int_0^{2\pi} e^{j2\pi\Delta/\lambda \cos(\theta)} G(\theta)\, d\theta \right| \\ &= \left(\left| \int_0^{2\pi} \cos\left(2\pi \frac{\Delta}{\lambda} \cos(\theta)\right) G(\theta)\, d\theta \right|^2 \right. \\ &\quad \left. + \left| \int_0^{2\pi} \sin(2\pi \frac{\Delta}{\lambda} \cos(\theta)) G(\theta) d\theta \right|^2 \right)^{1/2} \\ &= ((r_1)^2 + (r_2)^2)^{1/2} \end{aligned} \tag{6}$$

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where the diagonal elements of $\boldsymbol{R}$ are unity. Consequently, the capacity-maximising design is equivalent to the correlation-minimising design in isotropic scattering environments. The problem, which needs to be addressed is as follows: what is the optimum $G(\theta)$ which minimises $|r|$ constrained to $\int G(\theta)\mathrm{d}\theta = 1$ (total power radiated constraint)?

In order to answer this question, we propose a solution which first sets r_2 equal to zero and then minimises r_1. We restrict the choice of antenna patterns to those having a symmetrical backlobe

$$G(\theta + \pi) = G(\theta) \tag{7}$$

In other words, we look for antennas, which have the same backlobe and mainlobe patterns. By this assumption, we will have

$$r_2 = \int_0^{2\pi} \sin\left(2\pi\frac{\Delta}{\lambda}\cos(\theta)\right)G(\theta)\,\mathrm{d}\theta = 0 \tag{8}$$

which implies that the phase correlation vanishes by this method. Thus, the $|r|$ minimisation problem introduced in (6) reduces to the minimisation of

$$\begin{aligned}|r| &= |r_1| \\ &= \left|\int_0^{2\pi} \cos\left(2\pi\frac{\Delta}{\lambda}\cos(\theta)\right)G(\theta)\,\mathrm{d}\theta\right|\end{aligned} \tag{9}$$

where we have the assumption (7) on the antenna pattern. The pattern which minimises this expression is

$$G_{\mathrm{opt}} = \frac{1}{2}\{\delta(\theta - \theta_m) + \delta(\theta - (\theta_m + \pi))\} \tag{10}$$

where

$$\theta_m = \begin{cases} 0, & \dfrac{\Delta}{\lambda} < \dfrac{1}{4} \\ \arccos\left(\dfrac{1}{4(\Delta/\lambda)}\right), & \dfrac{\Delta}{\lambda} \geq \dfrac{1}{4}\end{cases} \tag{11}$$

and $\delta(.)$ is the Dirac Delta Function. Suppose we temporarily forget about the assumption in (7). The pattern function minimising $|r_1|$ is the one steered towards the angle minimising $\cos(2\pi\Delta/\lambda\cos(\theta))$, which is the one stated in (11). In other words, $G(\theta) = \delta(\theta - \theta_m)$ minimises $|r_1|$. Now by invoking the assumption in (7), which results in the pattern in (10), the minimisation of $|r|$ reduces to the minimisation of $|r_1|$, while the restriction does not harm the minimum value for $|r_1|$. That is because

$$\begin{aligned}|r_1| &= \left|\int_0^{2\pi} \cos\left(2\pi\frac{\Delta}{\lambda}\cos(\theta)\right)G_{\mathrm{opt}}(\theta)\,\mathrm{d}\theta\right| \\ &= \left|\frac{1}{2}\left[\cos\left(2\pi\frac{\Delta}{\lambda}\cos(\theta_m)\right) + \cos\left(2\pi\frac{\Delta}{\lambda}\cos(\theta_m + \pi)\right)\right]\right| \\ &= \left|\cos\left(2\pi\frac{\Delta}{\lambda}\cos(\theta_m)\right)\right|\end{aligned} \tag{12}$$

Therefore the pattern restriction introduced in (7) is just a mathematical trick and thus does not restrict the minimisation result. Equation (10) suggests that the optimum pattern is the one, which receives half of the power at θ_m and another half is received from direction $\theta_m + \pi$.

By employing this pattern, the correlation expression will be given by

$$r_{\mathrm{opt}} = \begin{cases} \cos\left(2\pi\dfrac{\Delta}{\lambda}\right), & \dfrac{\Delta}{\lambda} < \dfrac{1}{4} \\ 0, & \dfrac{\Delta}{\lambda} \geq \dfrac{1}{4}\end{cases} \tag{13}$$

However, the result in (13) is valid for antennas having unlimited directivity. For practical antennas having limited directivity, by using (9) we have the following expression for the correlation

$$r = \int_0^{2\pi} G_0(\theta - \theta_{\mathrm{opt}})\cos\left(2\pi\frac{\Delta}{\lambda}\cos(\theta)\right)\mathrm{d}\theta \tag{14}$$

where θ_{opt} is defined in (11) and $G_0(\theta)$ represents the pattern of the antenna with the main beam directed toward $\theta = 0$. This proposed pattern steered toward a specific angle can be provided by choosing appropriate antenna elements and proper elements orientation while installing the antennas on the transceiver. The schematic configuration of the proposed scheme is illustrated in Fig. 1.

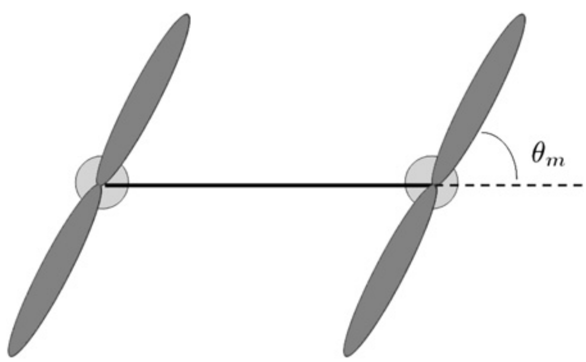

Figure 1 *Proposed element steering configuration*

It should be noted that the proposed scheme provides spatial diversity which is also enhanced by directional patterns. Therefore if we collocate the two antennas, they will be completely correlated because of their similar patterns. This is in contrast with pattern diversity solutions in which collocated antennas will benefit from the difference in their patterns in order to provide diversity.

4 Decorrelation analysis

In this section, we analyse the results of the previous section in the $\Delta/\lambda \ll 1$ regime. We have derived the correlation expression in (13) for optimal antennas (which we refer to as $r_{\rm opt}$), the correlation expression in (14) for practical antennas (which we refer to as r) and the expression $r_0 = J_0(2\pi\Delta\lambda)$ for the correlation of omnidirectional antennas in isotropic environment. In low Δ/λ regime, by Taylor expansion of (14) around $\Delta/\lambda = 0$, we will obtain ($f(x) = O(g(x))$ if $0 \le \lim_{x\to 0} f(x)/g(x) < \infty$.)

$$r_0 = 1 - \pi^2\left(\frac{\Delta}{\lambda}\right)^2 + O\left(\left(\frac{\Delta}{\lambda}\right)^3\right) \tag{15}$$

$$r_{\rm opt} = 1 - 2\pi^2\left(\frac{\Delta}{\lambda}\right)^2 + O\left(\left(\frac{\Delta}{\lambda}\right)^3\right) \tag{16}$$

$$r = 1 - \left\{2\pi^2\int_0^{2\pi} G_0(\theta)\cos^2(\theta)\,{\rm d}\theta\right\}\left(\frac{\Delta}{\lambda}\right)^2 + O\left(\left(\frac{\Delta}{\lambda}\right)^3\right) \tag{17}$$

By defining the decorrelation factor (denoted by α) as

$$\alpha = \lim_{\Delta/\lambda\to 0}\left(\frac{1-r}{(\Delta/\lambda)^2}\right) \tag{18}$$

we will have

$$\begin{cases} \alpha_0 = \pi^2 \\ \alpha_{\rm opt} = 2\pi^2 \\ \alpha = 2\pi^2\int_0^{2\pi} G_0(\theta)\cos^2(\theta){\rm d}\theta \end{cases} \tag{19}$$

It should be noted that

$$\pi^2 \le \alpha \le 2\pi^2 \tag{20}$$

where α reduces to α_0 and $\alpha_{\rm opt}$ in its extreme cases.

Therefore thanks to the definition of decorrelation factor in (18) in the case of closely spaced antennas, we obtain a measure of how much a directional antenna with pattern $G_0(\theta)$ can reduce the correlation.

We can further simplify α in the following way

$$\begin{aligned}\alpha &= \left\{2\pi^2\int_0^{2\pi} G_0(\theta)\cos^2(\theta)\,{\rm d}\theta\right\} \\ &= \pi^2\left(1 + \int_0^{2\pi} G_0(\theta)\cos(2\theta)\,{\rm d}\theta\right) \\ &= \pi^2(1 + 2\pi\mathcal{G}_2)\end{aligned} \tag{21}$$

where $\mathcal{G}_k = 1/2\pi\int_0^{2\pi} G_0(\theta)\cos(k\theta){\rm d}\theta$ are the Fourier series coefficients of $G_0(\theta)$, which is a periodic function with the period of 2π.

In order to emphasise the importance of the decorrelation factor in the MIMO system performance, we can use the Jensen's inequality for deriving a metric for the performance of MIMO systems, which has been previously proposed in the literature [16]

$$\begin{aligned} C_{\rm e} &\le C_{\rm J} \\ &= \log_2\det\left[I + \left(\frac{\rho}{n}\right)E_{\rm H}\{\boldsymbol{H}\boldsymbol{H}^{\rm H}\}\right] \\ &= \log_2\det\left[I + \left(\frac{\rho}{n}\right)\boldsymbol{R}\right]\end{aligned} \tag{22}$$

using the concavity of log det function and where $\boldsymbol{R} = E_{\rm H}\{\boldsymbol{H}\boldsymbol{H}^{\rm H}\}$ is the correlation matrix of the receiver. For the 2×2 MIMO system and small values of Δ/λ, we will obtain

$$\boldsymbol{R} \simeq \begin{pmatrix} 1 & 1-\alpha\left(\frac{\Delta}{\lambda}\right)^2 \\ 1-\alpha\left(\frac{\Delta}{\lambda}\right)^2 & 1\end{pmatrix} \tag{23}$$

By substituting (23) into (22) and assuming small values of Δ/λ, we will derive

$$C_{\rm J} \simeq \log_2\left(1 + 2\frac{\rho}{n} + 2\alpha\left(\frac{\rho}{n}\right)^2\left(\frac{\Delta}{\lambda}\right)^2\right) \tag{24}$$

demonstrating the important role of the decorrelation factor in the system performance.

5 Numerical illustrations

In this section, we numerically evaluate our results for some typical examples. Firstly, we consider two directional antennas namely the half-wavelength dipole and the vertical electric dipole antennas, and their field patterns. The half-wavelength dipole is a directional antenna with

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the following pattern

$$F(\theta) = \frac{\cos((\pi/2)\cos(\theta))}{\sin(\theta)} \qquad (25)$$

The field pattern of a vertical electric dipole antenna is also given by

$$F(\theta) = \sin(\theta)\left[2\cos\left(2\pi\frac{L}{\lambda}\cos(\theta)\right)\right] \qquad (26)$$

where L is the antenna size.

The main beam of these antennas is located at $\theta = \pi/2$, and consequently, we will have

$$G_0(\theta) = \frac{F(\theta - \pi/2)^2}{\int F(\theta)^2 \mathrm{d}\theta} \qquad (27)$$

Fig. 2 shows the correlation coefficient as a function of the distance between antennas normalised with respect to the carrier wavelength in isotropic scattering environment for the following four scenarios: omnidirectional antenna (resulting in the well-known Bessel-shaped correlation), the optimal antenna with unlimited directivity, the half-wavelength dipole with the field pattern given in (25) and the vertical electric dipole with the field pattern given in (26). In the case of omnidirectional antennas, the correlation function is Bessel with its first zero around $\Delta/\lambda = 0.4$. In the optimal antenna case the correlation is given in (13), where the antennas are completely uncorrelated if the distance between antennas is more than $\lambda/4$. As can be verified in this figure, the vertical electric dipole of the size 0.2λ has better correlation behaviour in comparison with the half-wavelength dipole. That is because of the higher directivity of the vertical electric dipole with respect to half-wavelength dipole as shown in Fig. 3.

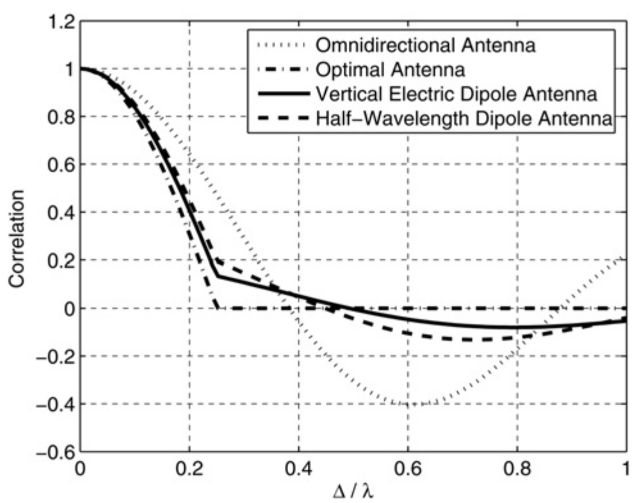

Figure 2 *Behaviour of correlation factor for four cases: omnidirectional antenna, the optimal antenna, half-wavelength dipole and the vertical electric dipole with the size of $L = 0.2\lambda$ against the distance of the antennas normalised with respect to the wavelength*

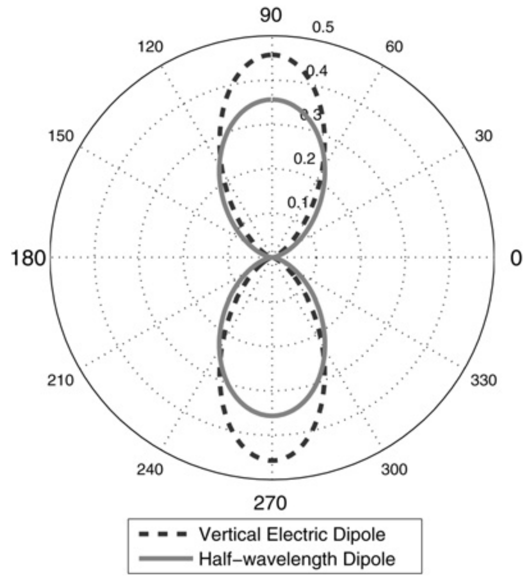

Figure 3 *Antenna pattern for the half-wavelength dipole and the vertical electric dipole*

The ergodic capacity (i.e. (3)) of the four cases mentioned above is depicted in Fig. 4. The transmitter is correlation-free with transmit SNR equal to 20 dB. As can be noted in this figure, the capacity of all cases is saturated at about 9.5 b/s/hz starting from about $\Delta/\lambda = 0.35$, despite the minor oscillatory behaviour of the omnidirectional antennas. This oscillation is because of the oscillatory behaviour of the Bessel function illustrated in Fig. 2. In addition, almost the same

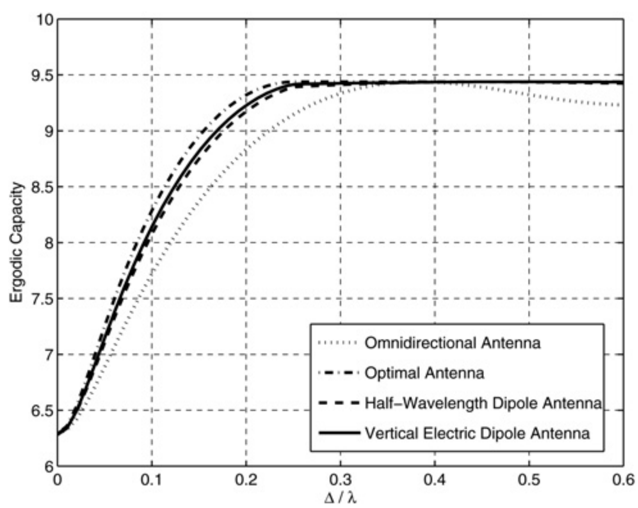

Figure 4 *Ergodic capacity for four cases: omnidirectional antenna, the optimal antenna, the half-wavelength dipole and the vertical electric dipole with the size of $L = 0.2\lambda$ against the normalised distance of the antennas*

The transmit SNR is 20 dB

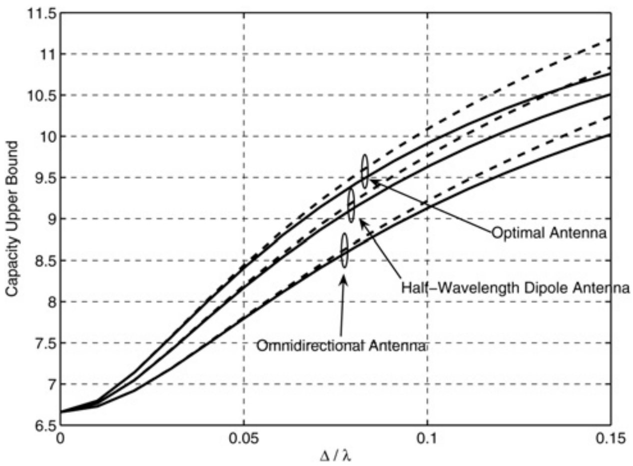

Figure 5 *Jensen upper bound (solid curves) and its approximation (dashed curves) against the normalised distance between antennas*

Three cases are illustrated: omnidirectional antenna, the optimal antenna and the half-wavelength dipole

performance can be observed for the half-wavelength and the vertical electric dipoles.

The Jensen upper bound for the capacity stated in (22) (solid curves) and the low Δ/λ regime approximation for it stated in (24) (dashed curves) are illustrated in Fig. 5. The simulation scenario is the same as in Fig. 4. This figure verifies that the approximate formula in (24) matches the bound in the case of closely spaced antennas.

6 Conclusion

This paper proposes a novel pattern design approach for the MIMO systems employing directional antenna elements. By employing identical directional patterns, we propose a steering scheme which significantly decreases the correlation of the elements. In fact, the decorrelation distance can be reduced from the 0.4λ value in the omnidirectional case to 0.25λ. In the case of very small distance between antennas, we define a new metric for measuring the decorrelation capability of the directional patterns, namely the decorrelation factor. The decorrelation factor is shown to play an important role in characterising system performance by providing an approximate expression for the capacity upper bound. Furthermore, the capacity enhancement of this method is verified through Monte Carlo simulations.

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