Maximum Stable Throughput of Network-Coded Multiple Broadcast Sessions for Wireless Tandem Random Access Networks

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Abstract—This paper presents an analytical study of the stable throughput for multiple broadcast sessions in a multi-hop wireless tandem network with random access. Intermediate nodes leverage on the broadcast nature of wireless medium access to perform inter-session network coding among different flows. This problem is challenging due to the interaction among nodes, and has been addressed so far only in the saturated mode where all nodes always have packet to send, which results in infinite packet delay. In this paper, we provide a novel model based on multi-class queueing networks to investigate the problem in unsaturated mode. We devise a theoretical framework for computing maximum stable throughput of network coding for a slotted ALOHA-based random access system. Using our formulation, we compare the performance of network coding and traditional routing. Our results show that network coding leads to high throughput gain over traditional routing. We also define a new metric, *network unbalance ratio (NUR)*, that indicates the unbalance status of the utilization factors at different nodes. We show that although the throughput gain of the network coding compared to the traditional routing decreases when the number of nodes tends to infinity, NUR of the former outperforms the latter. We carry out simulations to confirm our theoretical analysis.

Index Terms—Network coding, stable throughput, routing, queueing networks, random access, vehicular networks

1 Introduction

Network coding is known to significantly improve the performance of wireless networks by exploiting the broadcast nature of wireless medium [1], [2], [3], [4], [5]. Several network coding schemes have been developed based on the fact that in wireless environment nodes may overhear the packets destined to other nodes, resulting in new coding opportunities [4], [6], [7]. Several works have been performed both from analytic as well as simulation point of view to show the benefit of network coding over traditional routing in terms of improving throughput and average delay [8], [9], [10].

Although the improvement of network performance by network coding has been extensively investigated in the literature, the interaction of MAC and network coding has not been properly addressed. The theoretical studies addressing the network coding gain are based on simplifying assumptions such as no collision, no interfering link, idealized MAC (perfect scheduling) or random MAC with saturated nodes. Analytical study of network coding for random MAC is important since in many applications (such as vehicular networks) a random MAC is more preferable to scheduling due to lack of full coordinations among nodes and can be implemented in a distributed manner more easily. In this paper, the interaction of MAC and network coding and its effect on network performance has been investigated.

An important criterion to measure network performance is stable throughout, which is the successful packet delivery rate to destinations, while guaranteeing that network queues remain bounded. The present paper provides an analytical framework to evaluate the maximum stable throughput for network coding in multihop wireless tandem networks when nodes use a random access MAC to communicate with each other in a multiple broadcast scenario. This problem is inherently difficult mainly as a result of intractability of queue length distributions and collision probabilities. This difficulty arises as nodes interact with each other, i.e., *interacting queues* [11], [12], which is difficult to analyze. Even for the traditional routing with random access MAC, the analysis of stable throughput is still unclear in general and has only been addressed in special cases. For example, [11] analyzes a slotted ALOHA network with two nodes. [13] characterizes the stability region for a slotted ALOHA network with three nodes and Bernoulli arrivals. Recently, an approximate method has been proposed in [14] to analyze the stability of slotted ALOHA with an arbitrary number of users in a single-hop network which is proved to be exact when the number of nodes tends to infinity.

Earlier studies of interaction of MAC and network coding have avoided dealing with this problem assuming saturated queues, which leads to infinite queueing delay [5], [15] (a queue is called saturated if it always has a packet to send). We alternatively propose an approximate analytical

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model based on multi-class open queueing network to analyze the unsaturated scenario. In our approach, we focus on the rate of packet arrival and departure, rather than the detail of arrival and departure processes. Therefore, we consider the steady state long term behavior of the nodes. We exploit our analytical approach to evaluate the performance of network coding and compare it with the traditional routing. We also evaluate the performance of the network asymptotically when the number of nodes goes to infinity. In order to obtain a deeper sense of the difference between network coding and traditional routing, we define a new metric that indicates the amount of unbalance among nodes. In fact, network coding shows the importance of suitable traffic sharing among nodes in a tandem network. Finally, we will show the validity of our analytical approach by simulations.

Our work is motivated by many applications such as vehicular networks and ad hoc networks, where distributed methods are preferable and random access method is more applicable. One application of the proposed scenario is content distribution in vehicular networks in which an access point wants to distribute a content among vehicles. Recently, network coding has attracted much interest for content distribution in vehicular networks [16], [17], [18], [19]. For example, by using the benefit of both network coding and wireless symbol-level diversity, [16] devises a novel scheme to distribute a popular content among vehicles in a lossy wireless network.

The main contributions of this paper can be summarized as follows.

1. We propose an analytical framework based on multi-class queueing network. We model fundamental processes involved in network coding, i.e., packet combining and packet multicasting to be able to include them in a queueing network model.
2. We apply the above model to derive the maximum stable throughput of a tandem wireless network with random medium access. Based on the proposed analytical model, we compare network coding and traditional routing and show the performance gain of the network coding in our scenario.
3. We investigate the limiting behavior of network coding and traditional routing in our scenario and show that when the number of nodes goes to infinity two methods behave similarly.
4. We define a new metric for the network, which evaluates the efficiency of symmetric packet generation status at all wireless nodes. The new metric identifies when some nodes saturate (reach the border of instability) how far the other nodes are from saturation. We will show that in network coding this metric is higher, which indicates that by suitable asymmetric packet generation we are able to obtain higher throughput.

The remainder of this paper is organized as follows. A summery of related works is presented in Section 2. Section 3 describes the network scenarios and the main idea of our proposed method. Section 4 describes the basic modeling principles in detail. Section 5 provides the system model and performance analysis. The analysis of asymptotic performance is developed in Section 6. Section 7 addresses the extension of the proposed method to erasure channels. We present the numerical results and simulations in order to show the performance and accuracy of our analytical approach in Section 8. Finally, Section 9 summarizes the main results and discusses possible extensions of the proposed schemes.

2 Related Work

The problem of broadcasting from a source to multiple destinations is addressed by many researchers. For instance, an analytical model has been presented in [8] to derive average delay and throughput for random linear network coding (RLNC) with erasure channels. [20] and [21] investigate data broadcasting to multiple destinations over erasure channels. They propose a queue-based dynamic network coding policy and derive the maximum stable throughput. [22] considers a single source-destination communication over a time division duplexing (TDD) erasure channel, where a sender transmits coded data packets back to back using RLNC, before receiving feedback from the receiver. They have optimized the number of coded packets in order to minimize the delay.

The performance of network coding among different flows has also been investigated in the literature. Basically, a network of multiple nodes is considered, where one or more source nodes tend to send information to one or more destinations via multihop transmissions. [23] derives an upper bound for throughput when network coding is applied in a tandem network exploiting CSMA/CA. Their analysis is based on the assumption that all nodes are backlogged. Sengupta et al. [9] address the multiple unicast problem in a wireless network with arbitrary topology and error free channels with constant capacity and provide the throughput analysis. In [10], the authors characterize the butterfly network with constant capacity links and address achievable rate regions for data multicast under specific QoS requirements. In [15], the authors consider a similar topology as in [23] while the neighbor nodes are randomly deployed with Poisson point process and analyze the end-to-end throughput in a CSMA/CA network with saturated traffic. A wireless tandem network is also investigated in [5] where each node tends to send data to all other nodes via a multihop fashion. They derive maximum throughput region for saturated multicast traffic in a tandem network with random access MAC. They also provide analysis for both saturated and unsaturated cases when perfect channel access scheduling is applied.

Johnson et al. [24] have developed an analytical framework for a unicast session in a multi-hop wireless tandem network based on a random access MAC protocol. They have derived the end-to-end throughput by applying the stability condition of queueing networks. The scenario considered in our paper can be considered as an extension to [24] where multiple sources intend to send packets to all others. We apply network coding to combine multiple flows in order to enhance total throughput. It is worth mentioning that the proposed modeling approach can be easily adopted for multiple unicast sessions.

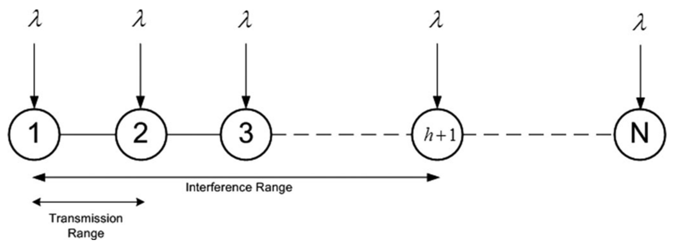

Fig. 1. Wireless tandem network of N nodes with multiple broadcast sessions.

3 Network Scenario

We consider a multihop wireless network comprised of N tandem nodes (Fig. 1) with regular topology. Let the packet generation process at each node be represented by an independent Poisson process with a rate, λ (i.e., symmetric packet generation). Each node wants to broadcast its packets to others. We apply a slotted ALOHA MAC scheme in which each node selects a random number of slots between 0 and $W-1$ to wait before transmission. The duration of each slot is equal to the transmission time of the fixed size packet, and all transmissions are synchronized. By associating the transmission of a packet to its waiting time, we model the service time of each packet to be a random number, between 1 and W, multiplied by the duration of the time slot. Without loss of generality, we assume the duration of the time slot equals 1. Whenever a node is backlogged, the transmission probability (p_{tr}) at each slot is equal to $\frac{2}{W+1}$ [25]. Consequently, the average service time (T) equals $\frac{(W+1)}{2}$. However, since the nodes are not always backlogged, the transmission probability is approximately equal to $p_{tr}\rho$, where ρ denotes the probability that the node is backlogged. In fact, ρ is the utilization factor of a node [26].

We assume the classical model for collision, in which a transmission is successful provided that there is no other transmissions in the interference range of the receiver. We have considered the transmission range equal to one adjacent node but the interference affects h adjacent nodes (h is defined as the *interference factor*). For the sake of simplicity, we ignore all physical channel non-idealities and consider the collision as the main hindering factor. The objective of this paper is to evaluate the maximum stable throughput, which is defined as the maximum data rate that can be received by nodes such that all nodes are throughput stable, i.e., balanced input-output property holds. We will demonstrate our modeling approach for both traditional routing and network coding in the next sections.

4 Basic Modeling Principles

In wireless networks, a packet can be sent to multiple destinations via a single transmission. In addition, in network coding a node can receive multiple packets and combine them into one coded packet. We refer to these two basic operations as the *packet multicasting* and *packet combining*, respectively. While the former is common in both traditional routing and network coding, the latter is restricted to network coding. In this section, we review the basic modeling principles, previously proposed in [27] and [28], as the building blocks of the analytical model proposed in this paper.

The modeling approach in this paper is based on multiclass queueing networks [29], in which, each wireless node is modeled as a queueing node and each packet plays the role of a customer. The transmission time of a typical packet is mapped onto the service time of the corresponding customer at the corresponding queueing node. Moreover, packet exchange among wireless nodes is mapped onto the routing probabilities of customers among corresponding queueing nodes.

In our model, it is necessary to consider the common constraints of queueing networks. In a conventional queueing network, a customer, departing a node, can be routed to only one node, which may be the node itself or may also be exogenous world (i.e., exit the network). Furthermore, a customer is neither destroyed nor merged with another customer (work-conserving property). Regarding the above constraints in a queueing network, in order to model the multicasting and packet combining features of network coding, we need to design an equivalent network such that the throughput of the queueing nodes in that network is equal to that of the corresponding nodes in the real scenario. Next, we discuss the packet multicasting and packet combining models.

4.1 Packet Multicasting

Consider a node multicasting a packet to two other nodes, and assume that the average transmission time of the packet is T seconds. In the equivalent queueing network, we consider two identical packets emitted by the transmitting node with the average service time $\frac{T}{2}$, where each packet is targeted to one of the receivers. To keep the input load of the queue constant, we also need to double the packet arrival rate (hence 2λ). The departed packets are routed to each of the two nodes with probability $\frac{1}{2}$. In this model, in the steady state, an equal number of packets reaches each of the two nodes within a sufficiently large time interval. In fact, duplicating the service rate is necessary to equate the packet departure rates in the real scenario and the corresponding queueing model. We consider extra packet arrivals to preserve the work-conserving principle. Fig. 2a illustrates the above model.

4.2 Packets Combining

Suppose that two arriving packets are combined at a node to create a new coded packet. We assign a new class to the coded packet to represent its new type and to keep track of it. Such as before two packets arrive to the node and two new packets of a different type depart. In order to preserve the work conserving principle, in our proposed model, one of the packets is routed to the exogenous world (outside the network). The class of the other packet is changed, and it is routed to another node in the network, as illustrated in Fig. 2b. Similar to the first model, consider $\frac{T}{2}$ as the packet transmission time for the new coded packet, and set the probabilities of transmission to another node or exiting the network equal to $\frac{1}{2}$. Note that if the coded packet is expected

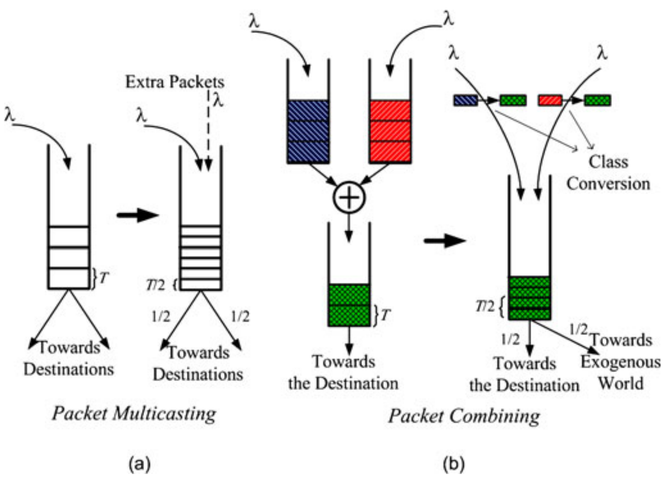

Fig. 2. Illustration of two modeling principles; packet multicasting and packet combining. (a) *Packet multicasting*. the real scenario (left), the equivalent queue (right). (b) *Packet combining*. the real scenario (left), and the equivalent queue (right) [27].

to be broadcast to two destinations, we route the departed packets to each destination with equal probabilities.

As noted earlier, our approach models the real system only from the throughput perspective in the average sense. In other words, the load of each queue in the real system is identical to the corresponding value in our model. As a result, we can derive the maximum stable throughput based on the proposed queueing network. However, the exact arrival and departure time instants of the packet in the proposed model might be different from those of the real network. Hence, our model does not necessarily represent the real scenario in terms of packet delay.

5 Analytical Modeling and Performance Analysis

Before investigating the queueing performance with traditional routing and network coding, we note that in a set of nodes in tandem, the end nodes are different from the intermediate nodes, in the sense that they do not participate in the traditional routing or network coding. Hence, the analytical models corresponding to the end nodes are slightly different from the models of other nodes. Our main focus in the following parts will be on the intermediate nodes. The list of symbols used in our analysis is summarized in Table 1.

5.1 Traditional Routing

In traditional routing, each node broadcasts its received packets from exogenous world, to its adjacent neighbors. Neighbors forward the packets to their neighbors until all nodes receive all packets. Each packet generated at an intermediate node i ($1 < i < N$) should be transmitted to two neighbors. However, due to collision, four cases may happen:

- Packet is received by both neighbors;
- Packet is only received by the left neighbor;
- Packet is only received by the right neighbor;
- None of the neighbors receives the packet.

TABLE 1
Description of Symbols

Symbol	Description
λ	Traffic generation rate at each node
α_{ij}	Total arrival rate of packet with class j at node i
ρ_i	Load of node i
p_{tr}	Transmission probability of packets
W	Back-off window size in slotted ALOHA MAC
N	Number of nodes in the wireless tandem network
k_{ij}	Probability of successful transmission from node i to node j
$r_{ij,kl}$	Routing probability of a packet from node i with class j to node k with class l
P_i^B	Probability of successful reception of a packet by both adjacent neighbors of node i
P_i^L	Probability of successful reception of a packet by only the left adjacent neighbor of node i
P_i^R	Probability of successful reception of a packet by only the right adjacent neighbor of node i

In the first case, the packet leaves node i and enters the queues in the neighboring nodes. In the second and third cases, the packet is received by one neighbor, but it should still be retransmitted to the neighbor that has not yet received the packet. In the fourth case, the packet should be retransmitted to both neighbors.

In our analysis, we adopt a two-part class to represent a packet in the equivalent queueing network. The alphabetic part indicates the direction(s) of the packet transmission and the numerical part of the class, which appears as the subscript of the alphabetic part, indicates the origin of the packet. In the traditional routing scenario, the numerical part is a number between 1 and N, indicating the source of the packet. The alphabetic part is B, R, or L. The letter B indicates that the packet is destined to both right and left neighbors, while the letter R (res. L) indicates that the packet should be only transmitted to the right (res. left) neighbor.

We consider three virtual queues at each intermediate node to represent the model discussed in Section 4, as illustrated in Fig. 3a. Packets enter the middle queue with rate 2λ and class B_i. The left and right queues contain the packets generated at the current node, which have not been received by both neighbors in earlier transmissions and have been queued for transmission to only one neighbor.

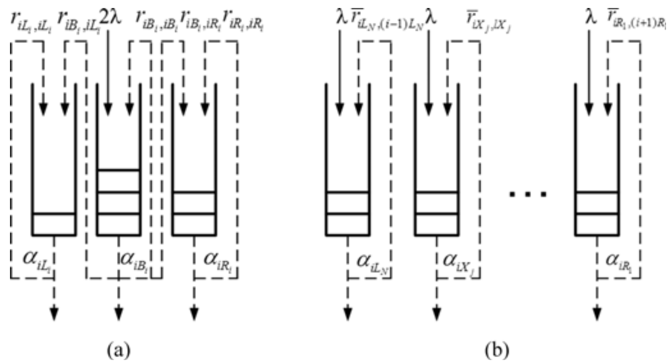

Fig. 3. Illustration of virtual queues for an intermediate node applying traditional routing. (a) three virtual queues represent source packets broadcasting to two neighbors (b) $N-1$ virtual queues represent relaying packets. $l \in \{i-1, i+1\}, X \in \{L, R\}$.

Note that the service rate of the middle queue is twice that of the left and right queues as the packets in the middle queue should be broadcast to two nodes (see Section 4). We change the class of packet, when routed to left or right queues, to a new class called X_i, denoting that the corresponding packet should now just be transmitted to one neighbor ($X = R, L$). Each node also holds $(N-1)$ virtual queues to handle packets of other sources (Fig. 3b). These are the packets originated in other nodes and will only be forwarded to one neighbor depending on the origin of the packet. In fact, the packets of all left nodes are forwarded to the right neighbor and vice versa.

Since each relayed packet at node i should be sent to only one of its neighbors, one virtual queue is assigned to the traffic of each source node. Consequently, each node has $(N+2)$ virtual queues. The end nodes, 1 and N, send their packets only to one neighbor and do not participate in relaying process. Hence, only one queue is needed to handle the traffic of end nodes.

Let k_{ij} be the probability of successful transmission of a packet from node i to node j. Regarding the interference factor to be h, k_{ij} can be found from:

$$k_{ij} = \prod_{l=\max(1,j-h),l\neq i}^{\min(j+h,N)} (1-\rho_l p_{tr});\ |i-j| = 1, \tag{1}$$

where p_{tr} and ρ_i denote the transmission probability of each node (nodes have the same contention window size) and the traffic intensity of node i, respectively. Moreover, the probability of successful reception of a packet by *both* adjacent neighbors of the intermediate node i is given by

$$P_i^B = \prod_{l=\max(1,i-1-h),l\neq i}^{\min(i+1+h,N)} (1-\rho_l p_{tr})\,\cdot \tag{2}$$

Also, the probabilities that the packet of the intermediate node i is *only* received by the left and the right neighbor, which are denoted by P_i^L and P_i^R, respectively are given by

$$P_i^L = \begin{cases} \displaystyle\prod_{l=\max(1,i-1-h),l\neq i}^{i-1+h} (1-\rho_l p_{tr}) \\ \displaystyle\times\left(1-\prod_{l=i+h}^{\min(i+1+h,N)}(1-\rho_l p_{tr})\right) & ; i-1+h < N, \\ 0 & ; i-1+h \geq N, \end{cases}$$

$$P_i^R = \begin{cases} \displaystyle\prod_{l=i+1-h,l\neq i}^{\min(i+1+h,N)} (1-\rho_l p_{tr}) \\ \displaystyle\times\left(1-\prod_{l=\max(1,i-1-h)}^{i-h}(1-\rho_l p_{tr})\right) & ; i > h, \\ 0 & ; i \leq h\cdot \end{cases} \tag{3}$$

Let $r_{ij,kl}$ represent the routing probability of a packet from node i with class j to node k with class l. Moreover, $\bar{r}_{ij,kl} = 1 - r_{ij,kl}$. The routing probabilities of the packets, generated or relayed, at intermediate node i are derived in the sequel. First, we consider the forwarding of packets originated at other nodes. For the packet of any node $j \in \{1,\ldots,N\}, j \neq i$, node i acts as a forwarder to its neighbor l, where $l = i+1$ or $l = i-1$, depending on the location of node j (see Fig. 3b). The routing probability for such a packet are given by

$$\begin{aligned} r_{iR_j,(i+1)R_j} &= k_{i,i+1}\ ; j < i \\ r_{iL_j,(i-1)L_j} &= k_{i,i-1}\ ; j > i\,\cdot \end{aligned} \tag{4}$$

Next, we find the routing probabilities for the packets generated at node i.

As discussed earlier, there are three cases that a transmitted packet returns to a virtual queue in node i. A transmitted packet of node i might only be received by one of the neighbors. We change the class of the packet to R_i or L_i depending on which neighbor failed to receive the packet. In this case, the packet leaves the main queue and enters the right or left virtual queues, with probability r_{iB_i,iR_i} and r_{iB_i,iL_i}, respectively, which is given by

$$\begin{aligned} r_{iB_i,iR_i} &= \frac{1}{2}P_i^L, \\ r_{iB_i,iL_i} &= \frac{1}{2}P_i^R\,\cdot \end{aligned} \tag{5}$$

The factor $\frac{1}{2}$ in (5) is based on our first modeling principle, discussed in Section 4. When a transmitted packet with class R_i or L_i is failed to be delivered to the corresponding neighbor, it is returned to its queue. The probability of this case can be written as

$$\begin{aligned} r_{iR_i,iR_i} &= 1 - k_{i,i+1}, \\ r_{iL_i,iL_i} &= 1 - k_{i,i-1}\,\cdot \end{aligned} \tag{6}$$

If the transmissions to both left and right neighbors fail, the packet does not exit the current queue. The probability of this event is given by

$$r_{iB_i,iB_i} = 1 - P_i^B - P_i^R - P_i^L\,\cdot \tag{7}$$

It is worth noting that in writing (1)-(7) we have assumed that the nodes are decoupled. This is true for product-form queueing networks, e.g., networks comprised of M/M/1 or M/G/1/PS (processor sharing) nodes [29]. In general, this is an approximation.

Next, we find the load of each queue. Let α_{ij} denote the total arrival rate of class j packets at node i. We first find the load of the queues at the end nodes 1 and N. At node 1, we have only one queue. The arrival rate at that queue is given by

$$\alpha_{1R_1} = \lambda + \alpha_{1R_1}\bar{r}_{1R_1,2R_1}\,\cdot \tag{8}$$

Using $r_{1R_1,2R_1} = k_{12}$, we have $\alpha_{1R_1}k_{12} = \lambda$. Regarding the slotted ALOHA MAC scheme (see Section 3), due to stability, we have $\alpha_{1R_1} = \rho_1 p_{tr}$, which results in

$$\rho_1 = \frac{\lambda}{p_{tr}k_{12}}\,\cdot \tag{9}$$

For node N, with a similar approach, we have

$$\alpha_{NL_N} = \lambda + \alpha_{NL_N}\overline{r}_{NL_N,(N-1)L_N}, \tag{10}$$

and

$$\rho_N = \frac{\lambda}{p_{tr}k_{N,N-1}} \cdot \tag{11}$$

Intermediate nodes have two traffic types: self-traffic and forwarding (relaying) traffic. The forwarding traffic is originated at the nodes to the left or right of node i and forwarded to the right or left nodes, respectively. The class of packet is not changed during forwarding. Network stability ensures that each node receives packets from other nodes with rate λ. The traffic originated in the left nodes has the total rate $\lambda(i-1)$. This traffic should be forwarded to node $(i+1)$. Using an approach similar to that of the end nodes, we find

$$\alpha_{iR_j} = \lambda + \alpha_{iR_j}\overline{r}_{iR_j,(i+1)R_j} \tag{12}$$

for $1 \le j \le i-1$. After simplification, we get

$$\alpha_{iR_j}k_{i,i+1} = \lambda \cdot \tag{13}$$

Summing the above equation for all values of j gives

$$\rho_i^{(R)} = \frac{\lambda(i-1)}{p_{tr}k_{i,i+1}}, \tag{14}$$

where $\rho_i^{(R)}$ is the total load of the traffic exiting node i and entering the right neighbor, i.e., node $(i+1)$. Similarly, for the traffic exiting node i and entering the left neighbor, i.e., node $(i-1)$, we have

$$\alpha_{iL_j} = \lambda + \alpha_{iL_j}\overline{r}_{iL_j,(i-1)L_j}, \tag{15}$$

for $i+1 \le j \le N$ and

$$\rho_i^{(L)} = \frac{\lambda(N-i)}{p_{tr}k_{i,i-1}}, \tag{16}$$

where $\rho_i^{(L)}$ is the total load for the traffic exiting node i and entering the left neighbor. Then the load due to relaying traffic is equal to $\rho_i^{(r)} = \rho_i^{(R)} + \rho_i^{(L)}$.

The traffic originated at intermediate node i (i.e., self-traffic) can be in the queues in Fig. 3a. The traffic equations for the queues are

$$\alpha_{iB_i} = 2\lambda + \alpha_{iB_i}r_{iB_i,iB_i}, \tag{17}$$

$$\alpha_{iR_i} = \alpha_{iB_i}r_{iB_i,iR_i} + \alpha_{iR_i}r_{iR_i,iR_i}, \tag{18}$$

$$\alpha_{iL_i} = \alpha_{iB_i}r_{iB_i,iL_i} + \alpha_{iL_i}r_{iL_i,iL_i} \cdot \tag{19}$$

This set of linear equations gives

$$\alpha_{iB_i} = \frac{2\lambda}{\overline{r}_{iB_i,iB_i}}, \tag{20}$$

$$\alpha_{iR_i} = \frac{2\lambda r_{iB_i,iR_i}}{\overline{r}_{iB_i,iB_i}\overline{r}_{iR_i,iR_i}}, \tag{21}$$

$$\alpha_{iL_i} = \frac{2\lambda r_{iB_i,iL_i}}{\overline{r}_{iB_i,iB_i}\overline{r}_{iL_i,iL_i}} \cdot \tag{22}$$

Denoting $\rho_i^{(s)}$ as the total load for the self-traffic at node i we get

$$\begin{aligned}\rho_i^{(s)} &= \frac{\alpha_{iB_i}}{2p_{tr}} + \frac{\alpha_{iR_i}}{p_{tr}} + \frac{\alpha_{iL_i}}{p_{tr}} \\ &= \frac{\lambda}{p_{tr}\overline{r}_{iB_i,iB_i}}\left(1 + \frac{2r_{iB_i,iR_i}}{\overline{r}_{iR_i,iR_i}} + \frac{2r_{iB_i,iL_i}}{\overline{r}_{iL_i,iL_i}}\right) \cdot\end{aligned} \tag{23}$$

Based on our discussion in Section 3, the service rate of middle virtual queue in Fig. 3 a is twice that of the left and the right virtual queues and is equal to $2p_{tr}$.

By using (14), (16) and (23) the total load of node i can be written as

$$\begin{aligned}\rho_i = \rho_i^{(s)} + \rho_i^{(r)} = &\frac{\lambda}{p_{tr}\overline{r}_{iB_i,iB_i}}\left(1 + \frac{2r_{iB_i,iR_i}}{\overline{r}_{iR_i,iR_i}} + \frac{2r_{iB_i,iL_i}}{\overline{r}_{iL_i,iL_i}}\right) \\ &+ \frac{\lambda(N-i)}{p_{tr}k_{i,i-1}} + \frac{\lambda(i-1)}{p_{tr}k_{i,i+1}} \cdot\end{aligned} \tag{24}$$

The maximum throughput can be obtained from the following optimization problem:

$$\begin{aligned}&\underset{\rho_i}{\text{maximize}} && \lambda \\ &\text{subject to} && \rho_i < 1;\ 1 \le i \le N, \\ &&& \rho_i \text{ satisfies } (9), (11), (24) \cdot\end{aligned} \tag{25}$$

Unfortunately, the above maximization problem can not be solved analytically. Instead, we provide a numerical recursive method to derive the maximum of λ, i.e., λ_{max}. The detail of our algorithm can be found in Algorithm 1, where δ_1 and δ_2 are typical small values and $\rho_i^{(old)}$ and $\rho_i^{(new)}$ are the values of ρ_i before and after update, respectively. It is worth nothing that the maximum stable throughput, defined in Section 3, is equal to $N\lambda_{max}$.

Algorithm 1 Numerical algorithm for deriving the maximum stable throughput

```
λ ← 0
ρ_i ← 0
while (ρ_i < 1,   1 ≤ i ≤ N) do
    λ ← λ + δ_1 λ
    λ_max ← λ
    ρ_i ← 0
    repeat
        Update k_i from (1)
        Update ρ_i from (9),(11),(24) for traditional routing
        Update ρ_i from (9),(11),(35) for network coding
    until (|ρ_i^(new) − ρ_i^(old)| / ρ_i^(new) < δ_2,   1 ≤ i ≤ N)
end while
return λ
```

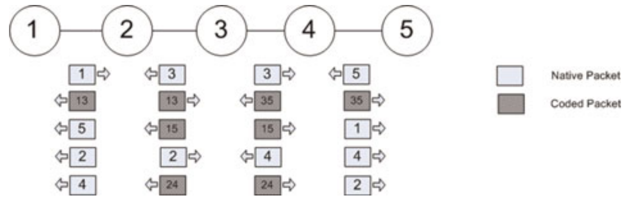

Fig. 4. Illustration of network coding method in a platoon of five nodes ($N = 5$).

5.2 Network Coding

In this section, we use a network coding strategy as follows:

1. node i XORs packets originated at nodes $(i - e)$ and $(i + e)$, for $1 \leq i \pm e \leq N$. Indeed, each node is only allowed to encode the packets of the two nodes that are located in equal distances (in terms of the hop counts) from the encoding node, where the encoding is the simple XOR operation. If a packet can not be encoded, i.e., packets originated at node j with $|j - i| > \min(i - 1, N - i)$, it is simply forwarded to the corresponding neighbor.
2. After reception of coded packets, each intermediate node decodes packets and re-encodes them according to the above scheme.

Fig. 4 illustrates our coding approach for the five-node tandem network. In the analysis of network coding scenario, we adopt a similar approach for defining the packet class. The alphabetic part has meaning similar to the traditional routing scenario, but the numerical part of the class is comprised of two numbers which indicates the elements of the coded packet, i.e., the origin of the two packets XORed with each other. In this example, node 2 receives packets from nodes 1 and 3, encodes them together and sends the coded packet to nodes 1 and 3. Also node 2 decodes the received coded packet 15 and 24 from node 3 and retransmits packets 4 and 5 to node 1. All packets are decodable because when a packet is received successfully at node j, it has one of the elements of the coded packet (according to our coding scheme). In order to maximize the coding combination rate, an incoming to-be-encoded packet is queued until its partner packet enters the node. This method ensures that encoding rate of each coding pair is maximum. Note that the maximum total coding rate at each intermediate node, i, is limited by the minimum of the left and right packet arrival rates, $\lambda \min(i - 1, N - i)$, which is achievable by the proposed method.

A header including the coding coefficients is inserted to each coded packet. The coding overhead ($2\lceil \log_2 N \rceil$ bits) is assumed to be negligible compared to the size of data packet. Hence, we assume that the length of coded and uncoded packets is the same. The main reason to adopt the proposed coding scheme is the simplicity of coding and decoding operations of XOR codes which facilitates implementation issues. This property motivates us to use this sub-optimal code in inter-session network coding, where designing optimal code is still an open problem. Even obtaining the optimal code within the class of linear codes is an NP-hard problem [30]. The proposed analysis in this section provides the maximum stable throughput for the specified network coding scheme. Similar to the traditional routing scenario, the end nodes do not relay or encode packets, rather they only transmit their own packets to the neighbor nodes. Hence, one virtual queue is assigned to each end node i ($i = 1, N$). The load of the first and the last nodes is derived from (9) and (11), respectively.

Each intermediate node takes part in three types of transmissions:

1. Broadcasting its own packets to its two neighbors;
2. Relaying a received uncoded packet to one neighbor;
3. Encoding packets of equidistant nodes and broadcasting coded packets to its two adjacent neighbors.

The first type resembles the traditional routing scenario discussed in the previous section and is modeled with three virtual queues. For such traffic, the load is given by

$$\rho_i^{(s)} = \frac{\lambda}{p_{tr}\overline{r}_{iB_i,iB_i}} \left(1 + \frac{2r_{iB_i,iR_i}}{\overline{r}_{iR_i,iR_i}} + \frac{2r_{iB_i,iL_i}}{\overline{r}_{iL_i,iL_i}}\right). \tag{26}$$

The second type of transmission is related to the traffic that is simply relayed (forwarded) at node i. This is the traffic generated at the nodes with more than $\min(i - 1, N - i)$ hops away from node i. Let us define $\theta_i = \min(i - 1, N - i)$ to simplify the notation. θ_i is the number of nodes to the left or to the right of node i, for which node i acts as an encoder. The traffic of any node j with $|j - i| > \theta_i$ should only be forwarded at i. For such traffic, node i maintains one queue per node and forwards the traffic to either the left or the right neighbor based on the location of the jth node. Given that the total number of such nodes to the left and right of node i is given by $(i - 1 - \theta_i)$ or $(N - i - \theta_i)$, the total load of the second type of transmission is

$$\rho_i^{(r)} = \frac{\lambda}{p_{tr}} \left(\frac{i - 1 - \theta_i}{k_{i,i+1}} + \frac{N - i - \theta_i}{k_{i,i-1}}\right), \tag{27}$$

where the superscript r denotes relayed (forwarded) traffic.

The third type of transmission is related to the one originated at the nodes j, $|j - i| \leq \theta_i, j \neq i$, which should be encoded at node i. Each two packets, which are originated at two equidistant neighbors, are XORed together to create one coded packet. The coded packet is then transmitted to the neighboring nodes. For such traffic, node i maintains three queues corresponding to the coded packets to be transmitted to both neighbors, the coded packets to be transmitted to the left neighbor, and the coded packets to be transmitted to the right neighbor. Therefore, the total number of such virtual queues at node i is equal to $3\theta_i$. Each three of the corresponding virtual queues have the same structure as the queues in Fig. 3a. For each coded pair, denoted by the class B_{xy}, packets originated at nodes x and y arrive at the corresponding middle virtual queue, each with rate λ. The coded packet, transmitted to two neighbors $i - 1$ and $i + 1$, may be received by none, one, or both neighbors. The packet leaves the middle queue and joins the neighbors' queues, if received by both neighbors. Our class assignment represents the nature of packet and simplifies the tracking of packet in the network. The packet will be decoded at the neighbors and may be re-encoded or simply forwarded to other nodes. On the other hand, if the packet of class B_{xy} is received by only the right (res. left) neighbor, it enters the left (res. right) virtual queue and its class is converted to L_{xy} (res. R_{xy}) to indicate that the coded packet is failed to be received by the left (res. right) neighbor, due to collision.

The traffic equations are

$$\alpha_{iB_{xy}} = 2\lambda + \alpha_{iB_{xy}} r_{iB_{xy},iB_{xy}}, \tag{28}$$

$$\alpha_{iL_{xy}} = \alpha_{iB_{xy}} r_{iB_{xy},iL_{xy}} + \alpha_{iL_{xy}} r_{iL_{xy},iL_{xy}}, \tag{29}$$

$$\alpha_{iR_{xy}} = \alpha_{iB_{xy}} r_{iB_{xy},iR_{xy}} + \alpha_{iR_{xy}} r_{iR_{xy},iR_{xy}}, \tag{30}$$

where

$$r_{iB_{xy},iB_{xy}} = 1 - P_i^B - P_i^R - P_i^L, \tag{31}$$

$$r_{iB_{xy},iL_{xy}} = \frac{1}{2} P_i^R, \tag{32}$$

$$r_{iB_{xy},iR_{xy}} = \frac{1}{2} P_i^L \cdot \tag{33}$$

The total load generated by such queues (corresponding to the transmission of coded packets) is given by

$$\begin{aligned} \rho_i^{(c)} &= \frac{\sum_{j \in U_i} \alpha_{ij}}{2p_{tr}} + \frac{\sum_{k \in V_i} \alpha_{ik}}{p_{tr}} \\ &= \frac{\lambda \theta_i}{p_{tr} \overline{r}_{iB_{xy},iB_{xy}}} \left(1 + \frac{2r_{iB_{xy},iL_{xy}}}{\overline{r}_{iL_{xy},iL_{xy}}} + \frac{2r_{iB_{xy},iR_{xy}}}{\overline{r}_{iR_{xy},iR_{xy}}} \right), \end{aligned} \tag{34}$$

where $U_i = \{B_{xy}; 1 \leq x < i, y = 2i - x, |x - i| \leq \theta_i\}$ and $V_i = \{Z_{xy}; 1 \leq x < i, y = 2i - x, Z = L, R, |x - i| \leq \theta_i\}$. The total load of node i can be obtained using (26), (27) and (34) as

$$\begin{aligned} \rho_i &= \rho_i^{(s)} + \rho_i^{(r)} + \rho_i^{(c)} \\ &= \frac{\lambda(1 + \theta_i)}{p_{tr} \overline{r}_{iB_{xy},iB_{xy}}} \left(1 + \frac{2r_{iB_{xy},iL_{xy}}}{\overline{r}_{iL_{xy},iL_{xy}}} + \frac{2r_{iB_{xy},iR_{xy}}}{\overline{r}_{iR_{xy},iR_{xy}}} \right) \\ &\quad + \frac{\lambda(N - i - \theta_i)}{p_{tr} k_{i,i-1}} + \frac{\lambda(i - 1 - \theta_i)}{p_{tr} k_{i,i+1}} \cdot \end{aligned} \tag{35}$$

Similar to the traditional routing scenario, maximizing the throughput is formulated as:

$$\begin{aligned} &\underset{\rho_i}{\text{maximize}} \quad \lambda \\ &\text{subject to} \quad \rho_i < 1;\ 1 \leq i \leq N, \\ &\qquad\qquad\quad \rho_i \text{ satisfies } (9), (11), (35) \cdot \end{aligned} \tag{36}$$

The maximum stable throughput can be obtained using the recursive numerical method illustrated in the Algorithm 1.

6 Asymptotic Analysis

In this section, we investigate the asymptotic total throughput for both network coding and traditional routing as the number of nodes tends to infinity. Our approximate approach indicates that the gain of the proposed network coding method decreases as the number of nodes increases and in the limit case, the performance of both methods becomes the same. We provide numerical results in Section 9 to verify the validity of our analysis.

6.1 Traditional Routing

Since the arrival rate and the window size of all nodes are equal, it is clear that the intermediate node, in the middle of the platoon, will be the bottleneck node as λ increases, because it has the maximum number of interfering nodes. Let $\lambda_T = \lambda N$ be the total traffic generated by the platoon and let the middle node's index be denoted by $\widehat{N} = \lfloor \frac{N}{2} \rfloor$. By substituting λ as $\frac{\lambda_T}{N}$ in (24) and assuming $N \gg 1$ the first term can be neglected, and we get:

$$\rho_{\widehat{N}} \approx \frac{\left(\frac{\lambda_T}{2}\right)}{p_{tr} k_{\widehat{N},\widehat{N}-1}} + \frac{\left(\frac{\lambda_T}{2}\right)}{p_{tr} k_{\widehat{N},\widehat{N}+1}}, \tag{37}$$

and

$$\rho_{\widehat{N}-h} \approx \rho_{\widehat{N}-h+1} \approx \cdots \approx \rho_{\widehat{N}+h} \cdot \tag{38}$$

Substituting (38) in (1) gives:

$$k_{\widehat{N},\widehat{N}+1} \approx k_{\widehat{N},\widehat{N}-1} \approx (1 - \rho_{\widehat{N}} p_{tr})^{2h}, \tag{39}$$

consequently

$$\rho_{\widehat{N}} \approx \frac{\lambda_T}{p_{tr}(1 - \rho_{\widehat{N}} p_{tr})^{2h}} \cdot \tag{40}$$

Since node $\widehat{N}$ is the bottleneck node, equating $\rho_{\widehat{N}}$ with 1, the maximum stable throughput can be derived as:

$$\lambda_T^\infty = p_{tr}(1 - p_{tr})^{2h} \cdot \tag{41}$$

6.2 Network Coding

Without loss of generality, we assume that the critical node (the first node to be saturated as λ increases) is in the range $[1, \frac{N}{2}]$. In contrast to the traditional routing scenario, the intermediate node is less likely to be the critical node in the network coding scenario since it has the maximum number of network coding opportunities (roughly speaking $\frac{N}{2}$ coding pairs). Indeed, the maximum number of coding opportunities decreases as we deviate from the intermediate node. On the other hand, the left-most h nodes also experience less colliding nodes. Hence, we expect the critical node, say node j, to be far from the middle node of the platoon. Adjacent nodes behave similarly as N tends to infinity:

$$\rho_{\widehat{j}} \approx \rho_{\widehat{j}+1} \approx \cdots \approx \rho_{\widehat{j}+h}, \tag{42}$$

where $\widehat{j} = \max(j - h, 1)$. Based on the above discussion, we infer:

$$\begin{aligned} \rho_j &= \frac{\lambda j}{p_{tr} \overline{r}_{iB_{xy},iB_{xy}}} \left(1 + \frac{2r_{iB_{xy},iL_{xy}}}{\overline{r}_{iL_{xy},iL_{xy}}} + \frac{2r_{iB_{xy},iR_{xy}}}{\overline{r}_{iR_{xy},iR_{xy}}} \right) \\ &\quad + \frac{\lambda(N - 2j + 1)}{p_{tr} k_{j,j-1}} \approx \frac{\lambda N}{p_{tr} k_{j,j-1}} \approx \frac{\lambda_T}{p_{tr}(1 - \rho_j p_{tr})^{2h}} \cdot \end{aligned} \tag{43}$$

Similar to the discussion following (40), equating $\rho_j = 1$ implies (41). Taking derivative of λ_T^∞ with respect to p_{tr} and equating to 0, the maximum asymptotic total throughput will be:

$$\lambda_T^{max} = \frac{1}{1+2h}\left(\frac{2h}{2h+1}\right)^{2h}. \tag{44}$$

7 Extension to Erasure Channels

To incorporate the non-idealities, in this section we incorporate i.i.d. erasure channel in our model where a transmitted packet may be corrupted at each destination with probability p_{err}. We assume that collision and erasure are two independent sources of packet loss. A packet is received successfully at each destination if it is neither collided by the other packets nor be erased by channel non-idealities like noise or fading. Considering both the collision and erasure effects, the probability of successful transmission of a packet from node i toward node j is obtained as

$$k_{ij} = p_{cor} \prod_{l=\max(1,j-h),l\neq i}^{\min(j+h,N)} (1-\rho_l p_{tr}), \tag{45}$$

where $p_{cor} = 1 - p_{err}$ is the non-erasure probability. The probability of successful reception of a packet by both adjacent neighbors of node i is given by

$$P_i^B = p_{cor}^2 \prod_{l=\max(1,i-1-h),l\neq i}^{\min(i+1+h,N)} (1-\rho_l p_{tr})\cdot \tag{46}$$

Also P_i^L and P_i^R (the probability that the packet of node i is only received by the left and right neighbors, respectively) are given by

$$P_i^L = \begin{cases} p_{cor} \prod_{l=\max(1,i-1-h),l\neq i}^{i-1+h} (1-\rho_l p_{tr}) & \\ \times\left(p_{cor}\left(1-\prod_{l=i+h}^{\min(i+1+h,N)}(1-\rho_l p_{tr})\right)+1-p_{cor}\right) & ;i-1+h < N \\ 0 & ;i-1+h \geq N, \end{cases}$$

$$P_i^R = \begin{cases} p_{cor} \prod_{l=i+1-h,l\neq i}^{\min(i+1+h,N)} (1-\rho_l p_{tr}) & \\ \times\left(p_{cor}\left(1-\prod_{l=\max(i-1-h)}^{i-h}(1-\rho_l p_{tr})\right)+1-p_{cor}\right) & ;i > h \\ 0 & ;i \leq h. \end{cases} \tag{47}$$

The maximum stable throughput for both traditional routing and network coding scenarios are obtained applying the same method introduced in Section 5.

8 Numerical Results

In this section, we illustrate our modeling approach to analyze a wireless linear network using network coding and traditional routing. Fig. 5 shows the effect of window size on the maximum stable throughput for a platoon of five nodes with the interference range $h = 1$. Note that there is an optimal size for the window that maximizes the throughput. On one hand, increasing window size lowers the collision probability for each node. On the other hand, it causes less transmission opportunities. The window size should be carefully selected to maximize the throughput. It is observed that network coding has a superior performance compared to traditional routing (approximately 20 percent throughput gain for optimal window size, i.e., $W = 4$).

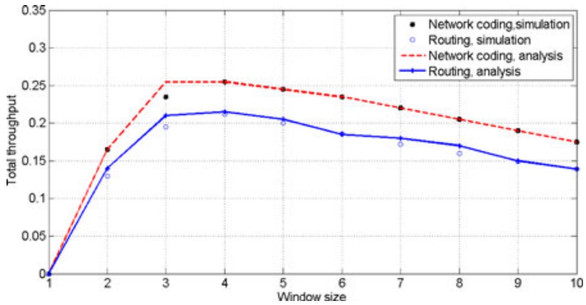

Fig. 5. Total throughput versus window size, $N = 5, h = 1$.

We have also simulated the original network (not the equivalent queueing network), where each node has a FIFO queue to buffer the incoming packets as well as its own source packets. The original packets enter the queue with rate λ packets per slot (generated by a Poisson process). The transmission may be unsuccessful due to collision, so the packet remains in the queue until successfully received by its destination(s). The packet type is incorporated in its header that shows the origin as well as the coding coefficients (in the network coding scenario). When a packet to be encoded enters an intermediate node's queue, it merges with an associated packet (if any exists) and forms a new coded packet (we simply dump the first packet and modify the header of the second packet). However, if the suitable partner packet is not present, the packet waits until its partner arrives. In the simulation, the interaction among queues has been considered, so the close agreement between our analysis and simulations show that decoupling assumption in our scenario is a good approximation. In our simulation, for each packet arrival rate, we compute the number of generated packets in a sufficiently large time interval and the number of absorbed packets (i.e., successfully received by their destinations) at the same interval, then find their ratios. If the total packet arrival rate is less than the maximum stable throughput, the network is stable and the ratio is equal to one. However, if it exceeds maximum stable throughput the ratio decreases, because some of the packets are not delivered to destinations in the same time interval. Hence, we start with a small packet arrival rate and gradually increase it until the ratio becomes less than 1. Thus, by comparing the ratio with unity we are able to find the maximum stable throughput.

We also investigate the role of interference factor on throughput in Fig. 6. Throughput decreases significantly as interference increases. As a result, it is needless to say that power control plays a critical role in achieving optimum throughput.

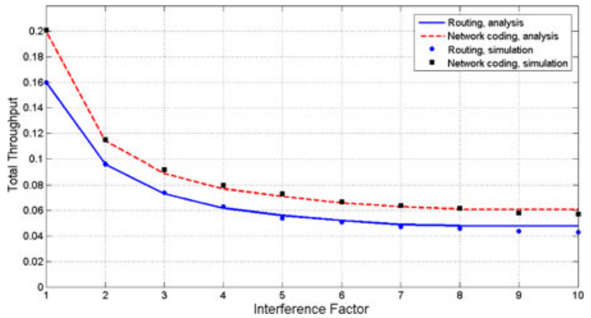

Fig. 6. Total throughput versus interference factor, $N = 10$. Window size (W) has been optimized to maximize the throughput.

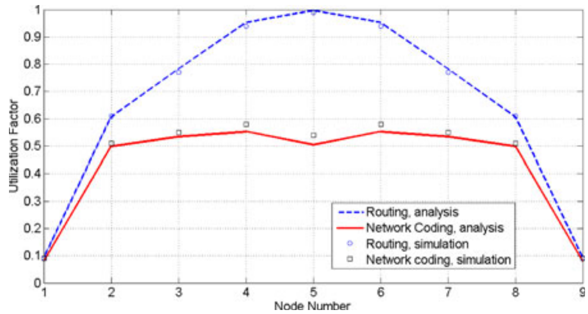

Fig. 8. Distribution of nodes' utilization factor, $h = 1$. In both traditional routing and network coding scenarios, $\lambda = \lambda_{max,rt}$: maximum stable throughput achievable by traditional routing.

Fig. 7 demonstrates the effect of the number of nodes on the overall throughput. As the number of nodes increases, the overall throughput for both network coding and traditional routing decreases and eventually tends to the same asymptotic value. It is worth mentioning that the total throughput, λ_T, is a decreasing function of the number of nodes. So decreasing the number of nodes results in higher total throughput.

Fig. 8 illustrates the impact of network coding on the utilization factors at different nodes, in a 9-node tandem network. Note that while the traditional routing-based network faces the infinite packet delay due to saturation of node 5, the utilization factors of nodes in network coding are relatively low. Moreover, the distribution of utilization factor is different for the network coding and traditional routing, as discussed in Section 6. In traditional routing-based network, the middle node has the highest utilization factor. However, in network coding case, the middle node has the maximum number of coding opportunities which results in lower traffic load compared with its adjacent neighbors.

The difference between the distribution of utilization factors is more apparent as the number of nodes increases. Fig. 9 demonstrates the traffic load pattern of a 200-node network at the packet generation rate equal to the maximum stable throughput (i.e., λ is equal to $\lambda_{max,rt}$ and $\lambda_{max,nc}$ for traditional routing and network coding scenarios, respectively). As mentioned earlier in Section 6, the critical nodes are placed in the middle of the platoon for traditional routing, while for network coding, the critical nodes deviate largely from the middle node for large network size. The utilization factors of nodes in network coding are highly unbalanced which shows that many nodes are under-utilized. Thus, we define a new metric that indicates when some of the nodes are saturated in a network, how far the other nodes are from saturation status. It is defined by $1 - \frac{\Sigma_{i=1}^{N} \rho_i}{N \rho^{max}}$ and called *network unbalance ratio (NUR)*, where $\rho^{max} = \max_i \rho_i$. Clearly, NUR is zero when all nodes are performing with maximum capacity. The NUR will be increased if load distribution among nodes is unbalanced. The more the load distribution is unbalanced, the more the NUR is. For network coding and traditional routing scenarios in Fig. 9, NUR is 0.36 and 0.03, receptively. We deduce that in maximum stable throughput obtained in symmetric packet generation network scenario, most of the nodes in network coding are under-utilized while for traditional routing scenario the load distribution is much more balanced. Intuitively, the packet arrival rate of each node, λ_i, should not be equal for network coding since the coding opportunities are different. Rather, λ_i should be increased as the position of a node is closer to the middle of the platoon to achieve more throughput gain. In other words, by optimizing packet generation rates (λ_i's) at different nodes, it is possible to exploit unused capacity of wireless nodes and obtain higher total throughput, i.e., $\Sigma_{i=1}^{N} \lambda_i$. We leave the analytical study of this problem to a future work. Fig. 10 illustrates the effect of erasure probability on the maximum total throughput. As the non-erasure probability increases, the maximum total throughput increases almost linearly for both traditional routing and network coding scenarios.

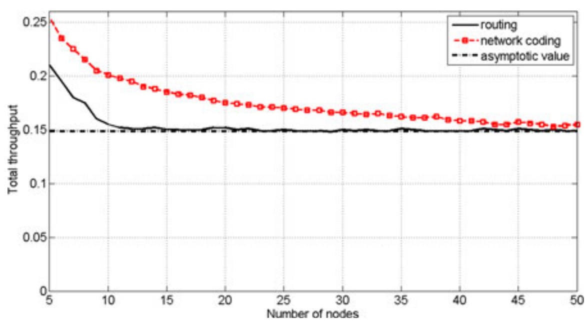

Fig. 7. Total throughput versus number of nodes, $h = 1$. Window size (W) has been optimized to maximize the throughput.

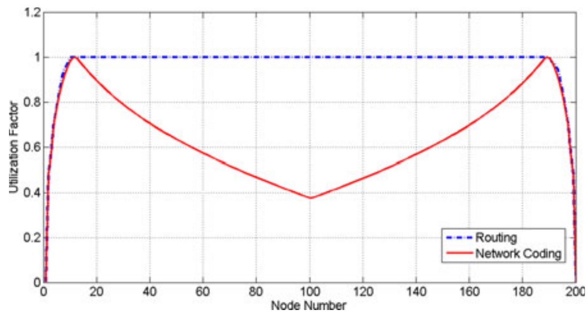

Fig. 9. Distribution of nodes' utilization factor in saturation point, $h = 1$. In traditional routing and network coding scenarios, $\lambda = \lambda_{max,rt}$ and $\lambda = \lambda_{max,nc}$, respectively.

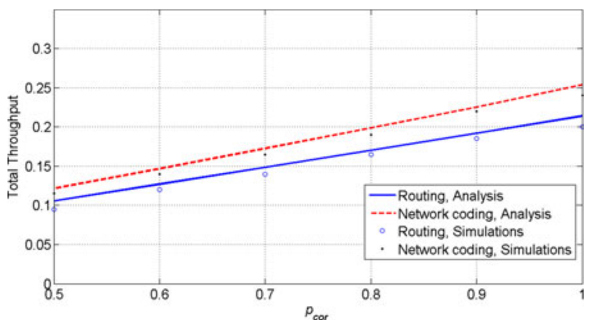

Fig. 10. Total throughput versus the probability of non-erasure, $N = 5, h = 1$. Window size (W) has been optimized to maximize the throughput.

9 Conclusion

In this paper, we presented a new approach for analyzing maximum stable throughput achieved by network coding. We applied our method to a linear random access wireless network with equidistant nodes in a multiple broadcast scenario. We derived the maximum stable throughput for both network coding and traditional routing scenarios and compared with the simulation results. Our fundamental results revealed that by using network coding, adjusting random access parameters (e.g., contention window size), and power control, throughput can be significantly improved. However, we observed when the number of nodes increases, the throughput gain of network coding will decrease. We also demonstrated that by applying asymmetric packet generation at nodes, it is possible to resolve this problem and reach higher throughput gain in network coding.

Our modeling approach was based on a new viewpoint, considering only the arrival and departure rates of the packets and their relations in general. This method can be applied to other scenarios like multi-source unicast scenarios or other medium access control methods like TDMA.

Acknowledgments

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