NLOS Identification in Range-Based Source Localization: Statistical Approach

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***Abstract*— Least squares estimation is a widely-used technique for range-based source localization, which obtains the most probable position of mobile station. These methods cannot provide desirable accuracy in the case with a non line of sight (NLOS) path between mobile station and base stations. To circumvent this drawback, many algorithms have been proposed to identify and mitigate this error; however, they have a large run-time overhead. On the other hand, new positioning systems utilize a large set of base stations, and a practical algorithm should be fast enough to deal with them. In this paper, we propose a novel algorithm based on subspace method to identify and eliminate the NLOS error. Simulation studies show that our algorithm is faster and more accurate compared with other conventional methods, especially in the large-scale cases.**

***Index Terms*— Non line of sight (NLOS) identification, source localization, subspace method, time of arrival (TOA).**

I. Introduction

RESEARCH on target localization has received significant attention in the last decades in such applications as communications [1], wireless sensor networks [2]–[7], and radar systems [8]–[15]. Source localization methods are usually based on different types of measurements, including received signal strength (RSS), time of arrival (TOA), and angle of arrival (AOA). Each technique may be preferable compared to others, depending on the applications, resources, and the environmental parameters; however, the TOA-based methods, which are also known as range-based methods, are the most common ones due to high accuracy and simplicity. Each TOA measurement induces a circular locus for the source to be located, with the associated base station (BS) as its center. The intersection point of all these circles gives the estimated position of the mobile station (MS). In the presence of noise, the circles will not have a unique point of intersection. This is why we need an algorithm such as Taylor series estimator to obtain the MS location under the least squares criterion as the most probable answer for the MS position [16], [17].

If the signal has traveled in a direct path from MS to BS, we can measure the exact distance between MS and BSs. However, the noise will limit the positioning accuracy. On the other hand, if a BS sees the MS through a reflector, the measured range will exceed its true value dramatically. This phenomenon, which is known as non line of sight (NLOS), leads to a large error in localization. NLOS error always adds a positive value to the true range, and in many cases, this additional length remains constant over a long period of time.

Generally speaking, localization accuracy can be improved by use of large number of sensors. Recently, various large-scale positioning systems have been developed. Specifically, with the advent of multi constellations global navigation satellite system (GNSS) receiver, available satellites for the users have been increased dramatically [18]. In [19], a large number of sensors has been employed to find the origin of an electrical pulse in an accurate way. Note that in the large-scale sensor systems, NLOS is considered as the main source of error. However, identification of NLOS stations based on the measured data has a large time overhead, usually in an exponential relation with the number of stations. That is why innovating an efficient method for NLOS identification is one of the most interesting fields of todays' ongoing research.

In the last few decades, many different algorithms have been proposed to mitigate or identify NLOS error [20]–[26]. In [20], a residual weighting algorithm (RWGH) has been proposed, which locates the source using all possible sets of sensors separately. Subsequently, a residual for the station is computed based on the estimated source position by each set. Using these residuals, a weighing matrix is formed, which can be employed in Taylor series estimator to obtain a better result in the case of NLOS measurements. It has been shown in [24] that RWGH provides good performance in dealing with small-scale positioning systems. Nevertheless, as the number of sensors increases, not only will the computational burden rise unbearably, but also the accuracy in NLOS mitigation will decrease. There exist some other methods, exploiting extra information in addition to range measurement to identify the NLOS error. In the proposed methods in [21] and [22], based on a digital map of urban environments, GPS receiver will be able to find the suspected satellite that might have NLOS. In [23], a method has been proposed which employs scattering model of the environment to mitigate the NLOS error. Despite of accuracy of these methods, they are not practical due to lack of this extra information for all environments. Furthermore, the structure of these environments is varying over time, and thus, it is hard to obtain digital map or scattering model frequently. Note that the above-mentioned methods are computationally-expensive and impractical. To reduce the complexity, we proposed an sparsity-based NLOS mitigation method in [24], which is called SRNI. SRNI employs the sparse recovery technique in [27] to solve an underdetermined system of equations which can address the NLOS problem.

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In this paper, we propose a novel algorithm to identify NLOS stations based on statistical features of range measurements, which is called Fast Identification of NLOS error using Eigenvector (FINE). We model the NLOS error as a deterministic additive term in addition to a white zero-mean Gaussian noise. FINE algorithm tries to find this value using the autocorrelation function of the error and its eigenvector. Since there are highly efficient algorithms for eigenvalue decomposition of special matrices, e.g., [28], the proposed method will have computational benefits in dealing with large-scale positioning problems. Simulation results demonstrate superiority of the proposed method in comparison with the state-of-the-art algorithms in terms of accuracy and complexity.

The notations used in this paper are as follows: We denote matrices by uppercase bold letter and vectors by lowercase bold letter. The notations $(*)^{-1}$, $(*)^T$, and $(*)^H$ stand for inverse, transpose, and conjugate transpose operations. $\boldsymbol{I}$ denotes the identity matrix, whose size can be found from context. The symbols $\mathbb{E}\{*\}$ and $\| * \|$ stands for statistical expectation and ℓ_2 norm, respectively.

The rest of this paper is organized as follows: The problem is formulated in Section II. The proposed method for identifying the NLOS stations is given in Section III. Simulation results are presented in Section IV. Finally, Section V concludes this paper.

II. Problem Statement

We consider a two-dimensional (2-D) localization scenario. We aim to locate a single mobile station (MS) using a set of N widely distributed base stations (BSs), whose positions are represented by $\mathbf{s}_i = [x_i, y_i]^T, i = 1, \ldots, N$. The true position of the to be located MS is denoted by $\mathbf{u} = [x, y]^T$.

In the presence of NLOS, the measured range in the ith BS can be represented as

$$\tilde{r}_i = r_i + n_i + w_i, \quad i = 1, 2, \ldots, N \tag{1}$$

where w_i's are measurement noises and assumed to be independent and identically distributed (iid) zero-mean Gaussian random variables with variance σ^2. n_i is a constant and non-negative parameter representing the NLOS value. In fact, n_i has non-zero value only for NLOS measurements. r_i denotes the true range between the MS and the ith BS, and can be represented as follows:

$$r_i = \|\mathbf{s}_i - \mathbf{u}\| = \sqrt{(x_i - x)^2 + (y_i - y)^2} \tag{2}$$

In the absence of NLOS, the MS position can be estimated, using least squares (LS) criterion, as follows [16]:

$$(\hat{x}, \hat{y}) = \operatorname{argmin}_{x,y} \sum_{i=1}^{N} \left(\tilde{r}_i - \sqrt{(x_i - x)^2 + (y_i - y)^2}\right)^2 \tag{3}$$

Problem (3) is highly nonlinear in terms of MS position. Thus, any deviation from NLOS-free model results in a large estimation error. Identifying the NLOS stations and disregarding the associated range measurements has been considered as a solution for circumventing the aforementioned problem. In the next section, we shall propose a novel solution for this problem, identifying NLOS stations using subspace orthogonality of the noise and NLOS.

III. Proposed Algorithm

In this section, we develop a novel algorithm based on statistical characteristics of error terms (noise plus NLOS). To this end, we first restate (1) in matrix form as

$$\tilde{\boldsymbol{r}} = \boldsymbol{r} + \boldsymbol{\rho} \tag{4}$$

where

$$\begin{aligned}
\boldsymbol{\rho} &= \boldsymbol{n} + \boldsymbol{w} \\
\tilde{\boldsymbol{r}} &= [\tilde{r}_1, \tilde{r}_2, \ldots, \tilde{r}_N]^T \\
\boldsymbol{r} &= [r_1, r_2, \ldots, r_N]^T \\
\boldsymbol{n} &= [n_1, n_2, \ldots, n_N]^T \\
\boldsymbol{w} &= [w_1, w_2, \ldots, w_N]^T
\end{aligned} \tag{5}$$

Taking autocorrelation on $\boldsymbol{\rho}$ and using (5) yield

$$\begin{aligned}
\boldsymbol{R}_\rho &= \mathbb{E}\{(\boldsymbol{n} + \boldsymbol{w})(\boldsymbol{n} + \boldsymbol{w})^H\} \\
&= \mathbb{E}\{\boldsymbol{n}\boldsymbol{n}^H\} + \mathbb{E}\{\boldsymbol{n}\boldsymbol{w}^H\} + \mathbb{E}\{\boldsymbol{w}\boldsymbol{n}^H\} + \mathbb{E}\{\boldsymbol{w}\boldsymbol{w}^H\}
\end{aligned} \tag{6}$$

where $\boldsymbol{R}_\rho$ denotes autocorrelation of $\boldsymbol{\rho}$. Since $\boldsymbol{w}$ is white and zero-mean Gaussian noise with the power σ^2, (6) can be expressed as

$$\boldsymbol{R}_\rho = \boldsymbol{n}\boldsymbol{n}^H + \sigma^2 \boldsymbol{I} \tag{7}$$

Multiplying (7) with $\boldsymbol{n}$ from right-hand side gives

$$\begin{aligned}
\boldsymbol{R}_\rho \boldsymbol{n} &= \boldsymbol{n}\boldsymbol{n}^H \boldsymbol{n} + \sigma^2 \boldsymbol{n} \\
&= \Big(\sum_{i=1}^{N} |n_i|^2 + \sigma^2\Big)\boldsymbol{n}
\end{aligned} \tag{8}$$

Expression (8) demonstrates that $\left(\sum_{i=1}^{N} |n_i|^2 + \sigma^2\right)$ is an eigenvalue of $\boldsymbol{R}_\rho$ with eigenvector $\boldsymbol{n}$. In a symmetric matrix, all of the eigenvectors are orthogonal to each other. Suppose that $\boldsymbol{e}$ represents another arbitrary eigenvector of $\boldsymbol{R}_\rho$. Since $\boldsymbol{e}$ is orthogonal to $\boldsymbol{n}$, multiplying (7) with $\boldsymbol{e}$ yields

$$\begin{aligned}
\boldsymbol{R}_\rho \boldsymbol{e} &= \boldsymbol{n}\boldsymbol{n}^H \boldsymbol{e} + \sigma^2 \boldsymbol{e} \\
&= \sigma^2 \boldsymbol{e}
\end{aligned} \tag{9}$$

By comparing (8) and (9), we realize that the eigenvector corresponding to maximum eigenvalue of $\boldsymbol{R}_\rho$ is $\boldsymbol{n}$. Therefore, if one can find $\boldsymbol{R}_\rho$, it will be possible to identify NLOS by finding its maximum eigenvalue and the associated eigenvector.

To calculate $\boldsymbol{R}_\rho$, we can measure $\boldsymbol{\rho}$ for M times and use an approximate approach for expectation operator as follows:

$$\boldsymbol{R}_\rho = \frac{\sum_{l=1}^{M} \boldsymbol{\rho}_l \boldsymbol{\rho}_l^H}{M} \tag{10}$$

where $\boldsymbol{\rho}_l$ represents the error vector, $\boldsymbol{\rho}$, in the lth observation. The time between observations should be large enough to let $\boldsymbol{w}$ be uncorrelated among $\boldsymbol{\rho}_l$'s. We also need the true position of the MS to calculate $\boldsymbol{\rho}$ via (4); however, this position is unknown. When there are a large number of BSs, we can find the MS position via all stations by neglecting the error due to the NLOS, and considering the result as the true MS position to calculate $\boldsymbol{\rho}$.

In order to find out whether there is an NLOS error or not, based on (8) and (9), we need to estimate σ^2 and compare it with the largest eigenvalue. If this eigenvalue exceeds σ^2, $\sum_{i=1}^{N} |n_i|^2$ term and consequently NLOS error should exist. According to (9), all the eigenvalues except the largest one should be equal to σ^2. However, since we use (10) instead of mathematical expectation to calculate $\boldsymbol{R_\rho}$, eigenvalues become random variables and non-constant. Thus, we need a method to assess existence of NLOS term in the largest eigenvalue of $\boldsymbol{R_\rho}$. Before describing this method, two lemmas are presented to introduce the eigenvalues distribution and relation among their sorted samples.

Lemma 1: The eigenvalues of an autocorrelation matrix for normal distribution can be described using Marcenko law [29]. Marcenko law introduces the following probability density function (PDF) for eigenvalues:

$$f_\lambda(\lambda) = \begin{cases} \frac{\gamma}{2\pi\lambda}\sqrt{(b-\lambda)(\lambda-a)}, & \text{if } \lambda \in [a, b] \\ 0, & \text{otherwise} \end{cases} \tag{11}$$

where γ denotes the ratio of the number of observations and samples, which controls the statistical feature of Marcenko distribution and

$$\begin{aligned} a &= (1-\gamma^{-1/2})^2 \\ b &= (1+\gamma^{-1/2})^2 \end{aligned} \tag{12}$$

Proof: Proof is given in [29]. ■

Remark 1: Cumulative distribution function (CDF) of (11) is not easy to work with. Whenever γ is close enough to one, we can fit an exponential function to the above-mentioned CDF as

$$F_\lambda(\lambda) = 1 - e^{-\alpha\lambda} \tag{13}$$

where α is a constant to adjust this function to the data. ■

Lemma 2: The function created by *sorting* the samples of a non-negative random variable can be approximated by the inverse of its CDF.

Proof: To illustrate this concept, let us define *sorting* as an operation on the samples, where each sample is larger than all of its left-sided samples. Moreover, without loss of generality, we normalize the samples to their largest value so that the range of samples becomes between 0 and 1.

Therefore, we can interpret *sorting* space as the probability that a sample is larger than all of its left-sided samples, which can be written via the following mathematical expression:

$$\tau = P(\lambda' < \lambda) = F_\lambda(\lambda) \tag{14}$$

or

$$\lambda = F_\lambda^{-1}(\tau) \tag{15}$$

where λ is the eigenvalue in position τ and λ' denotes all of its left-sided samples after sorting. ■

Let λ_i be the eigenvalue of $\boldsymbol{R_\rho}$ for $i = 1, \ldots, N$. We create a discrete function by sorting and relating eigenvalues to the sorting space as

$$F = \{(\lambda_1, \frac{1}{N}), (\lambda_2, \frac{2}{N}), \ldots, (\lambda_{N-1}, \frac{N-1}{N}), (\lambda_N, \frac{N}{N})\} \tag{16}$$

where

$$\lambda_1 < \lambda_2 < \ldots < \lambda_N$$

In this function, eigenvalues and the sorting space are intentionally inversed to result in CDF function. According to lemma 1 and 2, (13) is an appropriate function to be fitted on (16) by adjusting α for $i = 1 \ldots N-1$. To this end, we can use an initial guess for α and gradient descent method to improve our guess. Let the initial value of α be 0. We define the following vectors using data in (16).

$$\mathbf{h} = \begin{pmatrix} \lambda_1 e^{-\alpha\lambda_1} \\ \lambda_2 e^{-\alpha\lambda_2} \\ \vdots \\ \lambda_{N-1} e^{-\alpha\lambda_{N-1}} \end{pmatrix}$$

$$\mathbf{z} = \begin{pmatrix} \frac{1}{N} - (1 - e^{-\alpha\lambda_1}) \\ \frac{2}{N} - (1 - e^{-\alpha\lambda_2}) \\ \vdots \\ \frac{N-1}{N} - (1 - e^{-\alpha\lambda_{N-1}}) \end{pmatrix} \tag{17}$$

The error caused by this guess is calculated as follows:

$$\delta = [\boldsymbol{h}^T\boldsymbol{h}]^{-1}\boldsymbol{h}^T\boldsymbol{z} \tag{18}$$

By adding this error to the initial guess as (19), putting back the result in (17), and iterating this procedure, α converges to its final value.

$$\alpha \leftarrow \alpha + \delta \tag{19}$$

In this step all we know about σ^2, is its CDF and the fact that it is bigger than λ_{N-1}. Based on these data, the probability that σ^2 is less than an arbitrary constant Γ, is

$$\tau = P(\sigma^2 < \Gamma | \sigma^2 > \lambda_{N-1}) = \frac{F_\lambda(\Gamma) - F_\lambda(\lambda_{N-1})}{1 - F_\lambda(\lambda_{N-1})} \tag{20}$$

One can obtain Γ by substituting (13) into (20) and solving it for Γ as follows:

$$\Gamma = \lambda_{N-1} - \frac{\ln(1-\tau)}{\alpha} \tag{21}$$

With all these in mind, in order to identify the NLOS error, we can find eigenvalues of $\boldsymbol{R_\rho}$, sort them as (16), and fit (13) to it. Then, calculate Γ from (21) for an arbitrary τ. If the largest eigenvalue is greater than Γ, there should be an NLOS error with a chance of τ.

Returning to (8), we realize that the eigenvector corresponding to the largest eigenvalue is $\boldsymbol{n}$. The stations which deal with NLOS error have the have the largest components in $\boldsymbol{n}$. We decide to select the station with the largest component in $\boldsymbol{n}$ as the NLOS station in each run of the algorithm. The procedure to determine the existence of NLOS error and finding NLOS position is summarized in Algorithm 1.

Note that eigenvalue decomposition of $\boldsymbol{R}_\rho$ can be obtained by exploiting off-the-shelf algorithms such as MATLAB eig routine, which provides the eigenvalues in decreasing order. We employ resulting eigenvalue vector, $\boldsymbol{\lambda}$, and the eigenvector corresponding to maximum eigenvalue, $\boldsymbol{v}$, as the input of Algorithm 1.

Algorithm 1 NLOS Finder

Input:
- $\boldsymbol{\lambda}$:= A vector containing eigenvalues of $\boldsymbol{R}_\rho$
- $\boldsymbol{v}$:= Eigenvector corresponding to the maximum eigenvalue of $\boldsymbol{R}_\rho$
- τ := Chance of NLOS detection

Output:
- NSN := NLOS Station Index ▷ zero if there is no NLOS

```
procedure NLOS FINDER(λ, v, τ)
    F ← sort(λ)
    α ← fit function (13) to F
    Γ ← equation (21) using α
    if (Γ < λ_N) then
        NSN ← index of maximum(v)
    else
        NSN ← 0
    end if
    return NSN
end procedure
```

Algorithm 2 Fast Identification of NLOS error using Eigenvector (**FINE**)

Input:
- $\tilde{\boldsymbol{r}}_i$:= Range vector measured by BSs for the ith observation
- **S** := The set of the BS positions as $(\boldsymbol{x}_j, \boldsymbol{y}_j)$, where j is the BS index

Output:
- $(\hat{\boldsymbol{x}}, \hat{\boldsymbol{y}})$:= Estimated position of MS

```
procedure FINE(r̃, S)
    while ever do
        (x', y') ← LS(r̃_i, S)    ▷ Estimated position of MS with LS algorithm using all survived observation
        R_ρ ← Equation(10) using (x', y')
        λ ← Eigenvalue(R_ρ)
        v ← Eigenvector(R_ρ)    ▷ The eigenvector corresponding to maximum eigenvalue
        NSN ← NLOS finder(λ, v, τ)    ▷ Algorithm 1
        if (NSN = 0 or size(S) <= dimension + 1) then
            exit while
        else
            Eliminate the ith station in S
        end if
    end while
    (x̂, ŷ) ← LS(r̃_i, S)    ▷ Estimated position of MS with LS algorithm using the last observation and reminded part of S
    return (x̂, ŷ)
end procedure
```

We can find only one NLOS station in each run of Algorithm 1. To identify all NLOS stations, we should eliminate suspected station in each run, and repeat the algorithm for the survived stations. This procedure must be continued until the algorithm does not be able to find any other NLOS station. In a K-dimensional space, we need at least $(K + 1)$ stations to determine the MS position uniquely. Therefore, we cannot eliminate more than $(N - K - 1)$ stations. To illustrate the proposed method in a clear way, Algorithm 2 is presented.

We control a tradeoff between false alarm rate and successful NLOS detection rate using τ. Grater τ will result in more accurate NLOS detection, but some NLOS stations may be lost. Smaller τ leads to some false NLOS alarms, and subsequently we may lose some LOS stations.

Remark 2: As seen in Algorithm 2, the proposed method estimates the source position using survived measurements in a recursive structure, which is further employed to obtain an estimate of $\boldsymbol{R}_\rho$. The localization algorithm used here is nonlinear LS, which is sensitive to NLOS error. However, based on simulations conducted in Sec. IV-C, the proposed method is robust to increment in the number of NLOS stations. Furthermore, the proposed method can exploit robust localization methods such as [6] and [7] instead of LS algorithm to provide more accurate source location, and in such case, high sensitivity to NLOS error will not exist. ■

IV. Simulations

In this section, we use simulation study to demonstrate FINE algorithm characteristics and its advantages over other practical algorithms such as LS [17], RWGH [20], and SRNI [24]. We conduct four different scenarios which focus on time efficiency and accuracy of the algorithms.

Root Mean Square Error (RMSE) is employed to evaluate the accuracy of different methods. Assuming we have an M-trial Monte-Carlo experiment, RMSE is defined as

$$\text{RMSE} = \sqrt{\frac{\sum_{l=1}^{M} (\hat{x}_l - x)^2 + (\hat{y}_l - y)^2}{M}} \tag{22}$$

where $(\hat{x}_l, \hat{y}_l)$ denotes the positioning result for the lth trial.

In all scenarios, we consider the stations to be located uniformly on a circle with radius of 10000 meters centered at the origin, the number of which is noted in each simulation scenario. Fig. 1 depicts the aforementioned localization geometry for especial case of 10 stations. The MS is located at (200, −400) m. We have repeated the experiments for different localization geometries and the results are mainly the same. Additive white Gaussian noise is employed for modeling the range measurement noise. It is considered that the noises in all stations have the same power. In these simulations τ is selected as 0.95. We use 50 observations to compute $\boldsymbol{R}_\rho$ for FINE algorithm. SRNI parameters are considered as suggested in the simulation section of [24]. In subsections B, C, D, we have repeated the experiment for 1000, 100, 100 times, respectively. In the last simulation, we show that RWGH algorithm is highly computationally demanding, even for small-scale positioning problems. Moreover, we have shown in [24] that SRNI out-

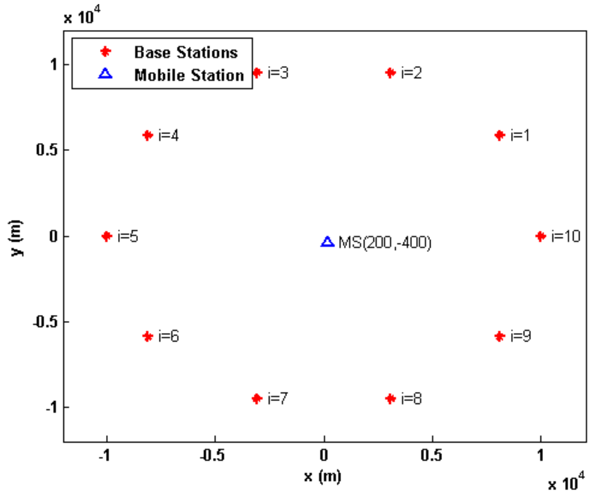

Fig. 1. Sample localization geometry for 10 BSs.

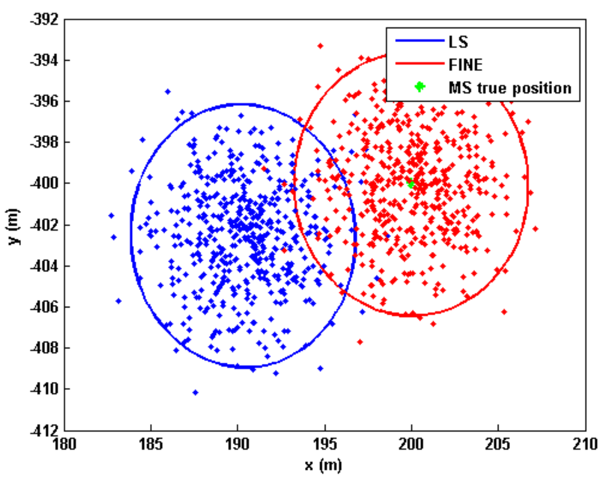

Fig. 2. Elliptical error probability for LS and FINE algorithms.

performs RWGH. Therefore, we have dropped RWGH from the simulations in subsections B and C.

A. NLOS Impact

This simulation is conducted to show how the NLOS error may affect positioning results even in the presence of many stations. We consider a set of 30 BSs, in which the first station experiences an NLOS error of 150 meters. Noise standard deviation is equal for all stations as 10 meters. The scatter plot for LS and FINE algorithms together with 95% Elliptical Error Probability (EEP) curves are depicted in Fig. 2. In this figure, the blue and red dots represent, respectively, the estimates provided by LS and FINE algorithms via a 500-trial ensemble run. The $p\%$ EEP denote an ellipse covering the estimations in a Monte-Carlo run with probability of p. As shown in Fig. 2, the realizations for LS algorithm have a large deviation from the true MS position, while the scatter points and the EEP curve for FINE algorithm are mainly around the actual value of MS position. This result demonstrates that the estimation will be biased in the case of NLOS error for LS algorithm, while FINE algorithm suppresses the error by identifying and eliminating the NLOS stations, and thus, its estimation is unbiased.

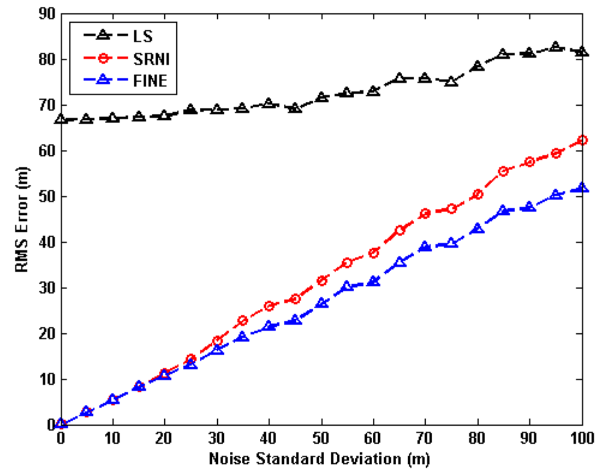

Fig. 3. Performance of different methods as a function of noise level.

B. Resistance Against Noise

A reliable algorithm should resist the increment of the noise level. We study this feature of LS, SRNI and FINE algorithms as well as their capability in NLOS mitigation. We consider 15 stations to be employed for localization. The first station measurement is corrupted by 500 meters of NLOS error. In Fig. 3, performance of the aforementioned algorithms are compared in terms of RMSE, when the measurement noise standard deviation varies from 0 to 100 meters with a 5 meter step. As expected, the LS algorithm cannot provide desirable performance in the presence of NLOS and demonstrates the worst accuracy. Furthermore, the proposed method (FINE) outperforms SRNI. This fact shows that when there exist a large number of BSs, it is better to eliminate the NLOS stations than to recover them.

C. Increment in the Number of NLOS Stations

It is important for an algorithm to stay operational over a different number of NLOS stations. We conduct this simulation to find out the ability of LS, SRNI, and FINE algorithms to deal with more NLOS stations. We consider 20 stations around the MS and a constant standard deviation of 10 meters for all station measurement noises. We suppose only the first station experiences the NLOS error in the first step. Then second station is added to NLOS set and this sequence continues until 12 first stations are corrupted by NLOS. NLOS values for these stations are considered as

$$\mathbf{n} = (350\ 300\ 500\ 100\ 450\ 200\ 600\ 200\ 500\ 800\ 450\ 500) \tag{23}$$

The results are summarized in Fig. 4. In fewer NLOS stations, SRNI and FINE algorithms perform similar to each other; however, in the case of more NLOS stations, SRNI loses its accuracy in recovering the NLOS error while FINE remains operational and accurate. Based on our experiments, when we have increased the number of NLOS BSs FINE will be accurate up to 14 out of 20 stations, and after that, all of the investigated algorithms cannot be accurate. However, such a high number of NLOS stations seldom occurs in practice.

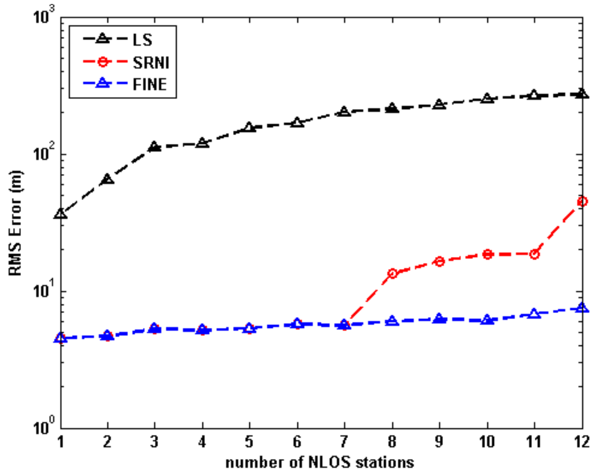

Fig. 4. Performance of localization algorithms versus number of NLOS BSs.

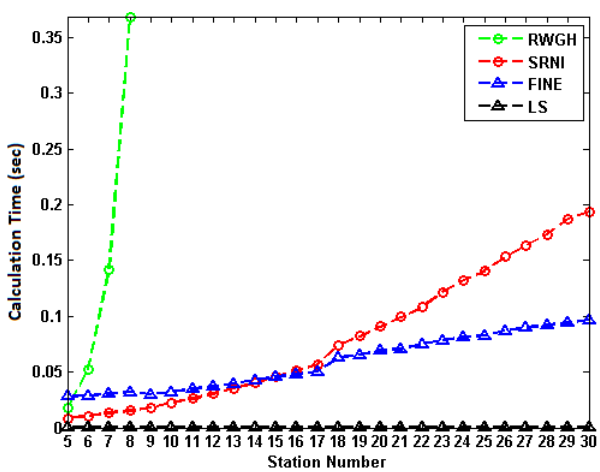

Fig. 5. Required time for different methods versus the number of BSs.

D. Time Complexity Analysis

The main requirement of NLOS identifier algorithm to be able to deal with many stations is the time complexity. In this part, we increase the number of stations from 5 to 30. Standard deviation of the noise is 10 meters. In this simulation, only the first station experiences an NLOS error of 500 meters. Fig. 5 illustrates the time consumption for LS, RWGH, SRNI and FINE algorithms on an Intel Core i5 2.4 GHz CPU laptop. RWGH needs a calculation time in an exponential relation with the number of stations. This fact limits the operability of RWGH for more BSs. SRNI and FINE algorithms both have acceptable time complexity, but FINE computing time grows in a lower rate.

V. Conclusion

Localization by range measurements corrupted by NLOS error will result in a large error in position estimation. Many algorithms have been introduced to identify or mitigate this error; they, however, are usually computationally inefficient. In this paper, we proposed a novel algorithm to identify NLOS stations using their statistical characteristics. We found a relation between the eigenvectors of the autocorrelation function of the error in range measurements and the NLOS vector. In each run of the proposed algorithm, we can find out whether there are NLOS errors or not, and if there is any, which station is the most probable one. Then, we employed a recursive method to identify all of the NLOS stations. Simulation results demonstrated that our algorithm has the best time efficiency and accuracy among stat-of-the-art algorithms. This algorithm can be used for a large set of stations with low complexity. Furthermore, simulations showed that the proposed algorithm remains stable in the increment of NLOS stations and also it has an acceptable resistance against noise.

To conclude, we provide a comparative analysis for the investigated algorithms. Despite low complexity of LS algorithm, it cannot provide desirable accuracy in the presence of NLOS. On the other hand, RWGH method is computationally burdensome, especially for large number of NLOS stations. Both SRNI and FINE have lower complexity and superior accuracy compared with RWGH. Specifically, FINE has the best performance in terms of accuracy and complexity among the investigated algorithms. It has been shown by numerical simulations that the proposed method has almost linear complexity with respect to the number of base stations as well as robustness to the number of NLOS stations, implying its efficiency over relatively large sensor networks.

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