Multiple-Access Performance Analysis of Combined Time-Hopping and Spread-Time CDMA System in the Presence of Narrowband Interference

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***Abstract*—We consider a combined time-hopping (TH) and spread-time (ST) multiple-access technique that uses an internal code. In this method, the duration of each bit is divided into N_s frames. The outputs of the encoder and a pseudorandom (PN) sequence specify the number of the frame in which the data bit is transmitted in ST code-division multiple-access (ST-CDMA) form using the second PN sequence. We consider the correlator receiver, followed by the channel decoder. We obtain the performance of the combined method in additive white Gaussian noise (AWGN) and fading channels in the presence of multiple-access interference (MAI) and narrowband interference (NBI). We also consider the conventional ST-CDMA system and compare its performance with our method. The results indicate that the proposed method has much better performance while it has the same bandwidth and spectral efficiency as the conventional ST-CDMA system. In the new method, like in the conventional ST-CDMA system, an external error-correcting code can be applied for further performance enhancement.**

***Index Terms*—Narrowband interference (NBI), spread-time (ST), superorthogonal code (SOC), time hopping (TH).**

I. Introduction

SPREAD-TIME (ST) code-division multiple access (CDMA) has recently received great attention. The ST-CDMA [1]–[6] technique is an alternative multiple-access scheme to direct-sequence spread spectrum (DS/SS) and is considered as the time-frequency dual of the spread spectrum (SS) CDMA technique. ST was originally introduced for optical CDMA systems [3]. It was shown that ST-CDMA could spectrally be more efficient and more resistant to various channel impairments than SS CDMA [1], [2]. ST-CDMA is also called spectral-encoding CDMA (SE-CDMA) [5], [6].

ST spreads the energy of the corresponding data pulses in the time domain, as opposed to SS, which spreads the signal energy in the frequency domain. In ST, the original spectrum of the data pulse representing one bit of information is multiplied by a pseudorandom (PN) sequence in the frequency domain, followed by the inverse Fourier transform of the resulting frequency signal. The resulting pulse is spread in the time domain. Since the resulting pulse is of infinite duration, it must be truncated by a time window. At the receiver, the desired data bit is obtained by sampling the output of a filter matched to the user pulse [1]. Multiple access is achieved by assigning different PN codes to different users.

The main advantage of ST is its ability in matching the transmitted spectrum with the channel characteristic, such as when the channel has support on disconnected frequency bands, which is rather difficult to achieve by chip shaping in SS [1]. Furthermore, it has been shown [1], [2] that the signal-to-interference ratio in the ST technique is equal to or better than the SS technique in additive white Gaussian noise (AWGN) and fading channels. It has also been pointed out that the ST technique can be advantageous for ultrawideband (UWB) systems, because the spreading in time reduces the effective instantaneous power of the transmitted signal [2].

In [5], the authors considered a particular implementation of a UWB communication system that uses the ST technique to suppress the narrowband interference (NBI) via spectral shaping. NBI can severely affect the performance of UWB systems. By using the ST technique, a significant improvement in the system performance is obtained. In [6], the effect of the imperfect channel estimation of UWB systems in the presence of the NBI was analyzed for ST and DS/SS systems. However, the performance analysis in [5] and [6] is just for single-user environments and does not take into account the effect of multiple-access interference (MAI).

Furthermore, in [7], a multichannel CDMA system, e.g., a hybrid multichannel time-division multiple access (TDMA)/CDMA, was proposed. It was proven that the multichannel TDMA/CDMA system could support more simultaneous users, in comparison with the single-channel system, in which all users send their data at the same time, with the two systems having the same bandwidth.

In this paper, we consider a multichannel CDMA system in which we combine the ST-CDMA and time-hopping (TH) techniques using a superorthogonal encoder [8], [9]. In our method, the bit duration is divided into N_s frames. The output of the encoder and a PN sequence select one of the N_s frames. The uncoded data bit is then transmitted transmitted over that frame in the form of an ST signal. At the receiver, we use a sliding correlator, followed by the Viterbi decoder.

We evaluate the performance of the proposed coded TH/ST-CDMA scheme in AWGN and fading channels. We obtain the upper and lower bounds on the bit error rate (BER) using the

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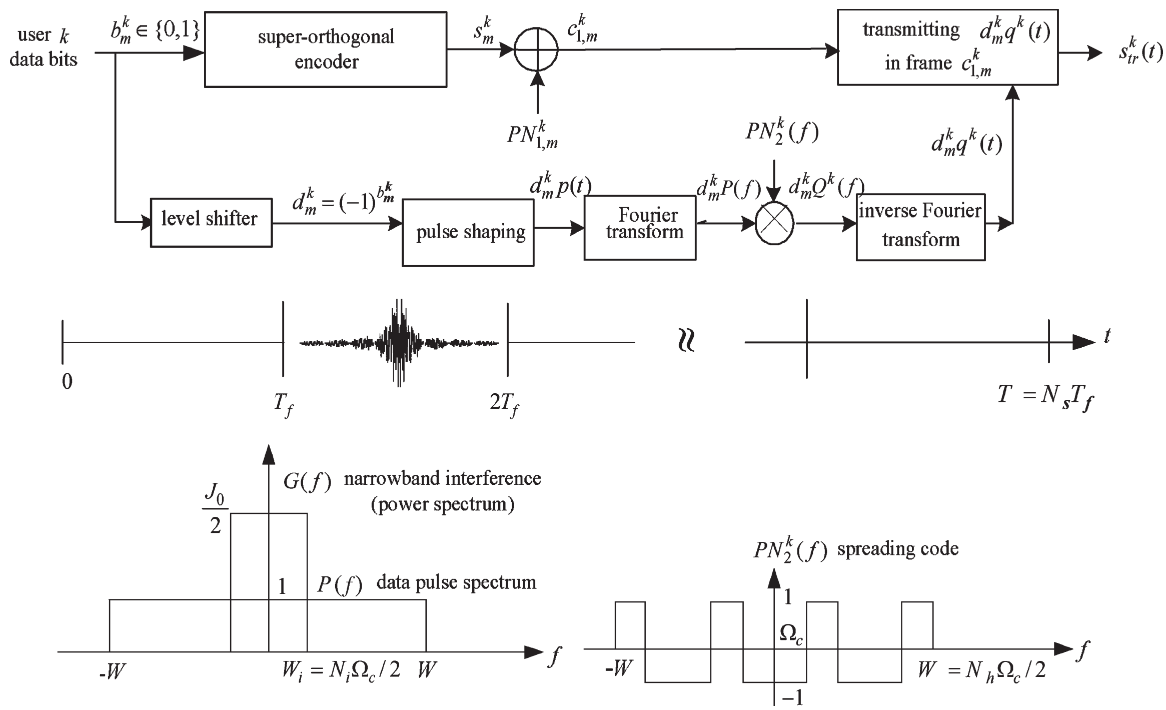

Fig. 1. Block diagram of the proposed system for user k.

Beaulieu series [10]. In addition to the MAI and thermal noise, we also consider the effect of NBI, in which the interference is received in part of the bandwidth of the desired user's signal. A similar internally coded scheme for the TH–UWB–CDMA and multicarrier frequency-hopping CDMA systems has been considered in [11] and [12]. However, the application, analysis, and, consequently, the results obtained in this paper are quite different from those in [11] and [12].

For comparison, we also consider the conventional ST-CDMA system, in which the data bit is transmitted over the whole bit time using the ST technique. The results indicate that the proposed method performs much better without requiring extra bandwidth. It is also demonstrated that the proposed method can significantly suppress the effect of the NBI. Furthermore, we provide some simulation results, which confirm our analytical results. In addition, we evaluate the performance of the proposed TH/ST-CDMA method and the conventional ST in UWB communication systems using the UWB channel model introduced in [13]. It is shown that the channel model considered in this paper well approximates the model in [13]. It must be noted that, in our method, like in any conventional multiple-access method, an external error-correcting code can be applied for further performance improvement.

This paper is organized as follows: In the next section, the model of the proposed system is described. In Sections III and IV, we obtain the performance of the proposed method in the fading and AWGN channels, respectively. In Section V, we provide some numerical results. Finally, Section VI concludes this paper.

II. System Description

Fig. 1 shows the block diagram, time axis division of the proposed method, and a typical spectrum of the data, NBI, and PN sequence. The duration of each bit T is divided into N_s frames, each with duration T_f; therefore, $T = N_s T_f$. Two PN sequences are assigned to each user. The data bit of the user k is applied to a superorthogonal encoder [8], [9] with a constraint length of K, which produces 2^{K-1} different symbols. The encoder output, which we consider as a symbol, is then added in modulo 2^{K-1} with the first PN sequence PN_1^k, where PN_1^k uniformly takes on the integer values between 0 and $2^{K-1} - 1$. The result determines one of the N_s frames; hence, $N_s = 2^{K-1}$. Then, in the selected frame, the data bit is transmitted in ST-CDMA form using the second PN spreading sequence $\mathrm{PN}_2^k(f)$, which is applied in the frequency domain. The spectrum of $\mathrm{PN}_2^k(f)$ has a bandwidth of W (the same as that of the data spectrum) and is segmented into $N_h/2$ equal-frequency chips, each with duration Ω_c as

$$\mathrm{PN}_2^k(f) = \sum_{i=-N_h/2}^{N_h/2-1} c_{2,i}^k rect\left(\frac{f-(i+1/2)\Omega_c}{\Omega_c}\right) \quad (1)$$

where $c_{2,i}^k$ is the spreading sequence of the kth user in the frequency domain, which takes on the values ± 1 with equal probability, and $rect(x)$ is the rectangle function defined in the interval $(-1/2, 1/2)$. Noting Fig. 1, if we choose $\Omega_c = 2/T_f$, then we have $W = \Omega_c N_h/2 = N_h/T_f$. We observe that the original spectrum of the data $P(f)$, which has a bandwidth of W, is multiplied by $\mathrm{PN}_2^k(f)$, i.e., spectral encoding is used. Then, the inverse Fourier transform is applied, i.e.,

$$q^k(t) = F^{-1}\left\{Q^k(f)\right\} = F^{-1}\left\{P(f)\mathrm{PN}_2^k(f)\right\}. \quad (2)$$

The resulting pulse $q^k(t)$ is sent to the selected frame. To have real $q^k(t)$, we must have $c_{2,i}^k = c_{2,-i-1}^k$. It is worth mentioning that, since the bandwidth is limited, the time duration of $q^k(t)$ is infinite. Thus, it is necessary to apply a time window so that its duration falls within one frame. However, it is shown [1] that, using a rectangular $P(f)$, 90% of the energy of pulse $q^k(t)$ falls within duration T_f and that intersymbol interference is zero at times hT_f, where h is an integer number. Of course, a $P(f)$ other than the rectangular $P(f)$ that confines more than 90% of the energy within this interval can be selected. Hence, we can neglect the effect of the window. Furthermore, it is demonstrated [4] that the signal-to-interference ratios with and without windowing are the same. Note that data pulse $p(t)$ has a duration of approximately $1/W = (T_f)/(N_h)$, and after time spreading, it is changed to the transmitted pulse $q^k(t)$ with a time duration of about $T_f = 2/\Omega_c$. Thus, the spreading of N_h times (the number of frequency chips) is obtained in the time domain. More details on the ST signal are presented in [1] and [2]. In the proposed system, we have

$$\text{Duty Cycle} = \frac{T_f}{T} = \frac{1}{N_s};\ N_s = 2^{K-1};\ \Omega_c = 2/T_f \quad (3)$$

$$\begin{aligned}&\text{Processing Gain}\\ &= \frac{T = \text{duration of one bit}}{(1/W) = \text{duration of original data pulse } p(t)}\\ &= \frac{N_s T_f}{T_f/N_h} = N_s N_h.\end{aligned} \quad (4)$$

The transmitted signal of user k can be written as

$$s_{\text{tr}}^k(t) = \sum_m d_m^k q^k\left(t - c_{1,m}^k T_f - mT\right) \quad (5)$$

where $d_m^k = (-1)^{b_m^k}$ and $b_m^k \in \{0,1\}$ is the mth data bit of the kth user, and $c_{1,m}^k$ determines the frame in which the desired signal is transmitted. $c_{1,m}^k$ takes on values ranging from 0 to $N_s - 1$, and it is determined based on the superorthogonal encoder output and PN1 sequence as

$$c_{1,m}^k = \left(s_m^k + \text{PN}_{1,m}^k\right) \text{ modulo } N_s \quad (6)$$

where s_m^k and $\text{PN}_{1,m}^k$ are the coded symbol and the first PN sequence component of user k at the mth bit interval, respectively (see Fig. 1).

A. Channel Model

We consider a downlink transmission. The channel is, in general, frequency selective in the whole bandwidth, i.e., $(\Delta f)_c \ll W = N_h \Omega_c/2$, where $(\Delta f)_c$ is the channel coherence bandwidth [14]. However, if $\Omega_c \ll (\Delta f)_c$, i.e., the chip frequency width is less than the channel coherence bandwidth, we can assume that the channel response is frequency nonselective in each frequency chip. Thus, the frequency response of the channel for user k can be modeled as [2]

$$H(f) = \sum_{n=-N_h/2}^{N_h/2-1} \alpha_n rect\left(\frac{f-(n+1/2)\,\Omega_c}{\Omega_c}\right);\ \alpha_n = \beta_n e^{j\theta_n} \quad (7)$$

where $\alpha_n = \beta_n e^{j\theta_n}$ is the complex coefficient of the channel in the nth frequency chip; β_n is the random amplitude (note that $\beta_n = \beta_{-n-1}$), which is assumed to have a Rayleigh distribution with $E(\beta_n^2) = \sigma^2$; and θ_n is the random phase with uniform distribution over $[0, 2\pi)$. The random variables β_n and θ_n are also stochastically independent. The accuracy of the preceding model depends on bandwidth W, the number of frequency chips N_h used in bandwidth W, and the shape of the channel frequency response [2]. Generally, there is a correlation between β_n and $\beta_{n'}$. However, here, the channel coefficients are assumed to be uncorrelated random variables. We also assume that the channel is slowly Rayleigh fading so that the channel parameters remain constant during at least one symbol interval and can correctly be estimated. The received signal can be written as

$$\begin{aligned} r(t) &= \sum_{k=1}^{N_u} s_{\text{tr}}^k(t) \otimes h(t) + J(t) + n(t)\\ &= \sum_{k=1}^{N_u}\sum_m d_m^k q^k\left(t - c_{1,m}^k T_f - mT\right) \otimes h(t) + J(t) + n(t)\end{aligned} \quad (8)$$

where $\otimes$ denotes the convolution operator, N_u is the number of active users, and $J(t)$ is the NBI. We assume that the NBI is a zero-mean Gaussian random process with bandwidth W_i, which is a fraction of the desired user's bandwidth W. In the preceding equation, $n(t)$ is the zero-mean AWGN with two-sided power spectral density $N_0/2$. We assume the same energy for all active users.

Without loss of generality, we assume that the first user is the desired user. Fig. 2 shows the block diagram of the receiver. It is assumed that the receiver knows the second PN code and can exactly estimate the channel characteristic. At the receiver, the received signal is convolved with $h^*(-t)$ (the superscript $*$ denotes the complex conjugate), and then, a sliding correlator with the base signal as

$$v_{i,h}(t) = q^1(t - hT_f - iT), \qquad h = 0, 1, \ldots, N_s - 1 \quad (9)$$

which moves over the N_s frames in each bit interval, is used. Note that $v_{i,h}^1(t)$ depends only on PN2. The correlator output in frame h during interval $(iT, (i+1)T]$ is obtained as

$$\begin{aligned} R_{i,h} &= \int_{iT+hT_f}^{iT+(h+1)T_f} \{r(t) \otimes h^*(-t)\}\, v_{i,h}(t)dt\\ &= \int_{-\infty}^{\infty} F\{r(t)\}\, H^*(f) Q^{1^*}(f) e^{j2\pi(iT+hT_f)f}\, df\\ &= S_{i,h} + I_{i,h} + J_{i,h} + n_{i,h}, \qquad h = 0, 1, \ldots, N_s - 1\end{aligned} \quad (10)$$

where $S_{i,h}$, $I_{i,h}$, $J_{i,h}$, and $n_{i,h}$ are the correlator output due to the desired user, MAI, NBI, and noise, respectively. These values, i.e., $R_{i,h}$, are then used for the branch metric calculations of the trellis diagram of the underlying convolution code. That is, let $M_{i,h}$ be the metric assigned to the branch with an output symbol of h at the ith time interval; then

$$M_{i,h} = R_{i,h}. \quad (11)$$

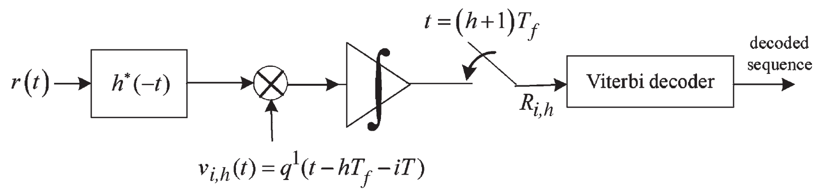

Fig. 2. Block diagram of the receiver for the first (desired) user.

Now, we obtain the correlator output due to the desired signal, interference, and noise in the fading channel. For the AWGN channel, i.e., $H(f) = 1$, the results obtained for the fading channel can be applied by setting fading coefficients $\alpha_n = \beta_n e^{j\theta_n}$ equal to 1. Without loss of generality, we consider interval $(0, T)$, and for simplicity, we drop index $i = 0$ in the rest of this paper.

1) Output Due to the Desired User: From (10), the correlator output due to the first (desired) user in interval $(0, T)$ and at frame h is easily obtained as

$$\begin{aligned} S_h &= \int_{iT+hT_f}^{iT+(h+1)T_f} \left\{ s_{\mathrm{tr}}^1(t) \otimes h(t) \otimes h^*(-t) \right\} v_{i,h}(t)\, dt \\ &= \int_{-\infty}^{\infty} S_{\mathrm{tr}}^1(f) H(f) H^*(f) Q^{1^*}(f) e^{j2\pi f h T_f}\, df \\ &= \int_{-W}^{W} \left(\sum_m d_m^1 e^{-j2\pi f m T} e^{-j2\pi f c_{1,m}^1 T_f} \right) \\ &\quad \times |H(f)|^2 \left|Q^1(f)\right|^2 e^{j2\pi f h T_f} df, \quad h = 0, 1, \ldots, N_s - 1. \end{aligned} \tag{12}$$

To compute the preceding integral, noting the channel model (7), we easily obtain

$$|H(f)|^2 = \sum_{n=-N_h/2}^{N_h/2-1} \beta_n^2 rect \left(\frac{f - (n + 1/2)\Omega_c}{\Omega_c} \right). \tag{13}$$

In addition, considering (1) and (2), and noting $c_{2,n}^1 c_{2,n}^1 = 1$, it can easily be verified that

$$\left|Q^1(f)\right|^2 = |P(f)|^2. \tag{14}$$

Therefore, by replacing (13) and (14) in (12), the output due to the desired user will be

$$\begin{aligned} S_h &= \int_{-W}^{+W} |P(f)|^2 \sum_{n=-N_h/2}^{N_h/2-1} \beta_n^2 rect \left(\frac{f - (n + 1/2)\Omega_c}{\Omega_c} \right) \cdot \\ &\quad \cdot \sum_m d_m^1 e^{-j2\pi f m T} e^{-j2\pi f c_{1,m}^1 T_f} e^{j2\pi f h T_f} df \\ &= \sum_m d_m^1 \sum_{n=-N_h/2}^{N_h/2-1} \beta_n^2 u_n \left(-mT - c_{1,m}^1 T_f + hT_f \right) \end{aligned} \tag{15}$$

where

$$u_n(t) \triangleq \int_{n\Omega_c}^{(n+1)\Omega_c} |P(f)|^2 e^{j2\pi f t} df. \tag{16}$$

For the rectangular data spectrum, $P(f) = \begin{cases} 1, & |f| < W \\ 0, & \text{else} \end{cases}$, and $\Omega_c = 2/(T_f)$, we obtain

$$u_n \left(-mT - c_{1,m}^1 T_f + hT_f \right) = \begin{cases} \Omega_c, & m = 0;\ h = c_{1,o}^1 \\ 0, & \text{else.} \end{cases} \tag{17}$$

This way, the desired output, noting $\beta_n = \beta_{-n-1}$, is

$$S_h = \begin{cases} 2d_0^1 \Omega_c \sum_{n=0}^{N_h/2-1} \beta_n^2, & h = c_{1,0}^1 \\ 0, & h \neq c_{1,0}^1. \end{cases} \tag{18}$$

2) Effect of the MAI: The correlator output due to the interfering user k, noting (12), can be computed as

$$\begin{aligned} I_h^k &= \int_{-\infty}^{\infty} S_{\mathrm{tr}}^k(f) H(f) H^*(f) Q^{1^*}(f) e^{j2\pi f h T_f} df \\ &= \int_{-W}^{+W} \left(\sum_m d_m^k e^{-j2\pi f \left(mT + c_{1,m}^k T_f \right)} \right) H(f) H^*(f) \\ &\quad \cdot Q^k(f) Q^{1^*}(f) e^{j2\pi f h T_f} df \\ &= \sum_m d_m^k \sum_{n=-N_h/2}^{N_h/2-1} c_{2,n}^k c_{2,n}^1 \beta_n^2 u_n \left(-mT - c_{1,m}^k T_f + hT_f \right) \end{aligned} \tag{19}$$

where $u_n(t)$ has been defined in (16). For the rectangular data spectrum, we obtain

$$I_h^k = \begin{cases} 2d_0^k \Omega_c \sum_{n=0}^{N_h/2-1} c_{2,n}^k c_{2,n}^1 \beta_n^2, & h = c_{1,0}^k \\ 0, & h \neq c_{1,0}^k. \end{cases} \tag{20}$$

The variance of the interference conditioned on the fading coefficients is computed as

$$\mathrm{Var}_{\mathrm{MAI}|\beta_{\{n\}}} = E\left[\left(I_h^k \right)^2 \right] = 4\Omega_c^2 \sum_{n=0}^{N_h/2-1} \beta_n^4 \tag{21}$$

where

$$\beta_{\{n\}} \triangleq \{\beta_0, \ldots, \beta_{N_h/2-1}\}. \tag{22}$$

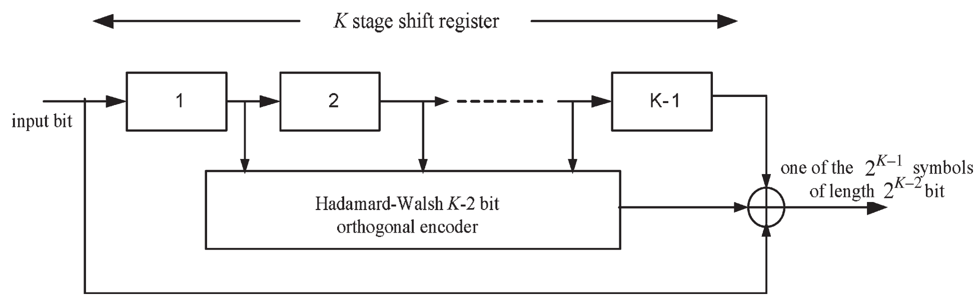

Fig. 3. Block diagram of the superorthogonal encoder.

3) Output Noise: The output noise in each frame n_h is computed by replacing $n(t)$, instead of $r(t)$, in (10). We can easily show that the output noise is also Gaussian with zero mean; its components are uncorrelated in different frames, and its variance in each frame is calculated as

$$\mathrm{Var}\left(n_h|_{\beta_{\{n\}}}\right) = \frac{N_0}{2}\left[\int_{-W}^{+W} |H(f)|^2 \left|Q^1(f)\right|^2 df\right] = \frac{N_0}{2}\sum_{n=-N_h/2}^{N_h/2-1}\beta_n^2 \int_{n\Omega_c}^{(n+1)\Omega_c} |P(f)|^2 df. \quad (23)$$

For the rectangular data spectrum, the variance and the moment-generating function of the output noise in each frame are

$$\mathrm{Var}(n_h|_{\beta_{\{n\}}}) = N_0\Omega_c \sum_{n=0}^{N_h/2-1}\beta_n^2 \quad (24)$$

$$\varphi_{n_h|\beta_{\{n\}}}(s) = \exp\left\{\frac{s^2}{2}N_0\Omega_c\sum_{n=0}^{N_h/2-1}\beta_n^2\right\}. \quad (25)$$

Noting (18) and (25), we define the average signal-to-noise ratio (SNR) as

$$\overline{\gamma}_b = E_{\beta_{\{n\}}}\left\{\frac{S^2}{2\mathrm{Var}(n_h)}\right\} = \frac{N_h\Omega_c\sigma^2}{N_0/2} \quad E\left(\beta_n^2\right) = \sigma^2. \quad (26)$$

4) Effect of the NBI: As stated, we assume that the NBI is a zero-mean Gaussian random process and occupies bandwidth W_i, which is a fraction of the bandwidth of the desired user W. Let $N_i/2$ be the number of frequency chips in which the NBI exists. We assume that the power spectrum of the NBI $G(f)$ is constant in bandwidth W_i, i.e., $G(f) = \begin{cases} J_0/2, & |f| < W_i \\ 0, & \text{else}\end{cases}$. Then, noting (23), we can easily obtain

$$\mathrm{Var}_{\mathrm{NBI}|\beta_{\{n\}}} = E\left(J_h^2\right) = \frac{J_0}{2}\left[\int_{-W_i}^{+W_i} |H(f)|^2\left|Q^1(f)\right|^2 df\right] = J_0\sum_{n=0}^{N_i/2-1}\beta_n^2\int_{n\Omega_c}^{(n+1)\Omega_c}|P(f)|^2 df. \quad (27)$$

Therefore, the moment-generating function of the NBI for rectangular $P(f)$ will be

$$\varphi_{J_h|\beta_{\{n\}}}(s) = \exp\left\{\frac{s^2}{2}J_0\Omega_c\sum_{n=0}^{N_i/2-1}\beta_n^2\right\}. \quad (28)$$

B. Lower and Upper Bounds on the BER

In the proposed method, like in [11] and [12], we use a convolutional error-correcting encoder. For a convolutional code, only the lower and upper bounds on the BER are analytically available. Here, we use a superorthogonal code (SOC), which is a near-optimal convolutional code; its path-generating function, which is required for the performance evaluation, is available. Fig. 3 shows the block diagram of the superorthogonal encoder [8], [9] considered in this paper. The encoder of SOC with constraint length K consists of a K-stage shift register, a $K-2$ bit orthogonal block encoder, and a modulo-2 adder. The contents of the $K-2$ inner stages of the shift register are applied to a Hadamard–Walsh orthogonal encoder, whose internal clock runs 2^{K-2} times as fast as the input bit rate. Thus, in one input bit time, one of the 2^{K-2} sequences with a length of 2^{K-2} bits is generated. Then, the first and last stages of the shift register are added modulo 2 to each of the 2^{K-2} bits of the orthogonal encoder output. Hence, depending on the first and last bits of the shift register, the final output will be the same as the orthogonal encoder output or its complement. That is, we obtain one of the 2^{K-1} possible symbols with a length of 2^{K-2} bits. The decoding of SOC is performed using the Viterbi algorithm with 2^{K-1} states. It is shown that, because of the special structure of SOC, the processing complexity of this decoder grows only linearly with K. Thus, the system is completely practical.

In the proposed method, the superorthogonal encoder determines the number of the frame in which the data bit is sent. That is, we use the output of the superorthogonal encoder as one of the 2^{K-1} symbols and not as a sequence of bits. Hence, the path-generating function in our application differs from that obtained in [8], in which the encoder output is considered as a sequence of 2^{K-2} bits. The path-generating function of the SOC in our application is computed as [11], [12]

$$T(D,N) = \frac{N(1-D)D^K}{1 - D\left(1 + N(1 - 2D^{K-2} + D^{K-3})\right)} \quad (29)$$

where, in the series expansion of the preceding equation, the power of D denotes the (symbol) weight of the encoder output, and the power of N represents the weight of the encoder input. The number of bit errors due to an error event with weight d, i.e., c_d, can be calculated as [8]

$$\left.\frac{\partial T(D,N)}{\partial N}\right|_{N=1} = \sum_{d=d_{\text{free}}}^{\infty} c_d D^d \tag{30}$$

where d_{free} is the free distance [8], [9] of the code, which is equal to

$$d_{\text{free}} = K = \log_2 N_s + 1. \tag{31}$$

Using the union bound, the upper bound [8] on the BER can be obtained as

$$P_b < \sum_{d=d_{\text{free}}}^{\infty} c_d P_d \tag{32}$$

where P_d is the probability of an error event with a Hamming distance of d [8], [9]. Note that the distance considered here is the symbol distance of the two paths (not the bit distance). We can also obtain a lower bound for the BER by considering only the first term of (32), i.e.,

$$P_b > P_{d_{\text{free}}}. \tag{33}$$

However, if we can obtain an expression for P_d as $P_d < q^d$, then the upper bound on the BER can also be computed as

$$\begin{aligned} P_b &< \sum_{d=d_{\text{free}}}^{\infty} c_d P_d < \sum_{d=d_{\text{free}}}^{\infty} c_d q^d \\ &= \left.\frac{\partial T(D,N)}{\partial N}\right|_{N=1,D=q} \\ &= \left.\frac{D^K}{(1-2D)^2}\left(\frac{1-D}{1-D^{K-2}}\right)^2\right|_{D=q}. \end{aligned} \tag{34}$$

Without loss of generality and for the performance evaluation, we assume that the desired user sends an all-zero sequence. Thus, P_d is the probability that the metric of a nonzero path with symbol weight d is larger than that of the all-zero path. As stated, the output of the correlator in each frame is used for the branch metric calculation of the Viterbi decoder. Thus, from (11), we easily obtain

$$\begin{aligned} P_d &= \Pr\left\{\sum_{k=1}^{d} R_{h_k\neq 0} > \sum_{k=1}^{d} R_{h_k=0}\right\} \\ &= \Pr\left\{\sum_{k=1}^{d} \left(R_{h_k\neq 0} - R_{h_k=0}\right)\right\} \\ &= \Pr\left\{Z = \sum_{k=1}^{d} Z_k > 0\right\}, \qquad Z_k \triangleq R_{h_k\neq 0} - R_{h_k=0} \end{aligned} \tag{35}$$

where $R_{h_{k\neq 0}}$ and $R_{h_{k=0}}$ denote the metrics of the branches corresponding to the nonzero and all-zero paths, at the instant of the kth different branches of the two paths, respectively, and Z_k indicates the difference in the metrics of these branches. Note that the two paths may have a length larger than d but may differ in only d branches.

Here, P_d cannot directly be obtained (except for the Gaussian probability density function (pdf) assumption for Z_k in the AWGN channel); therefore, we use the Beaulieu series [10], which is an infinite series for the computation of the cumulative distribution function of a random variable using the samples of its moment-generating function. From this series, we have [10]

$$P_d = P(Z > 0) = \frac{1}{2} + \sum_{\substack{m=1 \\ m\in N_{\text{odd}}}} \frac{2\text{Im}\left\{\varphi_Z(jmw_0)\right\}}{m\pi} \tag{36}$$

where N_{odd} is the set of odd natural numbers, $\text{Im}\{.\}$ stands for the imaginary part, $\varphi_Z(s)$ is the moment-generating function of Z, and ω_0 is the sampling rate. Note that, with a suitable value of ω_0, the series rapidly converges to an acceptable accurate value. Thus, it is sufficient to consider only the first few terms. To obtain $\varphi_Z(s)$, we note that the variables Z_k's are independent, because they belong to different bit intervals and have the same moment-generating functions. Thus, it suffices to find the moment-generating function of one of them. Hence, we can obtain

$$\varphi_Z(s) = \left(\varphi_{Z_k}(s)\right)^d. \tag{37}$$

In the succeeding sections, we will compute $\varphi_{Z_k}(s)$ for the fading and AWGN channels. Then, we obtain P_d from (36) for different values of d; next, an upper bound for the BER can be evaluated using (32). Note that P_d exponentially decays with d, so we need to consider only the first few terms of the series in (32).

III. Performance Evaluation in the Fading Channel

Without loss of generality and for simplicity, we assume that the desired user (user 1) sends its zero-sequence data in frame $h = (c_1^1 \triangleq 0)$. To compute the BER, we must first compute moment-generating function $\varphi_{Z_k}(s)$. We note that variables (metrics) $R_{h\neq 0}$ and $R_{h=0}$ are not independent, because, if there are U_0 and U_1 interfering users in the frames corresponding to the zero and nonzero branches, respectively, then U_0 and U_1 must satisfy $U_0 + U_1 \leq N_u - 1$, which makes the two aforementioned variables dependent. However, conditioned on U_0, U_1, and $\beta_{\{n\}}$, conditional variables $(R_{h\neq 0}|U_1, \beta_{\{n\}})$ and $(R_{h=0}|U_0, \beta_{\{n\}})$ are independent. Thus, the conditional moment-generating function of $Z_k = R_{h\neq 0} - R_{h=0}$ is obtained as

$$\varphi_{Z_k|U_0,U_1,\beta_{\{n\}}}(s) = \varphi_{R_{h\neq 0}|U_1,\beta_{\{n\}}}(s)\varphi_{R_{h=0}|U_0,\beta_{\{n\}}}(-s) \tag{38}$$

and the unconditional moment-generating function is computed as

$$\begin{aligned} \varphi_{Z_k}(s) = \sum_{U_1=0}^{N_u-1}\sum_{U_0=0}^{N_u-1-U_1} &\left(\int \varphi_{R_{h\neq 0}|U_1,\beta_{\{n\}}}(s)\varphi_{R_{h=0}|U_0,\beta_{\{n\}}}(-s)\right. \\ &\left.\times\, p(\beta_{\{n\}})d\beta_{\{n\}}\right) P(U_1,U_0) \end{aligned} \tag{39}$$

where $P(U_1, U_0)$ is the probability of having U_1 and U_0 interfering users in frames $h \neq 0$ and $h = 0$, respectively, which is equal to

$$P(U_1, U_0) = \binom{N_u - 1}{U_1} \binom{N_u - U_1 1}{U_0} \times A^{U_1} A^{U_0} (1 - 2A)^{N_u - U_1 - U_0 - 1} \quad (40)$$

where A is the probability that the two users send their data bits in the same frame, i.e.,

$$A \triangleq p\left(c_1^k = c_1^1\right) = 1/N_s. \quad (41)$$

Now, we compute $\varphi_{R_{h\neq 0}|U_1,\beta_{\{n\}}}(s)$ and $\varphi_{R_{h=0}|U_0,\beta_{\{n\}}}(-s)$. In frame $h \neq 0$, in which the first user does not send data, if there are U_1 interferers, then the correlator output consists of interference and noise. Thus, noting (20), we have

$$R_{h\neq 0|U_1} = 2\Omega_c \sum_{k=2}^{U_1+1} d_0^k \sum_{n=0}^{N_h/2-1} c_{2,n}^k c_{2,n}^1 \beta_n^2 + n_h. \quad (42)$$

It is easily verified that the pdf of the receiver output in frame $h \neq 0$ conditioned on the fading coefficients, U_1, data, and PN2 sequence is Gaussian (note that $R_{h\neq 0}|_{U_1,\beta_{\{n\}},d_0^{\{k\}},c_{2,\{n\}}^{\{k\}},c_{2,\{n\}}^1} = \text{constant} + n_h$), i.e.,

$$R_{h\neq 0}|_{U_1,\beta_{\{n\}},d_0^k,c_{2,\{n\}}^{\{k\}},c_{2,\{n\}}^1} \sim N\left(2\Omega_c \sum_{k=2}^{U_1+1} d_0^k \sum_{n=0}^{N_h/2-1} c_{2,n}^k c_{2,n}^1 \beta_n^2, \sigma_{n_h}^2\right) \quad (43)$$

where $N(\eta, \sigma_n^2)$ represents a Gaussian distribution with mean η and variance σ_n^2. Hence, the conditional moment-generating function of the receiver output, noting (25), will be

$$\varphi_{R_{h\neq 0}|_{U_1,\beta_{\{n\}},d_0^k,c_{2,\{n\}}^{\{k\}},c_{2,\{n\}}^1}}(s) = \prod_{n=0}^{N_h/2-1} \left[\left(\prod_{k=2}^{U_1+1} \exp\left\{2s\Omega_c d_0^k c_{2,n}^k c_{2,n}^1 \beta_n^2\right\} \right) \times \exp\left\{\frac{s^2}{2} N_0 \Omega_c \beta_n^2\right\} \right]. \quad (44)$$

Since, for different users k and frequency chips n, variables d_0^k, $c_{2,n}^k$, and $c_{2,n}^1$ take on the values $+1$ or -1 with equal probability, thus for a particular k and n, the variable $d_0^k c_{2,n}^k c_{2,n}^1$ takes on one of the values $+1$ or -1 with a probability of 1/2. This way, the moment-generating function of the output conditioned on the fading coefficients and U_1 is

$$\varphi_{R_{h\neq 0}|U_1,\beta_{\{n\}}}(s) = \prod_{n=0}^{N_h/2-1} \left[\left(\frac{1}{2} \exp\left\{2s\Omega_c \beta_n^2\right\} + \frac{1}{2} \exp\left\{-2s\Omega_c \beta_n^2\right\} \right)^{U_1} \exp\left\{\frac{s^2}{2} N_0 \Omega_c \beta_n^2\right\} \right]. \quad (45)$$

In frame $h = 0$, in which the desired user sends its data, there are also U_0 independent interferers, NBI, and noise. Hence, the correlator output will be equal to $R_{h=0} = S_{h=0|d_0^1=1} + \sum_{k=1}^{U_0} I_h^k + J_h + n_h$. Consequently, the conditional pdf of the correlator output, noting (18), (25), and (28), will be

$$R_{h=0}|_{U_0,\beta_{\{n\}},d_0^k,c_{2,\{n\}}^{\{k\}},c_{2,\{n\}}^1} \sim N\left(2\Omega_c \sum_{n=0}^{N_h/2-1} \beta_n^2 + 2\Omega_c \sum_{k=2}^{U_0+1} d_0^k \sum_{n=0}^{N_h/2-1} c_{2,n}^k c_{2,n}^1 \beta_n^2, \sigma_{n_h}^2 + \mathrm{Var}_{\mathrm{NBI}}\right). \quad (46)$$

Note that we have assumed in frame $h = 0$ that the first user sends an all-zero bit sequence; thus, $d_0^1 = (-1)^{b_0^1} = 1$, which results in $S_{h=0|d_0^1=1} = 2\Omega_c \sum_{n=0}^{N_h/2-1} \beta_n^2$. From (46), the conditional moment-generating function of the correlator output, noting the preceding discussions, is equal to

$$\varphi_{R_{h=0}|U_0,\beta_{\{n\}}}(s) = \prod_{n=0}^{N_h/2-1} \left[\left(\frac{1}{2} \exp\left\{2s\Omega_c \beta_n^2\right\} + \frac{1}{2} \exp\left\{-2s\Omega_c \beta_n^2\right\} \right)^{U_0} \times \exp\left\{ \left(\frac{s^2}{2} N_0 + 2s \right) \Omega_c \beta_n^2 \right\} \right] \cdot \prod_{n=0}^{N_i/2-1} \exp\left\{\frac{s^2}{2} J_0 \Omega_c \beta_n^2\right\}. \quad (47)$$

Then, by substituting (45) and (47) in (39) and computing the integral, moment-generating function $\varphi_{Z_k}(s)$ is obtained. In the Appendix, we have shown that the result of the integral in (39) is derived as

$$\int \varphi_{R_{h\neq 0}|U_1,\beta_{\{n\}}}(s) \varphi_{R_{h=0}|U_0,\beta_{\{n\}}}(-s) p\left(\beta_{\{n\}}\right) d\beta_{\{n\}} = \left[\left(\frac{1}{2}\right)^{U_0+U_1} \sum_{k=0}^{U_0+U_1} \binom{U_0+U_1}{k} \times \frac{1}{2s(U_0+U_1-2k+1)\Omega_c\sigma^2 - s^2\Omega_c\sigma^2\left(N_0+\frac{J_0}{2}\right)+1} \right]^{\frac{N_i}{2}} \cdot \left[\left(\frac{1}{2}\right)^{U_0+U_1} \sum_{k=0}^{U_0+U_1} \binom{U_0+U_1}{k} \times \frac{1}{2s(U_0+U_1-2k+1)\Omega_c\sigma^2 - s^2\Omega_c\sigma^2 N_0+1} \right]^{\frac{N_h-N_i}{2}}. \quad (48)$$

As a result, we can compute $\varphi_{Z_k}(s)$ from (39) and then obtain P_d and P_b from (32) and (35), respectively.

In the preceding results, we have considered the effect of the NBI. Since the ST-CDMA technique has the ability to match the transmitted spectrum with the channel characteristic, the NBI can be suppressed. This can be achieved by setting the part of the spreading code in the bandwidth where the NBI exists to zero. That is, we do not send data in that band or at the receiver $\mathrm{PN}_2^k(f)$ is set equal to zero in the band where the NBI is present. Since we have assumed that the NBI occupies $N_i/2$ frequency chips in bandwidth W, the results for the NBI-suppression case are as previously given, just by replacing N_h with $N_h - N_i$ and ignoring the NBI variance. This way, the result given in (48) changes to

$$\int \varphi_{R_{h\neq 0}|U_1,\beta_{\{n\}}}(s)\varphi_{R_{h=0}|U_0,\beta_{\{n\}}}(-s)p\left(\beta_{\{n\}}\right)d\beta_{\{n\}} = \left[\left(\frac{1}{2}\right)^{U_0+U_1}\sum_{k=0}^{U_0+U_1}\binom{U_0+U_1}{k} \times \frac{1}{2s(U_0+U_1-2k+1)\Omega_c\sigma^2 - s^2\Omega_c\sigma^2 N_0+1}\right]^{\frac{N_h-N_i}{2}}. \tag{49}$$

IV. Performance Analysis in the AWGN Channel

In this section, we first calculate the performance of the proposed method when the exact pdf of the correlator output is considered. Then, we compute the BER by the Gaussian pdf assumption for the MAI. Note that the analysis for the fading channel provided in Section III is an exact analysis.

A. Exact Analysis

The correlator output in the AWGN channel is easily obtained by setting $\alpha_n = 1 \Rightarrow \beta_n = 1$ in the results derived for the fading channel. Thus, noting (18), (20), (21), (25), and (27), we have

$$S = b_0^1\Omega_c N_h \tag{50}$$

$$\mathrm{Var}(n_h) = \frac{N_0}{2}\Omega_c N_h \tag{51}$$

$$\mathrm{Var}_{\mathrm{NBI}} = \frac{J_0}{2}\Omega_c N_i \tag{52}$$

$$I_h^k = 2d_0^k\Omega_c\sum_{i=0}^{N_h/2-1} c_{2,i}^k c_{2,i}^1, \quad \mathrm{Var}_{\mathrm{MAI}} = 2\Omega_c^2 N_h. \tag{53}$$

The SNR is defined as the energy of the desired signal to the noise variance, i.e.,

$$\mathrm{SNR} = \frac{N_h\Omega_c}{N_0/2}. \tag{54}$$

Noting (45) and (47), and using the binomial expansion, we obtain

$$\begin{aligned}\varphi_{Z_k}(s) &= \sum_{U_1=0}^{N_u-1}\sum_{U_0=0}^{N_u-1-U_1}\varphi_{R_{h\neq 0}|U_1}(s)\varphi_{R_{h=0}|U_0}(-s)P(U_1,U_0)\\ &= \sum_{U_1=0}^{N_u-1}\sum_{U_0=0}^{N_u-1-U_1}\prod_{n=0}^{N_h/2-1}\left(\frac{1}{2}\exp\{2s\Omega_c\}+\frac{1}{2}\exp\{-2s\Omega_c\}\right)^{U_0+U_1}\\ &\quad\cdot\exp\left\{-sN_h\Omega_c+\frac{s^2}{2}\left(N_0N_h+\frac{J_0}{2}N_i\right)\Omega_c\right\}P(U_1,U_0)\\ &= \exp\left\{-sN_h\Omega_c+\frac{s^2}{2}\left(N_0N_h+\frac{J_0}{2}N_i\right)\Omega_c\right\}\\ &\quad\cdot\sum_{U_1=0}^{N_u-1}\sum_{U_0=0}^{N_u-1-U_1}\left(\frac{1}{2}\right)^{\frac{(U_0+U_1)N_h}{2}}\exp\{-s(U_0+U_1)N_h\Omega_c\}\\ &\quad\times\left(\sum_{k=0}^{U_0+U_1}\binom{U_0+U_1}{k}\exp(4sk\Omega_c)\right)^{N_h/2}P(U_1,U_0).\end{aligned} \tag{55}$$

Then, we can compute P_d and P_b from (32) and (36), respectively.

B. Gaussian Assumption

We can also find another expression for the BER using the Gaussian approximation for the pdf of the MAI. Therefore, in frame $h \neq 0$, in which the first user does not send data, the conditional pdf of the correlator output, noting (51) and (52), is

$$p_R(x|h\neq 0, U_1) \sim N\left(0, U_1\mathrm{Var}_{\mathrm{MAI}} + \mathrm{Var}(n_h)\right). \tag{56}$$

In frame $h = 0$, the conditional pdf of the correlator output will be

$$p_R(x|h=0,U_0) \sim N\left(S|_{d_0^1=0}, U_0\mathrm{Var}_{\mathrm{MAI}} + \mathrm{Var}_{\mathrm{NBI}} + \mathrm{Var}(n_h)\right). \tag{57}$$

It is easily verified that the conditional pdf $p(Z_k|U_1,U_0) = p[(R_{h\neq 0}|U_1) - (R_{h=0}|U_0)]$ is also Gaussian, i.e.,

$$p(Z_k|U_0,U_1) \sim N\Big(-S|_{d_0^1=0}, (U_0+U_1) \times \mathrm{Var}_{\mathrm{MAI}}+\mathrm{Var}_{\mathrm{NBI}} + 2\mathrm{Var}(n_h)\Big). \tag{58}$$

Then, the unconditional pdf will be

$$p(Z_k) = \sum_{U_0=0}^{N_u-1}\sum_{U_1=0}^{N_u-1-U_0} p(Z_k|U_0,U_1)P(U_0,U_1). \tag{59}$$

It can be shown that Z_k can be approximated to have a Gaussian distribution with high accuracy [15]. Thus, it suffices to find its mean and variance. This way, we obtain

$$\begin{aligned}m_{z_k} &= \sum_{U_0=0}^{N_u-1}\sum_{U_1=0}^{N_u-1-U_0} E(Z_k|U_0,U_1)P(U_0,U_1)\\ &= -\Omega_c N_h\end{aligned} \tag{60}$$

$$\sigma_{z_k}^2 = \sum_{U_0=0}^{N_u-1}\sum_{U_1=0}^{N_u-1-U_0}\sigma^2(Z_k|U_0,U_1)P(U_0,U_1)$$
$$= \sum_{U_0=0}^{N_u-1}\sum_{U_1=0}^{N_u-1-U_0}\left\{(U_0+U_1)2\Omega_c^2N_h+\frac{J_0}{2}\Omega_cN_i\right.$$
$$\left.+N_0\Omega_cN_h\right\}P(U_0,U_1). \quad (61)$$

As the Z_k's are independent and have the same pdf, we conclude that $Z=\sum_{k=1}^{d}Z_k$ also has Gaussian distribution, where its mean and variance are d times of those of Z_k, i.e., $p(Z)\sim N(dm_{z_k},d\sigma_{Z_k}^2)$. Then, we can easily obtain

$$P_d = P(Z>0) = Q\left(\sqrt{\frac{dm_{z_k}^2}{\sigma_{Z_k}^2}}\right) \quad (62)$$

where

$$Q(u) = \frac{1}{\sqrt{2\pi}}\int_u^\infty \exp\left(\frac{-x^2}{2}\right)dx. \quad (63)$$

The upper bound on the BER can be obtained as

$$P_b < \sum_{d=d_{\rm free}}^{\infty}c_dP_d = \sum_{d=d_{\rm free}}^{\infty}c_dQ\left(\sqrt{\frac{dm_{z_k}^2}{\sigma_{Z_k}^2}}\right). \quad (64)$$

To find a closed form for the preceding relation, we apply the improved union bound [8] as

$$P_b < \sum_{j=0}^{\infty}c_{j+d_{\rm free}}Q\left(\sqrt{\frac{(j+d_{\rm free})m_{z_k}^2}{\sigma_{Z_k}^2}}\right)$$
$$< Q\left(\sqrt{\frac{d_{\rm free}m_{z_k}^2}{\sigma_{Z_k}^2}}\right)e^{\frac{d_{\rm free}m_{z_k}^2}{2\sigma_{Z_k}^2}}\sum_{d=d_{\rm free}}^{\infty}c_de^{\frac{-dm_{z_k}^2}{2\sigma_{Z_k}^2}} \quad (65)$$

where we have used $Q(\sqrt{x+y}))<Q(\sqrt{x})e^{-y/2}$ [8]. Noting (34), we obtain

$$P_b < Q\left(\sqrt{\frac{d_{free}m_{z_k}^2}{\sigma_{Z_k}^2}}\right)$$
$$\times\frac{1}{(1-2D)^2}\left(\frac{1-D}{1-D^{K-2}}\right)^2\Bigg|_{D=e^{-\frac{m_{z_k}^2}{2\sigma_{z_k}^2}}}. \quad (66)$$

We also obtain a lower bound by noting the first term of (64) as

$$P_b > Q\left(\sqrt{\frac{d_{\rm free}m_{z_k}^2}{\sigma_{Z_k}^2}}\right). \quad (67)$$

V. Numerical Result

For comparison, we also consider the conventional ST-CDMA [1], [2] and first analyze its performance. Then, we provide some numerical results.

A. Conventional ST

In this scheme, the transmitted pulse has a time duration of T, which is N_s times that in the proposed method. Thus, the chip frequency duration in the conventional system Ω_c' must be $1/N_s$ of that of the proposed method, i.e., $\Omega_c'=\Omega_c/N_s=2/T$. In addition, in order for the two systems to have the same bandwidth, the number of frequency chips in the conventional ST scheme must be N_s times that in our method. Let W' be the bandwidth of the conventional ST system; then

$$W' = N_sN_h\Omega_c'/2 = N_h/T_f = W \quad (68)$$
$$\text{Processing Gain} = \frac{T}{2/(N_hN_s\Omega_c')} = N_sN_h \quad (69)$$

which means that the two systems have the same bandwidth and processing gain.

1) Fading Channel: For the conventional ST system, we use a correlator receiver. The receiver outputs due to the desired user, MAI, and the variances of the NBI and noise for the rectangular spectrum, noting the results of the previous sections [see (15), (19), (25), and (27)], are easily computed as

$$S = d_0^1 2\Omega_c'\sum_{n=0}^{N_sN_h/2-1}\beta_n^2 \quad (70)$$
$$\text{Var}(n_0) = N_0\Omega_c'\sum_{n=0}^{N_sN_h/2-1}\beta_n^2 \quad (71)$$
$$\text{Var}_{\text{NBI}} = J_0\Omega_c'\sum_{n=0}^{N_sN_i/2-1}\beta_n^2 \quad (72)$$
$$I^k = 2d_0^k\Omega_c'\sum_{n=0}^{N_sN_h/2-1}c_{2,n}^kc_{2,n}^1\beta_n^2 \quad (73)$$
$$\text{Var}_{\text{MAI}} = 4\Omega_c'^2\sum_{n=0}^{N_sN_h/2-1}\beta_n^4. \quad (74)$$

The average SNR for the conventional ST system is obtained as

$$\overline{\gamma}_{b-\text{conventional ST}} = \frac{N_h\Omega_c'N_s\sigma^2}{N_0/2} = \overline{\gamma}_b. \quad (75)$$

We observe that the proposed method and the ST scheme have the same average SNR.

The receiver output is $R=S+\sum_{k=1}^{N_u-1}I^k+\text{NBI}+n_0$. The BER, noting the symmetry, can be computed using the Beaulieu series as

$$P_{b-\text{conventional ST}} = P\left(R=S+\text{MAI}+\text{NBI}+n_0<0|d_0^1=1\right)$$
$$= \frac{1}{2}-\sum_{\substack{m=1\\ m\in N_{\rm odd}}}\frac{2\text{Im}\left\{\varphi_R(jmw_0)|d_0^1=1\right\}}{m\pi} \quad (76)$$

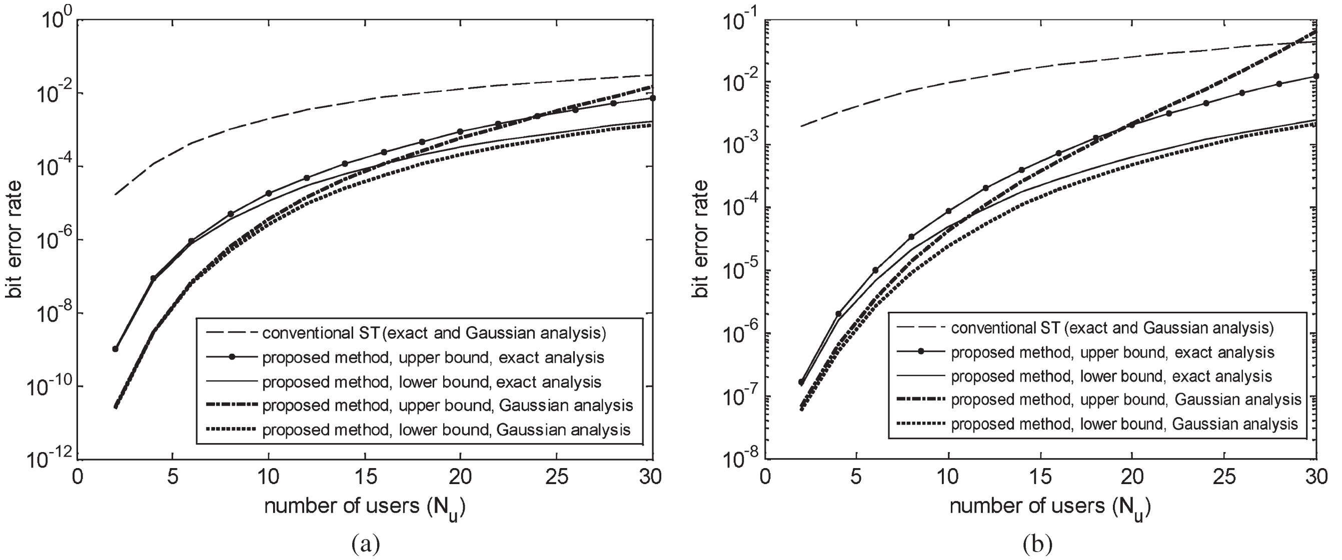

Fig. 4. Performance of the proposed method for the exact and Gaussian analyses in the AWGN channel, the processing gain of 256, and $N_s = N_h = 16$. (a) The NBI is not present, and SNR = 13 dB. (b) The energy of the NBI is the same as that of the desired user, $N_i = 2$, and SNR = 13 dB.

where $\varphi_R(s)$ is the moment-generating function of variable R (receiver output) as

$$\varphi_{\left(R=S+\sum_{k=1}^{N_u-1} I^k+\mathrm{NBI}+n_0\right)|d_0^1=1}(s)$$
$$=\left[\left(\frac{1}{2}\right)^{N_u-1}\sum_{k=0}^{N_u-1}\binom{N_u-1}{k}\times\frac{1}{2s(N_u-2k-2)\frac{\Omega_c}{N_s}\sigma^2-s^2\frac{\Omega_c}{N_s}\sigma^2\left(\frac{J_0}{2}+\frac{N_0}{2}\right)+1}\right]^{\frac{N_sN_i}{2}}$$
$$\cdot\left[\left(\frac{1}{2}\right)^{N_u-1}\sum_{k=0}^{N_u-1}\binom{N_u-1}{k}\times\frac{1}{2s(N_u-2k-2)\frac{\Omega_c}{N_s}\sigma^2-s^2\frac{\Omega_c}{N_s}\sigma^2\frac{N_0}{2}+1}\right]^{N_s\frac{N_h-N_i}{2}}. \quad (77)$$

In the case of the NBI cancellation, as stated before, we replace N_h with $N_h - N_i$ and ignore the NBI variance in (77).

2) AWGN Channel: The correlator outputs are obtained by setting $\beta_n = 1$ in the results of the fading channel [see (70)–(73)]. The exact BER, noting the analysis of the fading channel, is obtained as (76), where

$$\varphi_{\left(R=S+\sum_{k=1}^{N_u-1} I+\mathrm{NBI}+n_0\right)|d_0^1=1}(s)$$
$$=\left(\frac{1}{2}\right)^{\frac{(N_u-1)N_sN_h}{2}}\times\exp\left\{-s(N_u-2)N_h\Omega_c+\frac{s^2}{2}\left(\frac{N_0}{2}N_h+\frac{J_0}{2}N_i\right)\Omega_c\right\}$$
$$\cdot\left(\sum_{k=0}^{N_u-1}\binom{N_u-1}{k}\exp\left\{4sk\frac{\Omega_c}{N_s}\right\}\right)^{\frac{N_sN_h}{2}}. \quad (78)$$

The BER, assuming the Gaussian distribution for the MAI, is found as

$$P_{\mathrm{b-conventional\ ST}}=Q\left(\sqrt{\frac{S^2}{(N_u-1)\mathrm{Var_{MAI}}+\mathrm{Var_{NBI}}+\mathrm{Var}(n_0)}}\right)=Q\left(\sqrt{\frac{N_h\Omega_c}{2\frac{(N_u-1)\Omega_c}{N_s}+\frac{J_0}{2}\frac{N_i}{N_h}+\frac{N_0}{2}}}\right). \quad (79)$$

The SNR for the conventional ST system can be obtained similarly to (54).

B. Results

Now, we present some numerical results based on the analytical evaluations derived in the previous sections. Fig. 4(a) (absence of the NBI) and (b) (presence of the NBI) shows the performance of the proposed method and the conventional ST-CDMA system in the AWGN channel, using both the exact and Gaussian analyses. We observe that, in the proposed method, the Gaussian assumption overestimates the number of users, compared with the exact analysis. However, from this figure, we note that, when the number of users increases, the exact and Gaussian analyses have almost the same performance. In the conventional scheme, the exact and Gaussian analyses quite match. From Fig. 4, we also observe that the proposed method performs much better than the conventional ST scheme, particularly in the presence of the NBI, while the two systems have the same processing gain, spectral efficiency, and bandwidth. For example, in Fig. 4(a), in the absence of the NBI, to reach a BER of 10^{-4}, the number of users supported by the conventional ST-CDMA system is about four, while it is 15 in our method, which shows a great improvement. In Fig. 4(b), in the presence of the NBI, to reach a BER of almost 10^{-3}, the

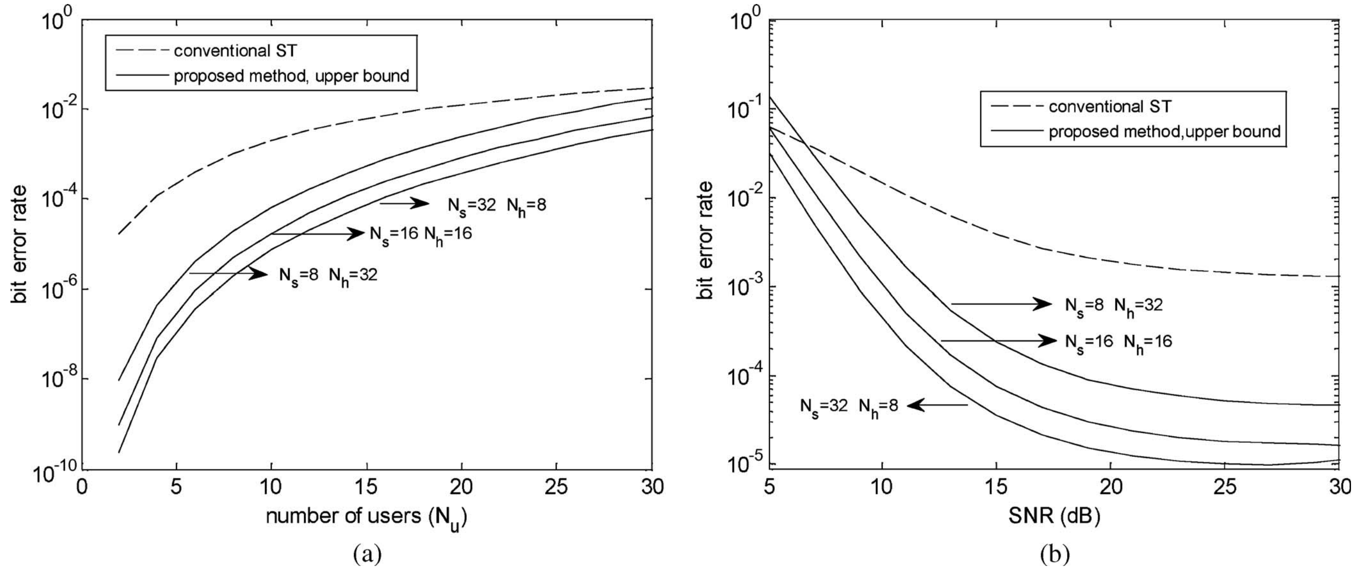

Fig. 5. Performance of the proposed method (upper bound) and the conventional ST-CDMA in the AWGN channel, $PG = 256$. NBI is not present. (a) SNR $= 13$ dB. (b) $N_u = 15$.

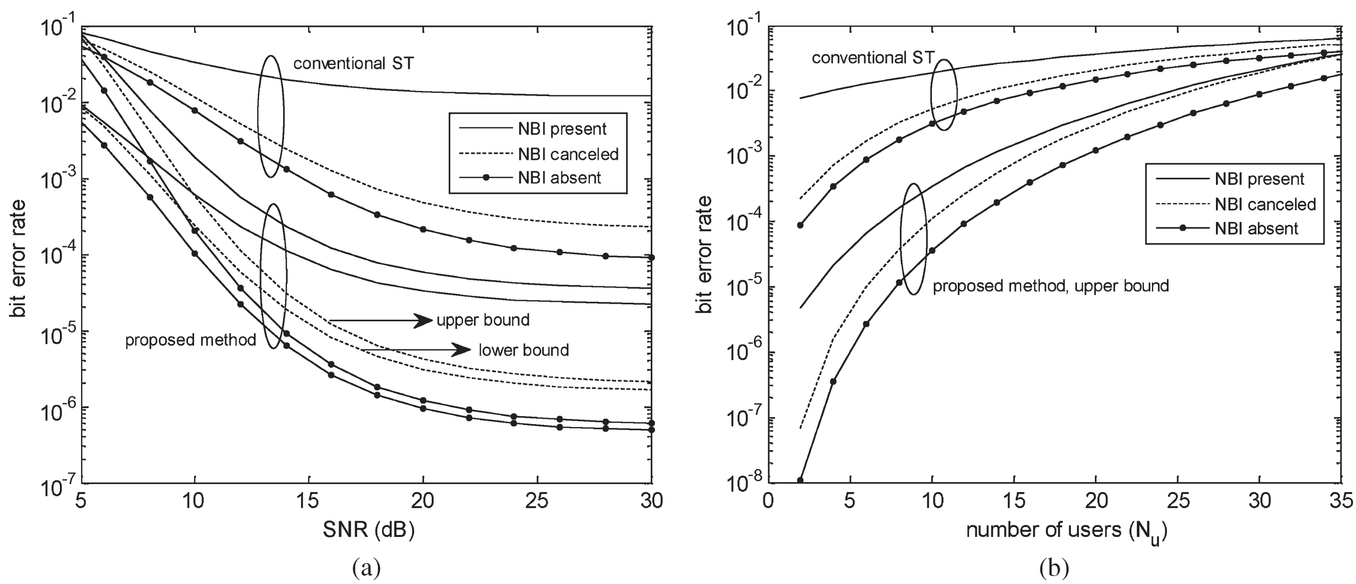

Fig. 6. Performance of the proposed method and the conventional ST-CDMA in the AWGN channel for the processing gain of 256, $N_s = N_h = 16$, and $N_i = 2$. (a) $N_u = 10$, and the energy of the NBI is 3 dB more than that of the desired user. (b) SNR $= 12$ dB, and the energy of the NBI is 2 dB more than that of the desired user.

numbers of users are two and 18 in the conventional ST and proposed methods, respectively.

Fig. 5 shows the performance of the proposed method and the conventional ST-CDMA system for the constant processing gain $PG = N_s N_h$ and various values of N_s and N_h in the AWGN channel based on the exact analysis. In this result, we have not considered the NBI. Note that the performance of the conventional ST-CDMA system depends only on the processing gain. We observe that our method significantly outperforms the conventional ST-CDMA system. We also observe that, for the constant processing gain, the performance of the proposed method improves for higher values of N_s and lower values of N_h due to the higher coding gain.

In Fig. 6, we consider the effect of the NBI and the MAI in the AWGN channel. In this figure, three curves for each scheme are shown. In the first curve, the BER is derived when the NBI exists, and it occupies one eight of bandwidth W, i.e., $N_i = 2$, $N_h = 16$. In the second curve, we have canceled the NBI by setting the PN sequence in the frequency chips where the NBI is present to zero. In the third curve, we have assumed that the NBI is absent. It is observed that the proposed method has much better performance than the conventional ST method and can also more significantly suppress the NBI effect. In fact, the proposed method in the presence of the NBI performs better than the conventional ST method in the absence of the NBI. The difference between the second and third curves is due to the fact that, for the second curve, some power of the desired signal will be lost at the receiver. That is, the second PN sequence at the receiver is set to zero in the part of bandwidth W where the NBI exists.

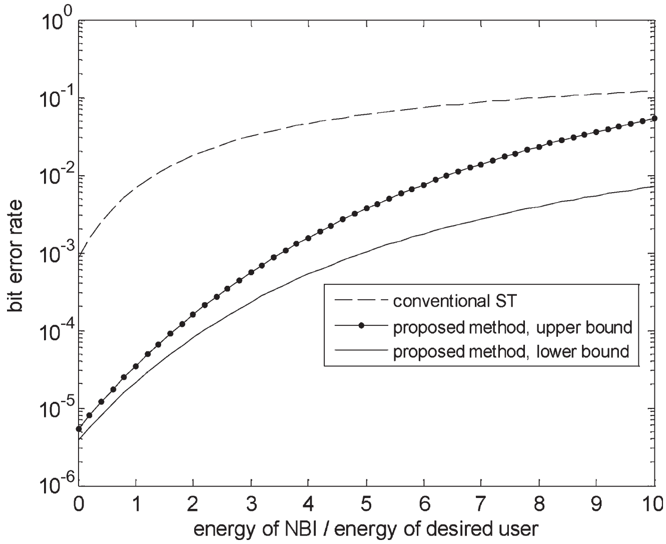

Fig. 7. Performance of the proposed method (lower and upper bounds) and the conventional ST-CDMA versus the ratio of the NBI energy to the desired signal energy in the AWGN channel ($N_s = N_h = 16$, $N_i = 2$, SNR $= 15$ dB, and $N_u = 10$).

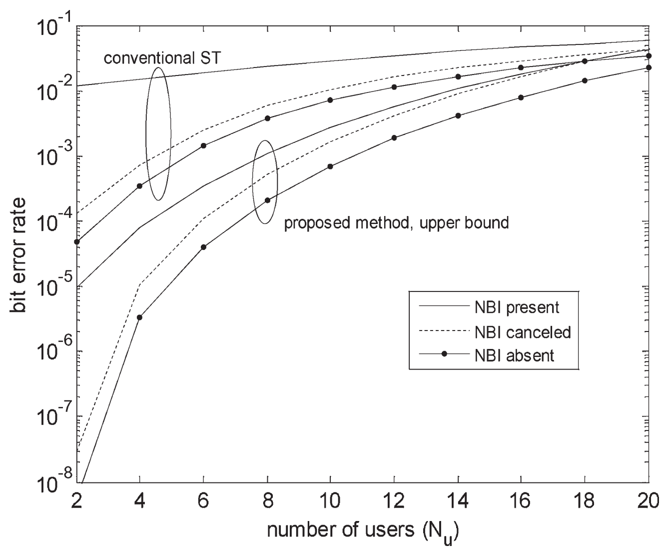

Fig. 8. Performance of the proposed method and the conventional ST-CDMA in the presence and absence of the NBI in the fading channel for the processing gain of 256, $N_s = N_h = 16$, and $N_i = 2, \bar{\gamma}_b = 15$ dB. The energy of the NBI is 2 dB more than that of the desired user.

In Fig. 7, the performances of the two systems are demonstrated versus the ratio of the energies of the NBI to the desired user. As expected, by increasing the energy of the NBI, the performance degrades. However, it is obvious that the new method outperforms the conventional ST.

Fig. 8 shows the performances of the conventional ST and the new method in the fading channel, in both cases of the absence and presence of the NBI. We observe that the performance of the proposed method in the fading channel is much better than that of the conventional ST, and it can more significantly suppress the NBI. From this figure and our further numerical results, it can be concluded that the results obtained in Figs. 4–7 for the AWGN channel also hold true for the fading channel.

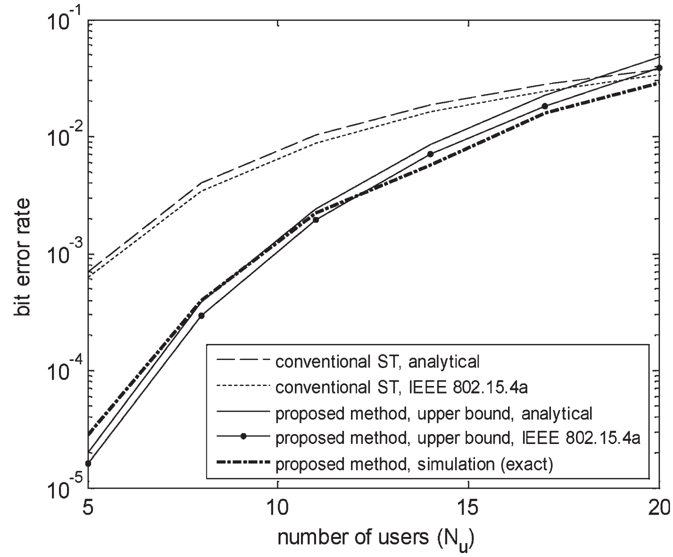

Fig. 9. Performance of the proposed method and the conventional ST-CDMA in the absence of the NBI in the fading channel using the analytical evaluations and simulation results, the processing gain of 256, $N_s = 8, N_h = 32$, and $\bar{\gamma}_b = 15$ dB.

In Fig. 9, we have provided the simulation results for the proposed coded scheme in the fading channel. We observe that the simulation results confirm the analytical evaluations. Note that the analytical results show the upper bound, whereas the simulation results demonstrate the exact BER. Furthermore, in this figure, we have included the performance of the proposed method and the conventional ST-CDMA method for the UWB application, with the UWB channel model specified by the IEEE 802.15.4a Group [13]. The center frequency and the bandwidth for this model are set to 6 GHz and 500 MHz, respectively. To evaluate the performance, we have considered the channel discrete model introduced in [16] to compute the channel coefficient for each frequency chip [i.e., β_n in (39)]. Then, we have numerically computed the integral in (39) by averaging over the 10^5 different values of β_n for different sample blocks. From Fig. 9, it is observed that the upper bounds on the BER for the channel model considered in this paper and the model specified by the IEEE 802.15.4a Group are almost the same for both the conventional and proposed ST schemes. Note that it can be shown that, in the frequency domain, each chip sample experiences a fading coefficient (which is denoted by β_ns) with a distribution that can be computed from the channel characteristic given in the time domain. In [16], the distribution of β_ns is computed for the UWB channel model in [13]. In the model considered in this paper, it has also been assumed that each frequency chip experiences a different fading coefficient but with an independent Rayleigh distribution. Thus, from Fig. 9, it can be concluded that the channel model considered in this paper well approximates the model in [13] for the UWB communications.

Fig. 10 shows the performance of the proposed method and the conventional ST system in the nonfading (AWGN) and fading channels in the presence of the NBI. It is observed that the proposed scheme in the fading channel substantially outperforms the conventional ST method, even when the latter method is operating in the nonfading channel.

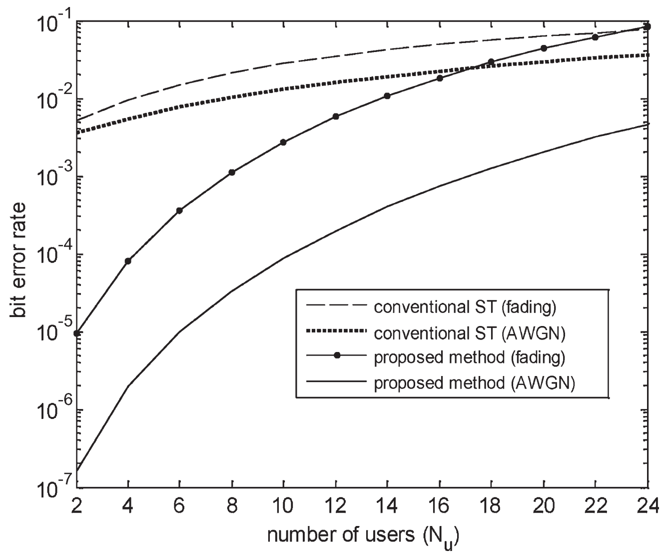

Fig. 10. Performance of the proposed method (upper bound) and the conventional ST-CDMA in the AWGN and fading channels, $N_s = N_h = 16$, and $N_i = 2$. The SNRs in the fading and AWGN channels are 15 dB, and the energy of the NBI is 2 dB more than that of the desired user.

VI. Conclusion

We have considered an internally coded TH system using the ST-CDMA technique. The performance of the correlator receiver, followed by the Viterbi decoder, in the AWGN and fading channels was analyzed. We considered the effect of the MAI and the NBI. The upper and lower bounds on the BER using the Beaulieu series were obtained. For comparison, we have also considered the conventional ST-CDMA system, in which the data are sent over the whole bit time with the same processing gain and bandwidth as those of our method. It was shown that the proposed method significantly outperforms the conventional ST-CDMA system. It was demonstrated that the Gaussian approximation for the pdf of the MAI in the AWGN channel has overestimated the number of users, compared with the exact analysis. In addition, in the proposed method, it was observed that, for the constant processing gain, the performance improves for higher values of N_s and lower values of N_h due to the increase in the coding gain by N_s. Furthermore, it was realized that the proposed method could substantially suppress the NBI. The simulation results confirmed the analytical evaluations. In addition, it was demonstrated that, when using the fading parameters recommended by the IEEE 802.15.4a Group for UWB for the performance evaluation, almost the same results are obtained.

Appendix
Calculation of the Integral in (39)

Here, we compute the integral in (39). Replacing (45) and (47) results in

$$\int \varphi_{R_{h\neq 0}|U_1,\beta_{\{n\}}}(s)\varphi_{R_{h=0}|U_0,\beta_{\{n\}}}(-s)p\left(\beta_{\{n\}}\right)d\beta_{\{n\}}$$
$$=\int_0^\infty\cdots\int_0^\infty \prod_{n=0}^{N_h/2-1}\left(\frac{1}{2}\exp\left\{2s\Omega_c\beta_n^2\right\}+\frac{1}{2}\exp\left\{-2s\Omega_c\beta_n^2\right\}\right)^{U_0+U_1}$$
$$\times\prod_{n=0}^{N_h/2-1}\exp\left\{\left(s^2N_0-2s\right)\Omega_c\beta_n^2\right\}\prod_{n=0}^{N_i/2-1}\exp\left\{\frac{s^2}{2}J_0\Omega_c\beta_n^2\right\}$$
$$\times p(\beta_0,\ldots\beta_n,\ldots,\beta_{N_h/2-1})d\beta_0,\ldots,d\beta_n,\ldots,d\beta_{N_h/2-1}. \tag{80}$$

Since the effect of the fading coefficients appears as β_n^2 in the equations, by a change in variable, we conclude that $\lambda_n = \beta_n^2$ has an exponential distribution [14], i.e.,

$$p_{\lambda_n}(\lambda_n)=\frac{1}{\sigma^2}\exp\left(-\frac{\lambda_n}{\sigma^2}\right)\quad E\left(\lambda_n=\beta_n^2\right)=\sigma^2. \tag{81}$$

We use the binomial expansion of the first term of the integral in (80), which results in

$$\left\{\frac{1}{2}\exp(2s\Omega_c\lambda_n)+\frac{1}{2}\exp(-2s\Omega_c\lambda_n)\right\}^{U_0+U_1}$$
$$=\left(\frac{1}{2}\right)^{U_0+U_1}\sum_{k=0}^{U_0+U_1}\binom{U_0+U_1}{k}\exp\{2sk\Omega_c\lambda_n)\}$$
$$\times\exp\left\{-2s(U_0+U_1-k)\Omega_c\lambda_n\right\}. \tag{82}$$

Replacing the preceding term in (80) gives

$$\int\varphi_{R_{h\neq 0}|U_1,\beta,\{n\}}(s)\varphi_{R_{h=0}|U_0,\beta_{\{n\}}}(-s)p\left(\beta_{\{n\}}\right)d\beta_{\{n\}}$$
$$=\int_0^\infty\cdots\int_0^\infty\cdots\int_0^\infty\left(\prod_{n=0}^{N_h/2-1}\sum_{k=0}^{U_0+U_1}\left(\frac{1}{2}\right)^{U_0+U_1}\binom{U_0+U_1}{k}\right.$$
$$\left.\times\exp\left\{-2s(U_0+U_1-2k)\Omega_c\lambda_n\right\}\right)$$
$$\times\left(\prod_{n=0}^{N_i/2-1}\exp\left\{\left(s^2N_0+\frac{s^2}{2}J_0-2s\right)\Omega_c\lambda_n\right\}\right)$$
$$\times\left(\prod_{n=N_i/2}^{N_h/2-1}\exp\left\{(s^2N_0-2s)\Omega_c\lambda_n\right\}\right)$$
$$\times\left(\prod_{n=0}^{N_h/2-1}\left(\frac{1}{\sigma^2}\right)\exp\left(\frac{-1}{\sigma^2}\lambda_n\right)\right)$$
$$\times d\lambda_0,\ldots,d\lambda_n,\ldots,d\lambda_{N_h/2-1}. \tag{83}$$

Since the preceding $N_h/2$ integrals are independent and have the same form, it suffices to calculate one of them. Then, we obtain

$$\int\varphi_{R_{h\neq 0}|U_1,\beta_{\{n\}}}(s)\varphi_{R_{h=0}|U_0,\beta_{\{n\}}}(-s)p\left(\beta_{\{n\}}\right)d\beta_{\{n\}}$$
$$=\left[\left(\frac{1}{2}\right)^{U_0+U_1}\sum_{k=0}^{U_0+U_1}\binom{U_0+U_1}{k}\frac{1}{\sigma^2}\right.$$

$$
\times \int_0^\infty \exp\left\{\left(-2s(U_0+U_1-2k+1)\Omega_c + s^2\Omega_c\left(N_0+\frac{J_0}{2}\right)-\frac{1}{\sigma^2}\right)\lambda_n\right\}d\lambda_n\Bigg]^{\frac{N_i}{2}}
$$

$$
\cdot\left[\left(\frac{1}{2}\right)^{U_0+U_1}\sum_{k=0}^{U_0+U_1}\binom{U_0+U_1}{k}\frac{1}{\sigma^2}\right.
$$

$$
\times\frac{1}{\sigma^2}\int_0^\infty \exp\left\{\left(-2s(U_0+U_1-2k+1)\Omega_c + s^2\Omega_c N_0-\frac{1}{\sigma^2}\right)\lambda_n\right\}d\lambda_n\Bigg]^{\frac{N_h-N_i}{2}}
$$

$$
=\left[\left(\frac{1}{2}\right)^{U_0+U_1}\sum_{k=0}^{U_0+U_1}\binom{U_0+U_1}{k}\times\frac{1}{2s(U_0+U_1-2k+1)\Omega_c\sigma^2-s^2\Omega_c\sigma^2\left(N_0+\frac{J_0}{2}\right)+1}\right]^{\frac{N_i}{2}}
$$

$$
\cdot\left[\left(\frac{1}{2}\right)^{U_0+U_1}\sum_{k=0}^{U_0+U_1}\binom{U_0+U_1}{k}\times\frac{1}{2s(U_0+U_1-2k+1)\Omega_c\sigma^2-s^2\Omega_c\sigma^2 N_0+1}\right]^{\frac{N_h-N_i}{2}}. \tag{84}
$$

REFERENCES

[1] P. M. Crespo, M. L. Honig, and J. A. Salehi, "Spread-time code-division multiple access," *IEEE Trans. Commun.*, vol. 43, no. 6, pp. 2139–2148, Jun. 1995.

[2] M. G. Shayesteh, J. A. Salehi, and M. Nasiri-Kenari, "Spread-time CDMA resistance in fading channels," *IEEE Trans. Wireless Commun.*, vol. 2, no. 3, pp. 446–458, May 2003.

[3] J. A. Salehi, A. M. Weiner, and J. P. Heritage, "Coherent ultrashort light pulse code-division multiple access communication systems," *J. Lightwave Technol.*, vol. 8, no. 3, pp. 478–491, Mar. 1990.

[4] M. G. Shayesteh, M. Nasiri-Kenari, and J. A. Salehi, "Spread-time CDMA performance with and without windowing," in *Proc. IEEE PIMRC*, Lisboan, Portugal, Sep. 2002, pp. 1166–1170.

[5] C. R. C. M. da Silva and L. B. Milstein, "Spectral-encoded UWB communication systems: Real-time implementation and interference suppression," *IEEE Trans. Commun.*, vol. 53, no. 8, pp. 1391–1401, Aug. 2005.

[6] C. R. C. M. da Silva and L. B. Milstein, "The effect of narrowband interference on UWB communication systems with imperfect channel estimation," *IEEE J. Sel. Areas Commun.*, vol. 24, no. 4, pp. 717–723, Apr. 2006.

[7] Q. Liu, R. A. Scholtz, and Z. Zeng, "Complexity of Verdu optimum multiuser detection algorithm in multichannel CDMA system," *IEEE Trans. Commun.*, vol. 47, no. 12, pp. 1913–1920, Dec. 1999.

[8] A. J. Viterbi, *CDMA: Principles of Spread-Spectrum Communications*. Reading, MA: Addison-Wesley, 1995.

[9] S. B. Wicker, *Error Control Systems for Digital Communication and Storage*. Englewood Cliffs, NJ: Prentice–Hall, 1995.

[10] N. C. Beaulieu, "An infinite series for the computation of the complementary probability distribution function of a sum of independent random variables and its application to the sum of Rayleigh random variables," *IEEE Trans. Commun.*, vol. 38, no. 9, pp. 1463–1474, Sep. 1990.

[11] A. Nezampour, M. Nasiri-Kenari, and M. G. Shayesteh, "Internally coded TH–UWB–CDMA system and its performance evaluation," *IET Commun.*, vol. 1, no. 2, pp. 225–232, Apr. 2007.

[12] M. G. Shayesteh and M. Nasiri-Kenari, "Internally coded MC–FH–CDMA communication system and its performance analysis," *IET Commun.*, vol. 2, no. 2, pp. 255–265, Feb. 2008.

[13] A. F. Molisch, C. C. Chong, B. Kannan, S. Emami, A. Fort, and U. G Schuster, *802.15.4a Channel Model Subgroup Final Report*, Sep. 2004, Germany. IEEE Std. 802.15-04-0535-00-004a.

[14] J. Proakis, *Digital Communications*. Englewood Cliffs, NJ: Prentice–Hall, 2001.

[15] A. Papoulis, *Probability, Random Variables, and Stochastic Processes*. New York: McGraw-Hill, 2002.

[16] E. Arikan, "Capacity bounds for an ultra-wideband channel model," in *Proc. IEEE ITW*, Oct. 2004, pp. 176–181.

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