Generalized Differential Transfer Matrix for Fast and Efficient Analysis of Arbitrary-Shaped Nonlinear Distributed Feedback Structures

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***Abstract*—A new, fast, and efficient approach based on the differential transfer matrix idea, is proposed for analysis of nonuniform nonlinear distributed feedback structures. The *a priori* knowledge of the most-likely electromagnetic field distribution within the distributed feedback region is exploited to speculate and factor out the rapidly varying portion of the electromagnetic fields. In this fashion, the transverse electromagnetic fields are transformed into a new set of envelope functions, whereupon the numerical difficulty of solving the nonlinear coupled differential equations is partly imparted to the analytical factorization of the fields. This process renders a new set of well-behaved nonlinear differential equations that can be readily solved. Strictly periodic, linearly tapered, and linearly chirped structures are analyzed to justify the accuracy and the efficiency of the proposed method.**

***Index Terms*—Bistability, nonlinear distributed feedback structures, transfer matrix, WKB method.**

I. Introduction

NONLINEAR optics is now a half-century old discipline, which; being the subject of intensive research in theory and in experiment, has opened up new horizons and broken new grounds in the realm of photonics and optoelectronics [1]. In particular, nonlinear distributed feedback (NLDFB) structures, whose input/output characteristics depend on both the intensity and the wavelength of the incident light at free space, are of interest as they can provide a wide range of applications in optical signal processing [2], digital optics [3], bistable lasing [4], beam shaping [5], and all-optical devices including hard limiters [6], memory and logic gates [7]. It is therefore highly desirable to have a fast yet sufficiently accurate technique to analyze and possibly devise NLDFB structures in the most general case, where the strict periodicity of the refractive index variation in the distributed feedback region is arbitrarily altered. The presence of nonuniformities within the NLDFB could be either due to the unintentional imperfections in the fabrication process, or to the intentional introduction of chirp, taper, and/or phase shift into the formerly fully periodic refractive index profile, which is by the way proved to be quite beneficial in the design stage [8].

Strictly periodic NLDFB structures have been already analyzed in the time [9], [10] and frequency domains [11]–[13]. Thanks to the nonlinear coupled mode theory, closed form expressions in terms of Jacobian elliptic functions are now available for the electromagnetic field distribution within the nonlinear distributed feedback grating in the continuous wave regime, whereupon the electromagnetic reflectance and transmittance are both obtained [12]. Non-periodic but arbitrarily inhomogeneous NLDFB structures; on the other hand, cannot be analyzed but through the numerical computation and/or approximative methods, which has been differently performed since at least the past two decades [14], [15]. The simplest and most systematic approach toward numerical simulation of the structure is however based on the transfer matrix method, where the whole structure is divided into a set of successive segments each represented by a transfer matrix [11], [13], [16].

The earliest attempt to generalize the transfer matrix method for analyzing nonlinear almost-periodic structures was made by Radic *et al.* They employed the analytic field distribution within the strictly periodic NLDFB to obtain the appropriate transfer matrix for every one of the consecutive strictly periodic segments that together makes up, or, to put it more accurately, approximates the whole. Each transfer matrix, originally being referred to as the generalized transfer matrix (GTM), then depends on both the wavelength and the intensity of the light and can be easily obtained by using the backward solution idea to avoid the necessity of applying any iterative procedure. This appealing strategy; together with most of the other similar techniques [16], however, has a two-fold weakness. First, the non-periodic structure is in real fact approximated by being decomposed into successive segments of fully periodic profiles. Second, electromagnetic fields within each of the approximate strictly-periodic segments are still further approximated by relying upon the standard assumptions of nonlinear coupled mode theory, i.e., moderate coupling strengths and slowly varying envelopes. These restrictions were later dismissed in [11], where analytical yet approximate distribution of electromagnetic fields was substituted for the exact yet more complicated numerical solution, i.e., the fourth-order Runge-Kutha method based on the backward difference stencil. This backward approach sidesteps the shooting procedure and requires but a single integration.

In the present study; however, a new semi-analytical algorithm based on the differential transfer matrix idea is proposed [17]. The transverse electromagnetic fields are then expanded in terms of WKB-like wave-functions. These functions can partly factor out the rapid phase and amplitude variations, and result in a new set of envelope functions with slowly varying spatial distributions, which surrogate the transverse electromagnetic fields. In this fashion, the *a priori* knowledge of the most-

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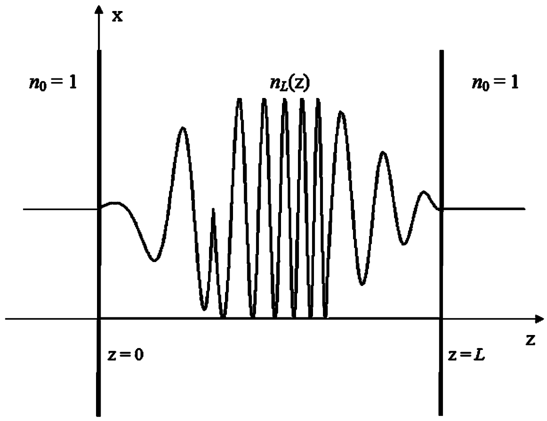

Fig. 1. Schematic of the considered NLDFB with arbitrary refractive index profile $n_L(z)$.

likely electromagnetic field distribution is exploited to heuristically factor out the numerically troublesome variations. This strategy results in a more appropriate formulation, which can relieve the numerical burden of solving the Maxwell's equations, particularly in the case of strongly nonlinearity.

The structure of this paper is as follows. In Section II, the basic formulation of the proposed technique is presented. In Section III, some plausible sets of functions are proposed to factorize the rapidly varying portion of the electromagnetic fields. In Section IV, different numerical examples are presented. Finally, conclusions are made in Section V.

II. Formulation

The schematic of the structure to be analyzed is shown in Fig. 1, where a non-magnetic one-dimensional longitudinally nonuniform NLDFB structure of thickness L (region 2) is sandwiched between two linear dielectric regions of refractive index n_0 (regions 1, and 3). It is assumed that the structure is uniform in the transverse x and y directions, and is illuminated by a normal incident plane wave with angular frequency ω.

The incident electric field phasor is polarized in the y direction and can be written as

$$\vec{E}_i = \hat{y} E_{i0} \exp(-j(k_0 z - \phi_i)) \tag{1}$$

where $k_0 = \omega(\epsilon_0 \mu_0)^{0.5}$ denotes the free space wavenumber, E_{i0} is the amplitude, and ϕ_i stands for the complex phase of the incident wave.

Similarly, the reflected and transmitted waves can be respectively expressed as

$$\vec{E}_r = \hat{y} E_{r0} \exp(-j(k_0 z - \phi_r)) \tag{2}$$

for $z < 0$ in region 1, and

$$\vec{E}_t = \hat{y} E_{t0} \exp(-j(k_0 z - \phi_t)) \tag{3}$$

for $z > L$ in region 3.

Along the same lines, E_{r0}, E_{t0}, ϕ_r, and ϕ_t, represent the reflected amplitude, the transmitted amplitude, the reflected phase, and the transmitted phase, respectively. Here, the incident phase $\phi_i = 0$ is taken as the reference value to determine the yet unknown ϕ_r and ϕ_t.

The intensity dependent refractive index profile in the Kerr-type nonlinear region, on the other hand, can be written down as

$$\varepsilon(z, |E_{2y}(z)|) = \varepsilon_0 \left[\varepsilon_{rL}(z) + n_0 n^{(2)} \left| E_{2y}(z) \right|^2 \right] \tag{4}$$

where $n^{(2)}$ is the effective nonlinear Kerr index, $E_{2y}(z)$ denotes the transverse electric field in the region, and $\epsilon_{r_L}(z) = n_L^2(z)$ represents the longitudinally nonuniform linear relative permittivity profile, which is here assumed to be an arbitrary function of z.

The transverse electromagnetic fields within the distributed feedback region, i.e., $E_{2y}(z)$ and $H_{2x}(z)$, are then subjected to the Maxwell's equations

$$\frac{d}{dz} \begin{bmatrix} E_{2y} \\ H_{2x} \end{bmatrix} = \mathbf{W}(z) \begin{bmatrix} E_{2y} \\ H_{2x} \end{bmatrix} \tag{5a}$$

$$\mathbf{W}(z) = \begin{bmatrix} 0 & j\omega\mu_0 \\ j\omega\varepsilon_0\varepsilon & 0 \end{bmatrix}. \tag{5b}$$

These transverse electromagnetic fields are however rapidly varying functions and should be obtained through meticulous numerical integration, e.g., fourth order Runge-Kuta method. Here, we substitute the transverse electromagnetic fields for a new set of unknown functions, $A^+(z)$ and $A^-(z)$, where the rapidly varying portion of the fields, $f^+(z)$ and $f^-(z)$, are factored out

$$\begin{bmatrix} E_{2y} \\ H_{2x} \end{bmatrix} = \mathbf{S}(z) \begin{bmatrix} A^+(z) \\ A^-(z) \end{bmatrix} \tag{6a}$$

$$\mathbf{S}(z) = \begin{bmatrix} f^+(z) & f^-(z) \\ \frac{1}{j\omega\mu_0} \frac{df^+(z)}{dz} & \frac{1}{j\omega\mu_0} \frac{df^-(z)}{dz} \end{bmatrix}. \tag{6b}$$

The most appropriate choice of $f^+(z)$ and $f^-(z)$ factor functions; which should be made by exploiting the *a priori* knowledge of the most plausible field distribution, is discussed in Section III. Once this choice is duly made; however, the $S(z)$ matrix is determined and can be employed to transform the Maxwell's equations into a new set of coupled ordinary differential equations, where

$$\frac{d}{dz} \begin{bmatrix} A^+ \\ A^- \end{bmatrix} = \mathbf{T}(z) \begin{bmatrix} A^+(z) \\ A^-(z) \end{bmatrix} \tag{7a}$$

$$\mathbf{T}(z) = \mathbf{S}^{-1}\mathbf{W}\mathbf{S} - \mathbf{S}^{-1}\frac{d\mathbf{S}}{dz}. \tag{7b}$$

In this fashion, a considerable part of the difficulty in numerically finding the rapidly varying transverse electromagnetic fields in (5a) and (5b) is imparted to the procedure of making the fitting choice of $f^+(z)$ and $f^-(z)$ factors, which can render the new set of unknown functions $A^+(z)$ and $A^-(z)$ into an easily obtainable set of slowly varying functions. This is numerically demonstrated in Section IV, where a couple of study cases are presented. It should be however noticed that no specific approximation is to be made in following the proposed approach as even an ill-suited choice of, $f^+(z)$ and $f^-(z)$ factors does not introduce any external error but further confounds the numerical integration. This is in contrast to the case of most of the other techniques, which are usually based on the standard nonlinear coupled mode theory and substantially employ the slowly varying envelope approximation.

As already mentioned, the backward approach in solving the nonlinear boundary value problem is adopted to avoid the necessity of multiple integrations. The $A^+(z)$ and $A^-(z)$ functions can be easily written as

$$\begin{bmatrix} A^+(L) \\ A^-(L) \end{bmatrix} = \mathbf{S}^{-1}(L) \begin{bmatrix} E_{2y}(L) \\ H_{2x}(L) \end{bmatrix}. \tag{8}$$

The transverse electromagnetic fields at $z = L$ then represent the transmitted wave in region 3

$$E_{2y}(L) = E_{t0} \tag{9}$$

$$H_{2x}(L) = \frac{-\beta_t}{\omega\mu_0} E_{t0}. \tag{10}$$

Once the amplitude of the transmitted wave, i.e., E_{t0}, is set, the backward approach can be employed to solve the propagation equations, find the electromagnetic field distribution for $0 < z < L$, and dodge the otherwise necessary iterative procedure [18]. Applying the boundary conditions at $z = 0$ then results in

$$E_{2y}(0) = E_{i0}\exp(-j\phi_t) + E_{r0}\exp(j(-\phi_t + \phi_r)) \tag{11}$$

$$H_{2x}(0) = -\frac{\beta_i}{\omega\mu_0} E_{i0}\exp(-j\phi_t) + \frac{\beta_r}{\omega\mu_0} E_{r0}\exp(j(-\phi_t + \phi_r)). \tag{12}$$

where upon the overall reflectance and transmittance can both be obtained

$$|R|^2 = \left|\frac{E_{r0}}{E_{i0}}\right|^2 \tag{13}$$

$$|T|^2 = \left|\frac{E_{t0}}{E_{i0}}\right|^2. \tag{14}$$

III. Plausible Factor Functions

In this section, three different sets of $f^+(z)$ and $f^-(z)$ factors are introduced to factorize the rapidly varying part of the transverse electromagnetic fields. These factor functions are plausibly based on the physical intuition of how electromagnetic waves propagate in the distributed feedback region. First, the static field approximation [19] is employed to anticipate the phase variation of the propagating fields within the distributed feedback region. The $f^+(z)$ and $f^-(z)$ functions are then respectively representing forward and backward propagating waves in the effective medium approximation of the original inhomogeneous structure. Second, linear WKB wave-functions are propounded to reasonably speculate and factorize the most rapid phase and amplitude variations of the transverse electromagnetic fields. Third, the WKB-based forward and backward propagating factors are substituted for quadrature phase standing waves, which practically show a similar performance.

Although each set of the abovementioned factor functions is originally based on a specific physical approximation, the resultant system of differential equations is mathematically equivalent to the system of Maxwell's equations. In other words, the inaccuracies of $f^+(z)$ and $f^-(z)$ factor functions are imparted to the slowly varying and easily obtainable $A^+(z)$ and $A^-(z)$ functions.

A. Static Field Factorization

In accordance with the zeroth order linear effective medium theory, i.e., static field approximation [22], an inhomogeneous structure whose refractive index profile has subwavelength nonuniformities can be effectively substituted for a homogeneous medium whose uniform permittivity profile is the spatial average of that of the original inhomogeneous structure. Therefore, at least in case the wavelength of light is larger than the bandwidth of the wavevector spectrum of the modulated refractive index profile of the DFB structure [20], the following functions can correctly represent the phase variation of the transverse electromagnetic fields

$$f^+(z) = \exp(-jk_0 n_{av} z) \tag{15a}$$

$$f^-(z) = \exp(jk_0 n_{av} z) \tag{15b}$$

where $k_0 = \omega\sqrt{\mu_0\varepsilon_0}$ and

$$n_{av}^2 = \frac{1}{L}\int_0^L n_L^2(z)dz. \tag{16}$$

This set of factor functions is quite simple and does not impose any analytical burden in calculation of the $T(z)$ matrix; yet, it does not result in a significant simplification of the numerical computation particularly if the wavevector spectrum has *a rich* harmonic content.

B. WKB Factorization

Insofar as the $f^+(z)$ and $f^-(z)$ factor functions are to *a priori* estimate the electromagnetic behavior in inhomogeneous media, the WKB wave-functions counts among the most reasonable candidates to analytically approximate the electromagnetic fields [21]. Therefore, the following set of factor functions can be proposed:

$$f^+(z) = \frac{1}{\sqrt{k_0 n_L(z)}}\exp\left(-j\int_0^z k_0 n_L(z')dz'\right) \tag{17a}$$

$$f^-(z) = \frac{1}{\sqrt{k_0 n_L(z)}}\exp\left(j\int_0^z k_0 n_L(z')dz'\right) \tag{17b}$$

which recasts the transverse electromagnetic fields as nonuniform plane waves with phase fronts which are properly shaped.

Invocation of the abovementioned WKB approximation is already proved to be numerically effective whenever linear but inhomogeneous structures are to be analyzed [22]. It is then expected to be useful in working out the nonlinear wave propagation within the NLDFB structure, where the nonlinear effect is by and large a weak perturbation that cannot significantly contribute in the rapid variation of the electromagnetic fields [23]. Consequently, the proposed set of factor functions is quite effective if its corresponding $\mathbf{T}(z)$ matrix is to be numerically integrated in place of the original $\mathbf{S}(z)$ matrix whose difficulty of integration is partly projected to the derivation of the $\mathbf{T}(z)$

matrix. This approach however incurs an analytical burden particularly if complicated refractive index profiles are involved.

C. Standing Wave Factorization

In both of the abovementioned methods, different forms of forward and backward propagating waves are proposed to anticipate the rapid variation of electromagnetic fields. It is however possible to employ quadrature phase standing waves, i.e., superposition of forward and backward propagating waves, to factorize the rapidly varying portion of the waveforms

$$f^{+}(z)=\frac{1}{\sqrt{k_0 n_L(z)}}\cos\left(\int_0^z k_0 n_L(z')dz'\right) \quad (18a)$$

$$f^{-}(z)=\frac{1}{\sqrt{k_0 n_L(z)}}\sin\left(\int_0^z k_0 n_L(z')dz'\right). \quad (18b)$$

Although it seems to be equivalent to the previously proposed WKB factorization, the numerical integration of its corresponding $\mathbf{T}(z)$ matrix may exhibit a different performance. This difference is due to the nonlinearity of the equations, which in most cases, however, is not strong enough to cause a considerable change in the performance of either of the last two factorization techniques.

IV. Numerical Examples

In this section, we consider a general NLDFB structure with a uniform effective Kerr index $n^{(2)}$, and the following linear refractive index profile

$$n_L(z)=n_0+n_1(z)\cos(2\beta_B z). \quad (19)$$

The intensity of light is also normalized to the so called critical intensity $|E_C|^2=n_0\lambda_0/(0.75\pi n^{(2)}L)$ [11].

As the first numerical example, a perfectly sinusoidal NLDFB structure operating at the free space wavelength λ_0 is considered. The constitutive parameters in accordance with Fig. 1, and (19), are: $L=100\lambda$, $n_0=1$, $\beta_B(z)=\beta_{B0}=2\pi/\lambda_{B0}$, and $n_1(z)=2n_0\kappa_0/\beta_0$, where $\Lambda=\lambda_0/2$ denotes the fundamental grating period and $\kappa_0 L=2.5$ is the average coupling strength within the distributed feedback region. The intensity dependent transmittivity of this structure at the normalized input intensity of $I_0=1.13$ has two stable values, viz. T_1, and T_2, which can be calculated by using either the here proposed WKB factorization technique, or the recently reported method of Yang *et al.*. [11], where the Maxwell's equations are directly solved to find the transverse electromagnetic field distribution. Either of the alternatives; however, results in a corresponding set of nonlinear coupled ordinary differential equations, which cannot but numerically be solved. Here, the fast yet erroneous second order Runge–Kutta method, hereafter denoted as RK2, and the slow yet accurate fourth order Runge–Kutta method, hereafter denoted as RK4, are both employed to see which of the aforementioned alternatives can yield the better behaved set of equations that can be accurately solved by using the simpler RK2 algorithm. The obtained results are tabulated in Table I,

TABLE I
Transmitivity of Perfectly Sinusoidal NLDFB Structures Computed by Using Different Methods

	Yang's Method [11] (RK2)	WKB factorization (RK2)	Yang's Method[11] (RK4)	WKB factorization (RK4)
T_1	not applicable	0.035	0.036	0.035
T_2	not applicable	0.928	0.927	0.927

where the proposed WKB factorization technique is proved to be the superior choice. Whilst the RK2 algorithm is accurate enough to handle the set of well-behaved nonlinear coupled ordinary differential equations rendered by following the proposed WKB factorization approach, it cannot be used to solve the nonlinear Maxwell's equations directly. The obtained results clearly demonstrate that whenever the rapidly varying functions are factored out of the numerical computation, the accurate results can be found at a lower computational cost.

This point is further elucidated in Figs. 2–3, where the real and imaginary parts of the rapidly varying transverse electromagnetic fields and that of the slowly varying $A^{+}(z)$ and $A^{-}(z)$ functions are respectively plotted. The former, which is to be found in following the Yang *et al.* method [11], is hard to be numerically solved, but the latter, which is the here introduced envelope function, can be more readily obtained.

As another example, a linearly chirped NLDFB structure is considered. The values of the constitutive parameters, referenced to (19) and Fig. 1, read as: $L=100\lambda_0$, $n_0=1$, $\beta_{B0}=2\pi/\lambda_0$, $n_1(z)=2n_0\kappa_0/\beta_0$, $\kappa_0 L=2.5$, and $S=2$, where

$$\beta_B(z)=\beta_{B0}+\frac{S(z-L)}{L^2}. \quad (20)$$

The linear chirp can be used to control the hysteresis width and the switching intensity of the NLDFB.

The transmittivity of the structure is here calculated by using both the proposed WKB factorization technique and the Yang *et al.* method [11]. The obtained results are depicted in Figs. 4–5, which show the transmittivity versus the normalized input intensity. In Fig. 4, the WKB factorization technique is followed and the resultant set of governing differential equations is solved by applying the RK2 algorithm with $N=1500$ intervals (dotted line), and the RK4 algorithm with $N=500$ (dashed line) and $N=1000$ intervals (solid line). In Fig. 5, the method of Yang *et al.*. is followed and the Maxwell's equations are directly solved by using the RK2 algorithm with $N=30000$ intervals (dotted line), and the RK4 algorithm with $N=3000$ (dashed line) and $N=1000$ intervals (solid line). A comparison of these figures shows that the proposed WKB factorization technique renders a better behaved set of equations, which can be solved at a lower computational cost. As already mentioned in Section III, applying the standing wave factorization in this example is as efficient as the WKB factorization technique, and does not make a conspicuous difference in the efficiency of the whole procedure.

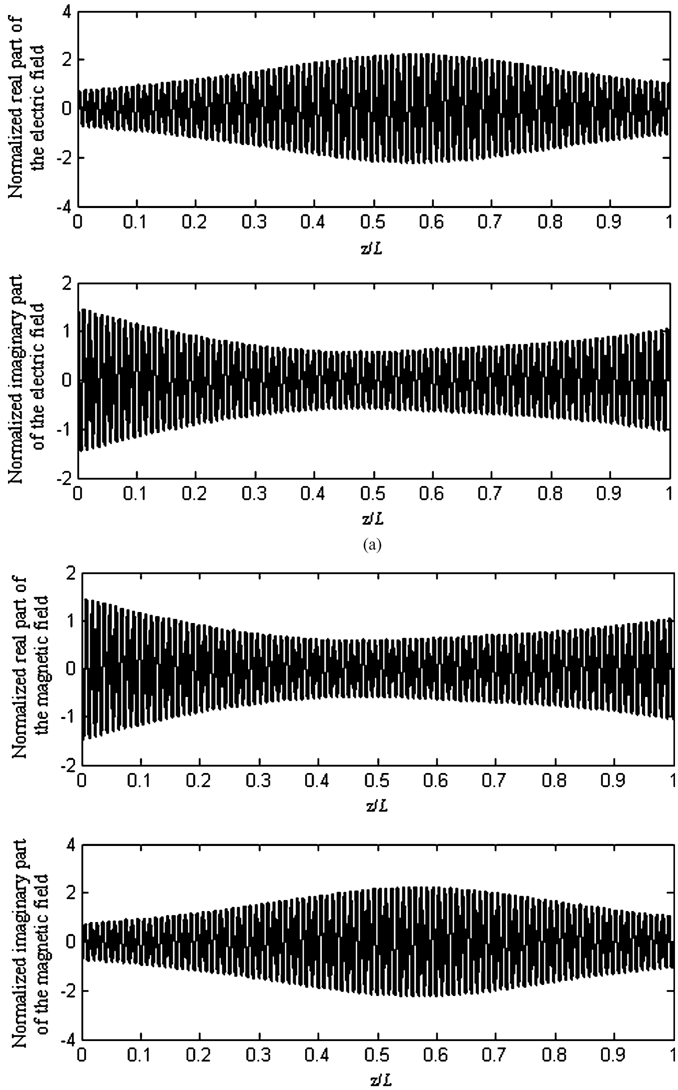

Fig. 2. (a) Real and imaginary parts of the normalized electric field versus the normalized coordinate in nonlinear region. (b) Real and imaginary parts of the normalized magnetic field versus the normalized coordinate in the nonlinear region.

The static field factorization; on the other hand, does not match either the WKB or the standing wave factorization approach.

Finally, a linearly tapered NLDFB is analyzed, where the following parameters are assumed: $L = 10\lambda_0$, $n_0 = 1$, $\beta_B(z) = \beta_{B0} = 2\pi/\lambda_0$, $\kappa_0 L = 2.5$, and

$$\kappa(z) = \kappa_0 \left[\frac{1 + \Delta\kappa \left(\frac{z-L}{2} \right)}{L} \right] = \frac{\beta_{B0} n_1(z)}{2n_0}. \tag{21}$$

Here, $\kappa(z)$ denotes the linear coupling strength and $\Delta\kappa = 1$ stands for the taper index. With these parameters, the transmittivity of the structure is once again calculated by using the proposed method with WKB factorization, and the direct approach of solving the Maxwell's equations [11]. Similarly, the linear taper of the abovementioned form can be used to adjust the hysteresis width and the transmission efficiency. Fig. 6 shows the transmittivity versus the normalized input intensity, where

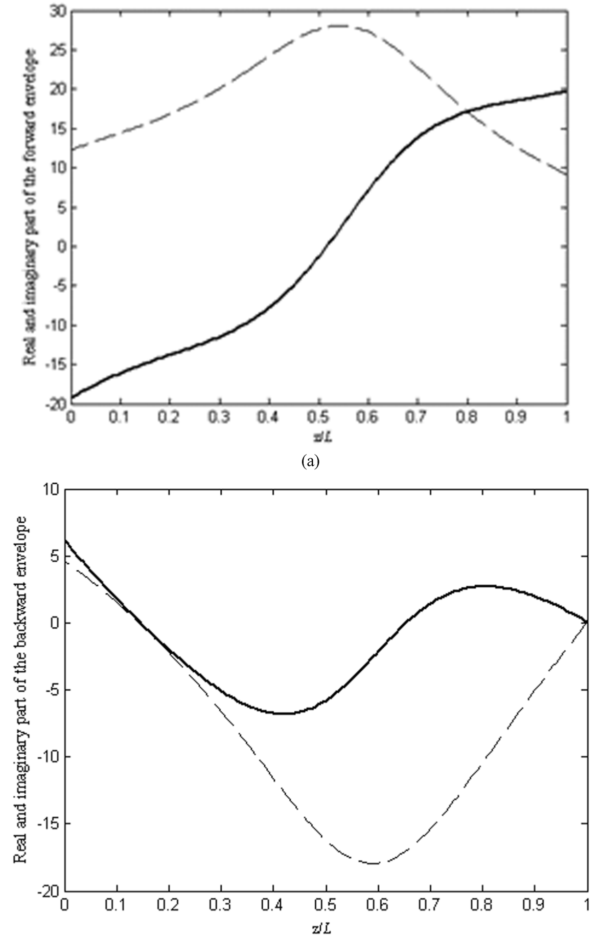

Fig. 3. (a) Real (solid line) and imaginary (dashed line) parts of the normalized $A^+(z)$, i.e., forward envelop function versus normalized coordinate in nonlinear region. (b) Real (solid line) and imaginary (dashed line) parts of the normalized $A^-(z)$, i.e., backward envelop function versus the normalized coordinate in the nonlinear region.

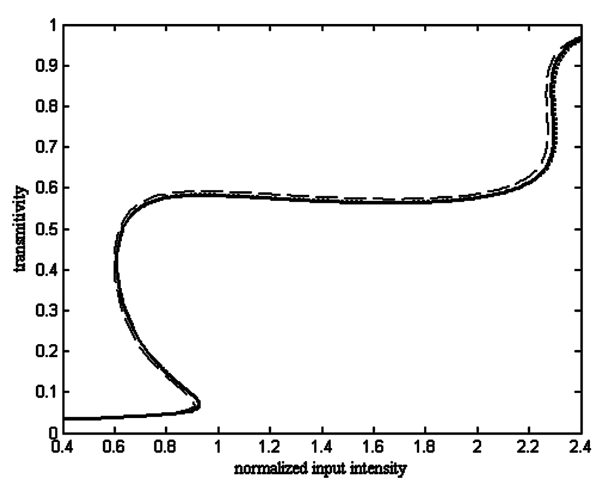

Fig. 4. Transmittivity versus the normalized input intensity: RK2 with 1500 intervals (dotted line), RK4 with 1000 (solid line) and 500 intervals (dashed line) applied to the proposed WKB factorization technique.

the Adams–Bashforth–Moulton algorithm [24] (solid line), the Gear's method [25] (dashed line), and the implicit Runge–Kutta scheme [26] (dotted line) are employed to solve the proposed

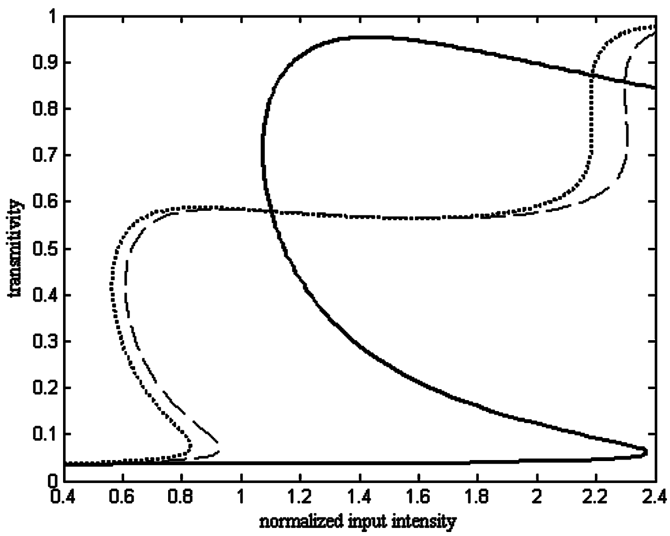

Fig. 5. Transmittivity versus the normalized input intensity: RK2 with 30000 intervals (dotted line), RK4 with 1000 (solid line) and 3000 intervals (dashed line) directly applied to the Maxwell's equations [11].

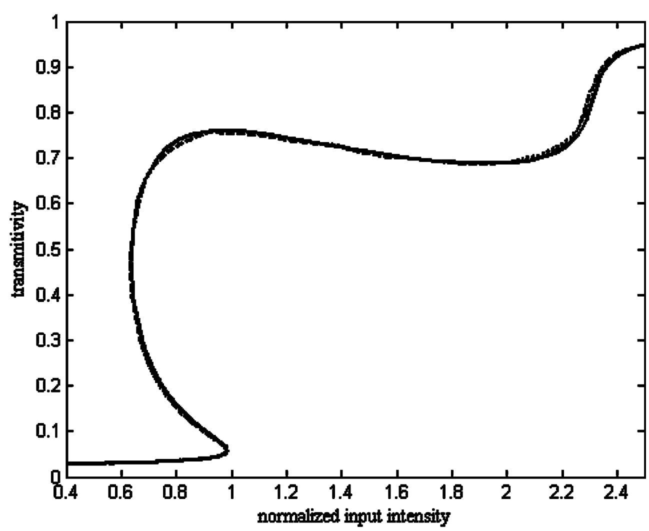

Fig. 6. Transmittivity versus the normalized input intensity: Adams–Bashforth–Moulton (solid line), Gear's method (dashed line) and Rosenbrock formula of order 2 (dotted line) applied to the proposed WKB factorization technique.

set of nonlinear coupled ordinary differential equations. Either of the abovementioned algorithms can be applied, as all the obtained results are in excellent agreement with each other. This is not true if the Maxwell's equations are to be solved directly. In Fig. 7, the transmittivity is plotted versus the normalized input intensity, where the nonlinear Maxwell's equations are directly solved by applying the Adams–Bashforth–Moulton algorithm [24] (solid line), the Gear's method [25] (dashed line), and the Rosenbrock formula of order 2 [26] (dotted line). Again, a comparison of these figures reveals that the proposed WKB factorization technique results in a better behaved set of equations, which can be readily solved by using any standard numerical approach.

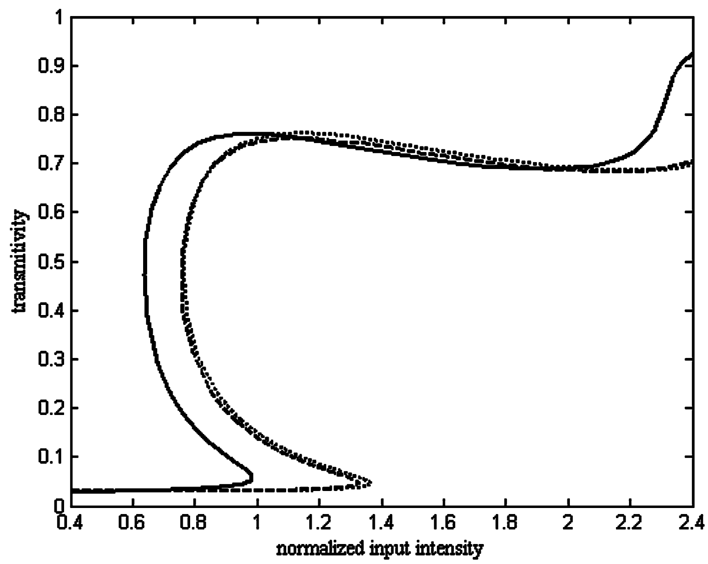

Fig. 7. Transmittivity versus the normalized input intensity: Adams–Bashforth–Moulton (solid line), Gear's method (dashed line) and Rosenbrock formula of order 2 (dotted line) applied to the Maxwell's equations [11].

V. Conclusion

A new and efficient matrix formulation for analysis of nonlinear wave propagation in NLDFB structures is presented. This method is based on the heuristic extraction of the rapidly varying factors of the transverse electromagnetic fields, which introduces a set of unknown envelope functions $A^+(z)$ and $A^-(z)$. In this fashion, the nonlinear set of Maxwell's equations is transformed into a new set of well-behaved coupled differential equations whose solution is usually a slowly varying function that can be readily found. Although the proposed factorization technique is originally based on the fairly accurate techniques like the WKB approximation, neither the much-used coupled mode theory nor the slowly varying envelope approximation is directly applied. In this way, a new set of coupled differential equations, which is mathematically equivalent to the original system of Maxwell's equations, is rendered. Different numerical examples, including the uniformly periodic, the linearly tapered, and the linearly chirped NLDFB structures, are presented and the superiority of the proposed method is numerically demonstrated.

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